We study sufficient degree conditions that force a host graph to contain a given class of trees. This setting involves some well-known problems from the area of extremal graph theory. The most famous one is the Erdős-Sós conjecture that asserts that every graph with average degree greater than k-1 contains any tree on k+1 vertices.

Our two main results are the following. We prove an approximate version of the Erdős-Sós conjecture for dense graphs and trees with sublinear maximum degree. We also study a natural refinement of the Loebl-Komlós-Sós conjecture and prove it is approximately true for dense graphs.

Both results are based on the so-called regularity method. The second mentioned result is a joint work with T. Klimošová and D. Piguet.