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Report on the doctoral thesis

by

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Logical Pluralism from Historical Perspective

I. Introduction and background

Arazim's work is, as the first two chapters already show, a work on the idea that formal logic can be a discipline on its own. But how can it be 'defined' in its topic? The proposal is: by answering the question which terms and which syntacto-semantic forms of sentences and inferences could be labelled "logical" – with some good reasons or intuitions, i.e. some good feelings. In some sense, Arazim takes up the outlook at the end of Gila Sher's book "The bounds of Logic" (1991). The task is to compare Sher's 'algebraic' proposal of defining logicality (on the roads of Mostowski, Tarski, Lindstöm etc) by invariant features with respect ot isomorphisms of models with a 'proof-theoretical' or (what is the same here) inferentialist point of view.

By putting the topic in this way, I already express some doubts about the whole enterprise – starting from Gila Sher's discussion of the logicality of certain generalized quantifiers. Her new 'logicism' defines formal logic as mathematical logic and puts invariance criteria with respect to isomorphisms between models into the focus. In doing so, she presupposes 'Tarskian' set-theoretical model theory as basic for logics. This is at the same time a standard position and anachronistic, at least in view of the label 'logicism': Frege's 'logicism' must be understood – despite of its failure – in opposition to Cantor's radically *non-logical,* i.e. merely mathematical, and this means: at the same time formal and merely intuitive 'definitions' of pure sets with its cardinal numbers. (Frege's predicative sets as courses of values in a pre-given domain are of a different form!)

Naive set theory provides (e.g. for Tarski) the universe of mathematical discourse, containing (an isomorphic exemplar of) any possible model as a local universe for variables and quantifiers.

The amazing – or perhaps annoying – point of Sher's approach is this: She bases her definition of logicality on allegedly 'philosophical' or even, like Tarski, 'metaphysical' grounds. This procedure is deeply problematic. As long as 'philosophical' and 'metaphysical' are just words for excuses or dogmatic belief, they are certainly misused. I would have wished as a reader that Arazim would have been a little more outspoken about this – and some other points in the above criticism.

A similar point holds for some superficial appeals to Kant. We find in Kant, for example, some casual and vague remarks on 'Aristotelian' logic as a settled and fixed system (of mereological relations between extensions and corresponding syllogistic deductions). In a sense, Kant is even right to say until today that the syllogisms and their semantical interpretation form a completed science – without really interesting improvements after Aristotle. Leibniz or Euler, for example, do not really add any new idea but only make some already implicit points more explicit.

Kant transcendental logic is interested in the question how names and variables of ordinary and scientific languages can refer to physical things – a question that goes beyond any merely formal (mathematical) logic, either in the form of Aristotelian (mereological) or Fregean (relational, functional) formal logic.

If this taken into account, the leading question of Arazim's interesting work gets even more pressing: What is 'formal' logic? Is it really, as Sher suggests (in content, i.e. without these words), nothing but a generalized 'algebraic' approach to mathematical theories and models? How should we understand the playground of producing and investigating diverse logical calculi – for example in the context of alternative 'definitions' of negation and other 'logical' operators in their 'pluralism'?

There is a rather institutional aspect to this whole debate: Is there a place for formal logics outside standard mathematics and its logico-linguistic foundations on one side, outside the structural models of computational linguistics and artificial intelligence on the other? This institutional question remains hidden behind Arazim's question of logical pluralism. It is, in other words, unclear if we can, as Arazim does, really assume that there is a fact of the matter to be elucidated when we ask for the logicality of terms and semantical rules. We rather might propose certain developments in formal logics understood as a sub-discipline of pure mathematics as we could read Gila Sher.

In contrast to both authors, quite some logicians and most mathematicians practically believe that after the achievements of Frege, Russell, Hilbert, Gödel, Tarski, Turing, Quine and Kleene, just to name a few, classical formal mathematical logic (of first order) is also, like Aristotle's syllogistics, a (more or less fully) completed discipline. We can leave the debate about intuitionistic logic out of view because it is, in the end, of absolutely no practical interest in mathematics. The technical methods of classical logics can be learned but not really improved – just as there is no second invention of the wheel. The results can be fine-tuned in mathematics only via set-theory and recursion theory.

As a result, standard logic looses any self-standing interest as a (still) fruitful field of scientific investigation and innovation – beyond the fact that its canonization in mathematics and its philosophical impacts might not be really understood until today. Gila Sher may not *intend* this result. But her identification of the logical with the mathematical ironically *supports* this view. Arazim wants to disagree.

II. The work in further chapters

Like Gila Sher, Arazim also starts (in Chapter 3) with the paradigm of standard Euclidean geometry and its relation to non-standard models for non-Eudlidean axiomatic 'geometrical' theories. I do not dwell on the usual hearsay on this topic, but name just one point: non-Euclidean systems are, at first, important only for understanding the difference between axiomatic systems of formal deduction on one side, truth-evaluative models on the other. Only when we measure distances by time we have to re-model the numbers of our time-measures in their dependence on the local points of measurement – such Einstein's and Minkowski's ideal space-time cannot have the structure of a 3 and 1/2 dimensional Euclidean vector space (with a directed time-line). Felix Klein's (in the end 'algebraic') analysis of invariances with respect to movements appears, under this perspective, also not really as a good paradigm for taking geometry as a model for logics. But I must leave these things as mere hints here.

Arazim begins with the question where to look for criteria of demarcation between formal logics and other contributions to the meaning of 'languagings,' as Sellars has coined the word. Parallel to Sher and influenced by Alberto Coffa, he takes Kant's view on logic as a starting point. This seems reasonable, at least at first sight. However, Alberto Coffa's book is no good guide in these matters, either. This is so especially because the constitution of point-spaces as models for formal geometrical axioms and theorems on the basis of so-called intuitions, i.e. observation of self-constricted diagrams, is totally neglected. Coffa's logical empiricism and his attack on Kant's synthetic apriori thus do not really put the focus on the relevant and central features in Kant's work. Arazim's historical recollection of this influential picture is, however, mainly correct and helpful – but only if we continue to think about the limits of the given view.

It is also true that Kant does not really care for formal, mathematical systems of logic and formal deductions. His 'formal' logical of non-phrase and verb-phrase, subject and predicate is utterly 'informal', non-schematic.

Chapter 4 discusses Quine's holism. This is indeed necessary because Quine's attack of Carnap's neo-Kantian distinction between 'analytical' axioms or theorems as parts or consequences of defining terms and 'synthetic' sentences with some 'empirical' content results, in the end, in a conventionalist attitude with respect to the question what belongs to logical semantics and what belongs to general empirical knowledge. For Quine, there is no fact of the matter of logicality beyond the decision to stick to some classical schemes of formal definition and formal inferences as far a possible as a core system of our conceptual network and its conceptual schemes. In a sense, for Quine, classical logics is good enough and decisions for non-standard theories, starting with his own non-standard set-theory, must be justified pragmatically and empirically.

Arazim is right to state that Quine often is evasive and hand-waving, for example in his vague distinctions between the logical centre and the empirical periphery of scientific theories. We could add, however, with respect to Carnap's tolerance principle, that this principle only looks tolerant because it contains the pre-judgement that semantical rules must take the exact form of mathematical rules – which might be a problem for the real context-dependent semantical dynamics and plasticity of world-adapted situation-sensitive forms of languagings.

Chapter 5 reviews Gila Sher's book. Gila Sher is unhappy with Kaplan's (or Montague's) understandings of noun-phrases as generalized quantifiers (following the ideas of K. Ajdukewicz) that proceed, at first, merely syntactical and leave the semantic interpretation wide open. As a result, such quantifiers are operators with ad-hoc interpretations – without clear demarcations between 'synthetic' ('empirical') meaning and 'logical' meaning. Sher thinks that operators like "there are n objects such that p" with cardinal numbers n (or less than n or more than n) can be seen as logical operators. The problem is not that we could not decide to follow her. The problem is that nobody (at least no mathematician) is really interested in arbitrary nestings and recursive definitions of the 'semantic' of such operators – outside logical games as merely theoretical examples without practical interests.

Arazim, too, presupposes that there is a thriving playground for producing all kinds of new systems of logic as formal language games, like modal logics, relevance logic, paraconsistent logic and so on. And he asks which of these systems can be rightly called "logic", such that we can put it into a particular field or scientific discipline outside of regular mathematics and technical disciplines as artificial intelligence and its calculi or automatic language processing or theoretical linguistics.

Unfortunately, there is no way to appeal to philosophy to justify formal logics if we leave the topic of Frege, Cantor, Hilbert or Brouwer behind and do not just deal with the philosophical, i.e. linguistic and technical foundations of mathematical truth and proof.

Chapter 6 asks for the value of logically, schematically 'valid' inferences. In the case of algebraic model theory, the situation is clear: formal deductions hold (i.e. 'are true') in all models of the axioms such that we can prove things for whole classes of models. In the case of material semantics of natural languages and reflective ideal models of their syntacto-semantical forms the case remains fairly unclear, just as the notions of truth and correctness in world-related assertions seem to be formal notions of approvability-in-context – with quite diverse content-related criteria. The same holds for the notion of canonized generic knowledge as an established learnable system of default rules of differentially conditioned normal inference – that must be adapted by speakers (and hearers!) dialectically to the particular situation of utterance just because of contingency (accidental chance, exceptions) and, hence, the ubiquitous possibility of error and privation or steresis.

Chapter 7 tries to present foundations for inferentialist demarcations of formal logic. Arazim supports, indeed, Brandom's idea that logic is a (or 'the') method of making implicit rules (‚norms') of inferences explicit, just as an implication sign $p \rightarrow q$ explicates the inferential form $p \rightarrow q$ via the 'modus ponens' that allows us to infer q from p and $p \rightarrow q$. The aspect of recursive definition of complex predicates $\varphi(x)$ is, however, in this approach somehow underrated – a point most prominent in Frege's truth-evaluative idealist mathematical logic.

In fact, there is some hearsay that Frege *rejected* "the grammatical subject-predicate distinction". It is true that Frege puts his fingers on the *semantical equivalence* of the active and passive voice, as we all do, grosso modo and cum grano salis. On the ground of such an *equivalence*, Frege *replaces* the *grammatical* notions "subject" and "object" by turning *both* into "arguments" of a *relation* expressed by an open sentence $\varphi(x,y,z\dots)$ with one, two or more free, 'unsaturated', places. Thus is the most important step in the direction of a relational mathematical logic with parameters and quantifiers as methods of reducing n-ary predicates to n-1-ary predicates. A sentence is 0-ary, i.e. saturated. These new notions and distinctions, introduced by Frege, can be summed up thus:

1. grammatical subject and object become *arguments,*
2. the predicates or verb-phrases turn into *n-ary open sentences* – which later are called *n-ary predicates,*
3. these predicates are treated as 'denotations' of *relations* resp. *truth-value-functions*. Especially in the case of only one free (i.e. unsaturated) variable, Frege and his followers call such truth-value-functions "concepts";
4. the last new distinction is that of force and content in an assertion.

Arazim does also not discuss the difference between the *formality* of *schematic rule*, defined on the level of syntax, and formality in the sense of *general validity* of inferences of a certain syntactic form. The varieties of schematic inferential rules for 'logical' connectives (words, terms, forms) thus contrast the varieties of 'formally valid' rules – with respect to some classes of models in truth-evaluative semantics.

Chapter 8 pledges for a replacement of the title "logical pluralism" by "logical dynamism". In fact, Arazim is right to say that meaning must be dynamic – but there is still some work to do to say how and why. The hint that the conceptual norms of implicit inferences, partially made explicit by rules or (conditioned) sentences are usually generic and no universal formal quantifications (as in § 4.1) is utterly correct, but perhaps not yet sufficient.

III. Assessment

Altogether, the dissertation represents the current state of the discussion after the works of Gila Sher, MacFarlane, Shapiro and, of course, Brandom and Peregrin. It contrasts the varieties of proof-theoretical systems for logical constants (for example in different versions of Gentzen's sequent calculus) to the logical operators (quantifiers) in Gila Sher's model-theoretical approach. It thus lays the ground for a future, perhaps more thoroughgoing, investigation of different rule-theoretical or inferentialist systems of ‘defining’ the logical contents by introduction and elimination-rules – with different structural rules for example in ‘classical’ and ‘intuitionistic’ logic and some possibilities of allowing for non-monotonic versions as in Ulf Hlobil’s and Robert Brandom’s approach.

Nevertheless, Arazim does not yet develop a structural order for ‘core meanings’ e. g. of negation sings and how additional rules ‘holistically’ can ‘change’ the ‘meaning’ (or, as Arazim says, the ‘logic’) of negation (or disjunction, or implication).

Here and at other places, the work is a little bit too defensive; it reconstructs the historical perspective and represents the state of the art in a very nice way, but leaves the directions of further developments and discussions wide open. This means that the work is very valuable and original starting point for such developments and discussions, as I have shown at some selected points here. As such, it is a good dissertation on the topic.

In conclusion, the thesis absolutely meets the standard customarily required for a doctoral dissertation all over the world. I therefore recommend the dissertation for a public defense. In other words, my assessment is a grade of “pass”, and I recommend the thesis to be accepted as a doctoral dissertation in Logic.

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