Charles University

Faculty of Social Sciences

Institute of Economic Studies

MASTER'S THESIS

Empirical evidence on pricing of contingent convertibles

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, December 28, 2017

Signature

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Abstract

The aim of this thesis is to shed more light into practical challenges related to pricing of contingent convertibles by empirically evaluating validity of two most crucial modelling assumptions of contingent convertible pricing framework.

First assumption is that contractually specified capital ratio can be proxied by stock price level. Second modelling assumption is that volatility smile characteristic for stock market can be also incorporated into the contingent pricing model.

First assumption is tested by comparison of probability of conversion implied by balance sheet figures with probability implied by market spreads. Analysis of our dataset indicates that probability implied by figures reported on balance sheet of issuer is statistically higher than probability estimated by market participants, suggesting that there is a confidence that reported figures do not fully represent the capital position of issuer and its ability to raise additional capital and revert the potential conversion. New information available on balance sheet also does not tend to immediately and fully materialize in contingent convertibles market.

Secondly, incorporation of volatility smile characteristic for stock market leads to very low and unstable trigger level compared to level implied by balance sheet.

Finally, findings collected throughout the thesis are utilized to suggest possible calibration setup of Credit derivatives model, which is again tested empirically on our dataset and evaluated based on various critieria.

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Abstrakt

Cílem této diplomové práce je poskytnout pomocí empirické analýzy dvou hlavních předpokladů modelu pro oceňování podmíněně konvertibilních dluhopisů vhled do praktického oceňování tohoto instrumentu.

První předpoklad modelu je aproximace kapitálového poměru specifikujícího level, při kterém dojde ke konverzi daného instrumentu, pomocí ceny akcie daného emitenta. Druhý předpoklad je pak začlenění volatility smilu, pozorovaného na akciovém trhu, také do modelu pro oceňování podmíněně konvertibilních dluhopisů.

První předpoklad je testován pomocí srovnání pravděpodobnosti konverze implikované finančními výkazy a pravděpodobnosti konverze implikované tržními spready. Z analýzy našeho datasetu vyplývá, že finanční výkazy emitentů indikují vyšší pravděpodobnost, než která by byla v souladu s pozorovanými spready na trhu s podmíněně konvertibilními dluhopisy. Toto zjištění naznačuje, že tradeři obchodující s podmíněně konvertibilními instrumenty předpokládají aktivní zájem a schopnost managementu emitenta zvýšit úroveň kapitálu v případě nouze a odvrátit tak konverzi. Analýza dále poukazuje na to, že nové informace dostupné ve finančních výkazech nejsou okamžitě a úplně reflektovány v tržním spreadu pro konkrétní instrument.

Začlenění volatility smilu pozorovaném na akciovém trhu do modelu pro oceňování podmíněně konvertibilních dluhopisů pak vede k velmi nestabilní a nízké úrovni konverze.

Zjištění učiněná v průběhu psaní diplomové práce jsou konečně použity při kalibraci modelu kreditních derivátů, který je následně empiricky otestován a jeho přesnost vyhodnocena pomocí rozličných kritérií.

Klasifikace	C51, C52, C58
Klíčová slova	oceňování podmíněně konvertibilních dluhopisů, model kreditních derivátů, empirická analýza podmíněně konvertibilních dluhopisů

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Master's Thesis Proposal

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Proposed Topic:

Empirical evidence on pricing of contingent convertibles

Motivation:

Pricing framework for contingent convertibles currently includes Credit derivatives approach, Equity derivatives approach and also Structural model approach. Vast amount of research, including empirical studies, has been devoted to each pricing approach in general. Empirical evidence on two crucial pricing variables in contingent convertible pricing framework – trigger level and volatility, is however still limited and little empirical analysis has been conducted on appropriateness of current approaches for their modelling.

Trigger level, specified usually in form of capital ratio, has to be approximated. Common approach is to approximate it with stock price level, however the estimation proces is not trivial. Rüdlinger outlines methodology of estimating stock price trigger level using figures reported on balance sheet of particular CoCo issuer. Little evidence is however available on empirical validity of this approach. Rüdlinger uses methodlogy only for initial calculation of trigger level which is kept constant aftwerwards and therefore study does not provide much evidence on empirical relationship between probability of conversion implied by reported figures and probability of conversion implied by market spreads.

Approximating trigger level based on capital ratio with stock price level then implicitly assumes that the volatility smile observable in stock market can also be incorporated into contingent convertible pricing framework. Again, little empirical evidence is available on possibility of this incorporation and on volatility modelling in contingent convertibles pricing in general. Authors of previous empirical studies mostly rely either on constant volatility or historical volatility and there is lack of studies examining incorporation of volatility smile.

Modelling of trigger level and volatility is crucial for good performance of either contingent convertible pricing model and I consider that extensive empirical study can help to tackle practical issues related to model setup or its calibration.

Hypotheses:

1. Hypothesis #1: Empirical evidence supports validity of stock price level approximation for contractually specified capital ratio based trigger
2. Hypothesis #2: Volatility observable in stock market can be incorporated into pricing framework for contingent convertibles
3. Hypothesis #3: Empirical evidence supports the validity of modelling contingent convertible using Credit derivatives model

Methodology:

After introduction to the pricing model for contingent convertibles and to the dataset used for the analysis, validity of two assumption will be tested. Using financial statements available, book implied trigger level will be calculated with method suggested by Rüdlinger, however with

relaxation of assumption of one to one relationship between book and market value of equity. This book implied trigger level will be basis for calculation of book implied probability of conversion implied by Credit derivatives pricing formula, which will be compared to probability of conversion implied by market CoCo spreads.
Secondly, volatility observable in stock market will be incorporated into the contingent pricing model using Gatheral functional form for volatility. Implied trigger level calibrated from the model will be again compared to book implied level and the effect of incorporating volatility smile into the pricing framework will be evaluated.
Finally, empirical findings gathered throughout the thesis will be utilized for setup of model calibration, and this setup will be again empirically tested based on various criteria.

Expected Contribution:

The thesis should provide so far neglected empirical evidence on two crucial pricing inputs in contingent convertible pricing framework – trigger level and volatility. Validity of approximating capital based ratio with stock price level will be tested and usefulness of methodology for its estimation suggested by Rüdlinger will be evaluated. Comparison of probability of conversion implied by reported figures and probability of conversion implied by market CoCo spreads can potentially reveal assumptions market participants have on behavior of CoCo issuer and serves as a basis for further model setup. Empirical evidence on volatility, the second most important pricing input, should then reveal whether it is possible to model volatility in the same way as in stock market. Validity of both assumptions is crucial for good performance of the model and providing empirical evidence on their validity should provide good stepping stone for eventual model calibration.

Outline:

1. Introduction: Introduction of contingent convertibles as instruments combining debt-like and equity-like characteristics. Focus mainly on properties and links important for the context of instrument valuation.
2. Literature review: Outline of previous research, focusing mostly on contingent convertibles pricing framework.
3. Model and data description: Introduction of the data set used for tests of validity of assumptions. General description of data sample and subsequent econometric analysis providing deeper insight into model dynamics.
4. Empirical tests: Evidence on validity of stock price level approximating capital based ratio. Evidence on incorporation of volatility smile observable in stock market to pricing framework for contingent convertibles.
5. Calibration: Empirical findings gathered throughout the thesis reflected in calibration setup. Calibration setup tested empirically using extensive dataset.
6. Conclusions: Summary of findings and their implications for valuation methodology, possibly recommendations for future research.

Core Bibliography:

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Acronyms

AT1	–	Additional Tier I capital
ATM	–	At the Money volatility
BS	–	Black Scholes model
CDS	–	Credit Default Swap
CET1	–	Common Equity Tier I
CE	–	Common Equity
CoCo	–	Contingent Convertible
FX	–	Forex exchange market
IV	–	Implied Volatility
OTC	–	Over the Counter
SABR	–	Stochastic Alfa Beta Rho model
SVI	–	Stochastic Volatility Inspired form

1. Introduction

Pricing framework for contingent convertible instrument is currently quite extensive and includes Credit derivatives approach, Equity derivatives approach as well as Structural model approach. Despite vast amount of research devoted to each particular approach in general, little examination was devoted to two crucial pricing inputs and their modelling – trigger level and volatility.

The first goal of this thesis is to shed more light into the estimation of trigger level at which conversion is assumed to occur. Contractually specified capital ratio needs to be proxied by stock price level. Previous empirical studies rely mostly on calibrated trigger level which is kept constant throughout the life of contingent convertible (CoCo) (Erismann, 2015) or use the proxy based on figures reported on balance sheet with assumption of one to one relationship between book and market value of equity (Rüdlinger, 2015). In this thesis, one to one assumption is abandoned and book approach to approximate trigger level modified from Rüdlinger´s version is extensivelly empirically tested. Relationship between probability of conversion implied by book figures reported for particular CoCo issuer and probability of conversion implied by market CoCo spread is examined with the goal of evaluation of usefulness of book figures in the estimation of trigger level.

The second goal is to evaluate how well does the incorporation of stock volatility smile fit into the pricing framework for contingent convertibles. Volatility used in the pricing formula massively affects the implied probability of conversion and appropriate modelling is therefore needed in order to obtain reasonable model spreads in line with market pricing. Using stock price trigger level as a proxy for capital based trigger level naturally entails incorporation of volatility observed in stock market also into contingent convertible pricing. Previous empirical studies rely mostly on historical volatility (Erismann, 2015) or test constant volatility assumption (Jung, 2012) and larger empirical analysis of volatility smile incorporation into CoCo pricing is still missing. This thesis sets out to provide an empirical insight into the effect of smile incorporation on implied trigger levels and shows its large impact on eventual model calibration.

Empirical evaluation of both trigger level estimation and volatility modelling assumption conducted in this thesis can be used as a guidance for practical implementation of the model. Calibration setup presented in last chapter incorporates some of the empirical findings gathered throughout the thesis and can serve as a stepping stone for calibration of the pricing model.

1.1 CoCo characteristics and introduction into pricing framework

Contingent convertible bond is a fixed income instrument with principal and scheduled coupons which is automatically converted into equity or written down when prespecified trigger event occurs. (Pazarbasioglu, et al., 2011) Such conversion or writing down provides quick capital injection and for that reason, it has been designed as an instrument bringing relief especially to banks in financial distress.

Although contingent convertibles have been introduced as a solution to excess debt problem in 1991 following junk bond crisis, it has become more prominent in the aftermath of Financial Crisis 2007-2008. (Harvard Law Review, 1991) Regulators vastly considered CoCos as an auxiliary mechanism in bank regulation, helping to align interest of bank management, shareholders and public and potentially eliminating necessity of bail out. (Dudley, 2009) Statistics also indicate broad acceptance of instrument within the industry, more than 150 banks issued contingent convertibles since 2009 and total amount issued reached $70 billion in 2015. (Avdjiev, et al., 2013)

Even though rising issuance of instrument can be linked to the development of the regulatory framework and increase in regulatory pressure, high industry acceptance stems hardly solely from the coercion of regulators. Issuance of contingent convertibles with appropriate conversion trigger simply provides bank with an opportunity to raise cheaper capital than through ordinary shares offerings. On the other hand, investors in contingent convertible bonds are likely to benefit from higher yield, reflecting reward for risk of possible conversion or complete write down.

Despite lasting unclarity about CoCo assets treatment on balance sheet from the regulatory viewpoint, demand for the instrument was driven by low interest rate environment. Investor base in 2015 consisted mainly of private banks and retail investors (52%), asset management groups (27%) and hedge funds (9%). (Avdjiev, et al., 2013)

Effectiveness of the CoCo as a loss absorbing instrument as well as appeal of the instrument to the investor side largely depends on the specific characteristics and the structure of CoCo. Two main distinguishing characteristics are trigger event activating mechanism and the loss absorption mechanism that follows. Trigger event can be either rule-based or entirely on discretion of the issuing institution.

Rule-based triggers typically entail that loss absorption mechanism is activated when the capital of the issuing institution falls below a predetermined ratio of risk weighted assets. Capital is represented mostly by Common Equity Tier I recorded in books and the level of the trigger primarily corresponds to the regulatory scheme under which bank operates.

Contrary to rule-based triggers, discretionary triggers are solely within the authority of issuing institutions and as such it is presumed that bank itself evaluates need of additional capital buffer. Discretionary triggers are theoretically deemed to provide more timely capital relief than mechanical triggers relying on book entries, however, as this discretionary power entails certain uncertainty and information asymmetry, investors typically demand much higher yields for CoCos with such trigger design and for that reason, discretionary triggers are rarely used.

Loss absorption mechanism refers to what ensues in the case of trigger event. The prevailing mechanism currently is principal write down – coupon payments are suspended as well as repayment of principal. Such write down might be temporary, meaning that payments are reinstated when bank restores its financial health. Write down is also not necessarily complete and some banks designed CoCo with partial cash payoff in case of trigger event. The second possibility of loss absorption mechanism is equity conversion. CoCo instrument in such case ceases to function as a debt instrument and its principal is converted into equity according to specified rule – either based on predetermined stock price or actual stock price at the time of trigger event. Below graphs summarizes frequency of particular CoCo loss absorption mechanisms and the predetermined trigger level based on 2016 data:

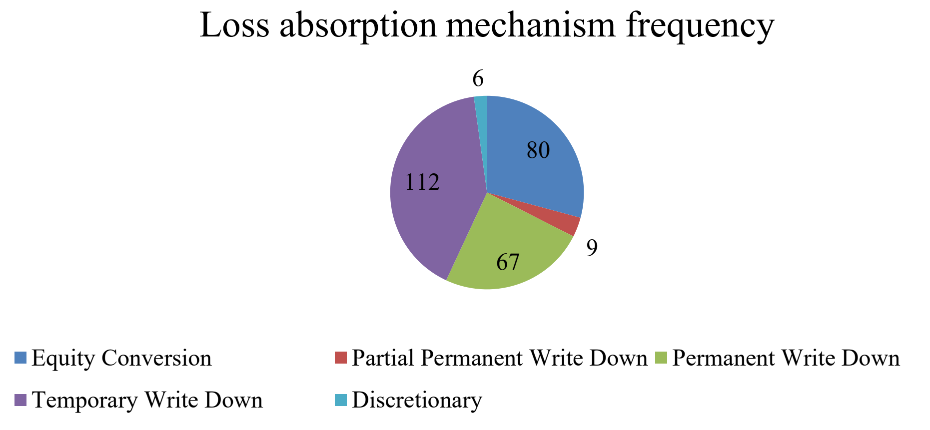

Figure 1: Frequency of different types of loss absorption mechanism (Bloomberg)

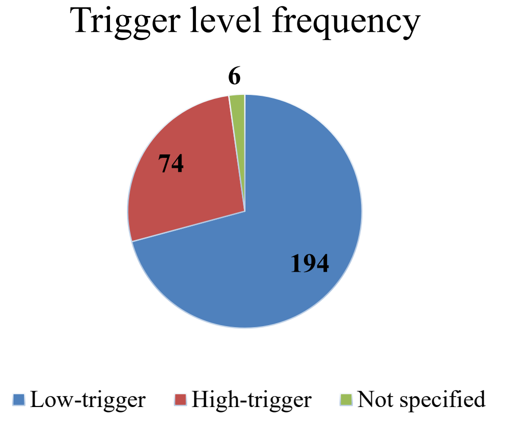

Figure 2: Frequency of trigger levels assorted by magnitude. Low-trigger CET1<= 6 % RWA, high-trigger CET1 > 6% RWA (Bloomberg)

While the design and structure of the CoCo instrument selected by the issuing bank are mainly reflection of the evolving regulatory framework and regulatory capital eligibility of such design and structure, attractiveness of instrument structure and characteristics for investors is reflected through yield. Issuers are therefore in primary markets trading off between desirable structure complying with the regulatory requirements and satisfying internal needs and the price offered by investors reflecting both bank stability and financial health as well as design of particular CoCo instrument. Table below shows coupon levels for our sample segmented based on loss absorption mechanism and trigger level of instrument:

Trigger.level	High.trigger		Low.trigger		Total	
Type	Average coupon rate	Count	Average coupon rate	Count	Average coupon rate	Count
Equity Conversion	7.653	36	6.280	40	6.931	76
Non Specified	0	0	7.436	6	7.436	6
Partial Permanent Write Down	3.5	1	4.588	8	4.467	9
Permanent Write Down	6.5	21	6.633	46	6.591	67
Temporary Write Down	6.996	11	6.544	100	6.589	111
Total	7.138	69	6.46	200	6.634	269

Figure 3: Average CoCo coupon divided by loss absorption mechanism and trigger level (Bloomberg)

Overall for our sample of CoCos, coupons for contingent convertible bonds which are in case of trigger event converted into equity are higher than coupons of both permanent and temporary written down. This however should not be interpreted as the effect of chosen mechanism on coupon as it completely disregards the position of the issuing bank. Incorporating bank specific variables into analysis, studies show that in primary markets, conversion to equity mechanism on average induces lower coupon than write down mechanism, and coupon required increases with trigger level (Tiainen, 2015) (Avdjiev, et al., 2013). These findings are consistent with the different outcomes of possible trigger event – conversion to equity mechanism provides investor with at least some payoff whereas holder of CoCo with permanent write down loss absorption mechanism is left with nothing. Higher trigger level then ceteris paribus increases the probability of trigger event happening.

While there are multiple studies on primary pricing of contingent convertibles, much less space was devoted to pricing in secondary market. Contingent convertibles are predominantly publicly traded and quoted instruments. Prices change daily based on market supply and demand, reflecting changes in bank fundamentals as well as shifts in market sentiment. Corresponding to hybrid structure of CoCos with prevailing debt-like features during periods of economic stability and equity-like features when approaching trigger level, two models have been presented. (Jung, 2012)

Firstly, Credit derivatives approach derives explicit formula by viewing instrument as bond with cash settlement in case of trigger event. In case of equity conversion, such settlement is deemed equal to the stock price times number of shares received as prespecified in contract. Less consideration has been devoted to actually more popular write down mechanism and especially troublesome temporary write down. For permanent write down contingent convertible final payoff is zero as investor loses all. However, temporary write down mechanism provides bank with the option to write down whole principal in the trigger event but potentially write principal up again when financial health is regained. This special option has been considered from regulatory and incentive point, but not from the valuation point and the increasing popularity of this loss absorption mechanism presents a challenge for CoCo valuation.

Secondly, Equity derivatives approach derives its pricing formula from resemblance of the CoCo payoff in conversion to knock in forward. Knock in forward is OTC contract known mainly in FX market, providing both up and down exposure as in case of regular forward. Transaction is however conditional to the breach of barrier. Following similarities with contingent convertibles, Equity derivatives approach treats CoCo as a defaultable bond and appropriate number of knock in forwards.

Resulting from differences in treatment of final cashflow in case of trigger event, both approaches generally lead to different pricing of CoCos unless the interest rate is equal to dividend rate.

Additionally, both approaches use quite restrictive assumptions to derive explicit formulas. One such strong assumption is that trigger event, which is in reality defined as capital ratio breaching predetermined level, can be modelled by assigning particular stock price to trigger level. This is very simplifying assumption, as CET1 ratio, most commonly specified as determinant of trigger event, is derived from book value of equity, not market value. More precisely, CET1 consists of common shares issued by bank, share premium resulting from the issue, retained earnings, minority interest and accumulated other comprehensive income and other disclosed reserves.

In order to estimate stock price level associated with CET1 level specified in the contract, Rüdlinger provides a methodology to derive the level based on CET1 ratio, risk weighted assets and tangible equity reported on the balance sheet of particular CoCo issuer. (Rüdlinger, 2015)

However, little research in general is devoted to closer examination of relationship between market value and book value of equity or examination of such relationship for banks specifically. Market value of equity and book value are hardly perfectly correlated and using such assumption without more extensive investigation however presents a threat to validity of pricing formulas. As trigger level is crucial pricing parameter in both Credit and Equity derivatives model, validity of using stock price level as a proxy for CET1 ratio obtained with Rüdlinger´s methodology will be evaluated empirically in this thesis.

The threat to validity of pricing formulas is then magnified by employing profoundly discussed Black – Scholes assumption, the constant stock volatility or alternatively, by incorporation of stock price volatility smile not necessarily consistent with pricing dynamics of contingent convertibles. Not only that constant volatility assumption has been increasingly considered too digressed from the real market behaviour and for real time trading stochastic volatility or local volatility models have been developed to overcome shortcomings of this BS assumption, in conjunction with the stock price level used as a proxy for CET1 trigger level, this assumption means presupposition of CET1 ratio with constant volatility. This assumption is likely to be breached especially when ratio is fluctuating close to trigger level, uncertainty about capital adequacy is increasing and CoCo investors are increasingly concerned about possibility of unfavourable equity conversion or worse, permanent write down.

Again, little research is available for appropriateness of constant volatility assumption or for appropriateness of incorporation of stock volatility modelling into CoCo pricing framework. The second goal of the thesis is therefore to empirically evaluate impact of volatility modelling, specifically in Gatheral functional form, on model prices and to compare model dynamics with dynamics observed in the market.

Previous section outlines possible shortcomings of both Credit derivatives and Equity derivatives approach stemming from quite restrictive and potentially overly simplifying assumptions. Using pricing formulas without closer look into the validity of underlying assumption could lead into substantial price distortion. Magnitude of such distortions might be in addition prone to increase when approaching trigger level, compounding investor uncertainty.

The aim of this thesis is to provide empirical insight into the validity of assumptions underlying previously presented pricing approaches. Specifically, assumptions are evaluated based on empirical examination of Credit derivatives model, but their evaluation is to large extent also applicable to Equity derivatives model approach as assumptions to be tested are shared among these two models.

Testing justifiability of using market stock value as a proxy for capital ratio as well as reality of modelling CET1 with volatility smile observed in stock market should provide a basis for deeper understanding of shortcomings of pricing models and evaluate to which degree these assumptions are oversimplifying. The empirical test should then serve as a stepping stone for model calibration and practical implementation of Credit derivatives model.

The subsequent analysis will be divided in four other chapters. Second chapter will briefly present previous research with focus on valuation model for contingent convertibles and its features important for pricing.

Third chapter will provide little more background for understanding Credit derivatives approach including final derived formula for pricing. Fundamental relationships between spread implied from formulas and underlying variables will be introduced as inferred from the price sensitivities deduced from models. Data set collected for empirical testing will be described and theoretical relationships derived from Credit derivatives model will be visualized against empirically observed relationships in order to provide some insights into further analysis. Chapter will be finalized by evaluation of ordinary least square model between CoCo spreads and variables of interest.

Fourth core chapter will empirically evaluate validity of underlying assumptions, that is assumption of stock market value as a proxy for CET1 ratio and incorporation of stock volatility smile into CoCo spread modelling dynamics. Probability of conversion of CoCo instrument estimated using figures reported on balance sheet will be compared to market implied probability of conversion. Volatility smile observed in stock market will be, in the form of Gatheral volatility, incorporated into CoCo pricing framework and its impact on implied trigger level and subsequently impact on calibration of the model will be evaluated.

Calibration of the model and overall test of Credit derivatives pricing formula will be conducted in fifth chapter based on comparing model implied spreads with observed spreads in the market. Last chapter will summarize findings and present the results of empirical testing.

2. Literature Review

Although concept proposing contingent claims as a solution for excessive corporate debt originated in 1991 (Harvard Law Review, 1991), pricing methodology started to develop much later with the spread of contingent convertibles following financial crisis 2008-2009. First comprehensive pricing models were introduced after 2010. George Pennacchi incorporated contingent convertibles into structural credit risk model of a bank and derived the value of CoCos by holding assumption that book value of equity, senior debt, deposits and contingent convertibles should equal the value of all assets (Pennacchi, 2010). Exploring interactions between the components of balance sheet, Pennacchi derives credit spread on contingent bonds for different levels of bank risk and enriches previous structural approaches by modelling banks´ assets with jump diffusion process, stochastic interest rates and allowing banks to adjust the amount of deposits to revert to desired capital ratio. Subsequent efforts regarding contingent convertibles resorted to more direct approach aimed for closed pricing formula enabling derivation of contingent value via plugging observable variables rather than via simulation.

De Spiegeleer and Schoutens were among the first to introduce pricing formulas having roots both in credit and equity derivatives mathematics (De Spiegeleer & Shoutens, 2011). The first approach, Credit derivatives pricing model, views contingent convertibles from the fixed income viewpoint. Contingent convertible price is there derived by adding the extra yield to risk free rate, credit and liquidity premium. This extra yield should compensate for the possibility of conversion or write down and should reflect probabilities of hitting the trigger level. The basis for determining this extra yield is already well-developed framework for pricing defaultable bonds and modelling of default intensity – trigger event is statistically modelled similarly using trigger intensity instead of default one. Loss given trigger event is computed using specified or implied conversion price. The final price under credit derivatives is than derived by De Spiegeleer and Schoutens correspondingly again to pricing scheme for defaultable bonds.

Equity derivatives approach, also introduced by De Spiegeleer and Schoutens, presents the price of contingent convertible as the sum of corporate bond, knock in forwards and subtracts the sum of binary down-in reflecting that coupons are not paid subsequently after trigger event.

Price sensitivities were outlined in De Spiegeleer and Schoutens, comparative statistics and dynamics were further analysed in Pricing of Contingent Convertibles by Jung who considered beside stock price sensitivities also volatility and maturity sensitivities (Jung, 2012).

It is important to note that significant simplifications were needed in order to arrive to closed pricing formula. Both approaches presuppose that stock price follows Brownian motion and methodology falls within Black-Scholes framework. Authors themselves however do acknowledge that CoCos are instrument with fat-tail risk and that assumption regarding constant price volatility does not correspond to reality and propose solutions overcoming these shortcomings of model. Stochastic volatility model (Heston model) is proposed by Jung and consequently tested via Monte Carlo simulations (Jung, 2012). De Spiegeleer and Corcuera devote their succeeding work to exploring applicability of exponential Levy process with jumps and heavy tails (Corcuera, et al., 2013). Jump Diffusion approach is additionally qualitatively evaluated by Teneberg (Teneberg, 2012). These more complex approaches however rely on simulation and parameter calibration.

Empirical examination of both closed formula models and simulation based model has been conducted but focused only on small sample of CoCo instruments. De Spiegeleer and Schoutens apply credit and equity derivatives models on contingent bonds issued by Lloyd and Credit Suisse. Authors however focus mainly on derivation of implied trigger level and then decomposing CoCo price to three building blocks: a risk-free corporate bond, knock-in-forward and short position in down-in binary options as stated in Equity derivatives approach (De Spiegeleer & Shoutens, 2011). Further examination of these two models is conducted by Jung on the CoCo issuance by Credit Suisse. Author shows that in case of interest rate equal to dividend yield credit and equity derivatives formulas result in the same CoCo price (Jung, 2012). Jung also concludes that Equity derivatives approach is more precise due to more realistic treatment of cash flows. Credit Suisse contingent convertible bond has been particularly popular – it has been used for empirical test of pricing models also by Erismann, Rüdlinger or in work of Veiteberg, Bysveen and Rosef (Erismann, 2015) (Rüdlinger, 2015) (Veiteberg, et al., 2012).

Rüdlinger examines market sensitivities even beyond the standard model by ordinary least square model. On the sample of twelve CoCo issuances, he concludes that CDS spread and stock returns are statistically significant for explanation of CoCo spread while interest rate is not. Erismann compares four models (Equity derivatives model, Credit derivatives model, structural model and CDS model developed by J.P. Morgan and concludes that all models tend to overestimate risk bound with contingent convertibles, but structural model shows the smallest pricing errors and should be therefore favoured.

All the previously mentioned studies however consider only relatively small sample and do not cover entire variety of possibilities for CoCo structuring; namely all types of loss absorption mechanism and both low and high trigger levels. Other issues with tests about model validity revolve around model assumptions for Equity derivatives and Credit derivatives approach, which are quite restrictive and far-reaching, but which are however not directly tested in studies mentioned beforehand.

The first and probably the most crucial assumption is that trigger level, which is in case of accounting trigger type usually expressed in terms of Core Tier 1 ratio, can be approximated by stock price. This assumption is deemed necessary as the CET1 ratio is not continuously available and therefore cannot be used within the pricing formula. The information about capital ratio is only available on quarterly or semi-annual basis and both current and potential investors are meantime kept in dark. Essentiality of variable which is non – observable on daily basis on the market invalidates use of closed from analytical formulas which are derived under replication theory – it is not possible to invest directly into underlying asset (Core Tier equity) and therefore position in contingent convertible cannot be replicated by position in underlying asset and risk-free asset as for options. (Teneberg, 2012)

Furthermore, trigger system relying on single accounting number is prone to manipulation attacks by both insider management and outside investors, potentially benefiting from conversion. (De Spiegeleer & Shoutens, 2011) Regulatory trigger, which typically means conversion at regulators discretion, e.g. declaring bank operation non sustainable without government support, presents the same need for approximation by linking the potential conversion to predetermined level of market stock price.

From the pricing viewpoint the validity of this assumption is crucial for theoretical validity of entire formula both in Credit derivative and Equity derivative approach. Not much evidence has been provided on the reality of this assumption and specifically on link between figures reported on balance sheet and estimated trigger level. Rüdlinger provides method for calculation of trigger level implied by book figures under assumption of one to one relationship between book and market value of equity and illustrates calculation on Credit Suisse 7.875% CoCo instrument. (Rüdlinger, 2015) The link however lacks further empirical analysis on larger sample of data and evaluation of validity of one to one assumption.

Second assumption underlying Credit derivatives and Equity derivatives approach is stemming inherently from the use of Black – Scholes model for derivation of probability of conversion. Black – Scholes model assumes that stock price is an infinitesimal random walk, more precisely geometric Brownian motion with constant drift and volatility. (Black & Scholes, 1973). This assumption, together with previously described assumption that CET1 level can be associated with the fixed level of stock price when conversion happens, enable model to evaluate the probability of conversion under such dynamics. Probability is subsequently used in pricing formula which calculates the price as a discounted expected value. The empirical observance of the volatility smile for options and different implied volatility for different moneyness of the option which is observed in the real market data is presumably passed on to contingent convertibles, including fat tail risk and possibly greater implied volatility for contingent convertibles close to the trigger. Some deliberation has been devoted to examination of the reality of this assumption and in addition, some more complicated models have been developed in order to overcome shortcomings of Black – Scholes dynamics for pricing of contingent convertibles.

Discussion and analysis of validity of constant volatility assumption for CoCos has been rather qualitative then quantitative. Spiegeleer and Schoutens follow their basic Credit derivatives and Equity derivatives approach with smile conform models replacing Brownian motion with drift and constant volatility with Levy process incorporating jumps and heavy tails (Corcuera, et al., 2013). Teneberg replaces standard geometrical Brownian motion with jump – diffusion process remarking that pricing of contingent convertibles is even more distorted than option pricing as stock return behaviour can hardly be approximated by normal distribution especially close to barrier (Teneberg, 2012). Jung also recognizes that implied market trigger is volatile over time and suggest Heston stochastic volatility model as an alternative model for price dynamics. (Jung, 2012)

Even though constant volatility has been criticized and alternative models not assuming necessarily constant volatility of implied trigger have been developed, little research was provided on more precise behaviour of stock volatility near trigger level. Together with widely applied linkage of stock price level with CET1 ratio level determined in the CoCo contract, these two crucial assumptions for Credit derivatives and Equity derivatives approach remain rather empirically untested.

3. Model and Data description

Next chapter is devoted to brief presentation of one of the most widely accepted theoretical models for contingent convertibles - Credit derivatives model. Crucial variables used in pricing model are introduced and pricing formula rather intuitively derived in order to provide closer look on theoretical dynamics and sensitivities.

After this necessary theoretical introduction, dataset prepared for the empirical analysis will be presented. Finally, relationships between contingent convertible spread and underlying pricing variables will be examined from the market data and compared to the theoretical predictions both visually and using ordinary least square model. This chapter should provide deeper insight into both theoretical and practical realm of the pricing and outline potential deviations between these for further investigation.

3.1 Credit derivatives model

Credit derivatives approach is building on default modelling theory and is using the modified intensity parameter together with CoCo specific recovery rate to derive the CoCo spread to risk free non – convertible bond (De Spiegeleer & Shoutens, 2011). Intensity parameter, typically denoted λ, is the probability of issuer defaulting in interval $[\mathrm{t}, \mathrm{t} + \Delta\mathrm{t}]$ for a small change of time $\Delta\mathrm{t}$. Survival probability, that is the probability of issuing institutions surviving up to time T (or equivalently probability of default time t being higher then T) is then

$$P(t > T) = \exp(-\lambda \mathrm{T}) \tag{1}$$

Recovery amount, the amount investor is expected to receive in case of default is recovery rate R and expected loss is then equal to $(1 - R)N$. Following that expected payoff should be the same for defaultable and non – defaultable, risk free bonds, following relationship for credit spread can be derived (De Spiegeleer & Schoutens, 2010):

$$Spread = (1 - R)\lambda \tag{2}$$

The credit spread is therefore the product of expected loss $(1 - R)$ and approximately the probability of such loss λ.

Credit derivatives approach extends previous derivation of credit spread to modelling trigger event and incorporating expected loss for contingent convertibles. There, λ^* is defined as intensity of trigger event defined parallel to the default intensity as probability of trigger event occuring within short time interval $[\mathrm{t}, \mathrm{t} + \Delta\mathrm{t}]$. Specific purpose of contingent convertibles then dictates that trigger event should occur before potential default and therefore trigger intensity λ^* should be higher than default intensity λ.

Expected recovery amount then depends largely on the loss absorption mechanism of underlying CoCo. In case of trigger event, write down type transforms contingent convertible bond of notional N into predetermined cash amount $\alpha.N$. Write down portion α is not necesarilly specified in each contract as one has to potentially estimate it in case of temporary write down with the possibility of issuer writing up the notional after regaining financial health. The lack of actually written down contingent convertibles in the past creates large uncertainty about consequent write up and although temporary write down CoCos are naturally considered to be a cheaper source of financing thus offering lower yield, actual size of the difference between permanent write down is unclear (McCunn, 2015).

Recovery rate R for CoCos with conversion to equity absorption mechanism is then dependent on market price of the stock at the time of conversion S_T and predetermined conversion price P_C at which bond is converted into equity via following relationship:

$$R = \frac{S_T}{P_C} \tag{3}$$

Regardless the loss absorption mechanism, equation for CoCo spread similarly to the previous equation for spread of defaultable bond states:

$$Spread = (1 - R_{CoCo})\lambda^* \tag{4}$$

As discussed previously, for equity conversion type R_{CoCo} can be calculated with estimated S_T and predetermined conversion price. Permanent or partially written down contingent convertibles also have quite predictable recovery amount. The crucial and more difficult part of the spread calculation then presents the estimation of trigger event intensity.

Modelling trigger event is generally difficult as the most of contingent convertibles have trigger level set for accounting ratio and not for market observable variable. Necessary approximation discussed previously is associating trigger event with particular market trigger. A trigger level based on accounting numbers, most often on Core Tier capital dropping below a minimum level, is replaced with a level for stock price. (De Spiegeleer & Shoutens, 2011) If stock price hits trigger level at any time before maturity, contingent convertible is deemed converted. Modelling of stock price then capitalizes on Black – Scholes model for option pricing, more specifically pricing model for barrier options. Probability p that particular barrier S^* is touched during the maturity of CoCo, denoted from now on as T is then derived as (De Spiegeleer & Shoutens, 2011):

$$p = N\left(\frac{ln\frac{S^*}{S} - \mu.T}{\sigma.\sqrt{T}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} N\left(\frac{ln\frac{S^*}{S} + \mu.T}{\sigma.\sqrt{T}}\right) \tag{5}$$

where

- $N(x)$ indicates cumulative standard normal distribution $N(x) = P(X \leq x)$ for $x \sim N(0,1)$
- S is current share price
- $\mu = r - q - \frac{\sigma^2}{2}$ for continuously compounded interest rate r , dividend yield q and stock volatility σ.

From equation (1) it can be subsequently obtained for trigger intensity:

$$\lambda^* = \frac{-\ln(1-p)}{T} \tag{6}$$

Spread for Contingent convertibles, reflecting the probability of suffering loss when trigger level is breached, combining (3), (4) and (6) and using the fact that $S_T = S^*$ is then:

$$Spread = -\left(1 - \frac{S^*}{P_C}\right) \frac{\ln(1-p)}{T} \tag{7}$$

Using Credit derivatives approach, spread can therefore be obtained using closed end formula. However, as not all variables used in calculation are directly observable in the market, prior estimation or approximation of some of these variables is needed:

- Interest rate r should be, as under Black Scholes risk neutral pricing framework, risk free rate corresponding to currency of the CoCo and with tenor matching the maturity of contingent convertible. Although some discussions about using stock repo rate, suggesting increased need of consideration for collateral agreements (Fung, 2011) have developed, using Libor rates or Treasury rates in corresponding currency is generally accepted approach.

- Dividend yield is commonly approximated using past dividends, which is referred as trailing dividend yield. Alternative approach is to estimate dividend yield from forwards or futures using spot – futures parity or the cost of carry model:

$$F_T = S_0 \exp[(r_f - q)T] \tag{8}$$

  The obvious drawback of this forward-looking dividend yield is that futures for stock are not always quoted on market.

- Maturity T is not necessarily as easy to determine as one might think. Perpetual contingent convertibles, lately so popular as a direct effect of Basel III regulation conditioning inclusion of instruments into AT1 Capital by perpetuity, require forecasting of future behaviour of the issuer. Perpetual CoCos are typically callable at multiple prespecified dates which adds the need to adjust the spread for such embedded option. As this option provides benefit for the issuer, spread of callable CoCo is expected to be higher than spread of otherwise equal, non – callable CoCo. Traditionally, value of callable fixed income instrument can be either calculated assuming that the instrument will be called at future call date (disregarding additional cash flows) or by substracting value of call option on such instrument with maturity equal to call date or with the use of a binomial tree, evaluating whether instrument is called at each node based on the development of future interest rates. (Rubio, 2005) Adding that behaviour of CoCo issuer is affected not only by interest rates development but also by development of capital structure and economic soundness, call option present a great challenge for CoCo spread calculation currently not possible to be empirically evaluated due to short existence of contingent convertible instruments.

- Volatility used in formula refers to volatility of stock price, not volatility of CoCo instrument price. Volatility in Black Scholes model is assumed to be constant, which mostly forces the user to rely on historical volatility. However, as all other variables in Black Scholes model are market observable, implied stock volatilities can be obtained reversely from the option prices. Such volatilities can subsequently be used in the formula for probability (5) and therefore, for the calculation of CoCo spread. Implied volatilities are usually available for several strike levels and differ based on option moneyness and one should use implied volatility accordingly with moneyness associated with trigger level. Various interpolation or modelling methods are available for calibration of whole volatility surface based on few available implied volatilities. Using this approach should reflect market reality more precisely than historical volatility.

- Current stock price is directly observable in the market, however barrier level for the stock price S^* is the probably the hardest input required for the pricing formula. Projected barrier level in addition has generally the highest relative impact on implied spread among all pricing inputs. Associating contract specified (mostly accounting, CET1 ratio based) trigger level with appropriate stock price seems therefore as the most crucial task in CoCo valuation. Despite that, it is commonly neglected, assigned to a specific level without further theoretical or empirical examination. Rather than arbitrarily choosing the barrier level, Erismann suggests calibrating such level using the initial CoCo price and implied trigger levels (Erismann, 2015). Analysis of implied trigger levels and general relation of capital ratios to stock price is further examined in subsequent chapter.

Previous section shows that despite that Credit derivatives approach is quite straightforward and provides a quick formula how to price contingent convertibles, input variables do need cautious measurement. Link between accounting trigger level and stock price level is crucial for the valuation and special care needs to be given to the association of stock price level with the CoCo barrier level. Consequently, and in parallel with the Black Scholes model and pricing of options, calibration might be necessarily used to first back out the implied trigger level using sub sample of contingent convertibles.

3.2 Dataset

Dataset collected for empirical testing comprises of eight contingent convertibles issued by eight different financial institutions – Deutsche Bank, HSBC, Barclays, UniCredit, Banco Santander, Societe Generale, Credit Suisse and BNP Paribas. Further in the text, names of the banks are used when referring to particular CoCo instrument.

Selection is driven partly by availability of variables serving as a pricing input – stock prices, volatilities or CDS spreads, partly by effort to include in sample different absorption mechanism (permanent/temporary write down, equity conversion) and contingent convertibles with both low trigger and high trigger levels. All the instruments have trigger event conditioned on CET1 ratio, as it has become standard in banking regulation. Maturities of selected CoCos are almost exclusively perpetual, as the tightening regulation now demands all Tier I capital instruments to be perpetual. (Basel Committee on Banking Supervision, 2013)

Appendix A1 contains the list of information for each contingent convertible in the sample including Issuer, Issue date, Maturity, Coupon rate, Loss absorption mechanism, Trigger level, Issue price, Currency, Coupon dates (frequency) and Callability of note.

3.2.1 CoCo Spreads

Variables observed for each contingent convertible note is closing price and mid z-spread, which are both available with daily frequency on Bloomberg. Period examined is ranging from the issue price of each instrument till the end of year 2016. Appendix A1 includes descriptive information about each time series and the graph of the series.

No extra cleaning of data is performed. Modification of CoCo input spread is only done in case of missing spreads/prices. If both spread and price is missing for business day, stale prices are assumed and closing spread from previous day is used. When only spread is missing for a day, but price is available, proxy spread is calculated based on the price and approximate sensitivity of spread change on the price change.

Highest number of observation is available for Barclays 7.625% CoCo which was issued early late 2012, smallest number of observation is 361 for Societe Generale 7.875% CoCo and BNP 7.375% CoCo which still provides quite a large period of observable spreads.

Lowest mean spread over the examined period is visible for low trigger Credit Suisse 6.25% CoCo, while Deutsche Bank 6.25% and Santander 6.375% CoCo is trading on average on highest spreads – reaching the mean value 7.32% (7.31% respectively).

Deutsche Bank 6.25% CoCo is also surpassing its peers in volatility – reaching standard deviation 3.33% over examined period. This value of statistic corresponds to hectic rally of the spread in early 2016, when spread exceeded 15%, while starting from spread below 4% at the time of issuance in May 2014.

Below is table with statistics for each CoCo instrument in the sample:

CoCo Instrument	Mean CoCo spread	Standard deviation of CoCo spread	Observations
Deutsche Bank 6.25%	7.32%	3.33%	682
Barclays 7.625%	4.23%	0.96%	1077
HSBC 6.375%	3.53%	0.45%	602
Credit Suisse 6.25%	3.45%	0.53%	667
UniCredit 8%	5.54%	1.11%	711
Santander 6.375%	7.31%	1.42%	608
BNP 7.375%	5.65%	0.65%	361
Societe Generale 7.875%	6.91%	0.90%	361

Figure 4: Summary statistics for CoCo Instruments included in the sample – CoCo spread

On top of calculating descriptive statistics for each CoCo instrument, tests for non – stationarity are run before delving into next sub - chapter devoted to testing basic empirical relationships between CoCo spread, Stock price, Volatility, Interest Rate and Credit Default Swap spread. Stationarity of each series of CoCo spreads is tested using Augmented Dickey – Fuller (ADF).

Results of the tests are included in Appendix A3. None of the CoCo spread time series in the sample shows Augmented Dickey – Fuller test statistic for which presence of unit root in a time series could be rejected. Because the non – stationarity of CoCo spread time series cannot be rejected, first difference of CoCo spreads is calculated and Augmented Dickey – Fuller test is again run for the first difference series. The first difference of the series is denoted d_CoCo spread. Results shown again in Appendix A3 now indicate that the null hypothesis of unit root presence can be rejected and therefore first difference CoCo spread series is used further in the regression.

3.2.2 Stock prices

To examine how the predicted relationship between CoCo spreads and changes in stock prices compares to empirical observations, closing stock prices for eight banks included in the sample are observed with daily frequency. Data is collected from Bloomberg source for each day since the issuance of the CoCo instrument to the end of year 2016. Summary statistics for stock price series can be found in Appendix A2. ADF test is again run on stock price series (results in Appendix A4) and similarly to CoCo spread series, first differenced series d_Stock price is used further in the analysis due to non – stationarity of original series.

3.2.3 Volatility

Stock price volatility is another crucial pricing parameter affecting implied fair spread both in Credit derivatives and Equity derivatives pricing model. Building upon assumption of contractually specified CET1 trigger level being proxied by specific stock price level, stock price volatility serves implicitly as a proxy for CET1 volatility. Stock price volatility then crucially affects implied probability of conversion under Black Scholes mechanics.

In order to examine empirical relationship between CoCo market spreads and market implied volatility for particular underlier, time series with daily frequency has to be collected for each of eight banks included in the sample. More precisely, whole volatility surface should be collected for each date, as implied volatility observed in equity markets is typically function of both strike and time to maturity. Both fluctuating current stock price and stock price trigger level estimated for particular instrument lead to different moneyness of the trigger level and therefore correspond to different implied volatility level consistently with option markets. Similarly, when considering time to maturity, different option maturities are likely to be characterized with different implied volatilities and this should also translate to different volatilities used as a pricing input for contingent convertibles – e.g. two coupons received in subsequent years should be associated with different implied volatilities calculated for each coupon date separately based on observed option prices.

Collecting whole implied volatility surface for each date in the data sample is however too cumbersome, as usually only few options are actively traded in the markets and surface therefore contains only few observable points. The rest of the surface has to be estimated using interpolation techniques of various degree of complexity.

The same applies for this thesis – implied volatility for few strikes respectively maturities is calculated each business date using available option prices. Available implied volatilities are then used for calibration of SABR stochastic volatility model which is subsequently used for obtaining implied volatilities for other moneyness levels (maturities).

Note that first differenced series d_Volatility is used again due to non – stationarity of level series.

3.2.4 Interest rates and dividend yield

Interest rate in Credit derivatives model should ideally be risk free rate. Risk free rate, together with dividend yield constitute drift of the Brownian motion and therefore have direct effect on probability of reaching trigger level. More precisely, drift is defined as: $\mu = r - q - \frac{\sigma^2}{2}$ where r is continuously compounded risk-free rate, q is dividend yield and σ represents volatility. Effect of drift parameters on probability and consequently on CoCo spread is more closely analysed in next subchapter, for now it suffices that under normal conditions, higher drift decreases the probability of reaching the trigger level and therefore would lead to lower CoCo spread predicted by the model.

For interest rate, swap rate in appropriate currency is taken as pricing input. Swap rate is rate at which two parties agree to exchange future cash flows, one party paying fixed rate and second party paying floating rate based on some interest rate benchmark. As swap market makers are typically large investment and commercial banks and swaps are now generally cleared through central counterparties, quoted swap rate should be close to risk free rate. (PIMCO, 2016)

Market mid swap rate is actively quoted and available for many tenors. For the purpose of pricing the particular CoCo instrument, either first call date or maturity is taken as a final date and for this final date interest rate is calculated using simple interpolation method. It shall be noted that taking single swap rate is certain simplification undertaken to control the amount of data collected and using set of interest rates associated with each coupon date would be preferable.

Interest rate (swap rate) is again first differenced and $\mathrm{d\_Interest\ rate}$ is actually used further in the analysis due to non – stationarity of original swap rate series.

Dividend yield is calculated from future prices using formula $F_T = S_0 \exp[(r_f - q)T]$, where F_T is future price with settlement at T, S_0 is stock price at the time of measurement, r_f is risk free rate as outlined above and q is the dividend yield we are interested in. Such implied dividend yield is commonly provided by Bloomberg for various maturities.

3.3 Sensitivity analysis

This subchapter is devoted to empirical evidence on relationship between CoCo spread, stock price, volatility, CDS spread and interest rate. Theoretical relationships implied by Credit derivatives model are first introduced and then compared visually to relationships observed on the market since the issuance of each CoCo instrument.

As each series included in the analysis is potentially non – stationary, Augmented Dickey Fuller tests are run and subsequently each series is differenced. Final ordinary least square model is run for each bank individually using differenced series. While OLS model is not primary goal of this thesis, it is natural and useful first step into the empirical analysis. It provides evidence on statistical significance of each variable in Credit derivatives model and it also serves as a good starting point in searching for deviations of model predicted spreads and observable spreads.

3.3.1 Sensitivities – Visualization

Theoretical sensitivities of CoCo spread on stock price, volatility and interest rate as a drift parameter are derived here using the Credit derivatives model formula.

Specifically, model predicted value for CoCo spread is calculated for different levels of variable of interest while keeping other variables constant.

For variables which are held constant throughout the calculation, average sample values during the observed period are taken. There are overall seven variables included in the Credit derivatives model – stock price, trigger level, volatility, maturity, dividend yield, interest rate and conversion price.

3.3.1.1 Sensitivity on Stock Price - Theoretical

Stock price together with estimated trigger level is crucial pricing input in Credit derivatives model for contingent convertibles. Probability of CoCo conversion under Black Scholes mechanics is dependent on the distance between current level of stock price and assumed trigger level and while trigger level is typically assumed to be quite stable on day to day basis, stock prices fluctuations will inevitably lead to fluctuations in implied probabilities of conversion and subsequently to changes in CoCo implied spreads.

Visualization below captures theoretical relationship between CoCo spread and stock price under Credit derivatives model. Average sample values for Deutsche Bank are used as input and kept constant throughout the calculation: average interest rate during the observed period was 2.03%, dividend yield implied from forward prices 1.53% and maturity (to first call date) 9.64. The stock price is then allowed to range between minimum sample value during observed period 10.55 and 33.20 and implied CoCo spread is plotted on y axis. Each line represents different level of volatility – red line is using maximum volatility during the observed period 42.16%, blue line average sample volatility 31.32% and green line minimum sample volatility 25.18%. Volatility here is using book implied trigger value as a strike, which will be explained more in detail later.

We see that there is inverse relationship between CoCo spread and stock price. Relationship is far from linear and we see that sensitivity is increasing when approaching trigger level – closer stock price gets to estimated trigger level, the larger is the impact on the implied CoCo spread. We can also notice that sensitivity is higher for higher level of volatility when the impact on probability of conversion is pronounced.

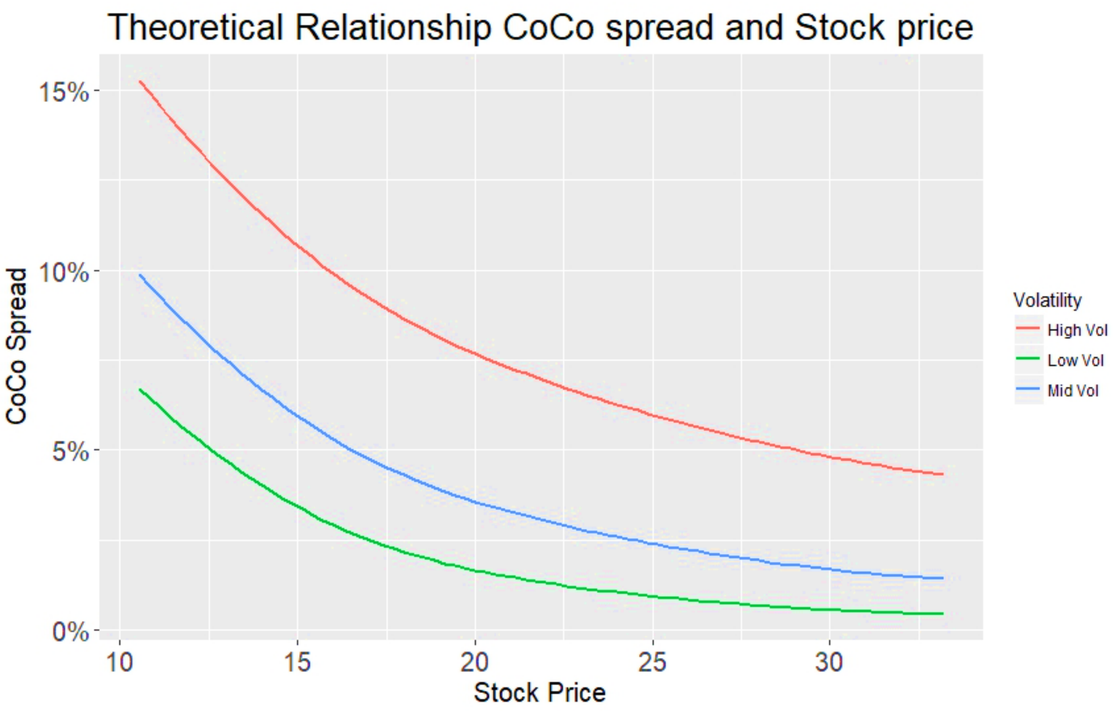

Figure 5: Theoretical relationship between CoCo spread and stock price derived from Credit derivatives model using average inputs for Deutsche Bank CoCo. Three different levels of volatility equal to maximum, minimum and average sample value.

3.3.1.2 Sensitivity on Stock Price – Empirical

We begin our empirical sensitivity analysis by plotting the levels of CoCo spread against each variable of interest. It is important to note that for now we ignore interactions between the explanatory variables – stock price, volatility and interest rate and variable levels are simply plotted against each other based on the sample we collected for each CoCo issue.

In order to increase readability, visualization for stock price variable is divided into two groups of four based on stock price range. Set of CoCo spread and stock price pairs for each CoCo issue is plotted and separate CoCo issues are differentiated by colour. As theoretical relationship above suggests that we should observe convex relationship, we use local regression line rather than simple OLS regression line to see whether the empirical relationship depends on the stock price level.

Empirical relationships mostly follow the theoretical relationship previously outlined – local regression line is mostly downward sloping and it appears that on average the slope of the curve is indeed decreasing when going further from the trigger level. We can also notice that for Deutsche Bank and Santander but also for other curves residuals tend to be quite high, representing sometimes difference 600bps in CoCo spread for the same stock price level. This is partly due to neglecting all other explanatory variables in visualization, partly simply because even the theoretical relationship is not polynomial and local regression does not fit very well. Still, visualization provides first picture of the empirical relationship observed during the period and can indicate first potential pricing anomaly for Barclays. Contrary to theoretical relationship, slope of local regression line for Barclays changes from negative to positive moving from low stock prices to higher.

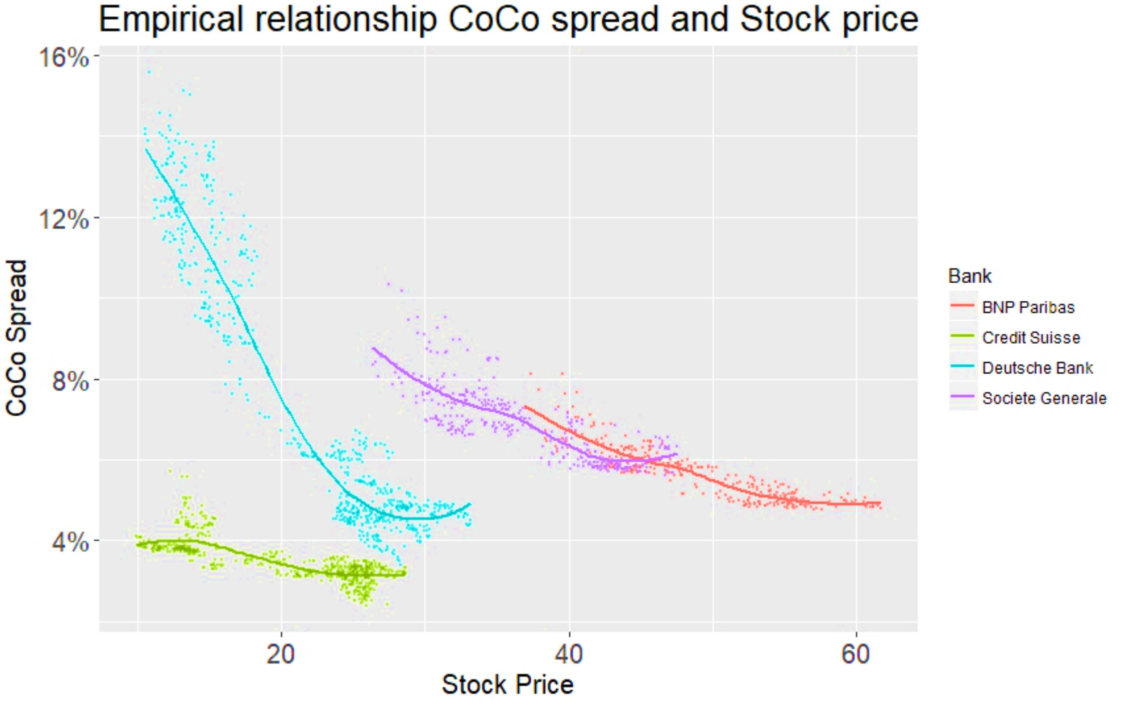

Figure 6: Empirical relationship between CoCo spread and Stock price observed for BNP Paribas, Credit Suisse, Deutsche Bank and Societe Generale

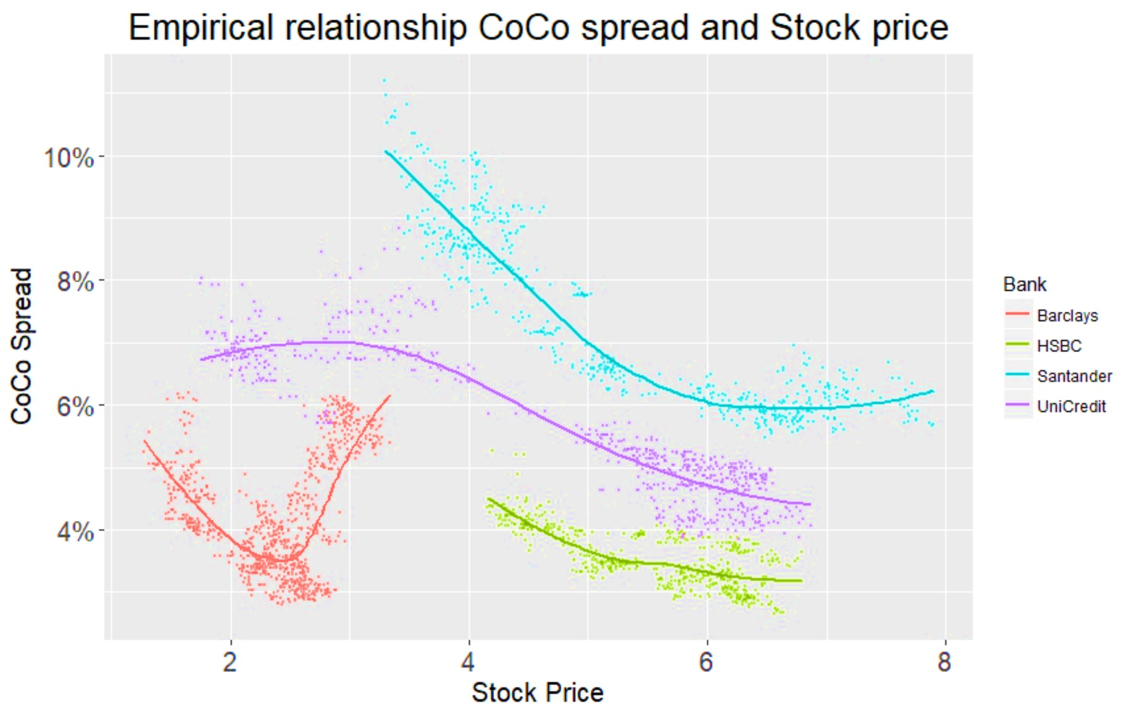

Figure 7: Empirical relationship between CoCo spread and Stock price observed for Barclays, HSBC, Santander and UniCredit

3.3.1.3 Sensitivity on Volatility - Theoretical

Beside stock price and estimated trigger level, probability of conversion under Credit derivatives model is also dependent on volatility – stock price volatility. Taking stock price level as a proxy for CET1 trigger level specified in contract naturally means that we take also stock price volatility as a second pricing input. Keeping the distance between stock price level and trigger level constant, volatility of stock price increases the probability of conversion and subsequently should lead to higher CoCo spread.

Theoretical CoCo spread implied from Credit derivatives model is visualized below. The graph shown is drawn for average sample values of Deutsche Bank 5.125% CoCo. Average stock price during the observed period was 22.24, interest rate 2.03%, dividend yield implied from forward prices 1.53% and maturity (to first call date) 9.64. Allowing volatility parameter now to range from sample minimum volatility 25.2% and sample maximum volatility 42.2% and drawing CoCo spread on y-axis leads to the upward sloping graph. Red line represents high trigger scenario, where estimated trigger level is set to 60% of minimum stock price for Deutsche Bank during the observed period - 13.34. Blue line represents mid trigger level 8.89 (40% of min stock price) and green line is for low trigger level 4.45 (20% of min stock price).

By comparing slope of each curve we can see that theoretical sensitivity to volatility parameter is higher for high trigger level – decreasing the distance between stock price level and trigger level increases the sensitivity of CoCo spread on volatility changes. We can also see that although relationship seems almost linear, sensitivity is slightly increasing in volatility – impact of volatility moves should therefore be slightly higher for higher volatility levels.

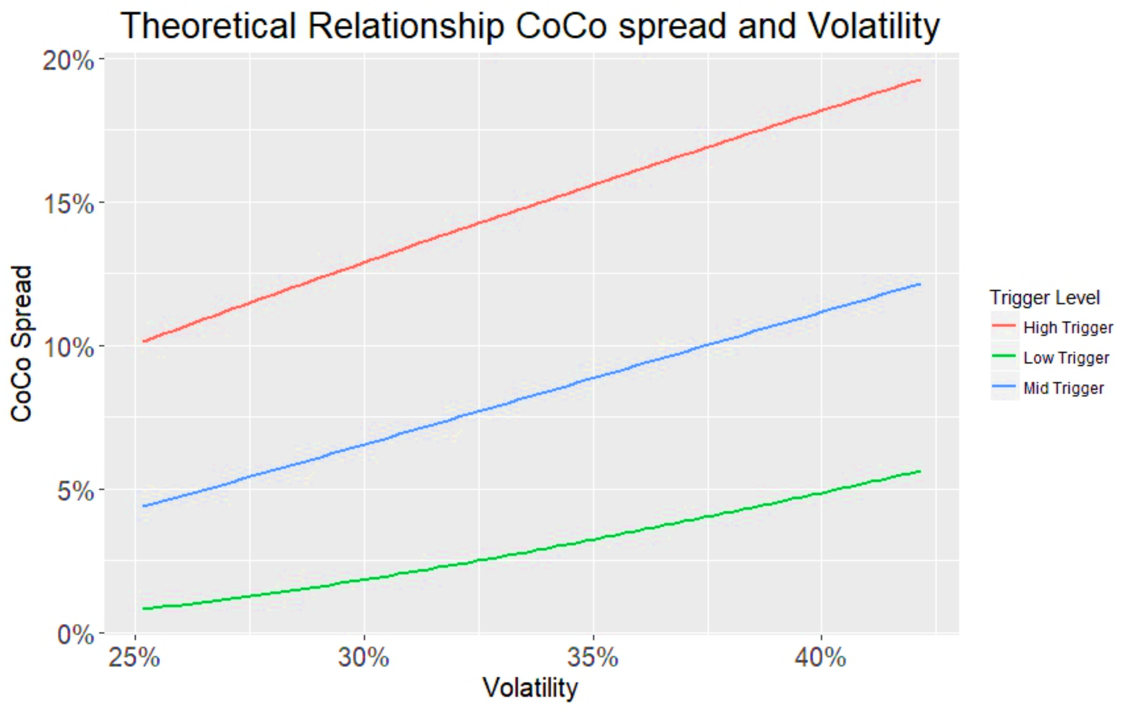

Figure 8: Theoretical relationship between CoCo spread and Volatility derived from Credit derivatives model using average inputs for Deutsche Bank CoCo. Three different levels of trigger equal to 60% (high trigger), 40% (mid) or 20% (low) of minimum stock price

3.3.1.4 Sensitivity on Volatility - Empirical

Similar to previous empirical visualization for CoCo spreads and stock Prices, volatility levels are plotted against CoCo spreads for each issue in the sample. Again, interactions between volatility, stock price or interest rates are ignored for now – this includes usually observed negative relationship between stock returns and volatility (periods with negative returns characterized with high volatility). CoCo issues are for visualization divided into two groups based on average CoCo spread during the observed period – low spread group contains Barclays, BNP Paribas, Credit Suisse and HSBC issue and high spread group contains Deutsche Bank, Santander, UniCredit and Societe Generale.

Visualization now employs simple regression line to provide the comparison to theoretical relationship, which resembles linear relationship for values in the sample. Plotted relationship is positive for all CoCo issues in the sample, while slope of the linear regression line differs quite significantly. Based on theoretical relationship deduced previously, we would expect higher sensitivity for high spread CoCos than for lower spread CoCos. Indeed, the regression line of the highest spread issue of Deutsche Bank has also highest slope (0.73 compared to second Barclays with slope 0.24), however we postpone further conclusions after the complete regression part.

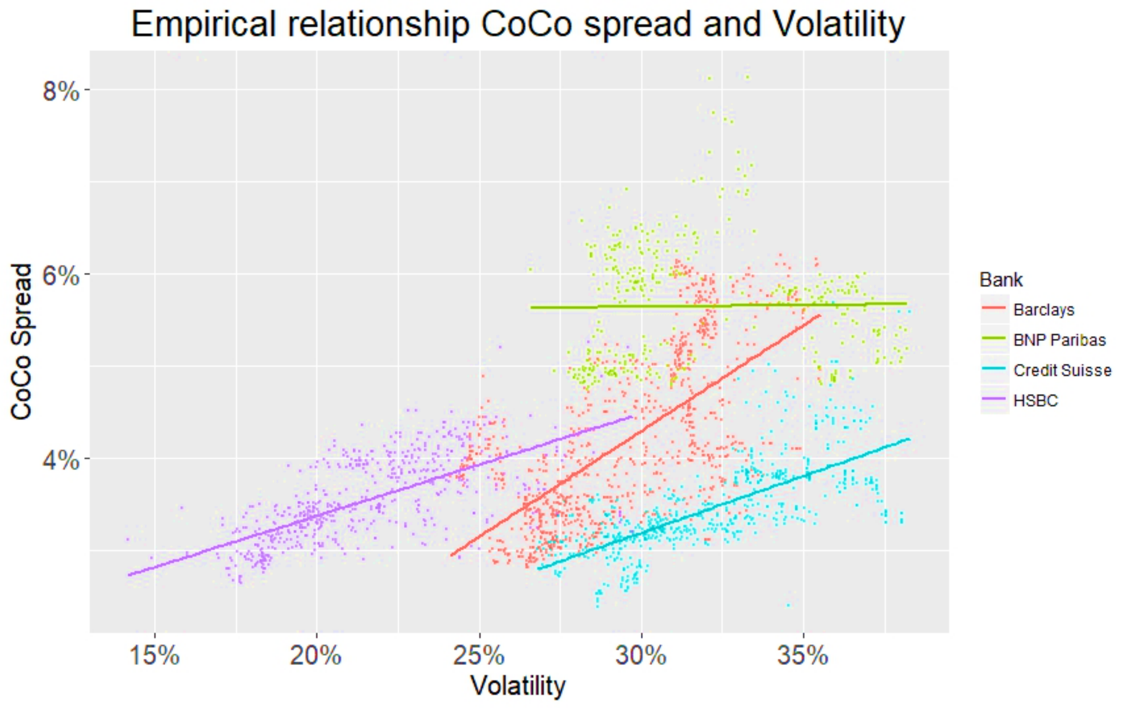

Figure 9: Empirical relationship between CoCo spread and Volatility observed for Barclays, HSBC, BNP Paribas and Credit Suisse

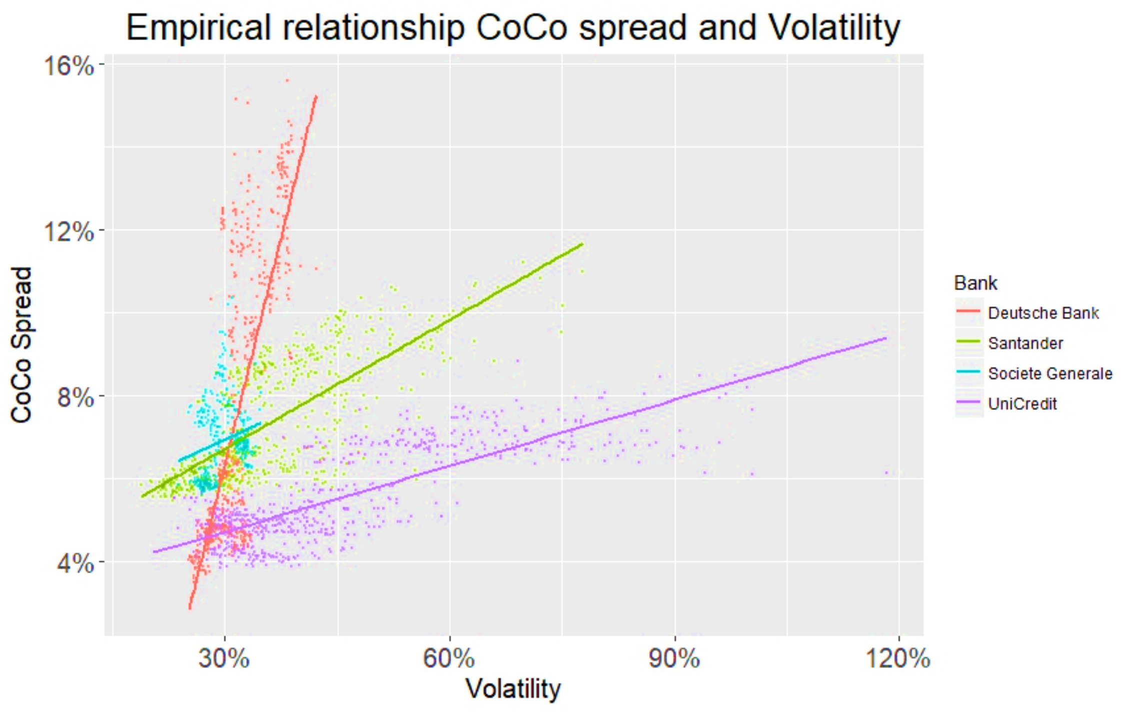

Figure 10: Empirical relationship between CoCo spread and Volatility observed for Deutsche Bank, Santander, Societe Generale, UniCredit

3.3.1.5 Sensitivity on Interest Rate - Theoretical

Interest rate together with dividend yield and adjustment for variance composes drift for Black Scholes process: $\mu = r - q - \frac{\sigma^2}{2}$. Ceteris paribus we would expect that probability of conversion, that is probability of hitting downward located trigger level, is decreasing in drift rate. Dividend yield is quite stable over time and focusing now solely on interest rate, we observe inverse theoretical relationship between interest rate and CoCo spread.

Relationship is again plotted for average sample values for Deutsche Bank: average stock price during the observed period was 22.24, average volatility 31.3%, average dividend yield 1.53. Interest rate is then allowed to range from minimum sample value in observed period 1.26% and maximum sample value 2.65% and is plotted against implied CoCo spread. Red line is obtained using the average parameters above and maturity 10 years, green line is for maturity 5 years and blue line represents short maturity 2 years.

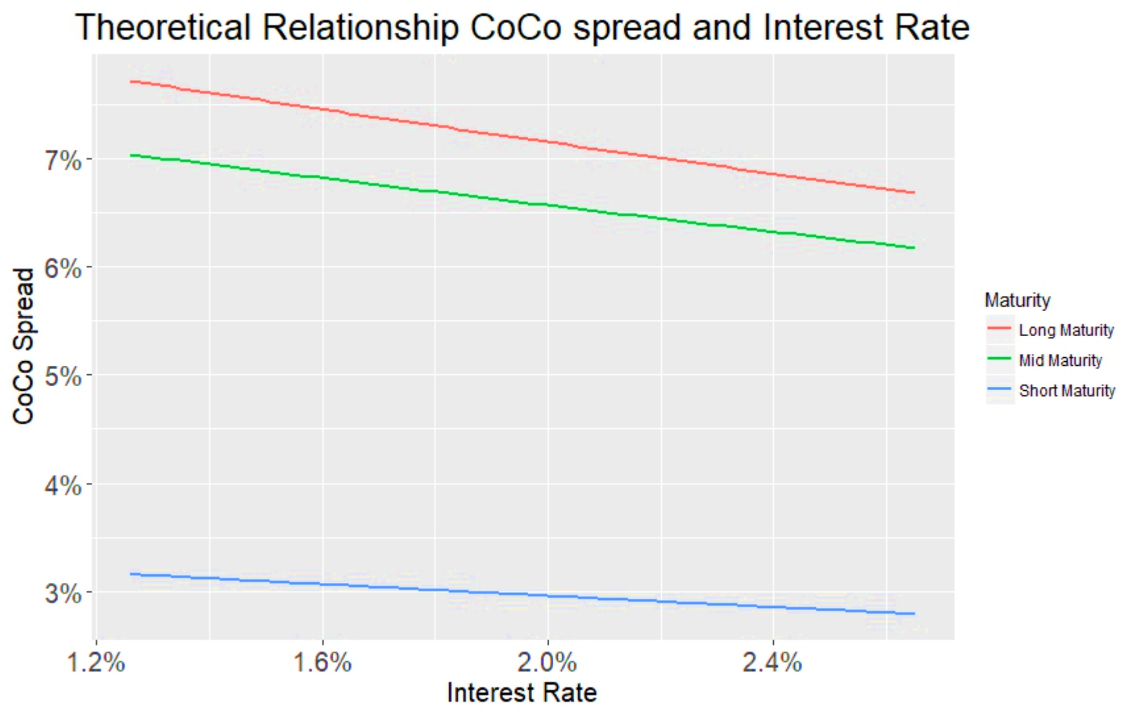

Figure 11: Theoretical relationship between CoCo spread and Interest rate derived from Credit derivatives model using average inputs for Deutsche Bank CoCo. Three different maturities equal to 10 years (long), 5 years (mid) or 2 years (short).

All curves are downward sloping as expected. What is also visible from graph is that the sensitivity of CoCo spread on interest rate is slightly higher for long maturity 10 years than for mid maturity line and even more higher than interest rate sensitivity for short maturity. Significance of the drift parameter and therefore the significance of interest rate is quite intuitively increasing in time to maturity.

3.3.1.6 Sensitivity on Interest Rate - Empirical

Last simple visualization is done for interest rate variable. Sample values for pair CoCo spread and interest rate are again plotted for each CoCo instrument in the sample. CoCo issues are again divided into two groups based on average CoCo spread. All regression lines plotted indicated inverse empirical relationship between CoCo spread and interest rate, which is in line with theoretical predictions. Regression line does not fit very well and we observe high residuals, which could again be result of omitted effect of stock price, volatility or other variables effecting CoCo spread in the visualization. In absolute terms, slope of regression line is again highest for Deutsche Bank CoCo issue with the highest average CoCo spread (7.37 compared to the second highest Santander 4.67) suggesting higher sensitivity to interest rate for issues with higher probability of conversion.

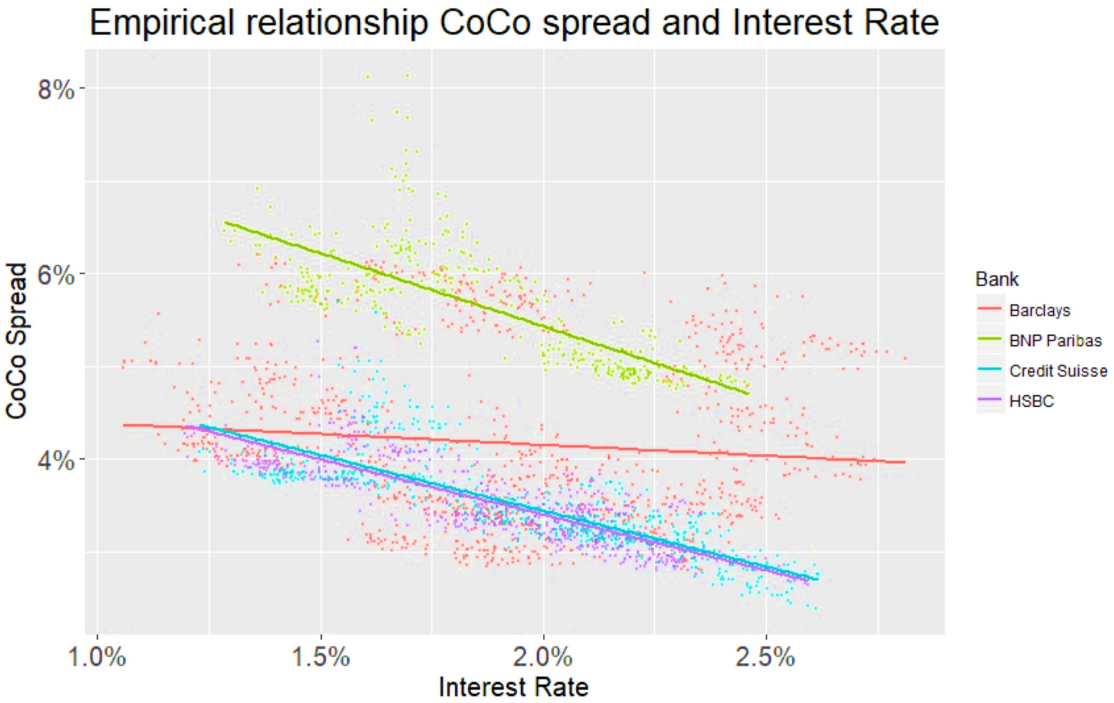

Figure 12: Empirical relationship between CoCo spread and Interest rate observed for Barclays, HSBC, BNP Paribas and Credit Suisse

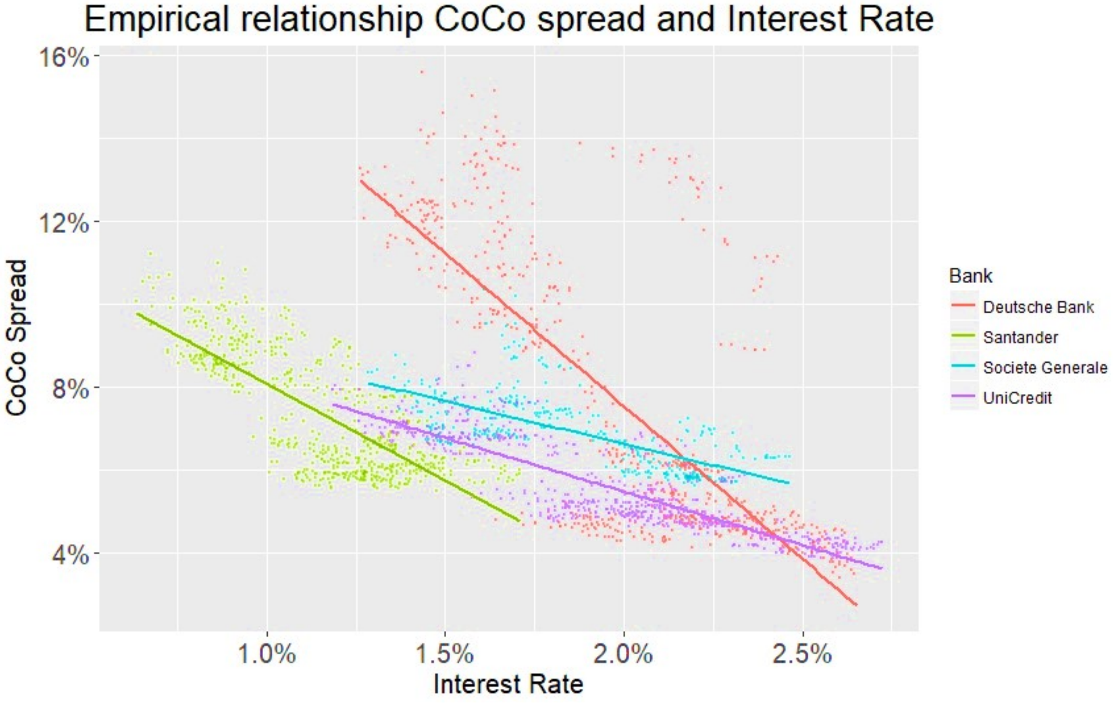

Figure 13: Empirical relationship between CoCo spread and Interest rate observed for Deutsche Bank, Santander, Societe Generale, UniCredit

3.3.2 Sensitivity Analysis - Regression

Stepping now from visual analysis, next natural step in the analysis is to run regression and examine empirical relationship between CoCo spread, stock price, volatility and interest rate more carefully. First, CDS Spread variable is added in order to lower the risk of erogeneity – CDS spread of particular CoCo issuer is quite likely strongly correlated with CoCo spread. After all, CoCo conversion is likely to precede potential default and both credit default swap and contingent convertible spreads are increasing when financial conditions of issuer deteriorate. CDS Spread, while not directly part of pricing formula for contingent convertible, is expected to have positive relationship with CoCo spread.

Next, each of the series included in regression is tested for stationarity. Augmented Dickey Fuller test is run and results are conclusive and high p- values indicate that non – stationarity of neither CoCo spread series, stock price series, volatility series, interest rate series or CDS Spread series can be rejected. Results of the tests for CoCo spread, stock price and CDS spread series can be found in Appendix A3 – A5. Each series is therefore first differenced for each CoCo issuer and following regression is estimated separately for each issuer in the sample and first differenced variables are denoted as $d_{Variable}$. The final equation to be estimated by OLS model therefore is:

$$d_{CoCoSpread_t} = \alpha + \beta_1 * d_{StockPrice_t} + \beta_2 * d_{Volatility_t} + \beta_3 * d_{CDSSpread_t} + \beta_4 * d_{InterestRate_t} + \varepsilon_t \quad (9)$$

Results of the regression are divided to two groups for greater readability – first group contains four CoCo issues with lower average spread over the examined period (Credit Suisse, Barclays, HSBC and BNP Paribas) and second group contains CoCo issues with higher average CoCo spread (Deutsche Bank, Santander, Societe Generale and UniCredit).

Overall, results of regressions for differences appear to be less conclusive than previous visual analysis for levels. Still, joint insignificance of variables can be rejected for each regression, as indicated by high F Statistics. R square ranges from lowest 0.066 for HSBC Bank to highest 0.689 for BNP Paribas.

Table 1: Low CoCo Spread group results

	Dependent variable:			
	d_CoCoSpread			
	Credit Suisse	HSBC	BNP Paribas	Barclays
d_StockPrice	−0.00008	−0.00074	−0.00018***	−0.00363***
	(0.00010)	(0.00046)	(0.00004)	(0.00091)
d_Volatility	−0.00281	0.00097	0.00340	0.02348***
	(0.00493)	(0.00200)	(0.00420)	(0.00631)
d_CDSSpread	0.67062***	0.24737***	1.20083***	0.18469***
	(0.08243)	(0.06928)	(0.09272)	(0.06069)
d_InterestRate	0.25810***	−0.24889***	−0.99197***	−0.60461***
	(0.08059)	(0.07364)	(0.08322)	(0.08735)
Constant	0.00002	0.00001	−0.00001	−0.00001
	(0.00004)	(0.00003)	(0.00003)	(0.00004)
Observations	621	570	358	1,014
R^2	0.12262	0.06583	0.69202	0.12002
Adjusted R^2	0.11692	0.05922	0.68853	0.11653
Residual Std. Error	0.00091 (df = 616)	0.00082 (df = 565)	0.00064 (df = 353)	0.00123 (df = 1009)
F Statistic	21.52179*** (df = 4; 616)	9.95355*** (df = 4; 565)	198.29760*** (df = 4; 353)	34.40501*** (df = 4; 1009)
Note:				*p<0.1; **p<0.05; ***p<0.01

Figure 14: Separate CoCo regression results for Credit Suisse, HSBC, BNP Paribas and Barclays

Table 1: High CoCo Spread group results

	Dependent variable:			
	d_CoCoSpread			
	Deutsche Bank	Santander	Societe Generale	UniCredit
d_StockPrice	−0.00198***	−0.00356***	−0.00006	0.00006
	(0.00022)	(0.00053)	(0.00009)	(0.00049)
d_Volatility	−0.03447***	0.00020	0.02128***	0.00032
	(0.01183)	(0.00439)	(0.00608)	(0.00094)
d_CDSSpread	1.09339***	1.95054***	1.54249***	0.66314***
	(0.10335)	(0.10617)	(0.14761)	(0.08155)
d_InterestRate	−0.72805***	−0.16903	−0.89010***	0.19769*
	(0.21751)	(0.12703)	(0.14454)	(0.11745)
Constant	0.00002	0.000002	0.000005	0.00001
	(0.00010)	(0.00005)	(0.00006)	(0.00005)
Observations	681	580	345	670
R^2	0.34177	0.59737	0.51176	0.11457
Adjusted R^2	0.33788	0.59457	0.50601	0.10924
Residual Std. Error	0.00254 (df = 676)	0.00119 (df = 575)	0.00110 (df = 340)	0.00132 (df = 665)
F Statistic	87.75089*** (df = 4; 676)	213.28000*** (df = 4; 575)	89.09416*** (df = 4; 340)	21.51162*** (df = 4; 665)
Note:				*p<0.1; **p<0.05; ***p<0.01

Figure 15: Separate CoCo regression results for Deutsche Bank, Santander, Societe Generale and UniCredit

Variable $d_{StockPrice}$ is significant at 1% significance level for BNP Paribas, Barclays, Deutsche Bank and Santander, while for the rest it is not significant even at 10% significance level. All significant coefficients for $d_{StockPrice}$ are negative as expected by theory. Higher stock price typically indicates improved financial position and capital structure of CoCo issuer and decreases the probability of conversion and therefore is ceteris paribus expected to decrease CoCo spread as well. Sensitivity to stock price is highest for Barclays (-0.00363 indicating average 36bps decrease of CoCo spread associated with increase of stock price by one) followed closely by second Santander (-0.00356) and third Deutsche Bank (-0.00198).

Variable $d_{Volatility}$ is significant at 1% level only for Barclays, Deutsche Bank and Societe Generale. Sensitivity is highest for Barclays (0.0235 indicating average increase of CoCo spread by 2.35bps associated with increase of volatility by one percentage point), followed by 0.0213 for Societe Generale. The sign of coefficient is negative for Deutsche Bank, contrary to theoretical expectations. Higher volatility is consequence of higher uncertainty among investors and ceteris paribus increases the probability of conversion and we would therefore typically expect positive relationship between volatility and CoCo spread.

Variable representing differenced CDS Spread $d_{CDSSpread}$ is statistically significant at 1% level in all regressions. Impact is positive for all CoCo issues included and the economic significance is also high – ranging from 0.18 for Barclays to 1.95 for Santander - indicating that 1bps increase in CDS spread for Santander is on average related to 1.95bps increase of this specific CoCo spread. Positive relationship between CDS spread is expected by theory and is also quite intuitive – both probability of default and probability of conversion are reflection of financial health of the particular issuer and increased implied probability of default captured in CDS spreads is therefore expected to also translate into higher estimated probability of CoCo conversion.

Variable $d_{InterestRate}$ is statistically significant at 10% for all regressions except for Santander. For all of them except for UniCredit it is also significant at 1% significance level. For all HSBC, BNP Paribas, Barclays, Deutsche Bank and Societe Generale the predicted effect is negative as expected by theory, for Credit Suisse and UniCredit the effect is positive contrary to theoretical prediction. Negative relationship in the context of pricing of contingent convertibles can be intuitively explained in following way – interest rate is part of the drift parameter and higher drift parameters means that future stock price increases more quickly and becomes more distant from trigger level. In a general context, negative relationship can be explained by liquidity risk. (Lin & Curtillet, 2007) Sensitivity is highest for BNP Paribas CoCo (-0.99 indicating average decrease by 99bsp of CoCo spread linked with 1% interest rate increase), followed by Societe Generale (-0.89) and Deutsche Bank (-0.73).

Although relationships obtained through complete regression are still quite in line with theoretical predictions (statistically significant negative relationship for CoCo spread and stock price for Deutsche Bank, Santander, BNP Paribas and Barclays, statistically significant positive relationship for CoCo spread and volatility for Societe Generale and

Barclays and statistically negative relationship for CoCo spread and interest rate for 6 out of 8 CoCo issues), results are perhaps not as strong as indicated by visualization. While we are still able to observe empirical relationships in line with the theory for most of the instruments, there are some anomalies (negative relationship between CoCo spread and Volatility for Deutsche Bank or positive between CoCo spread and interest rate for UniCredit). The highest significance is visible for CDS spread coefficients, which are also all positive as expected.

Both visualization and regression hopefully provided further insight into the extensive dataset. We were able to verify significance of most of explanatory variables which are further used in Credit derivatives model. In next chapter, we will examine crucial variable which was omitted in the analysis so far – estimated trigger level.

4. Empirical evidence on model assumptions

In this chapter we delve into examination of the crucial assumptions which are underlying for Credit derivatives pricing model for contingent convertibles and also Equity derivatives model. First part is related to trigger level modelling and assumption that CET1 ratio specified as a trigger level in contract can be approximated by specific stock price level. The second assumption builds on this approximation for CET1 Ratio and is very important for calibration purposes – it is the assumption that volatility observed in the stock market can be incorporated also in pricing framework for contingent convertibles.

4.1 Trigger level modelling

CoCo trigger level is specified mostly in the form of Common Equity Tier I capital. Common Equity Tier I capital based trigger, although quite transparent and not very prone to manipulation due to tight regulatory supervision, presents a challenge for CoCo pricing models. Common Equity Tier I capital is observable only after quarterly reporting of the financial institution and therefore needs to be proxied, in reality typically by stock price level. Such trigger level can be then easily incorporated into model with Black Scholes dynamics and probability of stock price hitting the trigger level leading to ensuing conversion of contingent convertible can be estimated. This probability is then essential for calculation of CoCo spread in Credit derivatives model or alternatively CoCo price in Equity derivatives model.

4.1.1 Motivation

Selection of conversion trigger has been extensively scrutinized in the recent literature. At the fundamental level, CoCo triggers can be categorized as either discretionary or mechanical, which can be further characterized by the variable which is chosen to potentially activate the conversion – Common Equity Tier I ratio, Tier I Capital ratio or perhaps Total Risk - based ratio. Alternatively, specification of contingent convertible trigger not on accounting based numbers, but rather market prices (stock prices, CDS spreads) have been discussed and these alternatives have been compared with each other based on clarity, objectivity, transparency or publicity. De Spiegeleer and Schoutens, Pazarbasioglu, McDonald and others all provide qualitative analysis of strengths and drawbacks of each particular option for trigger level both from the viewpoint of issuer and the financial stability of the institutions and from the viewpoint of the investor and his assurance regarding possible conversion (De Spiegeleer & Schoutens, 2010) (Pazarbasioglu, et al., 2011) (McDonald, 2013).

Selection of trigger event has large implications for predictability of contingent convertibles. Ability to correctly evaluate the probability of conversion is crucial to determine the spread – the reward for bearing the risk of conversion associated with the loss.

Since the expansion of market for contingent convertibles around 2012, Common Equity Tier I ratio has become a standard determinant for trigger event. Possible drawbacks of using such design have been discussed by many. Rüdlinger suggests that simpler ratio of book value of equity to book value of assets, that is treating asset associated risks equally, rather than risk weighting them as in computation of CET1 ratio, outperforms Common Equity Tier I ratio in terms of transparency (Rüdlinger, 2015). Kuritzkes and Scott make a valid argument about CET1 ratio being lagging indicator of the stability of financial institutions and support the claim with the data about capital ratios for failed or bailed out banks including Bear Sterns, Lehman Brothers and Merill Lynch – all of them reported Common Equity Tier I ratio larger than 12.3 prior to the crisis, indicating a large buffer for financial stability (Kuritzkes & Scott, 2009).

From the pricing perspective the most important characteristic of the trigger event is its observability and indisputability and Common Equity Tier I ratio strengths in this regard are questionable at least. CET1 ratio, available only after reporting date which is usually quarterly, is not suitable for the pricing model if striving to obtain CoCo spreads or alternatively prices on the continuous basis. Unobservability of CET1 ratio between quarterly reports calls for another variable, ideally closely approximating behaviour of the original Common Equity Tier I ratio. Finding a tradable underlying asset as a proxy for non – tradable and only sporadically observed CET1 ratio is crucial for derivation of the pricing formula using the replicating theory (Rüdlinger, 2015). From practical viewpoint, it is important to find a proxy variable which is available with high frequency in order to readily obtain updated prices dependent on latest market development and not to be limited by relying on only quarterly available input.

Using stock price level as a proxy for trigger level specified by CET1 ratio has been natural candidate and has become a standard for both Credit and Equity derivatives model. Its appeal consists in its basically continuous availability and importantly also in already well established Black Scholes framework for stock price modelled by Brownian motion. After association of stock price with the CET1 level, this framework enables straightforward computation of trigger event probabilities and subsequently CoCo spread. Such link between CET1 ratio and stock price, although nowadays a standard for CoCo pricing models, has been however rather presumed than deduced and examination of its empirical validity has been neglected.

Some authors resorted to usage of fixed and constant implied trigger stock price or using constant ratio between current stock price and trigger price. (De Spiegeleer & Shoutens, 2011) Such overlooking of the core pricing input could lead to massive mispricing, especially when probability of conversion increases and using static and now clearly unrealistic trigger level underestimates heavily fair value spread.

To overcome arbitrariness of chosen trigger level, others rely on calibrating the model with past observed CoCo prices/spreads and subsequently using trigger level implied by past prices to obtain real time CoCo prices. (Erismann, 2015) Such approach is appealing because it incorporates not only current pricing inputs such as stock price, volatility, dividend yield or interest rate but also last assumed trigger level not otherwise directly observable in the market. Using such calibration implicitly assumes that market participants were on average correct in trigger level estimation during last period and also assumes no change in this expectation from one period to another.

Potential drawback of the calibration approach is its zero usefulness in primary market, when past prices are not available and trigger level cannot be obtained through calibration. Additionally to issues in primary market, employing calibration method with past CoCo prices disregards information about the actual, contractually set trigger variable, available on quarterly basis in financial statements. Ratio such as Common Equity Tier ratio, upon which the potential trigger event is mostly based on, is available on balance sheet and it is surely worth investigating the link between accounting ratio and the market value of CoCo. However, not much of empirical examination has been given to this link.

This subchapter of the thesis therefore sets out to investigate the link between trigger level implied by reported figures and trigger level implied by observable CoCo spreads.

Probability of conversion indicated by figures on balance sheet is compared to the probability implied by market spreads. Additionally, size of fluctuations implied by book figures is compared to fluctuations observable in market for CoCo instruments.

4.1.2 Methodology

One of the two assumptions to be tested in this fourth part is the validity of using trigger level based on stock price for accounting trigger level specified in contract. In order to shed more light on the effect of accounting information for specific institution on the related CoCo prices, trigger level is not calibrated using past prices when calculating CoCo price for time t, but trigger level is estimated using available accounting information.

Rüdlinger provides the theoretical process of determining conversion level using accounting figures with assumption of one to one relationship between book and market value of equity. This assumption that ratio of market value of equity for particular CoCo issuer to its book value of equity is stable and is approximately one is rather strong – as indicated by the past ratios. Graph below shows market – to – book value since 2007 for all issuers in the sample:

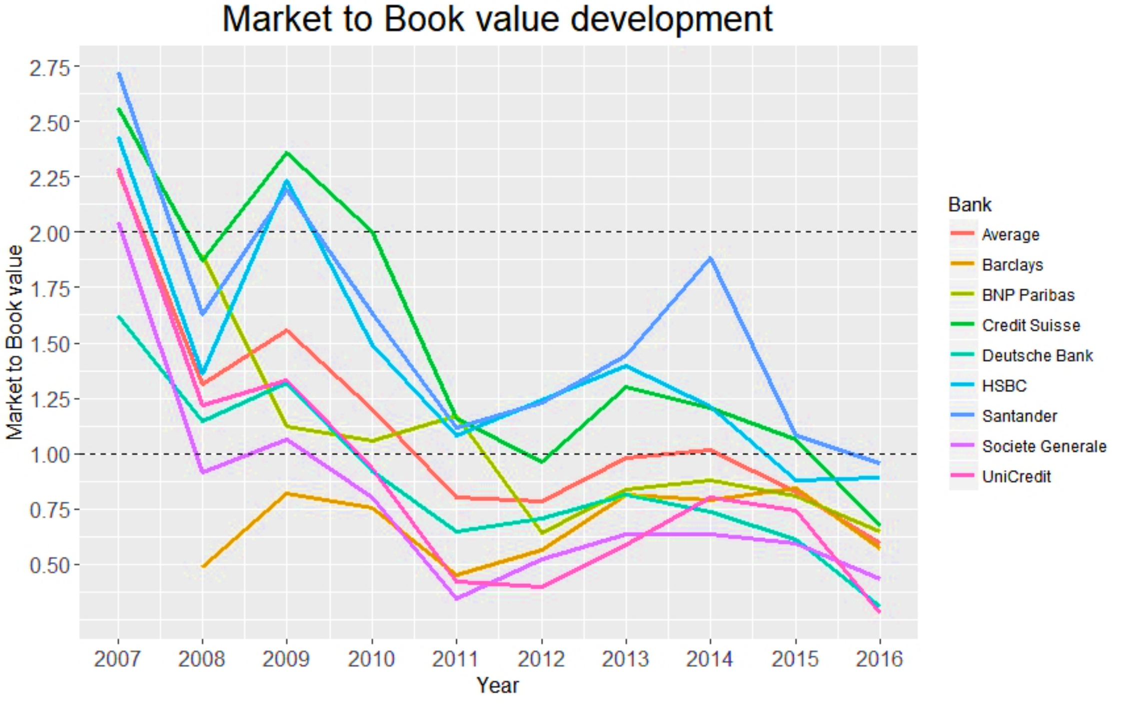

Figure 16: Development of Market to Book value for banks included in the sample (Bloomberg)

Graph shows pre – crisis ratios higher than two for most of the banks in the sample, indicating high market valuation compared to equity on balance sheet. The beginning of the crisis sharply lowers Market – to – Book value ratios and average of the sample falls below one in 2011. Since then, ratio stays below one for most of the banks in the sample, and in 2016 it is ranging from 0.26 UniCredit ratio to 0.96 Santander ratio. We can see that 1 to 1 ratio between Market and Book value of equity is indeed strong assumption. In order to reflect the changing nature of the ratio and loosen the strong assumption, final trigger level implied by book in this thesis is multiplied by current ratio between market and book value of equity. Let us first explain the mechanism suggested by Rüdlinger, the modification of his method is captured in equation 13 which reflects difference between book and market value. Illustratory graphs using Deutsche Bank figures are included to support explanation.

Firstly, amount of capital that can be absorbed before the trigger event occurs is calculated – this buffer is equal to reported CET 1 ratio minus the contractually specified trigger level for particular CoCo.

$$buffer = reported\ CET1\ ratio - contractual\ trigger\ level \tag{10}$$

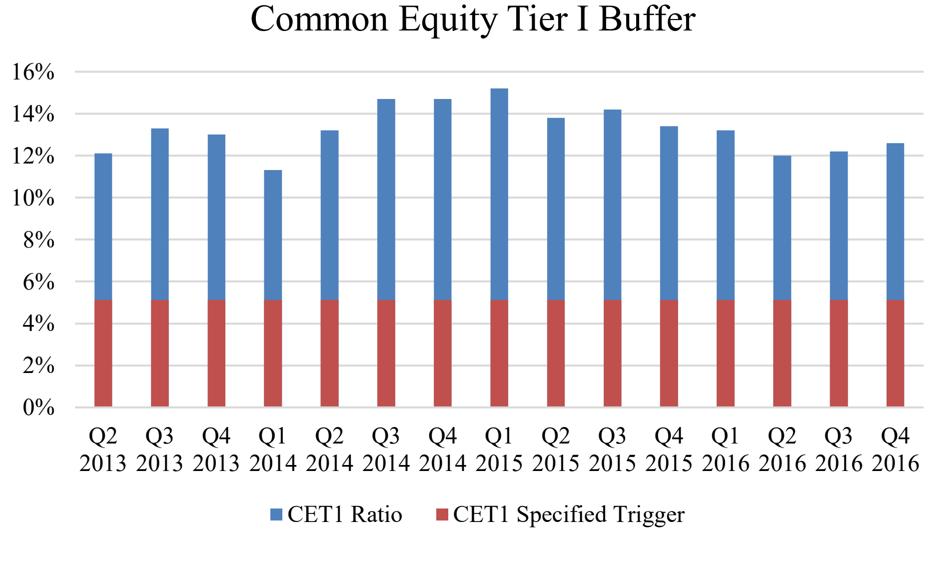

Figure 17: Common Equity Tier I Buffer of Deutsche Bank Q2 2013 – Q4 2016 (Bloomberg)

Multiplying the buffer with risk weighted assets gives the absolute amount of capital that corresponds to buffer against trigger event – loss absorbing capital.

$$loss\ abs.capital = RWA * buffer \tag{11}$$

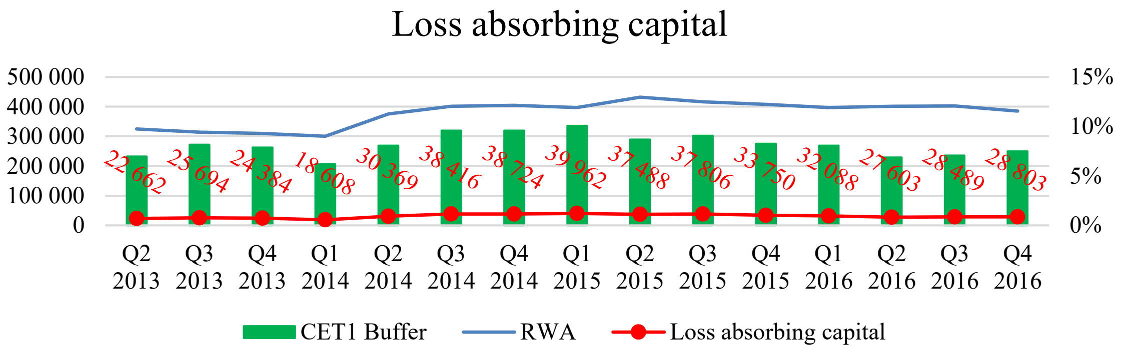

Figure 18: Loss absorbing capital of Deutsche Bank Q2 2013 – Q4 2016 (Bloomberg)

Book level of loss corrected capital is then obtained by substracting loss absorbing capital from total tangible common equity as included on balance sheet. (Rüdlinger, 2015)

$$book\ level\ of\ loss\ corrected\ capital = tangible\ CE - loss\ abs.capital \tag{12}$$

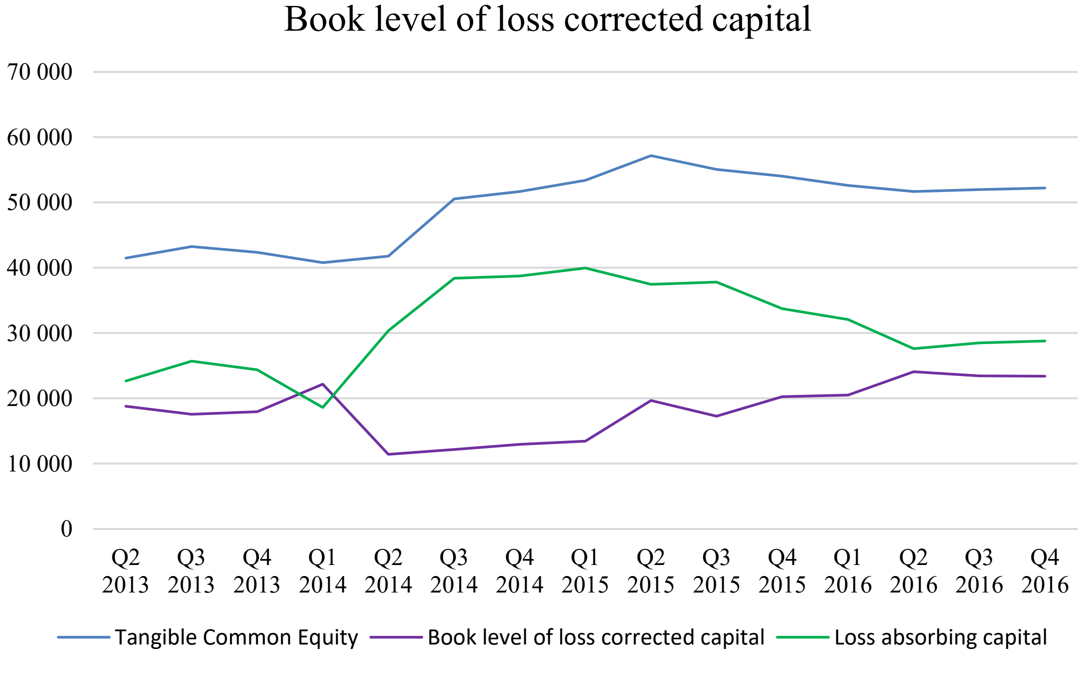

Figure 19: Book level of loss corrected capital of Deutsche Bank Q2 2003 – Q4 2016 (Bloomberg)

In order to recognize difference between Market value and Book value of equity previously outlined, book level of loss corrected capital is then multiplied by Market – to – Book ratio to obtain market level of loss corrected capital. This step is representing the modification of original approach suggested by Rüdlinger and is necessary to reflect the difference between market and book value of equity.

$$market\ level\ of\ loss\ corrected\ capial =$$

$$book\ level\ of\ loss\ corrected\ capital * \frac{MV\ of\ equity}{BV\ of\ equity} \qquad (13)$$

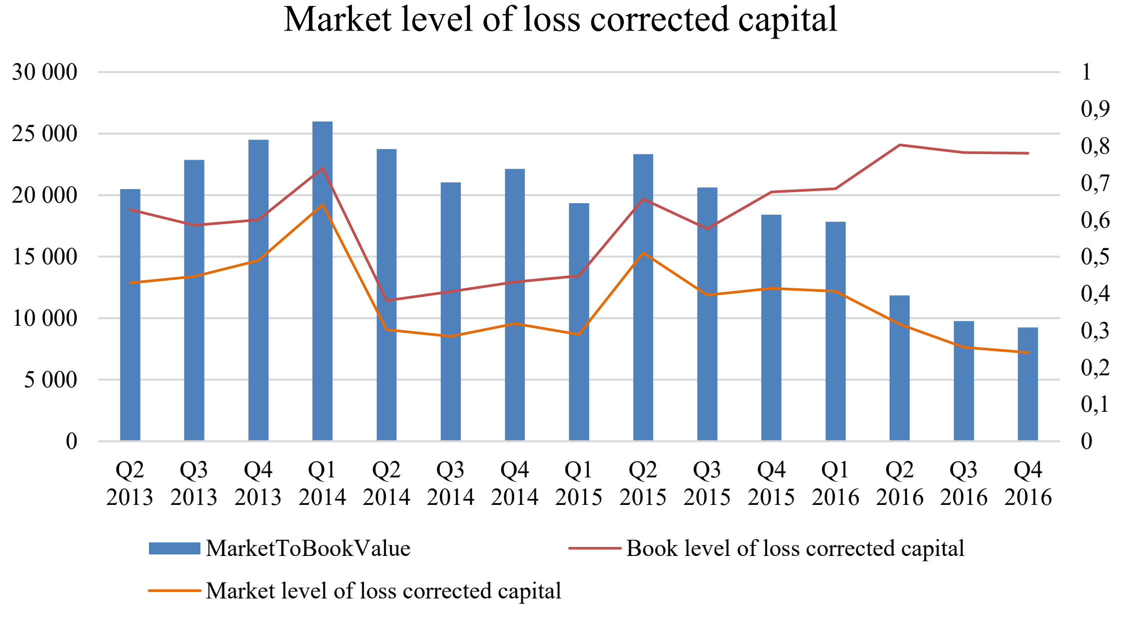

Figure 20: Market level of loss corrected capital of Deutsche Bank Q2 2013 – Q4 2016 (Bloomberg)

Finally, market level of loss corrected capital is divided by number of shares outstanding to arrive at book implied trigger level. This level represents stock price at which conversion is estimated to occur based on accounting figures.

$$book\ implied\ trigger\ level = \frac{market\ level\ of\ loss\ corrected\ capital}{shares\ outstanding} \qquad (14)$$

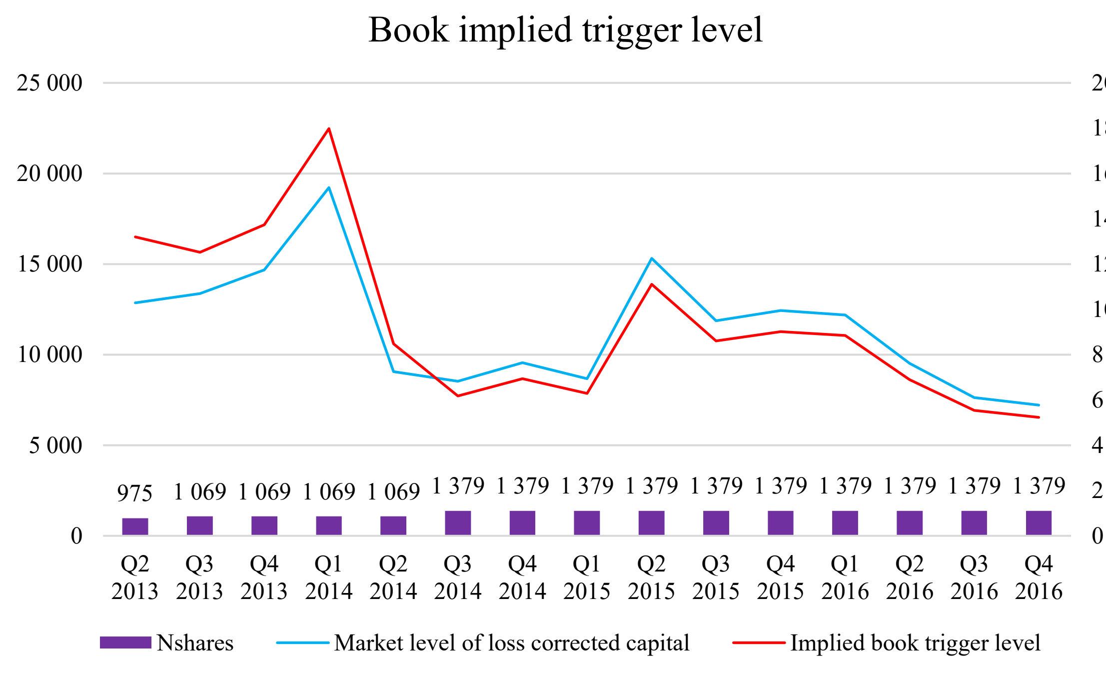

Figure 21: Book implied trigger level of Deutsche Bank Q2 2013 – Q4 2016 (Bloomberg)

In subsequent part, book implied trigger levels for 8 contingent convertibles, calculated using the outlined methodology and respective figures on balance sheet, are used to calculate probability of conversion associated with such trigger level – book implied probability of conversion. Book trigger level S^* together with stock price, volatility associated with the strike equal to S^* and drift determine under Black Scholes mechanics the probability of conversion, utilizing the previously introduced formula:

$$p = N\left(\frac{ln\frac{S^*}{S} - \mu.T}{\sigma.\sqrt{T}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} N\left(\frac{ln\frac{S^*}{S} + \mu.T}{\sigma.\sqrt{T}}\right) \tag{15}$$

This book implied probability is then compared with probability of conversion implied by the market spread of particular CoCo instrument using the previously introduced theoretical formula for CoCo spread in Credit derivatives model:

$$CoCoSpread = -(1 - R_{CoCo})\frac{\ln(1-p)}{T} \tag{16}$$

Solving for probability we obtain following formula, which gives us probability of conversion implied by market CoCo spread:

$$p_{market\ implied} = 1 - exp\left(\frac{-T * CoCoSpread}{1 - R_{CoCo}}\right) \tag{17}$$

Respective recovery rate is set to zero for contingent convertibles with write down mechanism and estimated using conversion price and deal specific details of conversion.

Focusing now on book and market implied probability levels enable us to compare different perception of issuer´s capital position and health estimated by market participants (captured in market CoCo spreads) and the one implied from reported figures without introducing volatility modelling, which is postponed to subsequent subchapter.

4.1.3 Results

Book and market implied probabilities of conversion for each CoCo instrument included in the sample are shown in Appendix 6. Market implied probability is drawn with red line and indicates which probability is implied by CoCo spread observable at each business date during the observed sample. Book implied probability is drawn with blue line and is calculated using observed pricing inputs – Stock price, Volatility, Interest rate, Dividend yield and Maturity and book implied trigger level calculated using methodology suggested by Rüdlinger modified by adjustment for market to book value ratio, as detailed previously.

The first noticeable characteristic of book implied probability of conversion is that it is not bound to interval (0, 1) as expected. In fact, probability implied by book exceeds one at some point in the observed period for Barclays and UniCredit. For Barclays this occurred at point when market implied probability was still below 40%. This means that at point when market CoCo spread implied 40% probability of conversion during the time to maturity of particular CoCo instrument, figures on balance sheet of Barclays indicated that conversion should had already happened.

This can only happen when at some point book implied trigger level is higher than stock price. For Barclays this occurred at the beginning of 2016 – trigger level calculated using last reported figures was 1.65 while stock price dropped below this level early in February 2016. Simple explanation could be that information arrived after the reporting date and caused market participants to lower the estimated trigger level while book implied trigger level remained stale until next reporting date. However, difference in book and market implied probability is simply too high to justify such explanation.

For UniCredit CoCo instrument, book implied probability actually exceeds one for the whole observed period – average stock price since the issuance of CoCo instrument in sample till the end of 2016 was 4.76 while trigger level implied by book figures averaged to 25.85. Still, no conversion of CoCo occurred during the period, indicating that book implied trigger level is certainly not suitable for all CoCo issuers.

Book implied trigger level for another bank in the sample HSBC reveals another issue – trigger level being higher than conversion price. Conversion price specified in the contract is 4.35578$ while trigger level implied by book figures exceeded this level for great majority of observed business dates. Trigger level higher than conversion price implies that in case of conversion, investor in contingent convertible instrument does not suffer loss but gains from the conversion. In the context of this thesis it then presents the issue with backing out probability of conversion using formula presented previously and results in non – sensical probabilities.

Focusing now only on CoCo instruments with reasonable probabilities limited to (0, 1) interval, we might be interested whether the difference between book implied and market implied probability is systematic – whether the difference in probability of conversion implied by market CoCo spread and probability implied by book figures in the sample is systematically different from zero (or alternatively, whether it is systematically positive/ negative). Therefore, we set up hypothesis to test the difference:

$$H_0: mean_{p_{book\ implied,\ bank}} - mean_{p_{market\ implied,\ bank}} = 0 \quad (18)$$

Table below presents mean book implied probability and mean market implied probability for the sample CoCo instruments. Table also includes T-statistics and both two-sided p-values and one-sided p-values for the test of hypothesis above (test is run without assuming the equality of standard deviations):

Bank	Mean book implied probability	Mean market implied probability	T-statistic	Two sided p-value	One sided p-value
Deutsche Bank	0.4656	0.4787	-1.362	0.1733	0.0866
Credit Suisse	0.3927	0.2706	19.425	1.465e-067	7.325e-068
Santander	0.2067	0.3003	-20.261	8.833e-074	4.417e-074
BNP Paribas	0.4631	0.3470	11.486	2.435e-028	1.218e-028
Societe Generale	0.4995	0.3820	10.024	1.99e-022	9.949e-023
UniCredit	1.1716	0.3799	313.725	0	0
Barclays	0.7855	0.2847	116.091	0	0

Figure 22: Comparison of mean book and market implied probability

We see that hypothesis holds only for Deutsche Bank CoCo, where we are not able to reject that mean of book implied probability and mean of market implied probability are different. For the rest of the sample, we are safely able to reject zero difference of means and can therefore conclude that there is systematical difference between the mean values. Additionally, mean book implied probability is significantly higher than mean market implied probability for all CoCo instruments except for Santander. This result indicates that market participants on average underestimate the probability of CoCo conversion compared to figures on balance sheets. This supports the hypothesis that market participants believe that bank management is both willing and able to avert potential conversion and in case of need is able to secure additional capital by other means before potential conversion of contingent convertibles. Significantly higher book implied probability compared to market implied probability also translates into low market CoCo spread compared to spread implied by balance sheet figures.

Secondly to book implied probability being on average higher than market implied probability, it is also noticeable that book implied probability is characterized by larger swings compared to market implied probability. As a result of larger change in book implied trigger level, there are drastic moves of implied probability following specific reporting dates. These drastic moves however do not seem to be present in the market implied probability, where such large jumps are not common.

To test whether this is consistent conclusion for all contingent convertibles in the sample, second hypothesis is set up and tested:

$$H_0: variance_{p_{book\ implied,\ bank}} - variance_{p_{market\ implied,\ bank}} = 0 \qquad (19)$$

Table below presents variance of book implied probability and market implied probability for the sample of contingent convertibles, together with F- statistics associated with tests for hypothesis above and corresponding p-values for both two sided and one-sided alternative.

Bank	Var. book implied probability	Var. market implied probability	F-statistic	Two sided p-value	One sided p-value
Deutsche Bank	0.04824	0.01478	3.26365	6.234e-051	3.117e-051
Credit Suisse	0.02571	0.00066	38.9256	0	0
Santander	0.01124	0.00171	6.56433	0	0
BNP Paribas	0.06249	0.00569	10.9844	0	0
Societe Generale	0.08044	0.01125	7.15258	0	0
UniCredit	0.00310	0.00143	2.17532	1.538e-024	7.691e-025
Barclays	0.01371	0.00633	2.16507	6.266e-036	3.133e-036

Figure 23: Comparison of variance of book and market implied probability

Results show that variance of book implied probability is indeed significantly higher than variance of market implied probability. This supports the hypothesis that market participants on average tend to underreact and changes in market implied probability and consequently also changes in market CoCo spreads are lower than those implied by figures on balance sheet, specifically by fluctuations in book implied trigger level. The general absence of large day to day moves in market implied probability of conversion also indicates that compared to "book approach", market participants absorb new information more gradually and large moves implied by newly reported figures and new book implied trigger do not materialize in the market.

In this subchapter we focused on trigger level. We compared market implied probabilities of conversion to book implied probabilities and concluded that book implied probabilities tend to be higher than those implied by CoCo spreads observable in the market. We also concluded that large moves in book implied trigger level generally do not translate into such large moves in market implied probabilities and market CoCo spreads.

Both tests revealed limitations of using book implied trigger level for practical calculation of model spreads. On average, relying on book implied trigger levels is prone to lead to too high model spreads when compared with observed market spreads. Large swings in book implied probability following reporting dates then do not seem to fully materialize in the CoCo market and relying on book implied trigger level therefore leads to overreaction of model in comparison to market. Issues with utilizing book implied trigger are then compounded for some CoCo issuers where associated book implied probability actually exceeds one and model does not lead to sensible spread.

These findings indicate that book implied trigger, even after abandoning one to one assumption between market and book value, might not be very useful for real pricing of contingent convertibles. Instead of relying on book implied level, calibration of the trigger level is potentially necessary. Such calibration requires volatility modelling, which is subject of following subchapter 4.2.

4.2 CoCo volatility modelling

Previous subchapter was dedicated to comparison of book implied probabilities and market implied probabilities. Empirical analysis indicated that book implied probability was too high when compared to market implied probability and suggested that rather than relying on book trigger level, calibration of trigger level might be more efficient – we might want to use past traded CoCo spreads to back out the trigger level instead of relying on reported figures. In order to obtain such calibrated trigger level, we have to delve into volatility modelling and in essence estimate which part of implied probability is attributed to position of trigger level and which part is attributed to its volatility. This subchapter is devoted to such endeavour and sets out to test the second assumption we decided to test – that volatility observable in stock market can also be incorporated in contingent convertible pricing framework.

4.2.1 Motivation

While providing market consensus for CoCo conversion probability during particular time period, market implied probability does not show us the whole picture. Focusing only on probability levels does not separate effect of trigger level and effect of volatility of trigger on the probability – one level of probability can represent different sets of trigger levels and volatilities.

Market implied probability on its own however cannot be easily translated into information about capital standings of the financial institution issuing CoCo instrument. High market implied probability could be possibly attributed to high volatility of CoCo trigger rather than high trigger level indicating deteriorating capital buffer.

To illustrate this, let us look on Deutsche Bank market implied probability of conversion. Figure 24 shows market implied probability backed out of CoCo spreads using previously introduced formula (17):

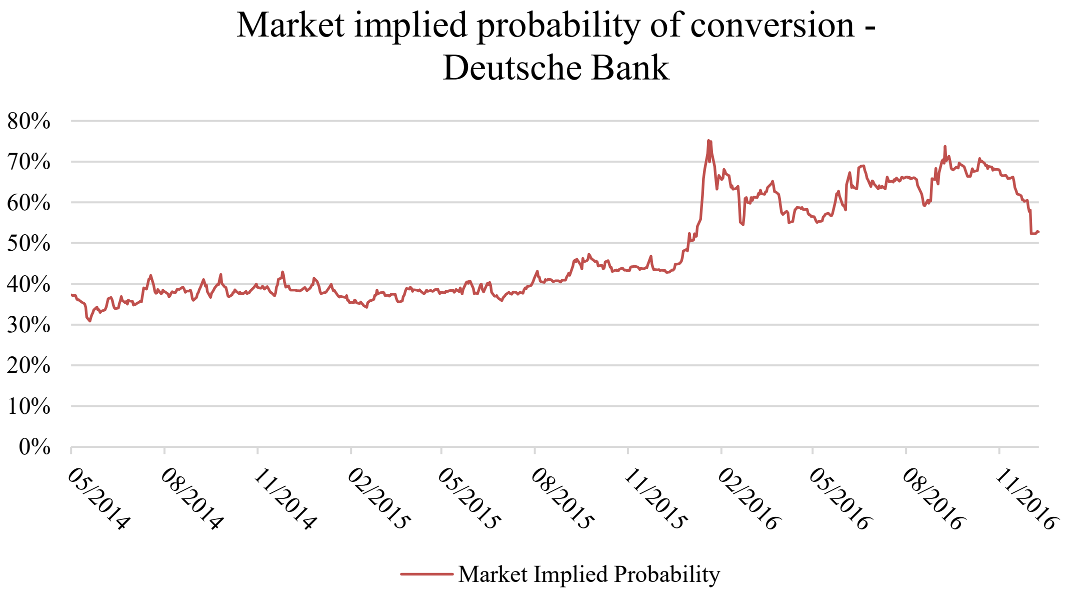

Figure 24: Market implied probability of conversion - Deutsche Bank

We see that the probability of conversion before maturity of CoCo instrument implied by market CoCo spreads soon after issuance is approximately 37%. This information alone does not lead directly to information about the trigger level implied by the CoCo spread. Depending on volatility, this probability could be translated into various trigger levels - for each market probability there exists a set of possible volatility and trigger level pairs. Figure 25 below shows such set which is matching the market implied probability for Deutsche Bank CoCo soon after the issuance of the instrument in the sample.

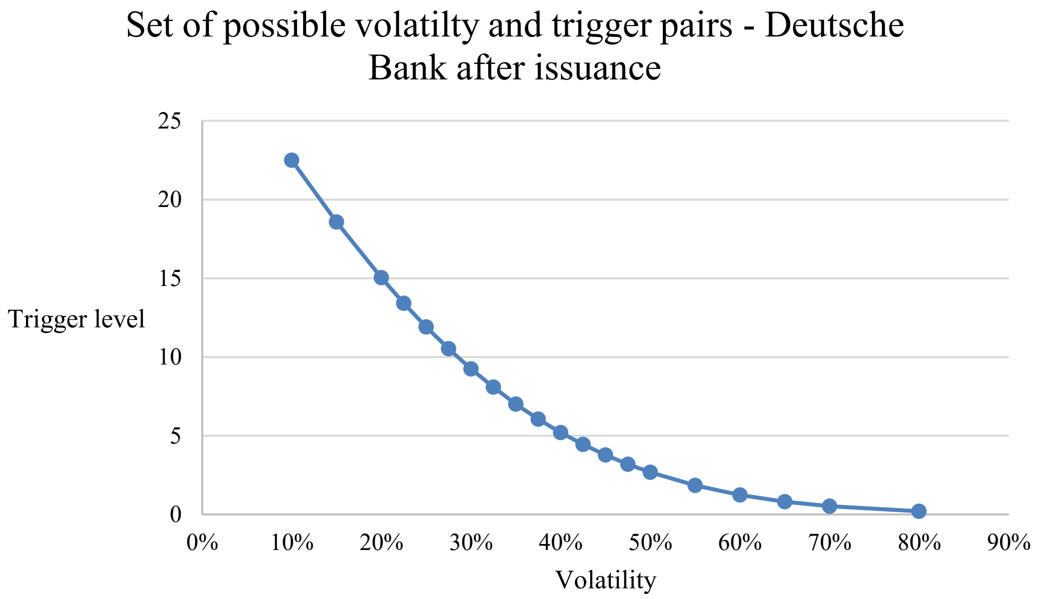

Figure 25: Set of possible volatility and trigger pairs - Deutsche Bank

Figure shows how sensitive implied trigger level is on its estimated volatility. Inverted shape of the curve reveals that conversion probability 37% could correspond both to high trigger and low volatility scenario (22.5 trigger level for 10% volatility) or low trigger and high volatility scenario (0.2 trigger level for 80% volatility) or any scenario located on blue line. This is a huge spread of possible trigger levels and indicates how essential modelling of the volatility of the trigger level is.

We would like to move from market implied probabilities to market implied trigger level to separate the effects. Estimating market implied trigger level can serve as a basis for comparison of different CoCo issuers and together with book implied trigger level introduced before it provides an important insight into difference between book implied and market implied information about the capital buffer of issuer.

Not only this allows us to draw some additional conclusions about the state of capital structure of the CoCo issuer, ability to estimate trigger level proves essential in calibration of Credit derivatives model. Previous analysis has shown significant difference between book implied and market implied probability and calibration of the model using the observed CoCo spreads is alternative way how to obtain trigger level which can be used in the model instead of book implied trigger level. This calibrated trigger level might consequently lead to more accurate CoCo spread implied by Credit derivatives model.

Previous empirical studies mostly avoid advanced volatility modelling and do not attempt to incorporate volatility modelling consistent with stock market into the CoCo pricing. Jung relies on constant volatility consistently with Black Scholes model assumptions (Jung, 2012), Erismann then employs historical volatility and therefore also does not incorporate volatility smile (Erismann, 2015). Rüdlinger does incorporate volatility smile observable in stock market, but he relies on constant book implied trigger level analysed in previous section (Rüdlinger, 2015). As our goal is now to move from book implied trigger level to calibrated one, we need to incorporate volatility smile modelling into calibration.

The natural approach for volatility modelling stems from implied volatilities observed in the option market – using stock trigger level as a proxy for CET1 ratio actually extends to assumption of same volatility characteristics. Implied volatilities are available for most of the CoCo issuers for various levels of moneyness, each backed out of the option price with corresponding strike. Specific interpolation technique or modelling assumption allows us to calibrate the whole volatility surface and such volatility surface or functional form of volatility together with dynamics of Credit derivatives model allows us to arrive at particular trigger level compatible with level of observed CoCo spread. Following section introduces such functional form of volatility, which is calibrated using the observed volatilities.

4.2.2 Methodology

There are currently many well developed complex approaches for calibration of the whole volatility surface based on few observed implied volatilities. For example, Heston model employing CIR process with mean reversion can be calibrated using the computational methodology developed by Yiran Cui, Sebastian del Baño Rollin and Guido Germano (Cui, et al., 2016). Similarly, SABR model with correlated stochastic process for asset price and volatility can be calibrated using various numerical techniques, as discussed in Finite Difference Techniques for Arbitrage-Free SABR (Le Floc'h & Kennedy, 2014).

For the purpose of this thesis we resort to more straightforward stochastic volatility inspired (SVI) parametrization presented by Gatheral. (Gatheral, 2004) Gatheral parametrization uses the closed form function for volatility characterized by set of parameters $\{a, b, \rho, m, \sigma\}$. For each expiration, functional form for volatility depending on strike k is (Gatheral, 2006):

$$\sigma^2(k) = a + b\{\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2}\} \tag{20}$$

The calibration of parameters $\{a, b, \rho, m, \sigma\}$ using the observed implied volatilities for set of strikes translates into solving of non-linear least square problem and is possible by either Newton method or Levenberg-Marquard (LM) method, which is used in this thesis. Figure 26 below shows the results of such calibration using LM method and set of following implied volatilities and strikes as an initial input:

Strike	6	12	18	24	27	30
Implied volatility	75%	57%	42%	29%	24%	20%

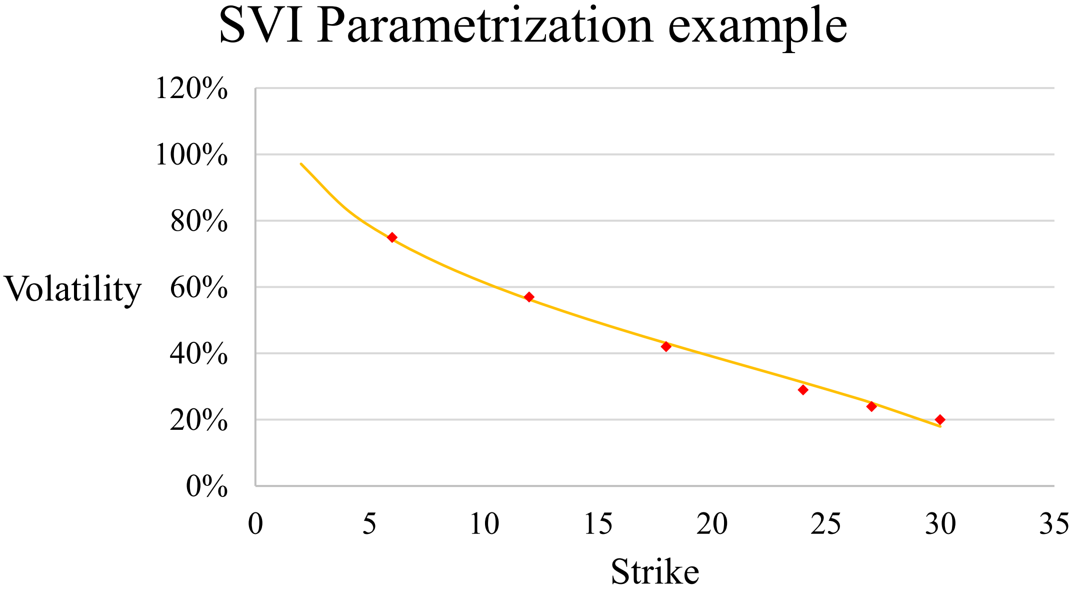

Figure 26: SVI Parametrization example using set of strikes and associated IVs in the above table as an input

For each CoCo issuer in the sample, Gatheral SVI Parametrization is used to calibrate set of parameters $\{a, b, \rho, m, \sigma\}$ each business date using the observable implied volatilities for various strikes. Functional form for volatility together with CoCo spreads then allows us to calibrate market implied trigger level by simultaneously solving two equations for variable k:

$$p = N\left(\frac{ln\frac{k}{S} - \mu.T}{\sigma.\sqrt{T}}\right) + \left(\frac{k}{S}\right)^{\frac{2\mu}{\sigma^2}} N\left(\frac{ln\frac{k}{S} + \mu.T}{\sigma.\sqrt{T}}\right) \tag{21}$$

$$\sigma^2(k) = a + b\{\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2}\} \tag{22}$$

Such that p equals to the market implied probability calculated before using formula (17). Time series of market implied trigger levels together with Gatheral fitted volatilities corresponding for particular trigger level resulting from the methodology outlined above is analysed. Next subchapter contains findings of the analysis.

4.2.3 Results

Implied market trigger and volatility associated with the trigger under Gatheral volatility functional form resulting from the calibration process outlined in previous subchapter is plotted in Figure 27 (Deutsche Bank CoCo) and Figure 28 (BNP Paribas). These two are selected based on the average volatility during the observed period far out of money – Deutsche Bank represents high volatility example with 20% moneyness volatility exceeding 100% most of the time and BNP Paribas represents example with lower average volatility far out of the money compared to the rest of the sample.

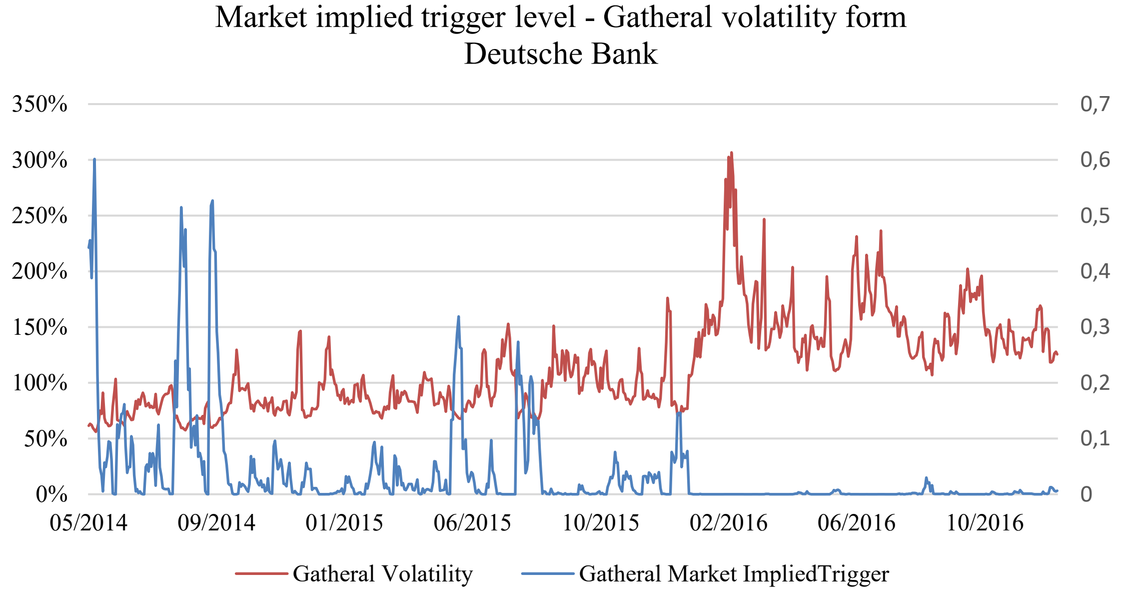

Figure 27: Market implied trigger level under Gatheral volatility form - Deutsche Bank

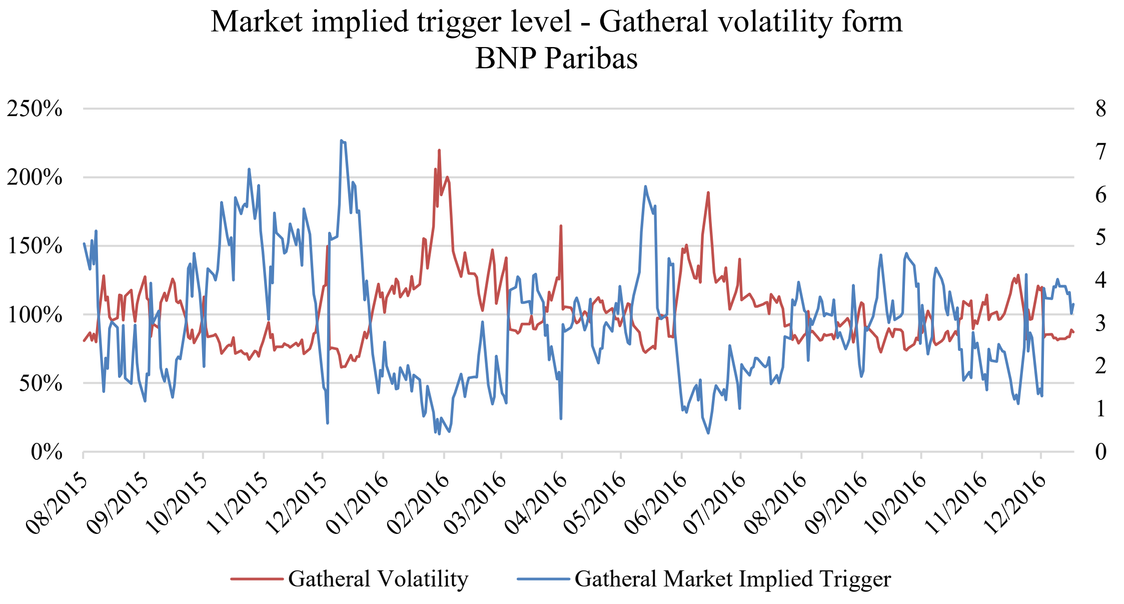

Figure 28: Market implied trigger level under Gatheral volatility form – BNP Paribas

We see that calibrated solution $(k, \sigma(k))$ which yields market CoCo spread under the assumption of Gatheral function form for volatility smile is very unstable. Possibility of such instability was already indicated by Figure 25 capturing the set of possible (k, σ) pairs matching the market implied probability – we have seen for particular Deutsche Bank CoCo that volatility exceeding 80% leads to very low implied trigger level.

Intuitively this can be interpreted in a following manner – high volatility dramatically increases probability of CoCo conversion leading to high CoCo spread implied by model. High volatility therefore must be offset by very low trigger value in order to match the observed market spread.

Secondly, implied market trigger under Gatheral volatility is very low. For BNP Paribas, average ratio of trigger level to stock price is 5.96% and for Deutsche Bank CoCo it is even lower, averaging to 0.13%. Such low trigger level is highly unrealistic when compared to book implied trigger level averaging to 49.97% (BNP Paribas) and 36.63% (Deutsche Bank). It is visible that for Deutsche Bank during 2016 period characterized with increased volatility of far out of the money options implied trigger level drops basically to zero.

Not only that incorporating Gatheral volatility smile into the calibration leads to unreasonably low trigger levels, implied trigger obtained through calibration is very unstable. Instability of trigger level would eventually lead to inaccurate model spreads obtained using such calibration. Previous section indicated large differences between book implied probability and market implied probability and we suggested that calibration of trigger level might be more efficient. We can see however that after incorporation of volatility smile observable in stock market, calibrated trigger level is characterized with high fluctuations. Large day to day changes in implied trigger translate into large changes in calculated CoCo spread under such calibration method. Such large day to day changes in CoCo spread are however hardly frequently observed on market and model is therefore likely to perform badly and predict too large changes.

This is illustrated in Figure 29. Market CoCo spread observable for BNP Paribas is compared to model spread with setup incorporating volatility smile. Model uses trigger level set to 5.96% of stock price – average ratio calculated before for BNP Paribas. Gatheral volatility associated with this moneyness is then used in calculation of model spread. This simplified example indicates how low trigger level and incorporation of volatility smile leads to high fluctuations in model spreads – these large swings in model spreads are not in line with fluctuations observable in the market.

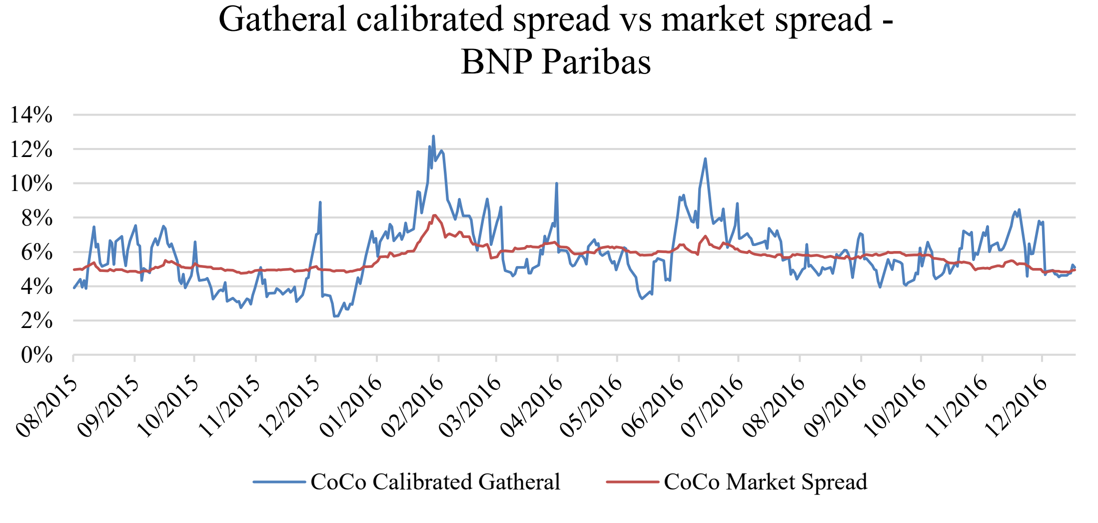

Figure 29: Comparison of Gatheral calibrated CoCo spread vs market CoCo spread - BNP Paribas

We have seen that incorporating volatility smile into calibration leads to highly unstable and unreasonably low implied trigger level compared to book implied level. This suggests that modelling CoCo trigger level with volatility observed for stocks is not very suitable and that lower and more stable volatility might be more suitable for calibration of the model. Volatility for far out of money options tends to be very high and large day to day swings occur frequently.

Analysis in this chapter suggests that market participants trading with contingent convertibles do not assume such behaviour for stock trigger level approximating CET1 ratio – otherwise we would observe much higher fluctuations in CoCo spreads reflecting these fluctuations in volatility. This finding is utilized in next chapter in order to obtain more stable calibration process more consistent with observable market for contingent convertibles.

5. Calibration of Credit derivatives model

Fourth chapter was dedicated to analysis of modelling assumptions crucial for Credit derivatives model – modelling of trigger level and interconnected modelling of volatility. This chapter will utilize findings gathered throughout the previous chapters and will build setup of calibration based on them. CoCo spread calibrated using Credit derivatives model will then be compared to CoCo spreads observed on the market. The model will be evaluated based on different criteria. Chapter is organized as follows – 5.1 subchapter is devoted to recollection of findings and presentation of calibration setup reflecting these findings, 5.2 presents results of calibration for each CoCo set and comparison of model implied CoCo spreads to observed spreads and 5.3 summarizes the conclusions of comparison and model evaluation.

5.1 Calibration setup

Two key pricing inputs into Credit derivatives model – trigger level and its volatility, are not directly observable and therefore require some analysis prior to calibration of the model. We will now utilize findings collected throughout Chapter 4 and present one calibration setup reflecting these findings.

Firstly, trigger level was approximated using book figures and probability of conversion implied by Credit derivatives model using this book trigger level was compared to probability of conversion implied by market CoCo spreads. We have seen that probability of conversion implied by book using methodology presented in Rüdlinger adjusted by market to book value of equity ratio is on average significantly higher than market implied probability of conversion. On top of that, methodology resulted in probability values exceeding one for the whole observed sample for UniCredit bank. Book implied probability was also prone to experience larger swings than market implied probability.

While derivation of book implied trigger level and probability might still have some merits in primary markets where calibration is not possible and can even serve as a check in secondary markets, we now resort to calibration of trigger level rather than relying on its book counterparty.

Calibration uses the past observable CoCo spreads and other observable pricing inputs to arrive at such trigger level S_T^* which gives CoCo spread calculated under Credit derivatives model equal to market CoCo spread. Such calibrated trigger level is then used at pricing date as independent pricing input. To ensure greater stability of trigger level used in Credit derivatives model, past 30 calibrated trigger levels are utilized and equally weighted average is calculated. It shall be noted that setup with different number of lags included or different weighting of the calibrated values is straightforward extension of the model and might even potentially lead to better results. Compared to empirical study conducted by Erismann who uses also calibration approach but after initial calibration settles for constant trigger level, we recalibrate trigger level at each pricing date (Erismann, 2015). This “rolling calibration” allows model to incorporate potential changes in trigger level into the calculation of CoCo spread and is therefore more flexible.

Chapter 4 also presented issue with modelling volatility in Credit derivatives model. This issue is interconnected with trigger level modelling as in order to get calibrated trigger levels we have to make assumptions about the volatility as well. Section 4.2 outlined that incorporating volatility smile into Credit derivatives model, in our case in form of functional form suggested by Gatheral, leads to very volatile and very low trigger level. Incorporation of smile resulted in very high volatilities for options far out of money being used and trigger level equal to small fraction of book implied trigger level. We have concluded that such low trigger level is unreasonable and in calibration scenario it would lead to very volatile implied CoCo spreads compared to market observable.

Due to this finding, lower, at the money volatility is used for calibration. Volatility time series estimated for different moneyness levels are highly correlated and the choice of using ATM volatility should therefore not affect direction of CoCo spread change implied by Credit derivatives model too much, but should rather decrease its magnitude. Even when ATM volatility is taken as pricing input, the sensitivity of CoCo spread to volatility changes is still too high and not consistent with sensitivity characteristic for CoCo markets. Volatility pricing input is therefore further averaged for past 30 trading days, consistently with averaging of calibrated trigger level.

To summarize the setup of the calibration – equations (21) and (22) are first used to get calibrated trigger level for time period t, denoted as $S_{t,cal}^*$. Average ATM volatility is used in the calibration, denoted as $\sigma_{ATM,avg}$.

$$p = N\left(\frac{ln\frac{S^*_{t,cal}}{S} - \mu.T}{\sigma_{ATM,avg}.\sqrt{T}}\right) + \left(\frac{S^*_{t,cal}}{S}\right)^{\frac{2\mu}{\sigma_{ATM,avg}{}^2}} . N\left(\frac{ln\frac{S^*_{t,cal}}{S} + \mu.T}{\sigma_{ATM,avg}.\sqrt{T}}\right) \quad (23)$$

$$Spread = -\left(1 - \frac{S^*_{t,cal}}{P_C}\right).\frac{\ln(1-p)}{T} \quad (24)$$

Calibrated trigger level is then averaged for last 30 days and used at subsequent pricing date – formulas (23) and (24) are used again to obtain CoCo spread implied by Credit derivatives model, now with $S^*_{t,cal}$ replaced by $\overline{S^*_t}$ where:

$$\overline{S^*_t} = \frac{\sum_{t-29}^{t-1} S^*_{i,cal}}{30} \quad (25)$$

The first subchapter hopefully provided insight into the calibration methodology and the outline of the reasons behind the choice of calibration setup. Following subchapter is devoted to presentation of results of calibrated Credit derivatives model and the comparison of CoCo spreads implied by the model and market observable CoCo spreads.

5.2 Results and evaluation of calibrated model

Time series of model CoCo spreads are presented in Appendix 7. For each CoCo issuer, market observable spreads are included in the figure as well to allow quick comparison of model implied and market implied spread. Results for Deutsche Bank are shown below in Figure 30.

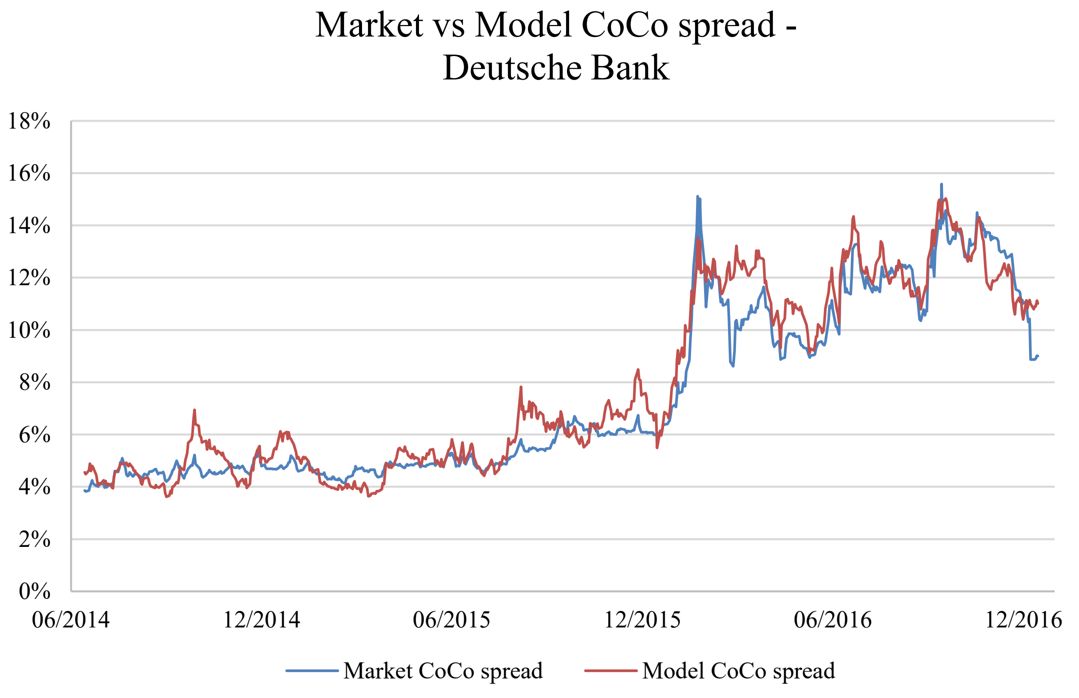

Figure 30: Market observable vs Credit derivatives model CoCo spread - Deutsche Bank

Figure for Deutsche Bank suggests that both market and model CoCo spreads tend to move together. Occasional relatively large deviations exceeding 100 bps have occurred couple of times, the gap is however always temporary and diminishes relatively quickly.

Credit derivatives model and the calibration setup outlined above will be evaluated based on two criteria – direction and size. To track how well changes in spreads implied by Credit derivatives model correspond to observed market data, three "direction" tests are conducted:

1) Spearman´s rank correlation is calculated between model and market spreads to evaluate how well are series monotonically related.

2) Same direction indicator is calculated as a ratio of periods when both model and market spreads moved in the same direction to the overall number of observations.

3) Engle – Granger two step test is conducted to test cointegration of model spreads and market spreads.

Beside direction, we also want to test whether the size of the change predicted by model on average corresponds to the size of change observed – to test whether the model is prone to systematically overshoot or undershoot the change in the spread. Two “size” tests are conducted:

1) Variance of the model spread and market spread is calculated and difference between them is tested for significance.
2) Series of absolute changes is calculated for both model and market spreads and the difference between mean of the model series and market series is tested.

5.2.1 Direction tests

The first direction test is Spearman´s correlation. Spearman´s rank correlation indicates how much model spreads tends to increase/decrease when market spreads increase/decrease and while Pearson´s correlation coefficient evaluates linear relationship between variables, Spearman´s correlation coefficient evaluates monotonic relationship. Spearman´s correlation equal to 1 indicates perfect increasing monotonic relationship. Results showing Spearman´s correlation coefficient between model and market spread series for each CoCo issuer are shown in Figure 31. Together with correlation coefficient, p-values are also reported to indicate whether the coefficient are statistically significant:

Spearman Correlation		
CoCo issuer	**rho**	**p-value**
Deutsche Bank	0.9415	0.0000
Credit Suisse	0.7065	0.0000
Santander	0.6969	0.0000
HSBC	0.5889	0.0000
BNP Paribas	0.8339	0.0000
Societe Generale	0.8416	0.0000
UniCredit	0.8968	0.0000
Barclays	0.8284	0.0000

Figure 31: Spearman´s rank correlation between model and market spreads

Results indicate high positive Spearman´s correlation for all CoCos included in the sample. Test for significance conducted for each coefficient rejects insignificance for all series as indicated by very low p-value. The lowest coefficient is 0.5889 for HSCB, highest then coefficient for Deutsche Bank 0.9415. Overall, high Spearman correlation indicates that spreads implied by Credit derivatives model do tend to move in the same direction as the market observed spreads.

The next evaluation method will provide a percentage of changes in the same direction. We define Same direction indicator (SDI) as:

$$SDI = \frac{\sum_{i=1}^{n} I}{n}$$

where I is indicator variable equal to one if model spread change and market spread change has same sign and zero otherwise and n is number of changes observed. Results showing SDI for each CoCo in sample are shown in Figure 32:

Same direction indicator	
CoCo issuer	**percentage**
Deutsche Bank	66.97%
Credit Suisse	68.92%
Santander	69.38%
HSBC	63.99%
BNP Paribas	71.90%
Societe Generale	80.00%
UniCredit	76.36%
Barclays	66.38%

Figure 32: Same direction indicator showing the percentage of changes in same direction for model CoCo and market CoCo spreads

Same direction indicator ranges from lowest 63.99% for HSBC to highest 80% for Societe Generale. This indicates that Credit derivatives model spread and market spread have co-movement in the same direction roughly 64%-80% of period observed. The Same direction indicator results are in line with Spearman´s correlation test and do not indicate any CoCo issue which would completely outlie the sample in terms of precision of Credit derivatives model.

The last test in this category is cointegration test. Cointegration allows us to see whether the difference between model spread and market spread is stable over time, that is, whether the series tent to revert back to each other. This is especially important for the evaluation of Credit derivatives model – cointegration between model and market spreads is desirable as the reversion prevents long deviations from average difference between model and market spreads (which is ideally zero).

Cointegration is tested using two step Engel-Granger method. First, each model and market CoCo spread series is tested for stationarity. P-values are reported – low p-value indicates that non-stationarity of the series (presence of unit root) can be rejected, high p-values indicate the opposite. In second step, residuals from regression of model spread on market spread are tested again for stationarity. If model spreads and market spreads are both I(1) but their difference (in test residuals) is I(0), we say that model spreads and market spreads are cointegrated. Results of cointegration test for each CoCo issue in the sample are included in Figure 33:

	Stationarity - p-value of unit root test		**Cointegration**
CoCo issuer	**Market Coco spread**	**Model CoCo spread**	**p-value**
Deutsche Bank	0.5680	0.6790	0.0000
Credit Suisse	0.0842	0.0505	0.0101
Santander	0.5938	0.0357	0.2090
HSBC	0.2692	0.0380	0.2254
BNP Paribas	0.4645	0.2781	0.1560
Societe Generale	0.2994	0.4543	0.0410
UniCredit	0.3462	0.1740	0.0154
Barclays	0.2313	0.0160	0.0023

Figure 33: Engel-Granger two step test for cointegration between model CoCo spread and market CoCo spread

Results for cointegration indicate that for half of the sample (Deutsche Bank, Credit Suisse, Societe Generale and UniCredit) model CoCo spread and market CoCo spread are conclusively cointegrated at 5% significance level. Non-stationarity was rejected for model CoCo spread of HSBC, Santander and Barclays and therefore no conclusion can be made for cointegration of these series. BNP Paribas market CoCo spread and model CoCo spread are both I(0), however p-value 0.1560 indicates that non-stationarity of difference cannot be rejected.

In conclusion, three direction tests indicated that on average model CoCo spread and market CoCo spread do tend to move together and changes in CoCo spread implied by Credit derivatives model correspond in direction quite well to market CoCo spreads. High and significant Spearman´s correlation indicates the presence of strong monotonic relationship between model spreads and markets spreads and Same direction indicator introduced shows that in sample, model implied spread changes and market changes had same sign 64% to 80% of observed periods. Cointegration test then suggests that for half of the sample, model and market CoCo spreads are cointegrated and that there should be reversion between them.

5.2.2 Size tests

Apart from evaluation whether Credit derivatives model implies the same direction of changes in CoCo spread, another desirable feature of the model is to have comparable variance. Model should not systematically imply larger/lower swings than observed market fluctuations in order to prevent overshooting or undershooting. Two tests are conducted to compare the size of changes. Firstly, variance of the model CoCo spread and market CoCo spread is calculated and the difference between them is tested for significance using F-test. Statistically significant difference would imply that the model is prone to have on average larger/lower variance than observed spreads and that sensitivities implied by the model are not in line with sensitivities implied by market. Results of F-tests for each CoCo in sample are shown in Figure 34 below:

CoCo issuer	**Variance Market**	**Variance Model**	**F-Statistic**	**p-value**
Deutsche Bank	0.001103	0.001176	1.0655	0.4148
Credit Suisse	0.000024	0.000112	4.6508	0.0000
Santander	0.000203	0.000438	2.1611	0.0000
HSBC	0.000019	0.000039	2.0977	0.0000
BNP Paribas	0.000041	0.000133	3.2625	0.0000
Societe Generale	0.000077	0.000224	2.9252	0.0000
UniCredit	0.000120	0.000357	2.9874	0.0000
Barclays	0.000084	0.000145	1.7174	0.0000

Figure 34: Comparison of Market variance of CoCo spread and Model variance of CoCo spread with F-Statistics testing the difference

Figure 34 shows that variance of market CoCo spread is lower than variance of CoCo spread implied by Credit derivatives model for all issuers in the sample. Difference is statistically significant for all issues except for Deutsche Bank. This means that for majority of sample, we are able to reject the hypothesis that model results in the same variance of CoCo spread – Credit derivatives model with our calibration setup tends to result in larger volatility of spreads than the volatility observed on CoCo market.

The second test of size of changes is testing the difference between mean of absolute change observable in the market and absolute change implied by the model. Systematic negative difference would corroborate the evidence from previous test of variances – that Credit derivatives model tends to overshoot and implies larger swings than swings observed on the market. Results containing mean for market and mean for model together with T-Statistic and corresponding p-value are presented in Figure 35:

CoCo issuer	**Market Abs. Change Mean**	**Model Abs. Change Mean**	**T-Statistic**	**p-value**
Deutsche Bank	0.001708	0.002289	-4.1649	0.0000
Credit Suisse	0.000518	0.001413	-12.8837	0.0000
Santander	0.001123	0.000192	-15.2664	0.0000
HSBC	0.000496	0.001245	-12.9955	0.0000
BNP Paribas	0.000745	0.000110	-9.4638	0.0000
Societe Generale	0.000964	0.002059	-7.5348	0.0000

UniCredit	0.000681	0.001864	-13.2735	0.0000
Barclays	0.000809	0.001539	-11.3540	0.0000

Figure 35: Test of difference between mean of absolute change in model CoCo spread and absolute change in market observable CoCo spread

Results are unambiguous and confirm conclusions from the test of variances. Mean of absolute change of CoCo spread implied by the model is statistically significantly higher than market observed CoCo spread for all the issues in the sample. This indicates that irrespective of direction, size of change implied by the model is too high compared to market observed changes and that the model on average is prone to overshooting. As an example, mean absolute change for Deutsche Bank CoCo spread observed on the market is 17 bps while mean absolute change for model spread is 22 bps. Comparing histograms for market absolute change (Figure 36) and model absolute change (Figure 37) provides more insight into the distribution of absolute changes:

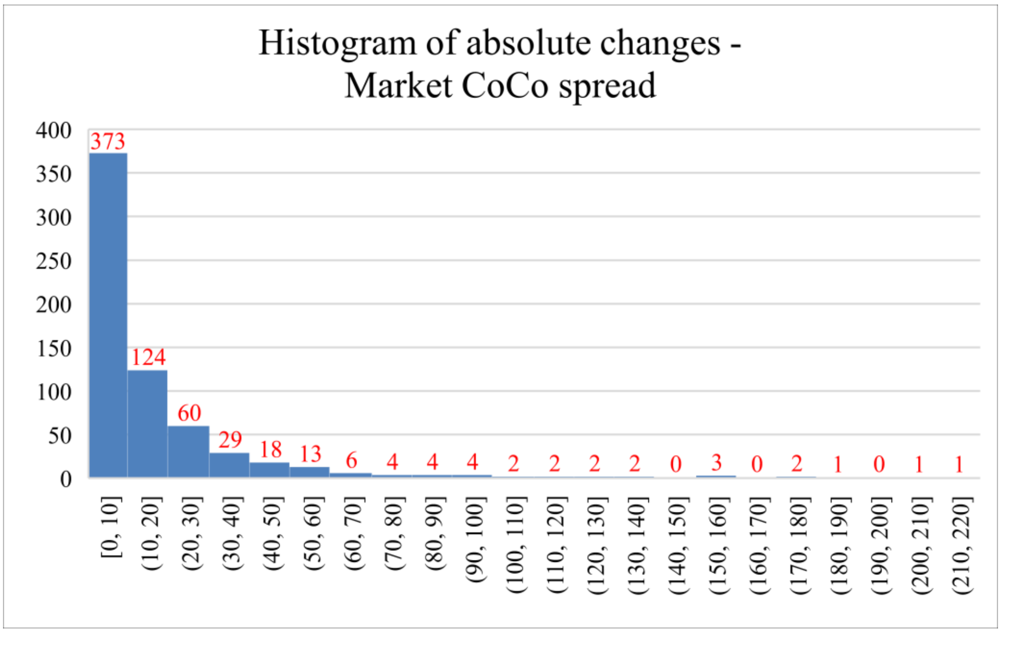

Figure 36: Histogram of absolute change in market CoCo spread – Deutsche Bank

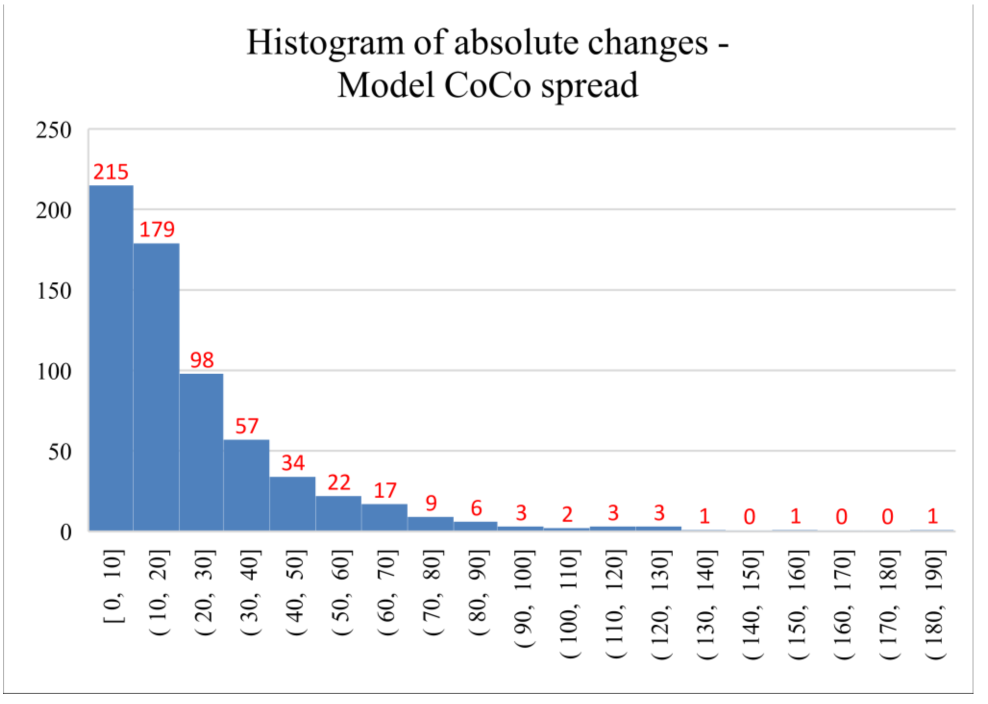

Figure 37: Histogram of absolute change in model CoCo spread – Deutsche Bank

Histograms show that the difference between means is not due to some outlier predicted by the model – largest absolute change implied by model is actually lower than absolute daily change observed on the market. Rather than that, whole distribution is quite different – for market absolute changes, more than 50% of the sample values is in the range 0 to 10 bps. This is not true for absolute changes implied by the model, where only about third of the sample value is within 0 to 10 bps and compared to market, much more absolute changes are in higher buckets.

Based on two size tests, we conclude that Credit derivatives model with our calibration setup leads to larger implied daily changes in CoCo spread compared to market changes. Difference seems not to be caused by outliers implied by the model, but rather by smaller, but systemtical overshooting of the model. Next subchapter will summarize conclusions of the model evaluation and will briefly suggest possible improvements in model calibration.

5.3 Conclusions of model to market comparison

Previous subchapter presented results of various tests focused on comparison of Credit derivatives model implied spreads and market observable spreads. Credit derivative model used calibrated trigger level, averaged for period of 30 trading days and lower, at the money volatility, also averaged for 30 trading days.

Both trigger level and volatility modelling have proven crucial. Chapter 4 already indicated that book implied trigger level often fails in real use as the probability associated with the book trigger level sometimes exceeded one and overall, implied probability was too high in comparison to probability implied by market spreads. This finding was then utilized in the setup of the Credit derivatives model where trigger level was calibrated instead of using the book counterparty.

Calibration of trigger level contributed to overall good results in cointegration test as calibrated trigger level utilized information contained in past spreads and as a result there was tendency of model spreads to revert to market spreads. It is however important to keep in mind that calibration will inevitably disregard new information available at the reporting date and will only slowly absorb the average impact on probability of conversion expected by the market participants – contained in past spreads.

Also, the calibration period of 30 days is rather arbitrary and open to more deliberation. Shorter period will allow new potential information to spread more quickly by allowing more dynamic trigger level. However, this more dynamic trigger level will translate into more unstable CoCo spread implied by the model and could potentially worsen the overshooting already observed for our calibration setup.

While direction tests were quite successful, size tests indicated that Credit derivatives model tends to overshoot the actual change in spreads. In other words, sensitivities implied by Credit derivatives model are larger than sensitivities observed in the CoCo market.

The potential explanation is that short term fluctuations in volatility or stock price are only partially considered by market participants and the full impact of changes in these two crucial variables on probability of conversion, as implied by Credit derivatives model, is not observed in market CoCo spreads. In this sense, market observed sensitivities to both stock price and volatility are moderate compared to Credit derivatives model.

On top of that, the overshooting of changes in CoCo spread we observed in Credit derivatives model was already with averaged ATM volatility used. Using Gatheral functional form for volatility or probably any incorporation of volatility smile into the model would mean using high volatilities characterized themselves with large volatility. This would likely translate into even higher overshooting of the model and diminished prediction power.

Still, averaging of ATM volatility over period of 30 days is only one of many sound solutions and other options, such as exponential weighting of volatility, could be preferable. Any averaging serves as a measure for greater stability by diminishing day to day changes in the pricing variable, while still allowing new information in the presence of volatility changes to be disseminated in the CoCo pricing.

Fifth chapter hopefully outlined potential pitfall of Credit derivatives model calibration and suggested potential solution. Next, final chapter, will summarize the findings gathered throughout all chapters and will conclude the thesis.

6. Conclusions

This thesis set out to empirically test validity of two assumptions underlying for pricing models for contingent convertibles. Apart from providing so far limited evidence on the reality of assumptions, goal of the thesis was also to outline possible calibration setup reflecting the findings gathered throughout the empirical analysis.

First assumption is the approximation of contractually specified CET1 ratio with stock price trigger level. In testing of this assumption, thesis build on methodology previously outlined by Rüdlinger, which suggests how to estimate stock price trigger level using figures reported on balance sheet. We however abandoned the assumption of one to one relationship between book and market value of equity and introduced more flexible solution.

Probability of conversion implied by Credit derivatives model using the book figures was then compared to probability of conversion implied by market CoCo spreads observable in markets. We concluded that even after modification of Rüdlinger´s approach, book implied probability is too high when compared to market implied probability. This suggests that market participants trading with contingent convertibles do not assume that information available on balance sheet of particular CoCo issuer fully reflects its financial position and health, but on contrary that they perhaps assume willingness and ability of management to raise capital in case of need by other means and revert the potential conversion. We also observed that large swings in book implied probability of conversion were not consistent with behaviour of market implied probability, which was characterized with lower volatility. This suggests that not all new information available on balance sheet fully materializes in CoCo market and that market is generally characterized with more gradual absorption of information.

The finding of this empirical test is that the trigger level calculated from reported figures is not very optimal for practical pricing of contingent convertibles. For some CoCo issues in the sample, book implied trigger level actually exceeded its stock price level, leading to book implied probability of conversion exceeding one. Large deviations between book implied probability and market implied probability would then translate into large deviations between model and market spreads for others. We concluded that calibration of trigger level might be necessary in order to obtain model spreads in line with market spreads.

As calibration of trigger level requires decision about volatility modelling, we moved to testing the second assumption. Assumption that volatility characteristic for stock market is also applicable in contingent convertibles pricing framework was tested by incorporating volatility smile in Gatheral function form into the pricing. We observed that the resulting trigger level implied by market spreads is very low and unstable. Incorporating volatility smile leads to high far out of money implied volatilities being used in the calibration and these out of the money implied volatilities are prone to high day to day fluctuations. We illustrated that incorporation of smile leads to model spreads characteristic with much larger volatility than volatility of observed market spreads and concluded that real observed sensitivity of CoCo spreads to volatility is lower than predicted by model.

Finally, we proposed a calibration setup reflecting the empirical findings gathered throughout the thesis. In this regard the model calibration presented in this thesis was novel as it incorporated both calibrated trigger level and volatility modelling – previous studies relied on constant or historical volatility, or incorporated volatility smile but used constant trigger level. Trigger level was recalibrated at each pricing date using past 30 market spreads and lower at the money volatility averaged for 30 days was used in order to reflect the finding from empirical analysis on smile incorporation.

Resulting model was evaluated based on several criteria. We concluded that Credit derivatives model with our calibration setup was performing quite well in "direction tests", that is that direction of changes in model spreads corresponded well to direction of market changes and there was general co-movement of model and market spreads with difference reverting to zero. We however observed that Credit derivatives model is, even after averaging of volatily, prone to overshooting the actual changes in the CoCo spreads. Absolute changes in model spreads on average exceeded changes in market spreads, indicating that the sensitivity implied by Credit derivatives model is higher than real market sensitivity of CoCo spreads and also model variance was significantly larger than market variance.

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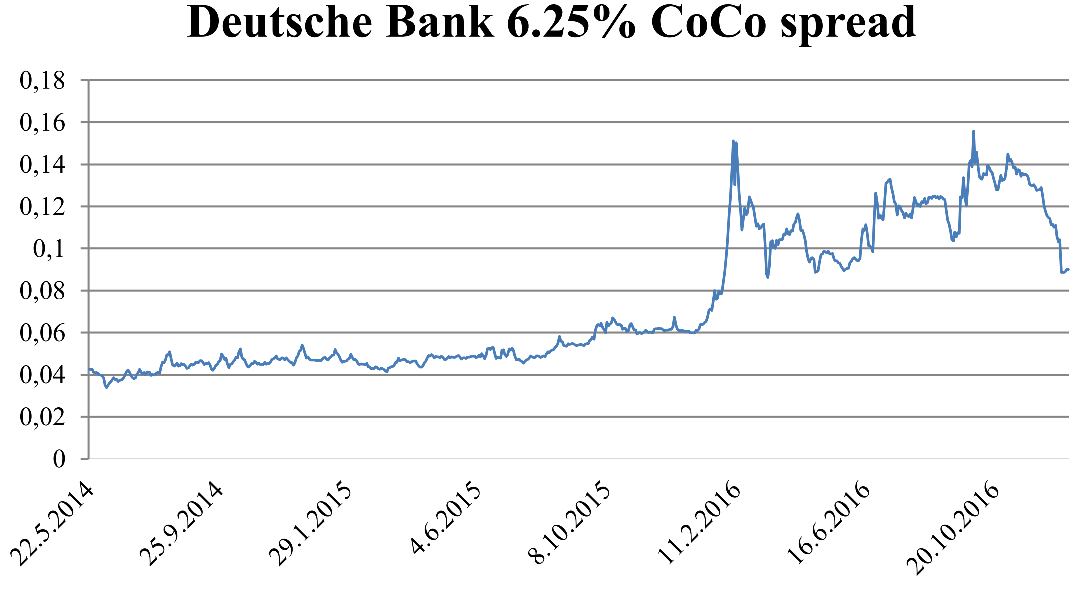
Appendix

A1: Contingent convertible instrument description and summary data

A1.1: Deutsche Bank 6.25% CoCo

Issuer	Deutsche Bank AG
Issue date	27.5.2014
Maturity	perpetual
Coupon	6.25%, from 2025 EUSA5 + 4.358%
Loss absorption type	temporary Write Down
trigger level	CET1 Ratio 5.125
issue price	100.065
currency	USD
coupon dates	30.4.2015, annual frequency after
callable	30.4.2025
	30.4.2030
	30.4.2035
	30.4.2040
	30.4.2045

A1.1.1: Deutsche Bank 6.25% CoCo spread graph



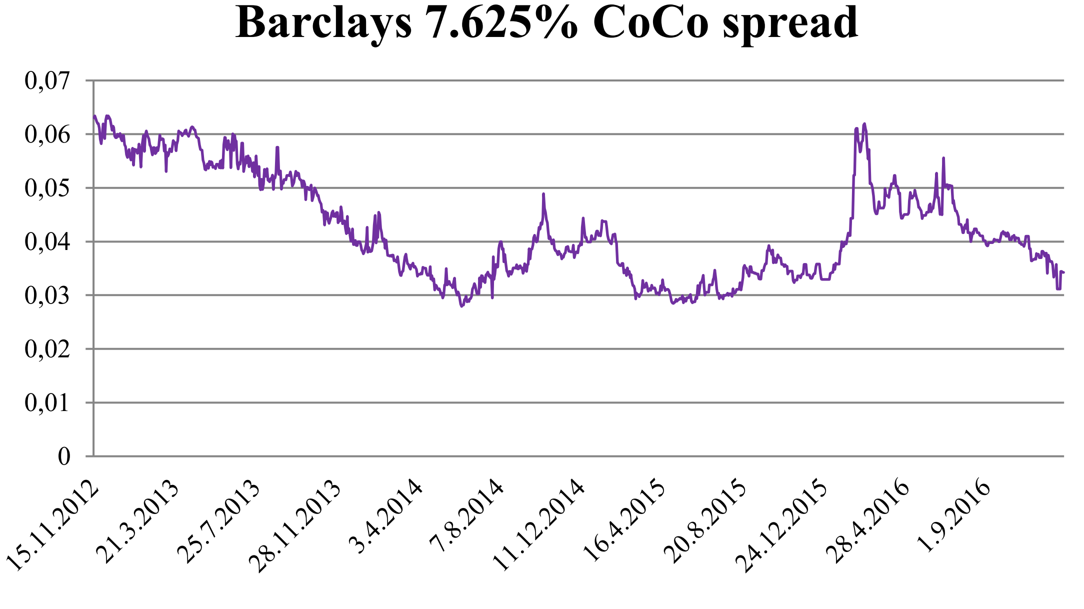
A1.1.2: Deutsche Bank 6.25% CoCo spread summary statistics

Deutsche Bank 6.25% Coco Spread	
Statistics Summary	
Mean	7.32%
Median	5.47%
Standard Deviation	3.33%
Sample Variance	0.11%
Kurtosis	-101.53%
Skewness	74.71%
Minimum	3.39%
Maximum	15.58%
Number of observation	682

A1.2: Barclays 7.625% CoCo

Issuer	Barclays PLC
Issue date	21.11.2012
Maturity	21.11.2022
Coupon	7.625%
Loss absorption type	pernament Write Down
trigger level	CET1 7%
issue price	100
currency	USD
coupon dates	21.5.2013, semiannual frequency
callable	No

A1.2.1: Barclays 7.625% CoCo spread graph



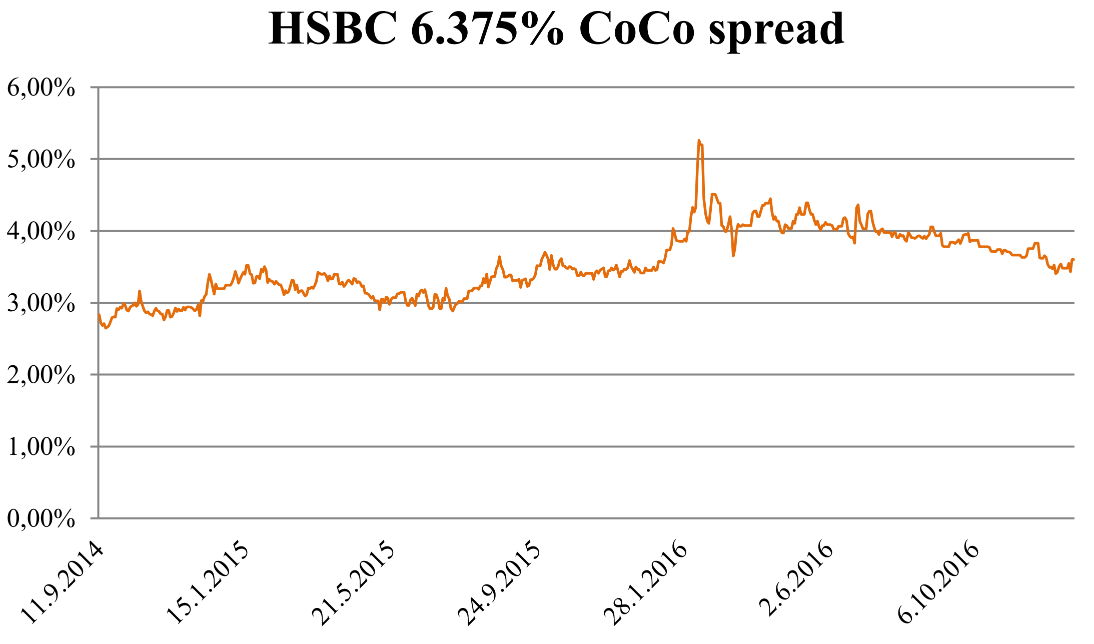
A1.2.2: Barclays 7.625% CoCo spread summary statistics

Barclays 7.625% Coco Spread	
Statistics Summary	
Mean	4.23%
Median	4.00%
Standard Deviation	0.96%
Sample Variance	0.01%
Kurtosis	-92.08%
Skewness	52.68%
Minimum	2.79%
Maximum	6.34%
Number of observation	1077

A1.3: HSBC 6.375% CoCo

Issuer	HSBC Bank PLC
Issue date	17.9.2014
Maturity	perpetual
Coupon	6.375%, USISDA+3.705% from first call date
Loss absorption type	equity conversion
Conversion price	4.35578$
trigger level	CET1 7%
issue price	100
currency	USD
coupon dates	17.3.2015, semiannual frequency
callable	17.9.2024
	17.9.2029
	17.9.2034
	17.9.2039
	17.9.2044
	17.9.2049

A1.3.1: HSBC 6.375% CoCo spread graph



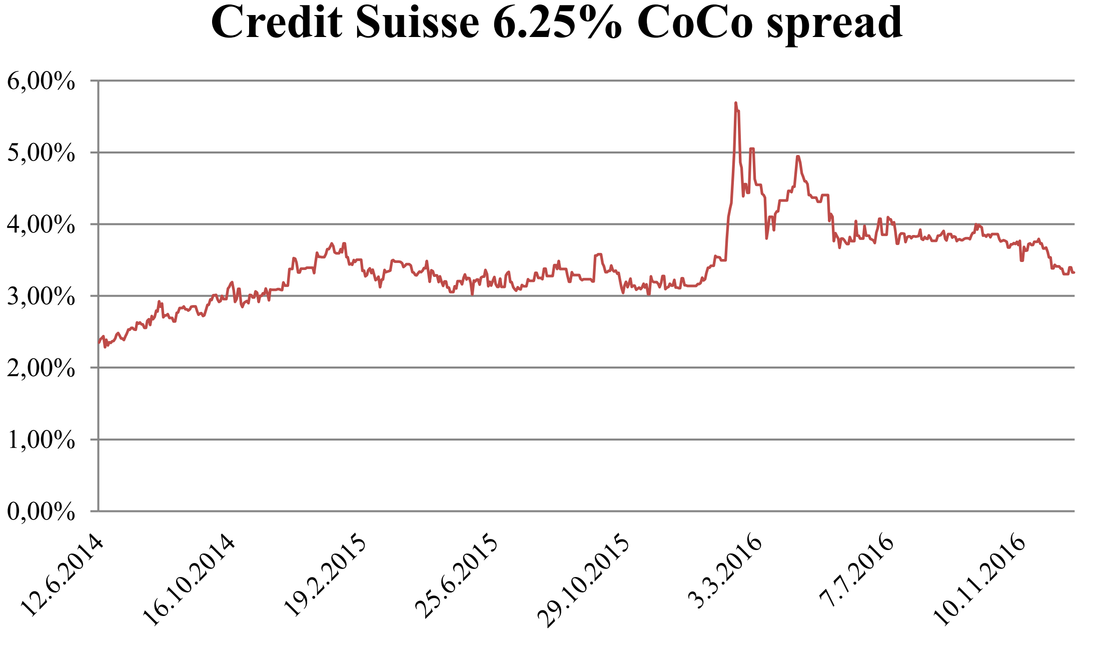
A1.3.2: HSBC 6.375% CoCo spread summary statistics

HSBC 6.375% Coco Spread	
Statistics Summary	
Mean	3.53%
Median	3.46%
Standard Deviation	0.45%
Sample Variance	0.00%
Kurtosis	-13.59%
Skewness	42.14%
Minimum	2.65%
Maximum	5.26%
Number of observation	602

A1.4: Credit Suisse 6.25% CoCo

Issuer	Credit Suisse AG
Issue date	18.6.2014
Maturity	perpetual
Coupon	6.25%
Loss absorption type	Pernament Write Down
Conversion price	-
trigger level	CET1 5.125%
issue price	100
currency	USD
coupon dates	18.12.2014 semiannual
callable	18.12.2024
	18.12.2029
	18.12.2034
	18.12.2039
	18.12.2044
	18.12.2049

A1.4.1: Credit Suisse 6.25% CoCo spread graph



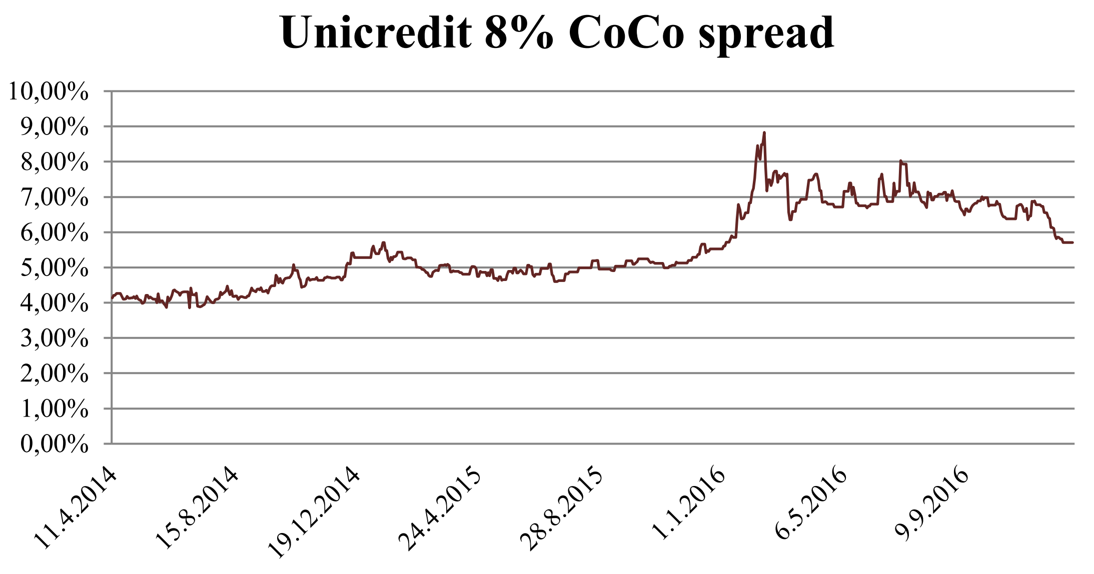
A1.4.2: Credit Suisse 6.25% CoCo spread summary statistics

Credit Suisse 6.25% Coco Spread	
Statistics Summary	
Mean	3.45%
Median	3.36%
Standard Deviation	0.53%
Sample Variance	0.00%
Kurtosis	128.35%
Skewness	70.97%
Minimum	2.28%
Maximum	5.69%
Number of observation	667

A1.5: UniCredit 8% CoCo

Issuer	Unicredit SPA
Issue date	4.3.2014
Maturity	perpetual
Coupon	8%, USSW5 + 5.180% from first call date
Loss absorption type	Temporary Write Down
Conversion price	-
trigger level	CET1 5.125%
issue price	100
currency	USD
coupon dates	6.3.2024
callable	6.3.2024
	12.3.2024
	6.3.2025
	12.3.2025
	semiannually up to 2048

A1.5.1: UniCredit 8% CoCo spread graph



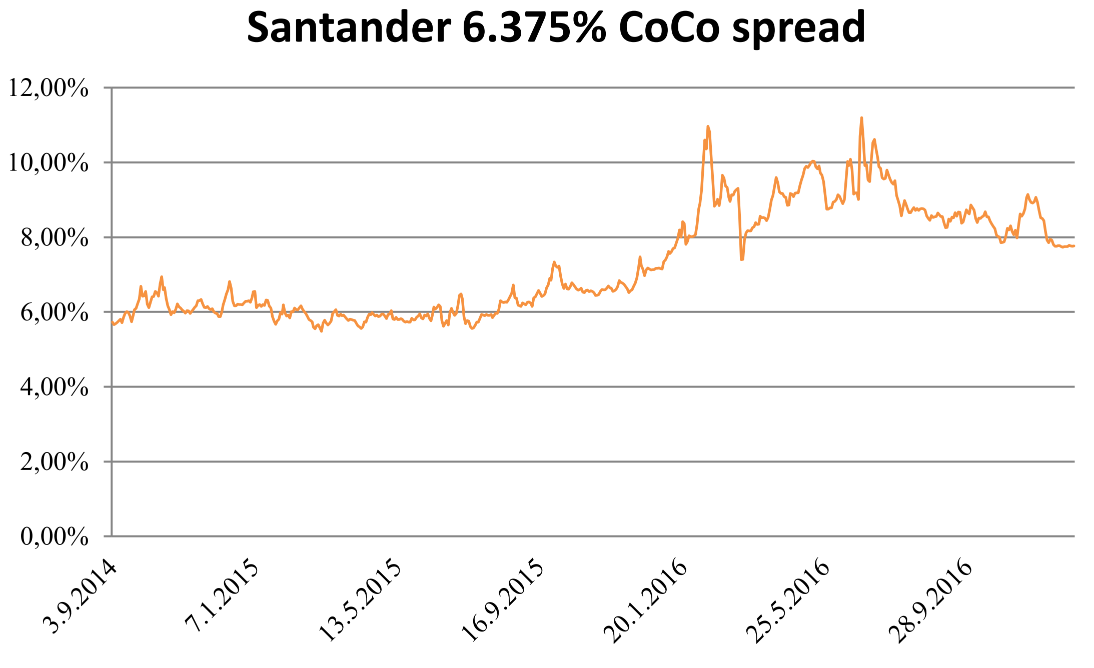
A1.5.2: UniCredit 8% CoCo spread summary statistics

UniCredit 8% Coco Spread	
Statistics Summary	
Mean	5.54%
Median	5.15%
Standard Deviation	1.11%
Sample Variance	0.01%
Kurtosis	-89.82%
Skewness	53.42%
Minimum	3.85%
Maximum	8.83%
Number of observation	711

A1.6: Santander 6.375% CoCo

Issuer	Banco Santander
Issue date	19.5.2014
Maturity	perpetual
Coupon	6.375%, USSW5+ 4.788% from first call date
Loss absorption type	equity conversion
Conversion price	MAX(St, 0.5$, 5.01$ per share)
trigger level	CET1 5.125%
issue price	100
currency	USD
coupon dates	19.8.2014 quarterly
callable	19.5.2019
	any time after

A1.6.1: Santander 6.375% CoCo spread graph



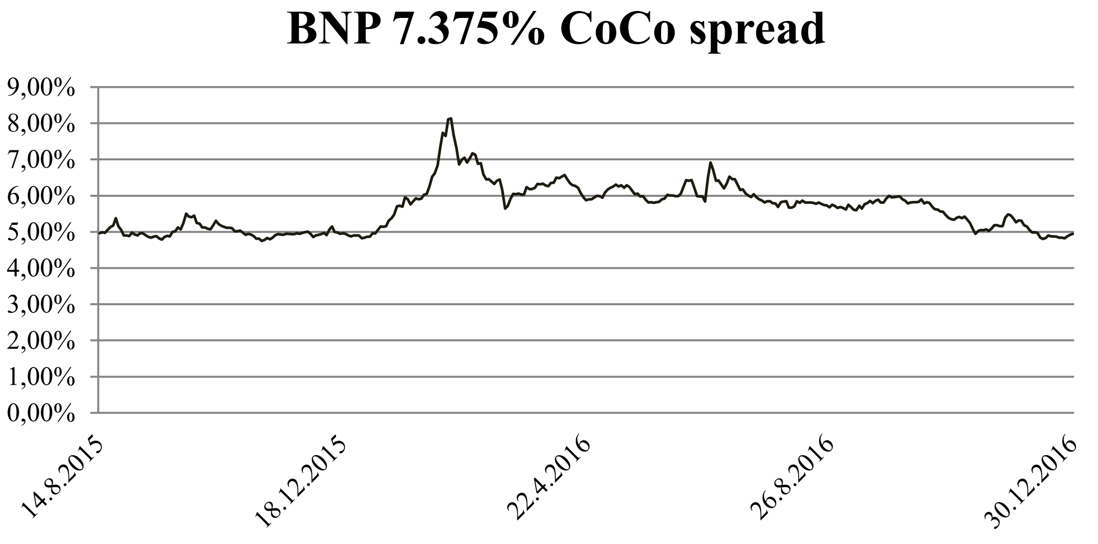
A1.6.2: Santander 6.375% CoCo spread summary statistics

Santander 6.375% Coco Spread	
Statistics Summary	
Mean	7.31%
Median	6.68%
Standard Deviation	1.42%
Sample Variance	0.02%
Kurtosis	-105.72%
Skewness	52.52%
Minimum	5.48%
Maximum	11.20%
Number of observation	608

A1.7: BNP 7.375% CoCo

Issuer	BNP Paribas
Issue date	19.8.2015
Maturity	perpetual
Coupon	7.375%, USSW5 + 5.150% from first call date
Loss absorption type	Temporary Write Down
Conversion price	-
trigger level	CET1 5.125%
issue price	100
currency	USD
coupon dates	19.2.2016 semiannual
callable	19.8.2025
	19.2.2026
	19.8.2026
	semiannualy up to 19.8.2049

A1.7.1: BNP 7.375% CoCo spread graph



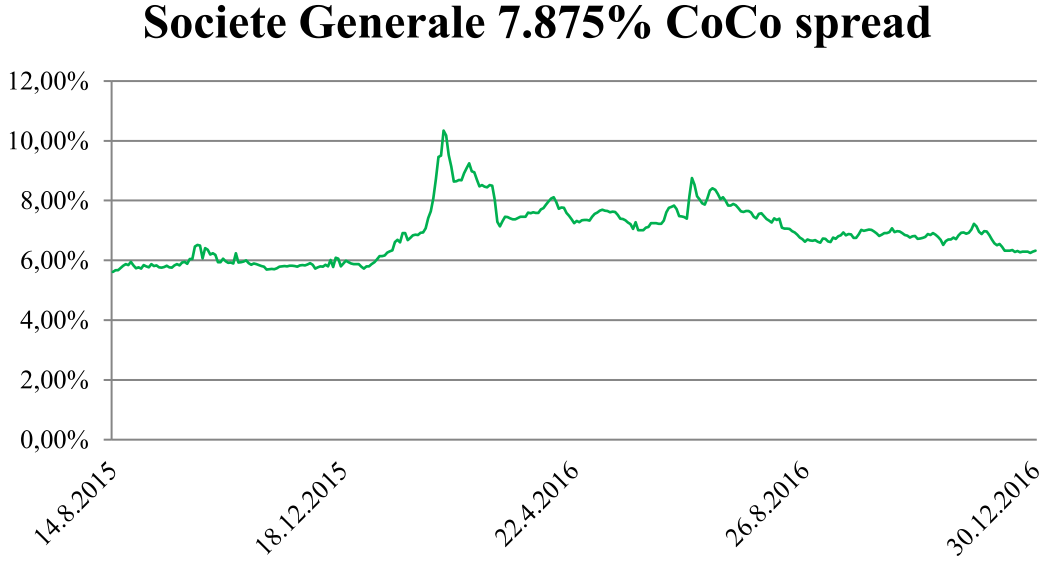
A1.7.2: BNP 7.375% CoCo spread summary statistics

BNP 7.375% Coco Spread	
Statistics Summary	
Mean	5.65%
Median	5.71%
Standard Deviation	0.65%
Sample Variance	0.00%
Kurtosis	67.15%
Skewness	74.13%
Minimum	4.75%
Maximum	8.13%
Number of observation	361

A1.8: Societe Generale 7.875% CoCo

Issuer	Societe Generale
Issue date	18.12.2013
Maturity	perpetual
Coupon	7.875%, USSW5 + 4.979% from first call date
Loss absorption type	Temporary Write Down
Conversion price	-
trigger level	CET1 5.125%
issue price	100
currency	USD
coupon dates	18.6.2014 semiannual
callable	18.12.2023
	18.12.2028
	18.12.2033
	18.12.2038
	18.12.2043
	18.12.2048

A1.8.1: Societe Generale 7.875% CoCo spread graph



A1.8.2: Societe Generale 7.875% CoCo spread summary statistics

Societe Generale 7.875% Coco Spread	
Statistics Summary	
Mean	6.91%
Median	6.87%
Standard Deviation	0.90%
Sample Variance	0.01%
Kurtosis	47.42%
Skewness	70.78%
Minimum	5.61%
Maximum	10.34%
Number of observation	361

A2: Stock price series – summary statistics

A2.1: Deutsche Bank

Deutsche Bank Stock Price	
Statistics Summary	
Mean	22.24
Median	24.68
Standard Deviation	6.41
Sample Variance	41.03
Kurtosis	-132.82%
Skewness	-32.33%
Minimum	10.55
Maximum	33.20
Number of observation	682

A2.2: Barclays

Barclays Stock Price	
Statistics Summary	
Mean	2.39
Median	2.47
Standard Deviation	0.45
Sample Variance	0.20
Kurtosis	-54.14%
Skewness	-41.58%
Minimum	1.27
Maximum	3.34
Number of observation	1077

A2.3: HSBC

HSBC Stock Price	
Statistics Summary	
Mean	5.54
Median	5.71
Standard Deviation	0.71
Sample Variance	0.50
Kurtosis	-118.44%
Skewness	-32.94%
Minimum	4.16
Maximum	6.80
Number of observation	602

A2.4: Credit Suisse

Credit Suisse Stock Price	
Statistics Summary	
Mean	20.52
Median	23.47
Standard Deviation	5.82
Sample Variance	33.89
Kurtosis	-151.66%
Skewness	-39.16%
Minimum	9.92
Maximum	28.70
Number of observation	667

A2.5: UniCredit

UniCredit Stock Price	
Statistics Summary	
Mean	4.76
Median	5.55
Standard Deviation	1.63
Sample Variance	2.66
Kurtosis	-127.01%
Skewness	-61.99%
Minimum	1.75
Maximum	6.87
Number of observation	711

A2.6: Banco Santander

Santander Stock Price	
Statistics Summary	
Mean	5.31
Median	5.07
Standard Deviation	1.29
Sample Variance	1.67
Kurtosis	-141.07%
Skewness	21.74%
Minimum	3.30
Maximum	7.90
Number of observation	608

A2.7: BNP Paribas

BNP Stock Price	
Statistics Summary	
Mean	49.07
Median	47.83
Standard Deviation	5.90
Sample Variance	34.80
Kurtosis	-102.85%
Skewness	18.00%
Minimum	36.91
Maximum	61.70
Number of observation	361

A2.8: Societe Generale

Societe Generale Stock Price	
Statistics Summary	
Mean	36.76
Median	35.41
Standard Deviation	5.48
Sample Variance	30.01
Kurtosis	-116.39%
Skewness	24.13%
Minimum	26.39
Maximum	47.50
Number of observation	361

A3: CoCo spreads – test of stationarity

A3.1: Deutsche Bank 6.25% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	9	2
test statistic	-0.756622	-13.2923
asymptotical p - value	0.3888	3.76E-28

A3.2: Barclays 7.625% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	1	8
test statistic	-0.308738	-6.82502
asymptotical p - value	0.5748	4.82E-11

A3.3: HSBC 6.375% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	12	0
test statistic	-0.615046	-11.2301
asymptotical p - value	0.4512	8.80E-23

A3.4: Credit Suisse 6.25% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	7	1
test statistic	-0.671021	-13.5616
asymptotical p - value	0.4267	8.14E-29

A3.5: UniCredit 8% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	0	0
test statistic	-0.886955	-22.7566
asymptotical p - value	0.332	9.14E-42

A3.6: Santander 6.375% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	5	0
test statistic	-0.681478	-17.154
asymptotical p - value	0.4221	1.96E-36

A3.7: BNP 7.375% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	5	2
test statistic	-1.31188	-8.75897
asymptotical p - value	0.1756	4.50E-16

A3.8: Societe Generale 7.875% Augmented Dickey Fuller

Dickey Fuller test	CoCo spread	d_CoCo spread
number of lags included	1	0
test statistic	-1.31188	-13.1686
asymptotical p - value	0.1756	7.66E-28

A4: Stock prices – test of stationarity

A4.1: Deutsche Bank stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	9
test statistic	-0.01837	-2.9063
asymptotical p - value	0.6767	3.56E-03

A4.2: Barclays stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	4	0
test statistic	-1.52858	-17.4644
asymptotical p - value	0.1188	5.85E-37

A4.3: HSBC stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	2
test statistic	-0.808451	-11.3167
asymptotical p - value	0.366	5.15E-23

A4.4: Credit Suisse stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	1	7
test statistic	0.0369873	-8.1579
asymptotical p - value	0.6946	1.81E-14

A4.5: UniCredit stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	1	2
test statistic	0.172963	-9.41779
asymptotical p - value	0.7365	7.41E-18

A4.6: Santander stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	3	2
test statistic	-0.118096	-3.95717
asymptotical p - value	0.6431	7.71E-05

A4.7: BNP Paribas stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	0
test statistic	-0.980769	-12.2191
asymptotical p - value	0.293	2.08E-25

A4.8: Societe Generale stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	2
test statistic	-0.8549	-7.05814
asymptotical p - value	0.3458	1.26E-11

A5: CDSSpreads – test of stationarity

A5.1: Deutsche Bank CDS Spread - Augmented Dickey Fuller

Dickey Fuller test	CDSSpreads	d_CDSSpreads
number of lags included	0	9
test statistic	-0.01837	-2.9063
asymptotical p - value	0.6767	3.56E-03

A5.2: Barclays stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	4	0
test statistic	-1.52858	-17.4644
asymptotical p - value	0.1188	5.85E-37

A5.3: HSBC stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	2
test statistic	-0.808451	-11.3167
asymptotical p - value	0.366	5.15E-23

A5.4: Credit Suisse stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	1	7
test statistic	0.0369873	-8.1579
asymptotical p - value	0.6946	1.81E-14

A5.5: UniCredit stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	1	2
test statistic	0.172963	-9.41779
asymptotical p - value	0.7365	7.41E-18

A5.6: Santander stock price - Augmented Dickey Fuller

Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	3	2
test statistic	-0.118096	-3.95717
asymptotical p - value	0.6431	7.71E-05

A5.7: BNP Paribas stock price - Augmented Dickey Fuller

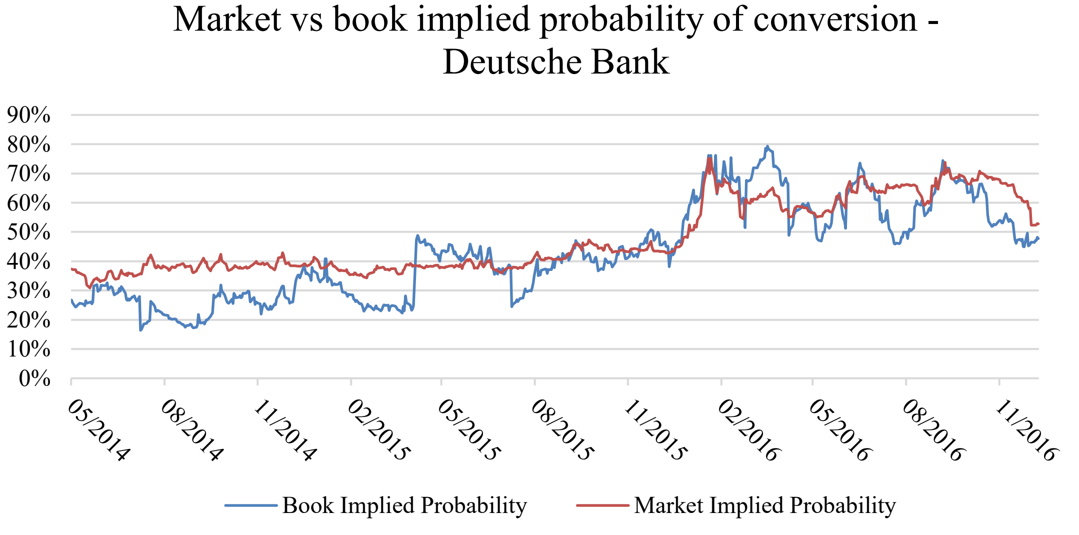
Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	0
test statistic	-0.980769	-12.2191
asymptotical p - value	0.293	2.08E-25

A5.8: Societe Generale stock price - Augmented Dickey Fuller

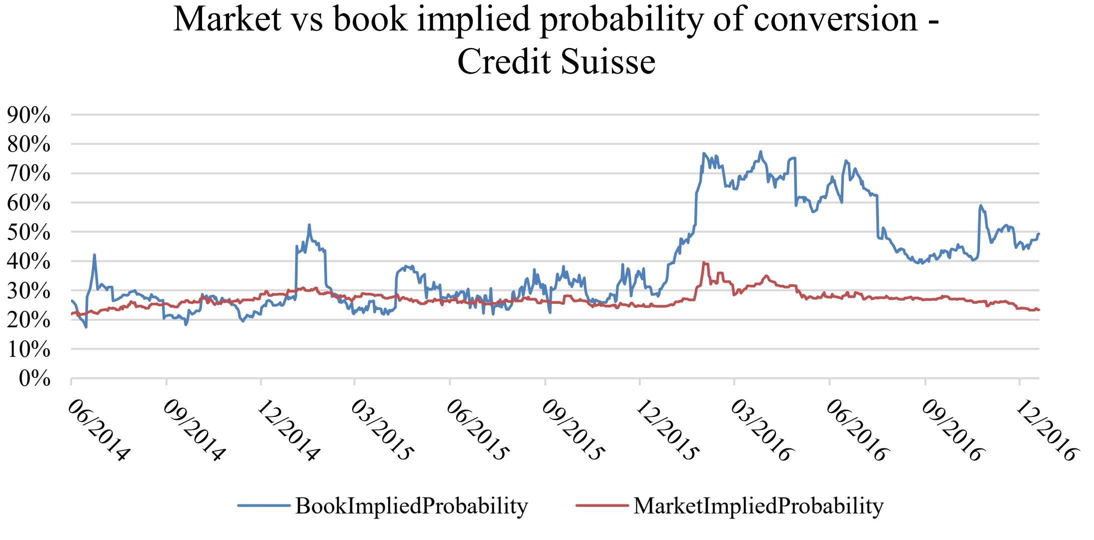
Dickey Fuller test	StockPrice	d_StockPrice
number of lags included	0	2
test statistic	-0.8549	-7.05814
asymptotical p - value	0.3458	1.26E-11

A6: Market vs book implied probability of conversion

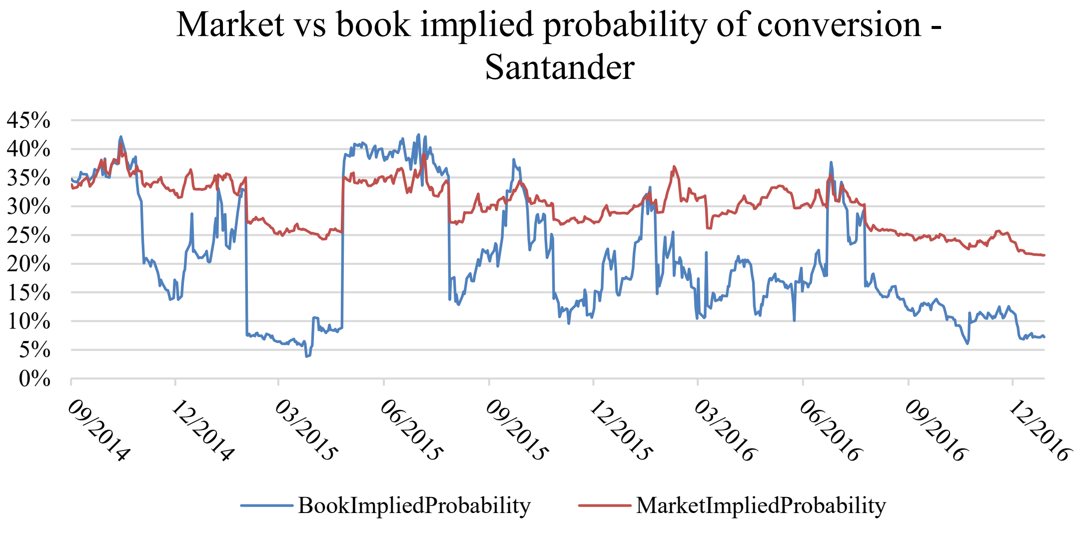
A6.1: Market vs book implied probability of conversion - Deutsche Bank



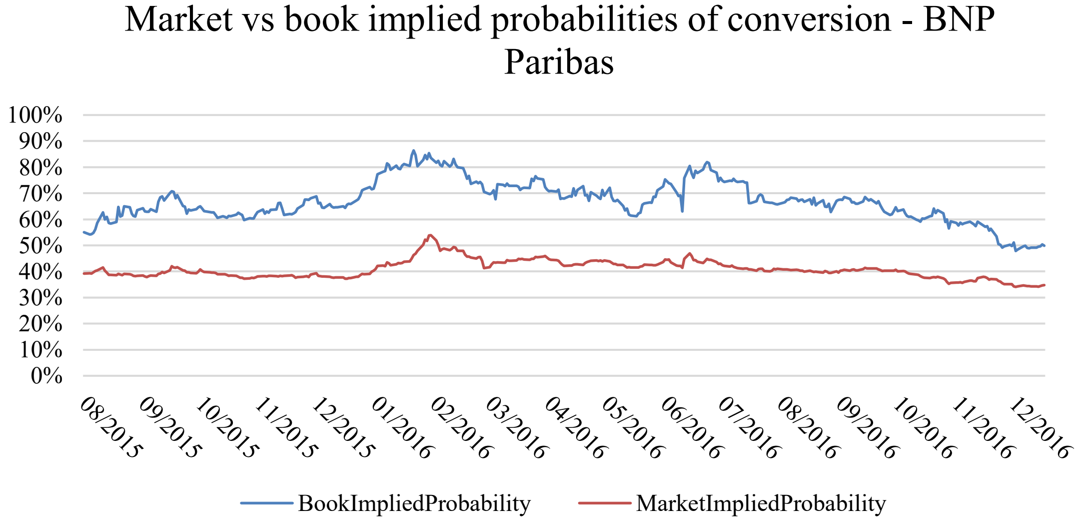
A6.2: Market vs book implied probability of conversion – Credit Suisse



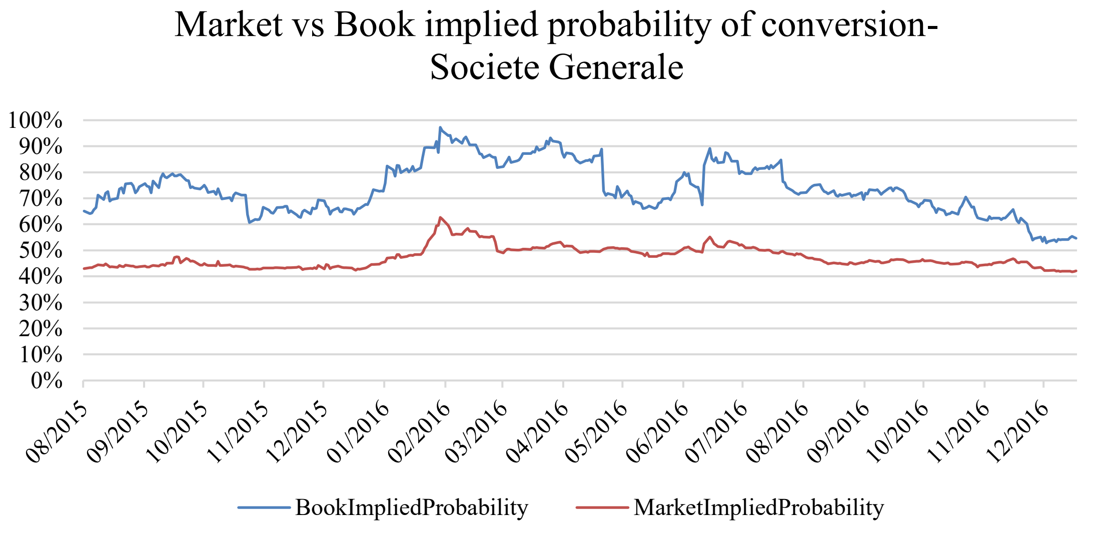
A6.3: Market vs book implied probability of conversion – Santander



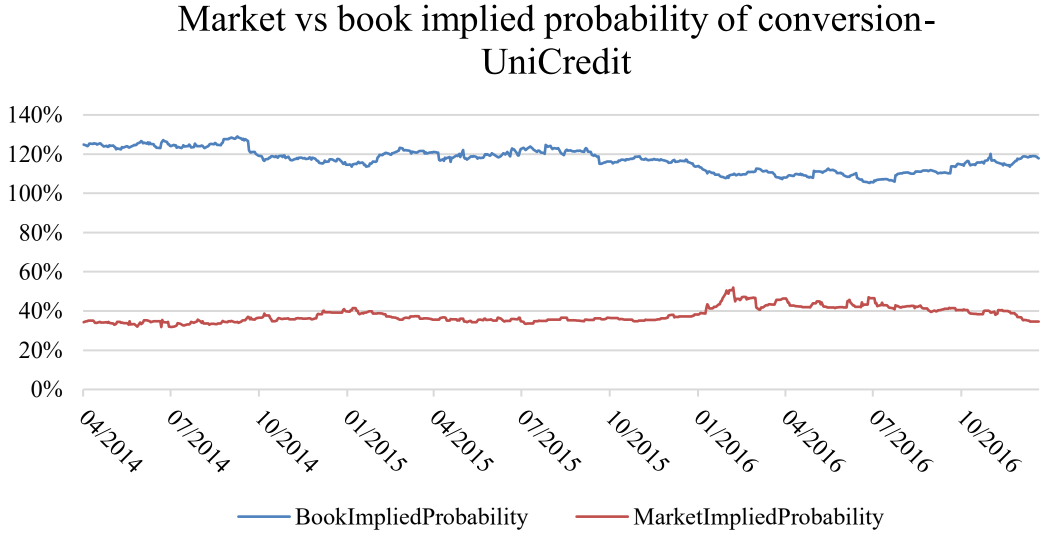
A6.4: Market vs book implied probability of conversion – BNP Paribas



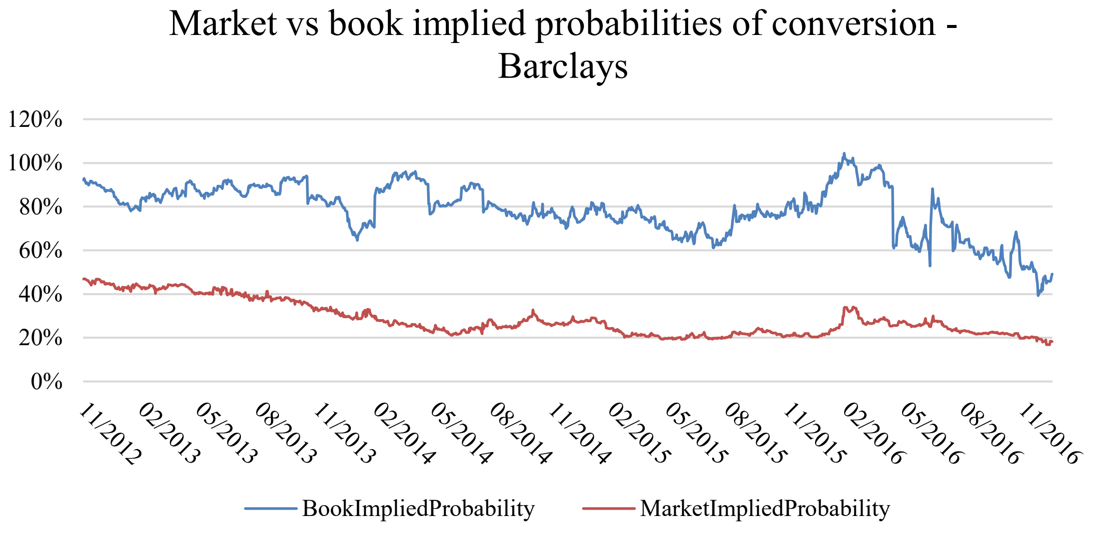
A6.5: Market vs book implied probability of conversion – Societe Generale



A6.6: Market vs book implied probability of conversion – UniCredit

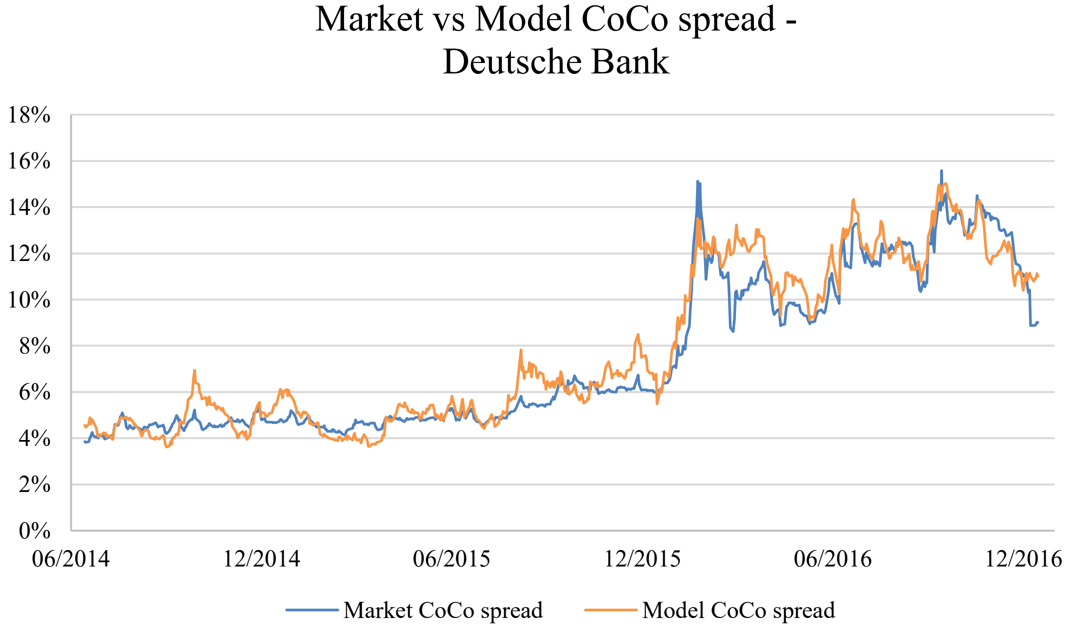


A6.7: Market vs book implied probability of conversion – Barclays

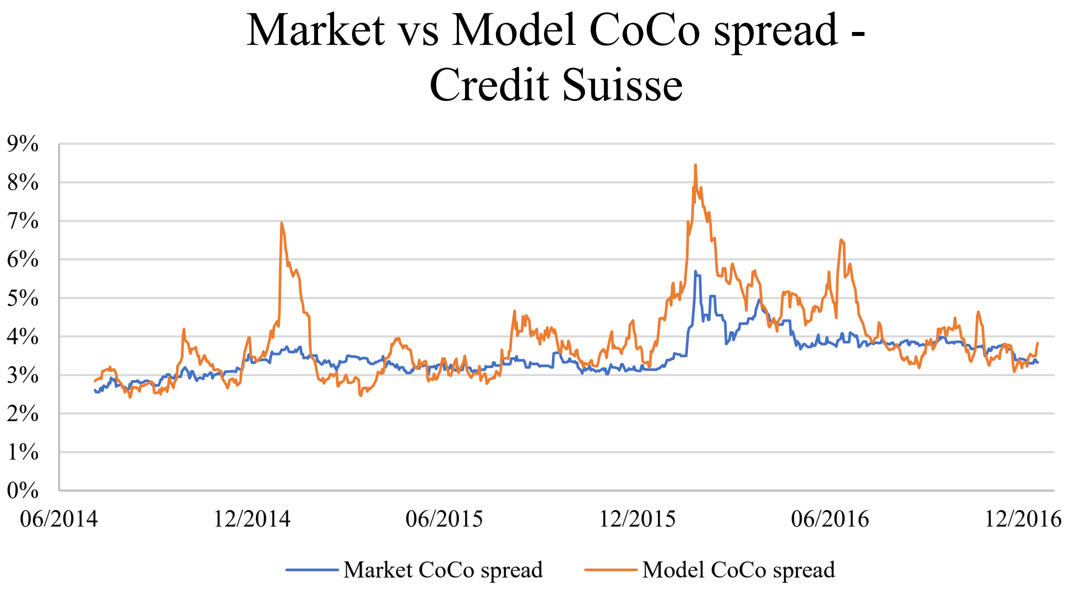


A7: Calibrated Credit derivatives model vs market CoCo spreads

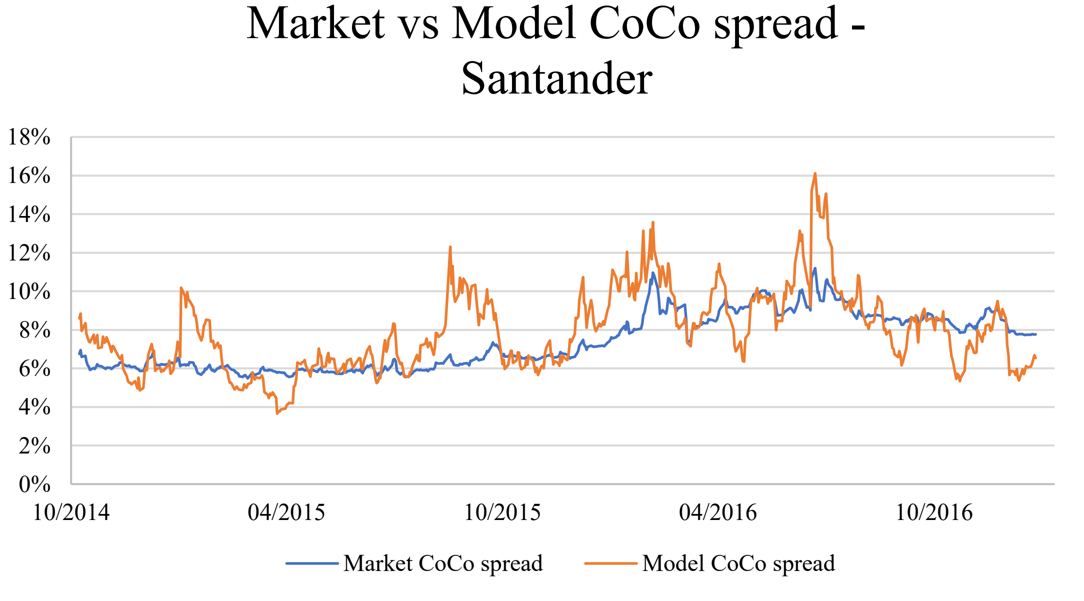
A7.1: Credit derivatives model CoCo spreads vs market CoCo spreads – Deutsche Bank



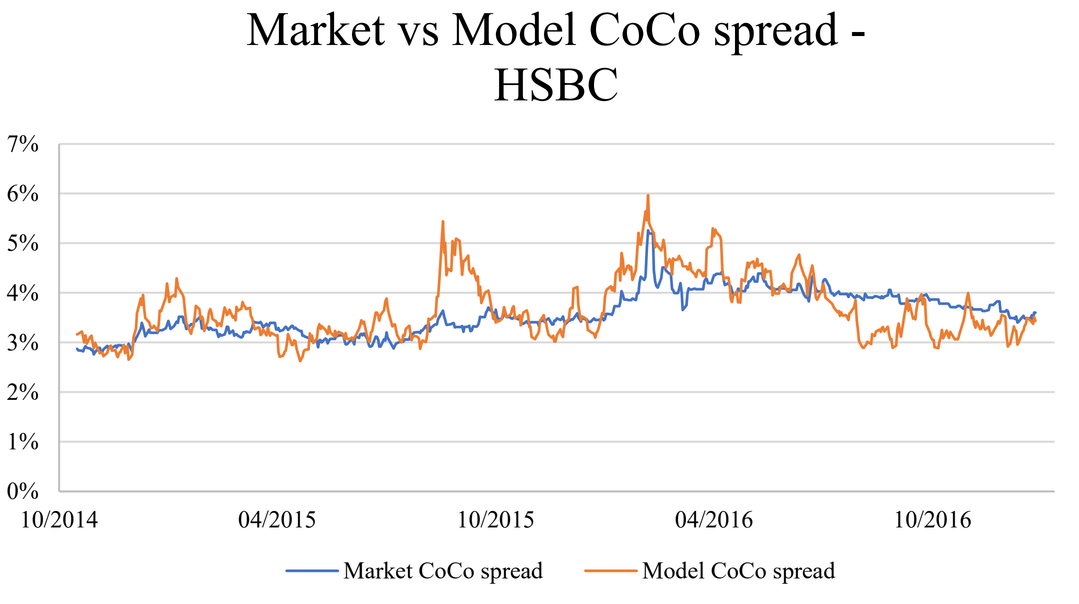
A7.2: Credit derivatives model CoCo spreads vs market CoCo spreads – Credit Suisse



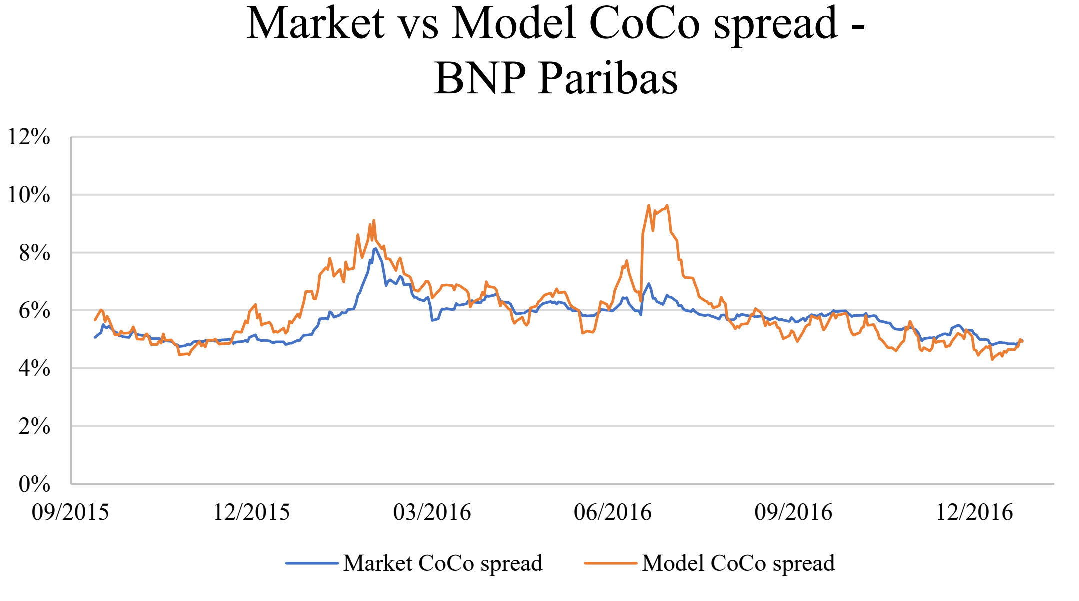
A7.3: Credit derivatives model CoCo spreads vs market CoCo spreads – Santander



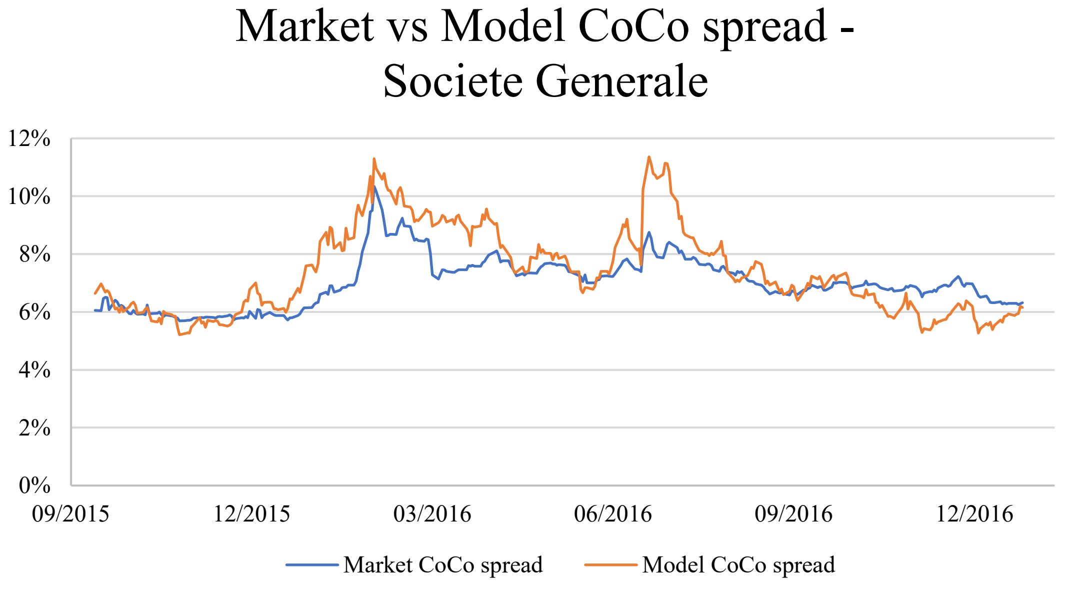
A7.4: Credit derivatives model CoCo spreads vs market CoCo spreads – HSCB



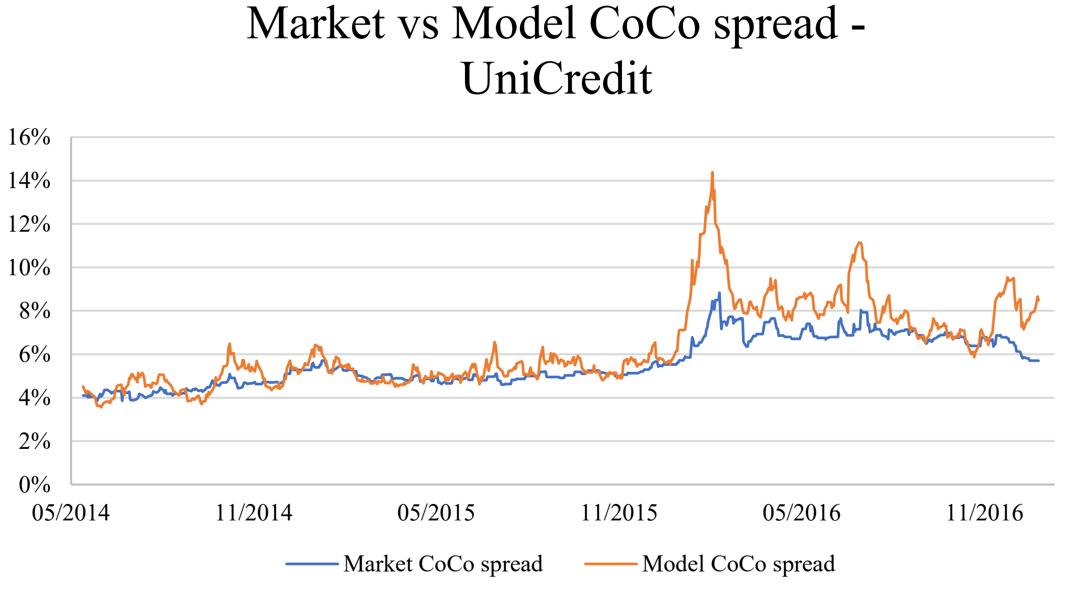
A7.5: Credit derivatives model CoCo spreads vs market CoCo spreads – BNP Paribas



A7.6: Credit derivatives model CoCo spreads vs market CoCo spreads – Societe Generale



A7.7: Credit derivatives model CoCo spreads vs market CoCo spreads – UniCredit



A7.8: Credit derivatives model CoCo spreads vs market CoCo spreads – Barclays