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MASTER'S THESIS

Monetary Policy and House Prices in the US: Evidence from Time-Varying VAR Model

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Signature

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Abstract

This thesis examines the effects of monetary policy shocks on the housing market. To this end, TVP-VAR model with dynamic dimension selection and stochastic volatility is estimated using monthly data for the United States over the period 1999-2017. Moreover, the model features estimating the optimal value of the Bayesian shrinkage coefficient in a time-varying manner. Since the sample covers the Zero Lower Bound period, Wu-Xia shadow rate is employed to measure the stance of monetary policy. To assess the link between housing variables and monetary policy, impulse responses and forecast error variance decompositions are provided. However, due to the time-varying nature of the model, they are estimated only for selected time periods that correspond both to the events that most likely influenced the path of macroeconomic and financial variables and to periods of low economic uncertainty. The main results are threefold. First, the model suggests that monetary policy shocks can contribute to developments in house prices. Second, the stimulative monetary policy positively affects residential investment and negatively affects mortgage rates, however, the effects are not significant due to the large confidence bands of the impulse responses. Third, higher values of the shrinkage hyperparameter are crucial for obtaining reasonable impulse responses. Those results are fairly robust to various specifications of the model.

JEL Classification C11, C51, E43, E47, R30

Keywords monetary policy, house prices, time-varying VARs, interest rates, zero interest-rate policy

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Abstrakt

Tato práce zkoumá vliv šoků do měnové politiky na realitní trh. K tomuto účelu byl odhadnut časově-proměnlivý VAR model s dynamickým výběrem dimenze a stochastickou volatilitou, který byl odhadnut na měsíčních datech pro Spojené státy v období 1999-2017. Model je dále charakterizován výběrem optimálního časově proměnlivého Bayesovského koeficientu smršťování. Protože model odhadujeme i v období nulové spodní hranice, byla použita Wu-Xia stínová úroková sazba, abychom mohli kvalifikovat postoj měnové politiky. K posouzení vztahu mezi realitními proměnnými a měnovou politikou byly odvozeny impulzní funkce a dekompozice rozptylu v chybě předpovědi. Avšak jelikož je model časově proměnlivý, tyto jsou odhadnuty pouze ve vybraných časech, které obsahují jak události, jež s nejvyšší pravděpodobností ovlivnily hodnoty makroekonomických a finančních proměnných, tak i období, ve kterých byla ekonomická nejistota minimální. Hlavní výsledky jsou následující. Zaprvé, model indikuje, že šoky do měnové politiky mohou ovlivňovat ceny nemovitostí. Zadruhé, stimulativní monetární politika pozitivně ovlivňuje realitní investice a negativně ovlivňuje hypoteční sazby, avšak tyto efekty nejsou signifikantní kvůli velice širokým pásmům okolo impulzních funkcí. Zatřetí, větší hodnoty Bayesovského koeficientu smršťování jsou nezbytné pro získání rozumných impulzních funkcí. Tyto výsledky jsou robustní vůči různým specifikacím modelu.

Klasifikace JEL	C11, C51, E43, E47, R30
Klíčová slova	monetární politika, ceny nemovitostí, časově proměnlivé VAR modely, úrokové sazby, politika nízkých úrokových sazeb

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Motivation Studying the effect of monetary policy on house prices has become one of the main concerns for monetary economists over the past years, especially after the burst of the U.S. housing bubble in 2007 which triggered the so-called mortgage crisis; the event considered by many as a major impetus to the outbreak of the 2008 financial crisis. Prior to these events, i.e. between 2001 and 2005, the key interest rate in the United States, the federal funds rate (which we refer to as the short-term interest rate or simply the interest rate), was maintained exceptionally low, well below what the Taylor rule would have implied. This coincidence motivated researchers to examine the impact of loose monetary policy on asset prices with the emphasis on the possible creation of asset price bubbles.

Recent crisis also showed that conventional monetary policy may not be sufficient to revive the economy - in December 2008, the federal funds rate was pushed to almost zero and the Fed also bought a huge amount of mortgage-backed securities to decrease mortgage rates and boost real estate sales, however, neither of those actions significantly helped to raise the aggregate demand. Moreover, with the policy rate near its effective lower bound, the Fed had to implement unconventional monetary policies aimed at lowering long-term interest rates. These policies consisted mainly of large-scale asset purchases and forward guidance, the term referring to central bank's signaling of the likely future path of federal funds rate to the public. Unconventional monetary policy paid off and the U.S. economy, although moderately, started to grow in June 2009. House prices (as measured by Case-Shiller U.S. National Home Price Index) increased by 19% from February 2012 to February 2014 following a clear upward trend since the beginning of that period.

The aim of this thesis is to investigate whether unconventional monetary policy implemented by the Fed during the crisis influences U.S. house prices more than conventional monetary policy due to the long-term policy of low interest rates. Although unconventional monetary policy undoubtedly helped the U.S. economy, it also created several potential issues we might have to deal with in the future. The low interest rate environment forces investors to demand more low-quality assets in order to achieve higher returns than they would have from buying e.g. the U.S. Treasury bonds. Investors may also invest more in real estate, which can create an upward pressure on house prices and lead to real estate mispricing - the issue that is also supported by the evidence of very high U.S. real estate prices relative to the yields coming from the rents on those properties. Raising the interest rate would then cause the rents to be less competitive, resulting subsequently in a price decrease of the properties. This environment also encourages banks to give mortgages to less reliable customers, a practice that partially led to the already mentioned U.S. housing bubble after 2000. In this thesis we will assess how traditional monetary policy influences house prices through the short-term interest rate and how this connection changes when unconventional monetary policy operating at the Zero Lower Bound (ZLB) takes place.

Hypotheses

Hypothesis #1: The federal funds rate (either effective or target) has negative effect on house prices in the United States.

Hypothesis #2: All of the following monetary policy measures have a significant effect on house prices: 1) shadow policy rate, 2) central bank's assets, 3) forward guidance, 4) deviation of the federal funds rate from that prescribed by the Taylor rule. Further, we expect that the shadow policy rate has a negative effect on house prices; the Federal Reserve's assets have a positive effect; and the last policy measure calculated as the difference between the short-term nominal interest rate and the Taylor rule implied rate negatively affects house prices in the sense that a negative difference (loose monetary policy) stimulates the increase in prices and the opposite holds for a positive difference.

Hypothesis #3: The impact of monetary policy on house prices is greater during the crisis, when the Federal Reserve operates at very low short-term interest rates and employs unconventional monetary policy measures, than before the crisis, when the Fed uses only conventional measures.

Methodology To assess the relationship between monetary policy and house prices as well as whether it changed during and before the crisis, we will use large time-varying parameter vector autoregressive models (large TVP-VARs). Regarding estimation and forecasting, we will follow Koop and Korobilis (2013), who proposed approximate estimation methods that do not require the use of MCMC methods and thus reduce computational burden. Their approach involves the use of forgetting factors, but instead of setting these factors to some constant value, as many authors do, they estimate them from the data. Estimated forgetting factors are then be needed in the dynamic model selection (DMS) - a method for choosing the optimal value of the shrinkage parameter at different points in time. This method hinges upon treating different values of the shrinkage parameter as defining different models for which we find the optimal shrinkage parameters. Another advantage of Koop and Korobilis' approach except the increased computational feasibility is that it allows for model switching: the algorithm uses past predictive likelihoods for the set of variables we would like to forecast to select a small, medium, or large TVP-VAR at each point in time. Koop and Korobilis (2013) highlight potential usefulness of such a procedure as it identifies which model forecasts the best and it might also improve the forecast performance of TVP-VARs of different dimensions.

Since this thesis aims to clarify the effect of monetary policy on house prices with the emphasis on the central bank's long-term policy of extremely low interest rates, we will face the choice regarding which monetary policy instruments we should include into the model. As a conventional monetary policy tool, we will use the short-term interest rate. However, during the crisis, the Fed experienced the so-called Zero Lower Bound period when the interest rate could not be pushed down further, and therefore it implemented unconventional policy measures. We will include the Federal Reserve's assets and the shadow policy rate to account for these measures in our model. The first variable is commonly used in the literature as a proxy for unconventional monetary policy and it also reflects the fact that large-scale asset purchases were extensively used by the Fed during the Zero Lower Bound period. The shadow rate, first estimated by Wu and Xia 2014, is another convenient measure of unconventional monetary policy: it normally follows the short-term interest rate, but when the interest rate gets stuck at the Zero Lower Bound, the shadow policy rate can go negative. It basically shows how the interest rate would have behaved if it could be negative. We will also compute the deviation of the federal funds rate from the rate implied by the Taylor rule, which is a suitable indicator for measuring the monetary policy stance as it can be used not only during the ZLB period. This indicator seems to be relevant as well: many researchers (most prominently Taylor, 2007, and 2009) argued that because the deviation of the federal funds rate from the Taylor rule implied rate was so great after 2000, these unusually low interest rates accompanied with the provision of large amounts of liquidity encouraged housing market imbalances prior to the crisis.

Lastly, we will also focus on forward guidance, which was one of the key practices implemented by the Fed during the crisis. We think that forward guidance plays an important role in the expectations formation of market participants about the future interest rates, and as such can eventually influence house prices. Quantifying forward guidance is however a challenging task; we will use a method developed in Swanson (2015), who estimated the forward guidance and large-scale asset purchases components of each announcement of the Fed's Open Market Committee and showed that these components have significant effects on asset prices. The variable we are mainly interested in, house prices, will be measured using Case-Shiller Home Price Index - U.S. national index - which is now being published by Standard & Poor's and is available on their web site.

Expected Contribution There is a large bulk of literature dealing with the impact of monetary policy on asset prices. However, any of the yet published studies did not consider employing large TVP-VARs for estimating the effect. Furthermore, to the best of our knowledge, any paper did not analyze how the relationship between monetary policy and house prices changes during the crisis when extraordinary monetary policy measures are implemented. One of these measures, which is frequently neglected in the literature, is forward guidance. This type of unconventional monetary policy was one of the key pillars of Fed's efforts during the crisis and we believe that analyzing its impact on house prices through the interest rate channel can provide further insights into the field. Finally, examining how the zero interest-rate policy affects the housing market could also help to explain the overpricing in the Czech housing market which can be seen nowadays. One of the reasons might be almost zero rates on savings accounts that can force rich individuals to seek other investment possibilities, some of them subsequently buying real estate.

Outline

1. Introduction - introducing the topic and the aim of the thesis, presenting the motivation, expected contribution and briefly presenting the main results, outlining the structure of the thesis

2. Literature review - review of all relevant literature, can have two subsections: one of them for monetary policy literature and the other for purely econometrics literature

3. Data - description of the dataset, motivation for using selected variables, data sources

4. Methodology - description of large TVP-VAR approach: first generally, and then with respect to the current study, description of the estimation and forecasting methods which were employed

5. Results - presenting and discussing the results

6. Conclusion - summary of the results and their relation to the overall aim of the thesis, discussion of the contribution, motivation for future research

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Supervisor

Chapter 1

Introduction

Over the past few years, developments in house prices have become increasingly important for monetary economists, especially in the context of monetary transmission mechanism. Evaluating the impact of monetary policy shocks on the housing market gained further attention after the U.S. housing bubble burst in 2007 which was considered by many observers as a trigger of the subsequent financial crisis. Following this event, central bankers were oftentimes accused (e.g., by Taylor (2007)) of promoting a too stimulative monetary policy in the early 2000's with the federal funds rate well below the rate suggested by Taylor rule, which was claimed to substantially contribute to housing market imbalances.

The aim of this thesis is to assess the effects of a monetary policy shock on the housing market. We examine the reaction of house prices to an expansionary monetary policy shock using a TVP-VAR model with stochastic volatility and dynamic dimension selection. We extend the Bayesian state-space model of Koop & Korobilis (2013) in order to generate impulse responses and forecast error variance decomposition, which is one of the main contributions of this thesis. Time variation in VAR coefficients allows us to compare the responses of house prices over time to examine if and how the link between monetary policy and house prices changed. We then study the behavior of house prices and residential investment following a monetary policy shock in six different time periods, including both periods of good economic conditions and the unstable times. Due to the stochastic volatility in the model, variance of shocks can change over time which better simulates the actual ongoings in the model. Dynamic dimension selection is used to select one TVP-VAR model in each time from the pool of possible candidates that all contain the variables of the small TVP-VAR.

We estimate the model on monthly data from January 1999 to April 2017. One of the potential problems when estimating a macroeconomic VAR on a sample that covers both the Zero Lower Bound and non-ZLB periods is that we can no longer use the federal funds rate to account for the changes in monetary policy, because it was pushed to nearly zero for the whole ZLB period. Therefore, we decided to use Wu and Xia shadow rate to approximate for the changes in monetary policy. This rate serves as an indicator of what would the policy rate be if it could go below zero. For house prices, we employed S&P/Case-Shiller U.S. National house price index which is a leading measure of residential real estate prices in the United States.

The results show that house prices and residential investment respond positively, though with a delay, to an expansionary monetary policy shock. Moreover, the response is generally more invariant to different model specifications and significant in stable times than in unstable times. Therefore, allowing for time variation in VAR coefficients seems to be important in quantifying the link between house prices and monetary policy and its changes over time. It is also shown that the choice of the shrinkage hyperparameter that controls the degree of variation in TVP-VAR coefficients around their prior means is crucial for obtaining reasonable results.

The remainder of this thesis is structured as follows. Chapter two provides overview of the related literature on the stance of monetary policy and house prices, while Chapter three describes the data and presents some preliminary analysis. Chapter four describes the model and the estimation procedure. Chapter five provides the empirical results and robustness checks. Finally, Chapter 6 concludes and suggests the directions for future research.

Chapter 2

Literature Review

This chapter summarizes relevant literature concerning the effects of monetary policy on the housing market. It includes all important studies irrespective of their modeling approach - some of them use VAR methods, while others employ DSGE models.

Taylor (2007) discusses and also provides an explanation for monetary policy actions surrounding the housing price boom in the early 2000s. Moreover, he advices central bankers what to do to prevent future crises. First, he argues that the volatility reduction in residential construction since the early 1980s was mainly the result of the more responsive monetary policy to changes in inflation and real GDP. Accordingly, monetary policy started to be also more systematic and predictable in 1980s, which helped to keep inflation steadier, and therefore reduced boom-bust cycles and subsequent interest-rate oscillations, which had caused volatile housing in the period before 1980. However, he points out that during 2003-2006, the federal funds rate was lower than it would have been if the central bankers acted as they did in the so called Great Moderation period during two previous decades as described above. He thinks that those unusually low interest rates further increased the (high) demand for housing leading to a huge house-price inflation. Thus, when the federal funds rate came back to normal (before-the-reduction) levels in 2006, demand for housing sharply fell, dragging down both the house-price inflation and residential construction.

To confront his theory with reality, he estimated the equation that links housing starts to the developments in the federal funds rate on quarterly data from 1959 to 2007, and found that the federal funds rate significantly affects housing starts, and that this effect is also of a high magnitude and occurs with a lag. He then used this model to simulate the path of housing starts from 2000 to

2007 under two different scenarios: first, that the federal funds rate follows its actual path, and the second, that the federal funds rate follows a Taylor rule with coefficients of 1.5 and 0.5 on inflation and real GDP, respectively. Under this specification, alternative paths of the federal funds rate split in the second quarter of 2002 and merge again in the third quarter of 2006. According to the results, housing boom under the Taylor-rule prescribed interest rate would have been of a much smaller magnitude than that occurring when the federal funds rate follows its actual path. He also argues that the boom would have been reduced (even with the federal funds rate following its actual path) if the response of the long-term interest rates to an increase in the federal funds rate would have been such that it was during the Great Moderation period. However, as the long-term interest rates adjust according to the expectations of the future short-term rates, Taylor claims that if, due to the exceptionally low short-term rates, the market participants believed that the monetary policy response to inflation has changed, their interest-rate expectations would have declined. Thus, the long-term rates would not increase similarly as they would have before 2000s following a rise in the short-term rates with these new expectations. This is further supported by the fact that policy rule estimates for 2003-2005 display a significant drop in the responsiveness of the federal funds rate to inflation. Based on this Taylor suggests to implement a policy that is predictable and systematic as the one that was active during the Great Moderation period, and to adjust the federal funds rate according to changes in inflation and real GDP, while being careful with adjustments based on other factors, as they may lead to unexpected changes in other responses in the economy because they are more difficult for market participants to follow up. Taylor also highlights the importance of clarity and transparency of the Fed's actions in resolving the crisis, which is also connected to the forward guidance policy that the Fed extensively implemented during recent crisis and that we want to include in the analysis as one of the unconventional monetary policy measures.

Iacoviello & Neri (2010) assess whether the developments in the housing sector can be one of the driving forces of the business cycles. To answer this question, they examined the nature of the housing market shocks and the relevance of spillovers from the housing market to the economy. They estimated a DSGE model with nominal and real rigidities on quarterly U.S. data from 1965 to 2006 and studied the combinations of shocks and frictions that can account for the dynamics of residential investment and housing prices observed in the data. According to the results, they attributed the increase in real housing prices to the slower technological progress in the housing sector and to the inclusion of a fixed factor (land) in the production function for new houses. They also acknowledged the three main forces that can explain a substantial part of the cyclical volatility in residential investment and housing prices: housing demand and housing supply shocks, and monetary factors, which can explain about 20 percent of this volatility. They conclude that the sharp increase in housing prices and residential investment in the early 2000s, including its reversal in 2005 and 2006, was in a great part driven by monetary factors, as opposed to a housing boom of the 1970s, which was most probably driven by a faster technological progress in the non-housing sector.

As for the transmission of the housing market shocks to the economy, they argue that nominal rigidities, particularly a wage rigidity, increase the responsiveness of residential investment to changes in housing demand and monetary policy, which in turn increases the sensitivity of output to aggregate demand shocks, because the fluctuations in residential investment directly affect output. Moreover, they divided the sample into two subsamples to be able to compare how the magnitude of housing market spillovers changed after the financial liberalization in the mortgage market in 1980s. Results show that the spillovers from the housing market to the rest of the economy are substantial and that they also became more important over time.

Smets & Jarociński (2008) also examined the role of housing market and monetary policy in the U.S. business cycles. After estimating a Bayesian vector autoregressive model using the data from 1987 Q1 to 2007 Q2, they tried to forecast the housing boom and its reversal in the early 2000s based on the observed real GDP, prices, and short- and long-term interest rate paths. However, with the benchmark VAR employing only real and nominal GDP developments, they were not able to explain housing boom in 2000 and its peak in 2006. To obtain a behaviour of housing prices and residential investment that would match the data more accurately, they needed to include the federal funds rate and the long-term interest rate to the information set, however, the prediction error was still quite severe. When they focused on the impact of housing demand shocks on the business cycles, they found that these shocks significantly influence house prices and residential investment (which is in line with most of the empirical literature), but their impact on the performance of the U.S. economy in terms of inflation and aggregate growth is only limited. Regarding the effects of monetary policy shocks, the results suggest that they can substantially affect house prices and residential investment. Thus, the authors conclude that the loose monetary policy of 2002-

2004 could have contributed to the housing boom in 2004 and 2005, but that its impacts on the overall economy are limited.

Negro & Otrok (2007) emphasize that it is important to differentiate between "local bubbles" (with respect to the United States, these are house price bubbles that emerged only in some states) and "national bubbles", because the local ones can be attributed to circumstances specific to each geographic market, whereas the national ones can be caused by a monetary policy, which is identical across all states. To examine which increases in the value of housing over mid-80s to the end of 2005 were idiosyncratic (state-specific) and which can be classified as a national phenomenon, they employ a factor model using state-level Office of Federal Housing Enterprise Oversight (OFHEO) house price indexes. They found that, historically, house price movements are attributable to local factors, while recently (from the perspective of their dataset) large increases in house prices that occurred in many states have been substantially driven by a nation-wide component, even if the local factors still have played an important role in the formation of those price bubbles. Nevertheless, the common component of the house price growth showed up to be significant, therefore they assessed to what extent monetary policy is responsible for this co-movement. To address this issue, they estimated VAR model with the common component in house prices being one of the variables, the others measuring monetary policy stance. According to impulse responses, loose monetary policy increases the housing factor. Moreover, they constructed a counterfactual scenario to examine how the housing factor and house-price growth across states would change if there were no monetary policy shocks from 2001. They conclude that monetary policy shocks seem to fairly influence house prices, but that this impact is rather small compared to the magnitude of the house price increase over 2001-2005. Overall, they conclude that expansionary monetary policy did not cause the housing boom, but they emphasize that they focused only on the low-interest-rates component that is due to monetary policy shocks (due to the Fed's deviations from its historical policy rule), and therefore the possibility that housing booms can be created in an environment of exceptionally low interest rates cannot be ruled out.

Eickmeier & Hofmann (2010) note that the previous four papers did not come to the consistent conclusion regarding the role of monetary policy in the housing boom, however, they explain those inconsistencies by the differences in sample periods. They studied the transmission of monetary policy shocks via financial conditions as well as whether these shocks contributed to the pre-crisis imbalances in housing and credit markets. Using 1987-2007 quarterly data for the United

States, they employed a factor-augmented vector autoregressive model (FAVAR) to be able to examine the interaction between monetary policy and more than 200 financial and asset variables. Their model includes three macro variables - GDP growth, GDP deflator inflation, and the federal funds rate - and a set of financial factors. To distinguish between macro and financial shocks, they imposed both contemporaneous zero restrictions and short-term sign restrictions on impulse response functions, which are in line with a large number of theoretical models and are thus frequently used in monetary transmission studies. Further, they allowed for contemporaneous interaction between policy rates and financial variables to be able to assess not only the effect of monetary policy shocks on financial variables, but also how financial shocks alter the path of policy rates. Their results suggest that monetary policy shocks significantly and persistently affect property prices, real estate wealth and the private sector debt, while the effect on the mortgage market, risk spreads in the money market, and the loan market is strong, but only short-term. Furthermore, monetary policy shocks, although at a late stage, noticeably contributed to housing and credit markets' imbalances prior to 2007. As for the effects of financial shocks on the path of policy rates, they found that negative financial shocks associated with the burst of the dot-com bubble could significantly contribute to the exceptionally low policy rates observed between 2001 and 2006, and that the feedback of those shocks via lower policy rates on property prices have been probably large.

Goodhart & Hofmann (2008) broadly examine the links between money, credit, house prices, and the economic activity using fixed-effects panel VAR estimated on 1973-2006 quarterly data from 17 industrialized countries. Results show that there is a significant multidirectional relationship between those variables, with money growth significantly affecting house prices and credit, credit influencing both money and house prices, and house prices having impact on credit and money. According to the impulse responses, shocks to money, credit, or to the house prices significantly affect price inflation and aggregate economic activity, and shock to either GDP, inflation, or to the interest rate significantly affects house prices, money and credit. Furthermore, the link between house prices and monetary variables appears to be stronger in a more recent period from 1985 to 2006, which, the authors believe, is caused by the financial system liberalizations during the 1970s and the early 1980s. Also, shocks to money or to credit are intensified when house prices are booming. However, the intensification of both the link between house prices and monetary variables and of the effects of money and credit shocks is found statistically insignificant, as the authors report large confidence bands around the estimated impulse responses. The authors also suggest to use a counter-cyclical regulatory ceiling on loan-to-value ratios in mortgage lending that would stimulate housing during the periods of poor mortgage growth and low house price inflation, and hinder the creation of housing bubbles in the opposite periods. The authors believe that such an instrument could partly solve the problem of regional differences in house prices and credit dynamics when implementing a monetary policy that is uniform across more regions. However, they found only a minor correlation between those loan-to-value ratios and the differences in the episodes of house price increases or booms across countries.

Hloušek (2013) applied a DSGE model of Iacoviello & Neri (2010) to the Czech economy to analyze the links between developments on the housing market and the macroeconomy. He used quarterly data from the beginning of 1998 to the end of 2012. Financial frictions are modelled using collateral constraint that restricts borrowing capacity and consumer spending. Relaxing the constraint increases the response of consumption and output to the monetary policy shock, therefore, monetary policy can have higher impact on real economy if houses are better collateralizable. From this, Hloušek (2013) concludes that the ability of monetary policy to influence consumption and output highly depends on the loan-to-value ratio, and with more accessible loans, the impact of monetary policy on those variables is stronger, while this does not hold for the impact on inflation. His results indicate that shocks to consumption, housing technology or housing preferences are important determinants of fluctuations in real variables, whereas shocks in inflation target or the cost-push shocks affect mostly nominal variables. He further argues that the Czech house price boom and bust of 2000s was primarily caused by housing preference shocks, which are the demand-side shocks, and that the supply shocks also contributed to the housing market turmoil, but their effect was much lower.

Another application of DSGE models to housing market and monetary policy is proposed by Darracq Paries & Notarpietro (2008), but, contrary to the previous DSGE studies, this one uses an open-economy framework which enables to include international factors and cross-country spillovers and study the transmission of housing market and monetary policy shocks both domestically and internationally. There are two goods in the model: nondurable consumption goods that can be traded internationally, and residential goods that are non-tradeable. The authors also included credit market frictions faced by households, because they found them important for the conduct of monetary policy, as the evidence in the literature shows that they can significantly influence households' consumption and home-purchasing decisions. With this model at hand, they analyze monetary policy implications of housing-related shocks and the credit frictions, and the importance of them for closed and open-economy fluctuations. The model involves the euro area and the US and is estimated on quarterly data from 1981 to 2005.

Results indicate that there are substantial spillovers from housing market shocks to the non-residential consumption, which exist because of the collateral channel, and are further affected by the share of borrowers in the economy. Furthermore, these shocks are the main drivers of the negative cross-country comovement of both real house prices and residential investment. The authors also document that while the collateral channel causes housing shocks to significantly affect domestic economic activity, it performs relatively poor in transferring those shocks internationally, as compared to the shocks affecting tradeable consumption goods. Finally, the authors considered the optimal monetary policy response to housing shocks and for this exercise they augmented the traditional Taylor rule to reflect also the house price developments. This new Taylor rule enhanced the empirical fit of the model, and it also showed up to be welfare-improving compared to the traditional Taylor rule, at least for the U.S. economy.

Williams (2015) focuses on the tradeoff between macroeconomic and financial stability goals, from which he specifically selects an objective of keeping stable house prices. Numerous empirical studies report that higher interest rates generate a decline in house prices, but also in GDP and inflation that can offset the benefits from lower housing prices. Of course, there does not need to be a conflict between macroeconomic and financial stability targets if, for example, both house prices and the overall economy are booming, then higher interest rates (tighter monetary policy) can help in fulfilling both targets simultaneously. However, if this is not the case and macroeconomic and financial stability goals differ, Williams states that the quantitative assessment of benefits and costs from using monetary policy to regulate house prices should be performed. For researchers in drawing any conclusions regarding the role of monetary policy for house price developments, he highlights the importance of proper separation of policy changes that respond to economic developments from those that are driving those developments. In this regard, he employs a method developed in Jorda *et al.* (2014) and Jorda *et al.* (2015) who took only countries with exchange rates fixed to some foreign currency, because those countries are also those that cannot respond freely to changes in their economic conditions by altering their short-term interest rates, and therefore their interest rates' changes are reactions to some other country's economic developments and not to domestic economic conditions. This way it is possible to examine only the "pure" effect of monetary policy when the effects of interest rate changes for domestic variables in those countries are examined. Williams analyzed 17 such countries using annual data from 1870 to 2013 (excluding the interwar period of 1914-15 and the period of 1973-1980 oil crisis). In the pooled setting, he found that house prices and real GDP per capita respond negatively to an increase in the short-term interest rate. Inflation response initially shows a price-puzzling behaviour, and then, after more than one year, inflation declines. However, Williams mentions that the response of an inflation would probably be different with a sample consisting of countries which can freely alter their interest rate paths, because the effect on inflation most probably depends on each country's monetary policy regime. He also did not find any evidence that the monetary policy would have stronger effects during housing or real estate debt booms. Moreover, he considered a possibility of a structural break that could occur in such a long sample, however, by replicating the analysis using only the post World War II data, he obtained only a more negative effect on house prices and a smaller (but still negative) effect on output after a positive monetary policy shock. Finally, Williams examined numerous studies focusing on the monetary policy-house prices link, and reports that the ratio between the magnitude of the effect on house prices and the magnitude of the effect on GDP appears to be robust to different sample countries or period specifications, especially across studies employing a large sample. Based on those empirical studies, he assessed that to offset an increase of over 50 percent in house price-to-rent ratios that occurred in the US between 2001 and 2006 with a monetary policy, real GDP per capita would have to decline by more than 12 percent, which is much more than the 5.5 percent peak-to-trough drop that the U.S. economy experienced during the Great Recession. He concludes that there is a costly tradeoff between macroeconomic and financial stability goals in advanced economies when these goals do not coincide - positive interest rate shock reduces real house prices but at the expense of lower output and inflation (for most of the studies, one percent loss in GDP is accompanied by a four percent reduction in house prices) - and that this result is robust to all examined studies.

Before the Great Recession, many authors (e.g. Bernanke & Gertler (2001)) argued that the monetary policy should not react to asset price movements. However, after the huge swings in asset prices during the crisis and a substantial role of house price developments preceding the crisis, the debate on the role of asset prices in the monetary policymaking reopened.

Notarpietro & Siviero (2014) focused on house prices and they took a deeper look on the role of financial frictions in determining the optimal policy rule. In particular, they examined if there exists a monetary policy rule that would respond also to house price movements among other variables and would be social-welfare-maximizing. Responsiveness of the monetary policy to house price fluctuations is modelled using a New Keynesian model with a housing sector and households exposed to financial frictions in the form of collateral constraints that limit the maximum amount that households can borrow by the value of existing collateral. Therefore, a degree of financial frictions is determined by the average loan-to-value ratio (LTV) and by the share of borrowers in the economy. Moreover, there are two types of households characterized by a different discount factor: patient and impatient ones, where the latter type of the households has perpetually binding collateral constraint. According to the results, welfare-maximizing monetary policy rule entails a reaction to house prices, whose sign and size noticeably depend on the degree of financial frictions in the economy. Relatively small proportion of constrained agents implies a welfare-maximizing rule with central bank moving the policy rate in the opposite direction than the house prices move, irrespective of the value of the average loan-to-value ratio. However, when the share of borrowers in the economy increases, the average LTV becomes important in determining the sign of the policy rate response to house price fluctuations, and with the average LTV around 90 % or more, it becomes welfare-maximizing to offset house price increases by setting a higher policy rate. Therefore, according to the authors, financial frictions play an important role in the assessments of optimal monetary policy rules.

Lim & Tsiaplias (2016) build on the criticism of previous studies along the following lines: first, they highlight the importance of regional heterogeneity in assessing the link between house prices and monetary policy; they argue that a substantial amount of potentially useful information may be lost if an aggregate house price indicator is used (e.g., a national house price index) because regional house price differences might be significant. For example, if housing investors make their investment decisions according to those regional differences, then certain regions can be more prone to changes in the interest rate conditions, and those effects can even offset each other. Therefore, an aggregation may result in misleading conclusions about the impact of interest rate changes on house prices. Moreover, regional differences in labour demand can be potentially relevant for the house price dynamics. Second, the authors point out that the overly assumed linearity between interest rates and house prices needs to be questioned, and they support this view with the evidence of Himmelberg *et al.* (2005)

and Kuttner (2012). To overcome those problems, they constructed a model allowing for potential nonlinearities between a house-price-to-income ratio and the interest rate and also for regional spillovers. Regional house-price-to-income ratios are constructed using Australian regional house price indices that account also for changes in the housing mix and its quality. Nonlinearities are examined using a vector-autoregressive model and logistic transition function. This way, smoothness and threshold parameters are estimated, from which the presence of an 'over-reaction' point is examined, where the response of house prices to the interest rate significantly changes, and if this point can be classified as 'hard' or 'soft' depending on whether the change in dynamics upon reaching it is abrupt or smooth. Estimation revealed that such a 'transition' point is present and it can be considered as being 'soft'; the transition reflects both how much below the point interest rates were and how long they stayed below the threshold. Moreover, regional spillovers are contingent on the interest rates being above or below the threshold, indicating that housing conditions change from stable to unstable. Below the 'transition' point, housing bubble can occur as unstable dynamics create conditions for housing boom and bust. According to the authors, this has important implications for monetary policy easing, which, after exceeding a certain point and for a sufficiently long time, creates a non-negligible risk of housing market instability. Finally, the authors document that the results are not robust to the use of aggregate data (either Australian or the U.S. data), which further supports their assumption that regional heterogeneity should be taken into account when examining the house prices-interest rate relationship.

Brito *et al.* (2016) took a different perspective on the highly-examined relationship and analytically described global house price dynamics under different monetary policy scenarios. They used overlapping generations general equilibrium model to which they included a housing market to be able to consider housing-wealth effects on aggregate consumption. Their setting assures that aggregate demand responds to changes in housing wealth and transparently models house price variations under the presence of rules-based monetary policy (Taylor-rule reflecting policy). They showed that the policy based on Taylor rule cannot burst the housing market bubbles that are generated by self-fulfilling upward trajectories in house prices along with the optimal behaviour of forward-looking agents. Further, boom (or bust) in house prices is accompanied by the monetary policy being more (or less) active. They also demonstrates that either the boom or the bust cannot be mitigated by the monetary authority's interest-rate feedback rule that reacts to both inflation and house price developments. Moreover, if such a rule would respond more to the house price increases than to the consumer price inflation, the model solution would be attracted to a liquidity trap, and would result in the local indeterminacy at the steady-state equilibrium in other than liquidity trap fixed points.

McDonald & Stokes (2013) represent another paper which aims to shed light on the causes of the pre-crisis housing bubble, now, however, using the same house price index - S&P/Case-Shiller index - which we will be using in our analysis. The authors use a simple VAR approach and perform Granger causality tests to assess the link between the fluctuations in the fed funds rate and those in the U.S. house prices, and investigate to what extent monetary policy created the housing bubble. To this end, they use both 10-city and 20-city monthly aggregate housing price index and effective federal funds rate from January 1987 to August 2010. Granger causality tests indicate that there exists a link coming from the fed funds rate to house price indices, and that this Granger causality is much stronger after 2000. This corresponds to the view that by keeping the fed funds rate artificially low during 2001-2004, the Federal Reserve significantly contributed to the creation of the housing price bubble. Similarly, sharp rise in the fed funds rate in 2004-2006 is argued to be a key driving force of a subsequent house price decline. Findings also suggest that the relationship between the fed funds rate and house prices changed after 2000, that, the authors say, could be due to the lack of regulation or changes in the mortgage-market credit standards.

Smith (2013) uses a DSGE model with housing and financial sector in which housing market and real economy are connected through a housing-secured debt, and house price fluctuations are amplified through borrowers and banks' balance sheets, implying a self-fortifying credit/liquidity crunch. The main purpose of Smith's paper is to evaluate how quantitative easing programmes and equity injections into big banks are successful in reducing the house price troughs. To uncover the transmission mechanism of unconventional monetary policies, he divided banks to two categories: simple and complex banks, and shows how movements in house prices can significantly affect financing premiums and therefore the production. House price movements are then amplified because of an asset redistribution between simple and complex banks. He found that the effectiveness of unconventional monetary policies is highly dependent on the level of heterogeneity in the financial sector. Therefore, the channel through which house price disturbances are amplified within the financial sector is the same as the one through which unconventional monetary policies are transmitted.

Chapter 3

Data

This chapter provides a description of the data set and data adjustments. Furthermore, lag length selection is discussed and the results from Granger non-causality tests that were performed according to Toda & Yamamoto (1995) procedure are presented.

3.1 Data description

We use monthly data for the United States covering the period from January 1999 to April 2017. The sample is limited because the data on one of the key variables - real private residential investment - are not available before 1999. Due to the fact that we partly adopt the estimation procedure of Koop & Korobilis (2013), we construct three housing-oriented TVP-VARs[1] that differ in the number of included variables. Based on the number of variables - 'dimension' of each VAR - we estimate small, medium and large model. The first one comprises three standard macroeconomic variables: real GDP, consumer price index (CPI)[2] and a short-term nominal interest rate, plus nominal house prices. The second model adds real residential investment and mortgage rate to the four previous variables and as such resembles Rahal (2016) VAR model of housing and at the same time contains 'lens of fundamentals' for forecasting house prices: residential

[1] In the following we use the term VAR or TVP-VAR interchangeably when we talk about our model, but it always means a TVP-VAR, i.e., the VAR with time-varying parameters. "VAR" was sometimes used to save space but we stress that all the estimated models are those with time-varying coefficients.

[2] We prefer using CPI over other aggregate price level indicators to measure inflation, because CPI is available in monthly frequency (as opposed to e.g. GDP deflator) and central banks usually target CPI inflation.

Variable	Small VAR	Medium VAR	Large VAR
Real GDP	✓	✓	✓
S&P/Case-Shiller U.S. National HPI	✓	✓	✓
Capacity Utilization: Manufacturing			✓
Civilian Unemployment Rate			✓
Real Disposable Personal Income: Per Capita			✓
Real Personal Consumption Expenditures			✓
Real Private Residential Fixed Investment		✓	✓
CPI: All Items	✓	✓	✓
Mortgage Debt Outstanding			✓
Housing Starts: Total			✓
Total Reserves of Depository Institutions			✓
M2 Money Stock			✓
10-Year Treasury Constant Maturity Rate			✓
30-Year Fixed Rate Mortgage Average		✓	✓
Spot Crude Oil Price: WTI			✓
S&P 500 Index			✓
Producer Price Index for All Commodities			✓
Real Broad Effective Exchange Rate			✓
CBOE Volatility Index: VIX			✓
Wu-Xia Shadow Rate	✓	✓	✓

Table 3.1: Variables employed in all TVP-VAR models. The ordering of variables in a small, medium or large baseline model matches the ordering in this table.

investment, mortgage rates and house prices (Gattini & Hiebert (2010)). Finally, large VAR includes all medium-VAR (and therefore small-VAR) variables plus some additional macroeconomic and financial variables that should capture the overall economic outlook and help to predict house prices. A complete list of variables included in each TVP-VAR model is in Table 3.1. Below, we briefly describe main variables used to assess the link between house prices and monetary policy.

House prices

Changes in nominal house prices are gauged by S&P/Case-Shiller U.S. National Home Price Index which is a leading measure of residential real estate prices in the United States. The index is a composite of nine U.S. Census division home price indices and it intends to measure changes in the market value of all existing[3] single-family houses at the national level. Home price indices for all U.S. Census divisions are calculated based on the repeat sales methodology, which is considered to be the most reliable method for assessing house price movements.

[3]Meaning that newly built houses are excluded from the index calculation.

This method creates the so called 'sale pairs' from (single-family) house price records by matching the original sale price of a house to its new price when the same house is resold. The difference between those two prices that constitute one sale pair thus represent the change in the market value of the same house, while keeping the quality and size of the house unchanged. Only the data for houses that have been sold at least twice are taken into account and the sale pairs are further weighted to alleviate the impact of extreme price changes[4] on the resulting index. The index is calculated monthly using the sale pairs for the current month (i.e., the month for which we want to calculate the index value) and the two preceding months that are included to offset delays in data records and to make the sample size large enough to produce reliable price change averages[5]. Value of the index is adjusted to equal 100 in January 2000 and the data are available from January 1975 at e.g. the Federal Reserve Bank of St. Louis database (FRED)[6].

Other house price indices for the United States also exist; in empirical research, the house price index (HPI) published by the Federal Housing Finance Agency (FHFA) is oftentimes used. FHFA computes monthly and quarterly HPI figures for each U.S. Census division, state and Metropolitan Statistical Area, and the nationwide HPI. The last one has however a slightly different methodology than the S&P/Case-Shiller U.S. National HPI, although it is again based on repeat sales of all existing single-family properties. Dissimilarities include using only the data on repeat mortgage transactions for mortgages purchased or securitized by Fannie Mae[7] or Freddie Mac[8] to construct FHFA HPI as opposed to collecting the data on sales prices from county assessor and recorder offices for the calculation of S&P/Case-Shiller HPI. Moreover, FHFA national HPI uses data from 50 U.S. states and the District of Columbia, but the S&P/Case-Shiller HPI misses information on purchase prices from 13 states[9]. Still, we decided to use the S&P/Case-Shiller HPI because it covers a broader range of mortgages including those that do not satisfy the loan purchasing guidelines determined by Fannie Mae

[4]Non-market, idiosyncratic price changes can occur, e.g., if the original house owner needs to sell really quickly it results in an abnormally high price which increases the value of the house relative to the market.

[5]For more information about the algorithm used to calculate the index, see S&P CoreLogic Case-Shiller Home Price Indices methodology available at `http://us.spindices.com/index-family/real-estate/sp-corelogic-case-shiller`.

[6]`fred.stlouisfed.org/series/CSUSHPINSA`

[7]The Federal National Mortgage Association

[8]The Federal Home Loan Mortgage Corporation

[9]Source: `www.fhfa.gov`.

or Freddie Mac (the so called non-conforming mortgages) and also the sub-prime mortgages.

Residential investment

Another key housing variable is residential investment, which contains expenditures on construction or purchasing of new dwellings. We used the series on real private residential fixed investment from FRED, available from Q1 1999 with quarterly frequency. The composition of residential private fixed investment is displayed in Figure 3.1, and we can see that single family structures make up the most important part of it implying that changes in single-family housing stock as measured by S&P/Case-Shiller HPI are likely to be important for residential investment. There is one feature of Figure 3.1 that is worth noting: all recessions were preceded by declines in residential investment into single-family dwellings and followed by strong increases[10]. This could be partly attributed to the more favorable conditions that can emerge right after the recession has ended, but the central bank's measures designed to offset the recession are still stimulating the economy. However, it appears interesting that residential investment into single-family homes (as a percentage of GDP) did not experience similar growth after the most recent recession as it did after almost all the other recessions. Shleifer *et al.* (2015) explains the slow recovery by the 'investment hangover' defined as a situation in which housing capital is overbuilt which leads to lower investment in such capital. They document an overbuilding of housing capital by 2005 and argue that it occurred because there was also an investment boom in addition to house price boom and since this capital is highly durable, it accumulated. Overbuilt capital then prevents investment in it because an excess of housing stock compensates for new investment.

Residential investment is mainly affected by the demand for houses, therefore any factors affecting the demand also affect residential investment. Income is among the most prominent ones making the residential investment to vary procyclically and be more volatile, while changes in the interest rate are also important since lower interest rates encourage potential home buyers to take a mortgage, and thus a monetary policy that affects the interest rate can also influence residential investment. Of course, house prices are presumably the most relevant factor that influences the demand and therefore residential investment.

[10] Except for the recession at the beginning of 1960s after which the increase was not so significant as compared to other recessions.

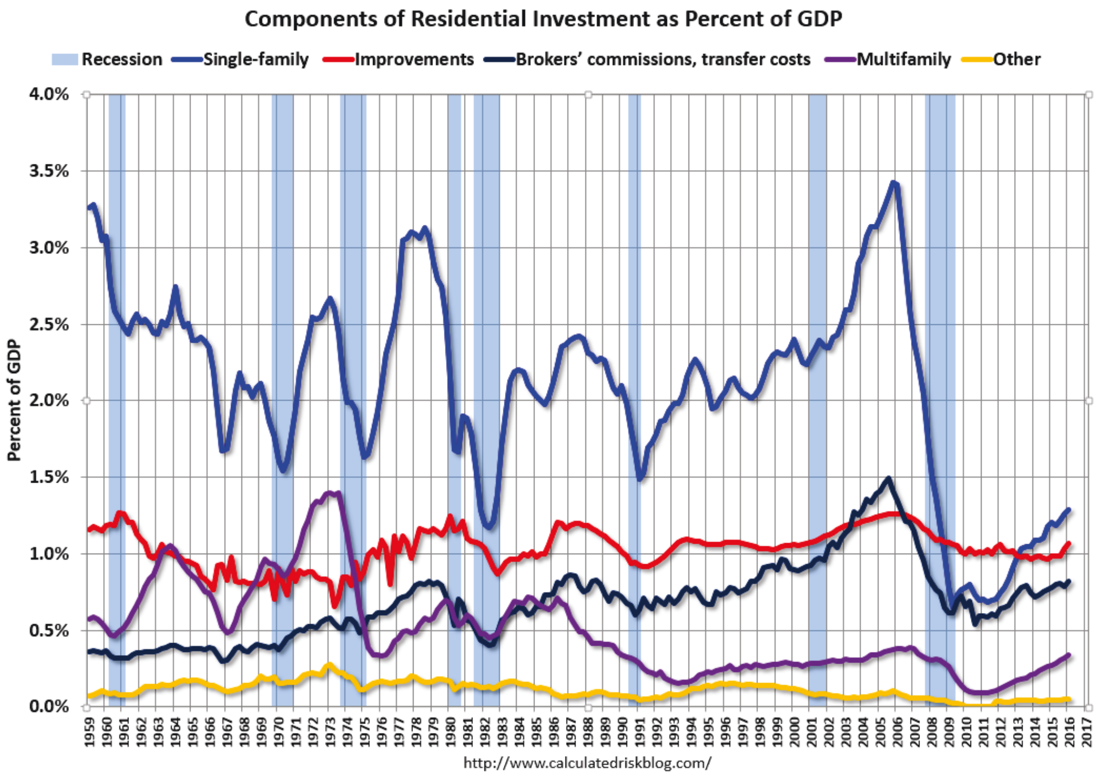

Figure 3.1: Components of residential private fixed investment in structures, 1959-2017, each as a percentage of GDP. Shaded areas mark U.S. recessions as defined by the National Bureau of Economic Research.
*Source:*www.calculatedriskblog.com/2016/05/q1-2016-gdp-details-on-residential-and.html

Mortgage rate

Mortgage rates also respond to monetary policy that can alter the path of interest rates and at the same time they substantially affect the housing market because they determine the overall cost of a mortgage and the amount of regular payments associated with it. Lower mortgage rates make mortgages more attractive to borrowers, boosting the demand for houses and subsequently residential investment and, since housing supply is more rigid, lower rates also boost house prices. Mortgage rates tend to move in the same direction as long-term interest rates - they mainly follow the 10-year Treasury bond yield. Lower rates usually occur during a recession, because investors seek safe investment opportunities and thus increasingly invest in bonds which pushes bond yields down, and as mortgage rates respond to the 10-year Treasury bond yield, they go down too. Moreover, as central bank undertakes actions to stimulate the economy during a recession, short-term (and also longer-term) rates decline which can further contribute to lower mortgage rates. This could partly explain the increase in residential investment observed after every recession[11] because cheaper mortgages can encourage residential investment through channels discussed above. For mortgage rates, we use the 30-Year Fixed Rate Mortgage Average series retrieved from FRED and available weekly from April 1971. However, it should be noted that FRED's mortgage rate data come from Freddie Mac and thus include only "first-lien prime conventional conforming home purchase mortgages with a loan-to-value of 80 percent"[12].

Wu-Xia rate

As an indicator of the stance of monetary policy, we use Wu & Xia (2014) shadow federal funds rate[13]. In standard macroeconomic VAR literature, the federal funds rate is usually employed as a measure of monetary policy, because it is the Federal Reserve's ("the Fed's") main monetary policy instrument. It can be lowered to stimulate the economy or raised when the economy is growing too fast. However, in December 2008 the Federal Open Market Committee (FOMC) lowered a target range for the federal funds rate to nearly zero, and therefore moving it down further to stimulate the economy was no longer an option and the Fed had to rely on unconventional monetary policy measures such as large-scale asset purchases and forward guidance. In this zero-interest-rate environment, commonly referred to as the Zero Lower Bound (ZLB) period, assessing the impact of monetary policy has become difficult since we cannot use the federal funds rate to evaluate the effects of monetary policy. This issue was overcome by employing Wu and Xia rate that is identical to the effective federal funds rate in the non-ZLB period (i.e., when the target federal funds rate is at least 25 basis points), but in the ZLB period (from January 2009 to November 2015) it differs from the effective federal funds rate because it is not lower bounded by zero, suggesting what the path of the federal funds rate would be if it could evolve to negative values. In December 2015, the FOMC decided to raise the target range for the federal funds rate to 25 to 50 basis points, therefore from this period on the shadow rate is equivalent to the effective federal funds rate.

There are also alternative measures of the monetary policy stance during the

[11]See Figure 3.1.

[12]`www.freddiemac.com/pmms/about-pmms.html`

[13]Data are available at `sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates`.

ZLB period, for example, Krippner (2013) shadow short rate (SSR)[14]. This rate is based on a two-factor shadow/lower-bound term structure model (SLM), whereas Wu and Xia rate emerges from a three-factor SLM. We decided to use Wu and Xia rate in a baseline model because it is the most common choice in the literature, however, we use Krippner's SSR as a robustness check. The paths of the effective federal funds rate, Wu and Xia rate and Krippner's SSR during our sample period are plotted in Figure 3.2. We can see that Krippner's SSR is more negative from the beginning of the ZLB period to the end of 2014 than Wu and Xia rate, but then it starts to rise sharply, while the lift off in Wu and Xia rate is more gradual. Wu and Xia rate is identical to the effective federal funds rate in the non-ZLB period, but Krippner's SSR is slightly different because Krippner provides SSR estimates also in non-ZLB period and claims that his SSR is "essentially equal to the policy interest rate" (Krippner (2014), p. 3) during this period. Henceforth, we will use the terms Wu and Xia rate (or Krippner's SSR) and policy rate interchangeably because the shadow rate serves as a reasonable indicator for what the level of the policy rate would be if it could go below zero during the ZLB period.

[14]Data are available at `www.rbnz.govt.nz/research-and-publications/research-programme/additional-research/measures-of-the-stance-of-united-states-monetary-policy`.

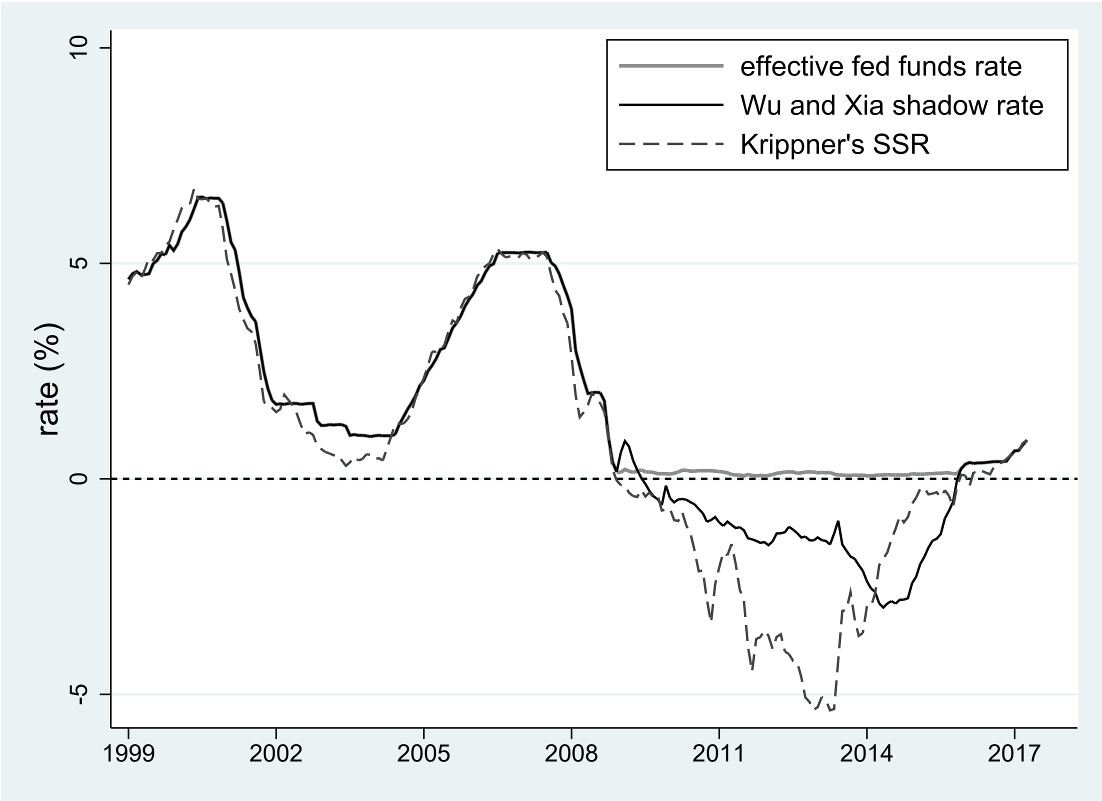

Figure 3.2: Effective federal funds rate and the two shadow rates. Effective federal funds rate is different from Wu and Xia shadow rate only in the ZLB period from January 2009 to November 2015.

3.2 Data manipulation

Table A.1 provides us with the description of downloaded data. Several adjustments had to be done to construct the final data set. First, since the variables that are represented by some index have different base periods (e.g., January 2000 for S&P/Case-Shiller HPI and 2012 for industrial production index), we rescaled all indices to have 2009 as a base year, so that the average of 2009 observations is equal to 100 for each index. We chose 2009 as a base year because variables that appear in real terms are already in 2009 dollars. Moreover, monthly nominal variables (total reserves of depository institutions, M2 money stock) were deflated by CPI: All items[15] because CPI has also monthly frequency, and quarterly nominal variables (mortgage debt outstanding) were deflated by GDP deflator[16] which is

[15]Now with base year 2009

[16]GDP deflator does not appear among variables in Table A.1 because it was not used in the estimation but only to deflate nominal variables; it was retrieved from FRED and is already based in 2009.

available only quarterly. Whenever it was possible, the seasonally adjusted series were downloaded.

3.2.1 Cubic spline interpolation

Next, we needed to unify the frequency of the data because some series were available only monthly, some only quarterly, and we have also one that is published weekly (mortgage rate) and one with daily intervals (market volatility). We decided to convert all series to monthly frequency as it is the most common one among our data. Daily and weekly series were adjusted by taking monthly averages and quarterly data were converted into monthly by cubic spline interpolation. This technique can provide estimates between known data points which should be smoother and more precise than those obtained using other interpolations. Suppose that we have $n+1$ data points (x_i, y_i) while it holds that $x_0 < x_1 < \cdots < x_n$. We are searching for a function $P(x)$ that connects all data points:

$$P(x) = \begin{cases} S_1(x), & x_0 \leq x < x_1 \\ \dots & \\ S_i(x), & x_{i-1} \leq x < x_i \\ \dots & \\ S_n(x), & x_{n-1} \leq x \leq x_n \end{cases}$$

by fitting a cubic polynomial $S_i(x) = a_i + b_i x + c_i x^2 + d_i x^3$, $d_i \neq 0$, in each of the n intervals. Therefore, we are solving for $4n$ coefficients and we know that it holds that:

$$\begin{aligned} &S_i(x_{i-1}) = y_{i-1} \quad \text{and} \quad S_i(x_i) = y_i, \quad i = 1, \dots, n \\ &S_i'(x_i) = S_{i+1}'(x_i), \quad i = 1, \dots, n-1 \quad \text{(inner points)} \\ &S_i''(x_i) = S_{i+1}''(x_i), \quad i = 1, \dots, n-1 \quad \text{(inner points)} \end{aligned}$$

Those are $2n + (n-1) + (n-1) = 4n - 2$ conditions, so we need additional two conditions to be able to compute the coefficients of cubic polynomials. Commonly used are these two boundary conditions:

$$\begin{aligned} &S_1'(x_0) = S_n'(x_n) = 0 \quad \text{(clamped boundary conditions), or} \\ &S_1''(x_0) = S_n''(x_n) = 0 \quad \text{(natural boundary conditions)} \end{aligned}$$

In our application, x_i are different quarters and y_i, $i = 0, 1, \ldots, n$, are values of a given quarterly variable in those quarters. For a purpose of cubic spline interpolation, value in each quarter will represent value of the corresponding variable in monthly frequency in the first month of that quarter, therefore each x_i will denote the first month of some quarter. We need to determine the remaining two monthly values for each quarter by calculating a_i, b_i, c_i and d_i for each polynomial between the two subsequent quarters, and then evaluating $S_i(x)$ for the two remaining months x in that quarter. Those calculations were done in MATLABusing *spline* function.

3.2.2 Transformation of variables

Since we are working with Bayesian models, (non-)stationarity of variables is not an issue. However, in the model of Koop & Korobilis (2013) which we use, (at least approximate) stationarity and standardization of variables is required in order to produce positive-definite covariance matrices. Positive definiteness of a matrix is necessary for computing its Cholesky decomposition that is needed for drawing from a normal distribution with this covariance matrix and for calculating impulse responses using a recursive ordering of variables[17]. To transform the data, we first determine the order of integration of all variables to assess which of them need to be differenced. To this end, two complementary tests were employed; the Augmented Dickey–Fuller test (ADF test) with the null hypothesis that a series contains a unit root and Kwiatkowski–Phillips–Schmidt–Shin test (KPSS test) for the null of stationarity. Brief description of both tests follows.

ADF test estimates a regression

$$\Delta y_t = \tau y_{t-1} + \delta + \gamma t + c_1 \Delta y_{t-1} + \cdots + c_k \Delta y_{t-k} + \epsilon_t \tag{3.1}$$

to test the hypothesis that $\tau = 0$ (unit root) against the alternative that $\tau < 0$ (stationarity). Number of lags k can be determined by using a procedure of Campbell & Perron (1991)[18]. We can exclude intercept or a deterministic time trend if we believe that δ or γ are equal to zero. Usually, a drift term is included because it would be highly restrictive to test with zero intercept, as Davidson & MacKinnon (1993) noted, and we can always test for the joint hypothesis that

[17] This is described in Chapter 4.

[18] We used the number of lags determined by R for unit root tests according to the length of the series; the number of lags was chosen to be 6.

$\tau = 0$ and $\delta = 0$ (unit root without drift). The test statistic for the null $\tau = 0$ is the usual t-statistic computed by dividing the estimated τ by its standard error, however, critical values of the standard t-distribution cannot be used because the series is non-stationary under the null. Correct critical values were simulated by Dickey & Fuller (1979) and they are available in any software that conducts the test.

KPSS test for the null hypothesis of level stationarity uses the equation

$$y_t = r_t + \epsilon_t \tag{3.2}$$

where $r_t = r_{t-1} + u_t$ is a random walk with fixed initial value and ϵ_t a stationary error, while u_t is assumed to be normal i.i.d. with zero mean and variance σ_u^2. Null hypothesis is that $\sigma_u^2 = 0$[19] which indicates stationarity. If we want to test for a trend stationarity, we just include the term γt into the regression 3.2. The statistic for the test is LM statistic that is computed using the formula that can be found in many time series textbooks, e.g., Kočenda & Černý (2014).

Table B.1 shows the results of ADF and KPSS tests for variables in original levels. For variables that are clearly growing or declining[20], we included a deterministic trend component γt into the regressions 3.1 and 3.2 to consider a potential trend stationarity of the series, and for those that apparently do not exhibit any long-run trend (e.g., market volatility), we estimated regressions without a deterministic trend. This is in line with Elder & Kennedy (2001) and Kočenda & Černý (2014) who suggest to choose a regression with or without trend based on a visual inspection of data and economic intuition and employ trend only if the series is steadily growing or declining over time. There are also series whose growth status is unknown and cannot easily be inferred from graphing the series against time; for those series, unit-root testing was performed by estimating both regressions[21].

According to Table B.1, most series in levels appear to be non-stationary[22], as

[19]The alternative is, of course, that $\sigma_u^2 > 0$.

[20]By visual inspection of data

[21]There are two exceptions from the above procedure - in case of 10-year treasury constant maturity rate and 30-year fixed rate mortgage average, there is seemingly a downward trend, but we test for stationarity using both trend and non-trend regressions because we believe that the downward trend is only because of the relatively short sample period used, and therefore it should not appear in the long-run. However, this has no influence on the results since both tests conclude that both variables are non-stationary.

[22]Results were qualitatively identical if we took the logarithm of series that are steadily growing over time, e.g., real GDP, to linearize the trend.

we cannot reject the null of a unit root by ADF test at the 5% significance level (which is usually taken as a threshold), and at the same time we can strongly reject the null of level (or trend) stationarity by KPSS test. Those series were transformed by taking log-differences except for the civilian unemployment rate and the interest rates[23] which, although they are clearly non-stationary, remain in levels as in Bernanke *et al.* (2004) or Eickmeier & Hofmann (2010)[24]. There are also three series for which the ADF and KPSS tests conflict: capacity utilization, real private residential fixed investment and industrial production index. For capacity utilization, ADF test strongly rejects the null of a unit root suggesting a stationarity of the series, but KPSS test rejects level stationarity at the 5% level. Following Bernanke *et al.* (2004) and Koop & Korobilis (2013), we decided to use this series in levels. Besides the consistency with the literature, we can argue that because the ADF test has low power, rejecting the null can be taken as a severe evidence against the unit root, and therefore we can make an assumption that the series is stationary. In case of an industrial production index, we can reject the null of a unit root at the 5% level by ADF test, however KPSS test strongly rejects trend stationarity[25]. Despite this, we decided to take log-differences because it makes the industrial production index stationary, as confirmed by both ADF and KPSS tests, and suggests that the original series in levels has a stochastic trend. The last conflicting case includes residential investment that was steadily growing from the beginning of our sample (1999), but then it experienced a large decline following a burst of the housing bubble, and hence we cannot consider it as a trending variable during the period covered by the data. Following the procedure of Elder & Kennedy (2001) for variables whose growth status is unknown, we first include a deterministic trend to regressions 3.1 and 3.2. The result is that although the ADF test can reject the null of a unit root at the 5% level, KPSS test strongly rejects trend stationarity. However, if we believed that residential investment is trend-stationary, as suggested by ADF test and detrended it, we would again get a mixed evidence from both stationarity tests. Taking log-differences does not help either, but if we further difference the already log-differenced series, stationarity is eventually induces, so residential investment appears to be integrated of order 2. Despite this fact, we decided to avoid those rather extreme transformations due

[23] 10-Year Treasury Constant Maturity, 30-Year Fixed Rate Mortgage Average, Wu-Xia Shadow Rate and Krippner's SSR.

[24] Another reason for using the interest rates in levels is a better interpretation of an interest rate shock.

[25] This variable is clearly growing.

to the difficult interpretation of the results and use the series in levels for which we obtained the evidence of stationarity at least from ADF test[26]. Moreover, transforming all variables so that the resulting series is stationary based on both ADF and KPSS tests would require taking second log-differences of the S&P/Case-Shiller house price index because, as will become clear in section 3.3.2, this variable is apparently I(2). Such transformation would make the interpretation of impulse responses of this key variable cumbersome, which is why we decided to use first differences at most when transforming the variables. Transformation codes for all variables used in the empirical part can be found in Table A.1.

Apart from the transformations, we followed Koop & Korobilis (2013) and standardized each variable by subtracting off its mean and dividing it by its standard deviation[27][28]. Table B.2 shows the results of ADF and KPSS tests applied to the transformed and standardized variables. For most of the variables, both tests now indicate stationarity. However, for house price indices, civilian unemployment rate, residential investment and interest rates there is still an evidence of a unit root. Therefore, we set the prior mean[29] such that for those variables the coefficients on their first own lags are 1 to express the prior belief that they follow a random walk. There are also three variables for which the tests contradict - capacity utilization, real personal consumption expenditures and mortgage debt. For those variables, the coefficients on the first own lags are set to 0.95 to reflect a high degree of persistence while still acknowledging the stationarity indicated by ADF test.

Table 3.2 presents descriptive statistics of transformed and standardized variables. We can see that all variables that are still non-stationary have notably different means from zero than those that were confirmed to be stationary. Moreover, their standard deviations are higher as compared to stationary variables. The highest standard deviation can be observed for residential investment because it remained in levels and was only standardized. Almost all variables appear to be

[26]Whether or not the resulting series is stationary is not of immense importance for the estimation because we can always adjust the prior mean of VAR coefficients accordingly to equal 1 for the first own lags of non-stationary variables. However, the transformations help in producing reasonable covariance matrices in terms of positive definiteness. Also, Koop & Korobilis (2013) transformed the series to be only "approximately stationary" (Koop & Korobilis (2013), p.30).

[27]This is usually done in factor augmented vector autoregressive models, see e.g., Eickmeier & Hofmann (2010).

[28]Following Koop & Korobilis (2013), means and standard deviations were computed using a training sample of the initial 40 observations.

[29]Prior mean is described in detail in Chapter 4.

non-normal as the null hypothesis of normality can be rejected by Shapiro-Wilk test even at the 1% significance level. However, there exist few exceptions to this: for housing starts, we cannot reject the null of normality at any possible significance level, and for Macroeconomic Advisers' real GDP index[30], we can reject the null of normality at the 10% level (p-value = 0.061) but we cannot reject it at the 5% level, therefore this variable could be considered normally distributed. Interestingly, transformed and standardized monthly estimates of real GDP are closer to normality than (similarly transformed and standardized) quarterly GDP which was, however, converted to monthly figures using a cubic spline interpolation before it was transformed and standardized. Except for this, values of all statistics are close to one another for the two GDP measures. Finally, our key variables for assessing the stance of monetary policy, Wu and Xia shadow rate and Krippner's shadow short rate, are also comparable, even though Krippner's SSR has a larger minimum which is also apparent from Figure 3.2 and caused probably by differences in shadow/lower-bound term structure models used to generate the shadow rates.

[30]Monthly estimates of real GDP.

Variable	mean	std.dev	min	max	median	Shapiro-Wilk statistics
Real GDP	−0.192	0.944	−4.263	2.306	−0.085	0.919***
S&P/Case-Shiller U.S. National HPI	−2.140	4.412	−16.009	6.249	−0.846	0.919***
Capacity Utilization: Manufacturing	−0.499	0.992	−3.780	1.020	−0.397	0.916***
Civilian Unemployment Rate	2.694	2.923	−1.082	9.195	1.902	0.878***
Real Disposable Personal Income: Per Capita	−0.150	1.382	−11.327	7.483	−0.090	0.647***
Real Personal Consumption Expenditures	−0.208	0.709	−2.626	4.020	−0.223	0.937***
Real Private Residential Fixed Investment	−3.641	11.495	−21.379	18.406	−3.722	0.953***
CPI: All Items	−0.152	1.369	−9.072	5.243	−0.117	0.891***
Mortgage Debt Outstanding	−0.509	2.540	−13.400	11.408	−0.545	0.886***
Housing Starts: Total	0.010	1.811	−4.824	5.096	−0.003	0.997
Total Reserves of Depository Institutions	0.272	1.486	−3.128	14.382	0.113	0.439***
M2 Money Stock	−0.024	1.074	−2.533	5.711	−0.125	0.908***
10-Year Treasury Constant Maturity Rate	−3.296	2.367	−7.257	2.067	−3.065	0.962***
30-Year Fixed Rate Mortgage Average	−3.620	2.478	−7.513	1.982	−3.167	0.955***
Spot Crude Oil Price: WTI	−0.142	1.034	−4.124	2.266	−0.018	0.964***
S&P 500 Index	0.152	0.912	−3.872	2.277	0.264	0.968***
Producer Price Index for All Commodities	0.052	1.249	−5.891	2.913	0.154	0.935***
Real Broad Effective Exchange Rate	−0.360	1.500	−4.932	6.487	−0.374	0.983***
CBOE Volatility Index: VIX	0.039	1.198	−2.800	5.481	−0.053	0.938***
Wu-Xia Shadow Rate	−2.120	1.737	−4.997	1.195	−2.399	0.941***
Industrial Production Index^{RC}	0.014	1.391	−9.337	2.989	0.140	0.865***
Macroeconomic Advisers' Real GDP Index^{RC}	−0.091	0.905	−3.171	2.709	0.022	0.988*
FHFA House Price Index^{RC}	−1.193	2.773	−11.950	3.295	−0.311	0.870***
Krippner's SSR^{RC}	−2.191	1.899	−6.000	1.294	−2.424	0.967***
$N = 220$						

Table 3.2: Descriptive statistics for all variables used in the empirical part. Superscript RC denotes variables used for a robustness check. The last column reports test statistics from Shapiro-Wilk test for normality (null hypothesis is that the data come from a normally distributed population). The asterisks indicate significance at the 10% (*), 5% (**) and 1% (***) levels.

3.3 Preliminary analysis

3.3.1 Lag order selection

Frequentists usually choose VAR order based on information criteria including Akaike, Schwarz and Hannan-Quinn information criterion, final prediction error, or based on the sequential likelihood ratio and Lagrange multiplier tests. However, those lag order selection criteria mostly do not agree on the optimal lag number, so it is important to decide which criterion should be trusted based on a particular application and purpose of VAR. In this regard, Ventzislav & Kilian (2005) showed that for constructing the most accurate structural and semi-structural impulse response estimates, Akaike information criterion (AIC) for monthly data and Hannan-Quinn (HQC) criterion for quarterly data[31] is the most preferred criterion because it yields the lowest ratio of mean-squared error (MSE) of the impulse response estimates to the MSE obtained by using the true lag order.

Bayesians, on the other hand, typically avoid using information criteria and select the lag length based on data frequency[32] or their prior belief about the actual ongoings in the system. The selected lag length is conservative and rather high when the frequency of the data is higher or the variables are persistent[33]. This would usually not be possible in frequentist framework due to the issue of overparameterization. Bayesians deal with overparameterization by imposing a structure on the prior variance of lagged coefficients such that the coefficients are more tightly centered around zero[34] at higher lags. This actually makes sense because for most economic time series, recent observations convey more information about their future values than the historical ones. Considering the Bayesian approach and the periodicity of our data, we should employ 12 lags and select the prior with diminishing importance of VAR coefficients as the lag length in-

[31]With the exception of small samples (up to 120 quarters) for which Schwarz information criterion (SIC) should be used.

[32]E.g., 4 lags for quarterly data and 12 lags for monthly data.

[33]For example, Leeper (1997) used 18 lags when estimating a Bayesian VAR with monthly data.

[34]If the prior mean of the coefficient is zero, 'tightly centered' means that its prior variance is lower.

creases. However, since we are using a TVP-VAR model, we would have to draw $20 \cdot (1+12\cdot 20) = 4820$ coefficients for the large VAR for each t, $t = 1, \ldots, T$, which would result in an immense computational burden. Therefore, we opted for 4 lags which is also the optimal lag length selected by SIC for the medium VAR[35][36]. As demonstrated in Schwarz (1978), SIC can be interpreted as a large-sample version of Bayes procedures that choose a model based on the posterior model probabilities[37], so that our selection is compatible with Bayesian framework. Banbura *et al.* (2010) also utilized SIC in the context of large Bayesian VARs with monthly frequency and showed that the specification with the lag order selected by SIC performs well in their forecasting exercises.

3.3.2 Granger causality

To gain more insight into the underlying process, Granger (non-)causality[38] tests were performed. This concept of causality was developed by Granger (1969) and centers around the idea that a cause has to come prior to its effect. Therefore, if variable x affects variable y, past values of x should help in predicting y. Formally, x is said to Granger-cause y if in a model

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q}$$

we can reject[39] the null hypothesis that $\beta_i = 0, i = 1, \ldots, q$. Rejecting the null implies that the past values of x contain useful information about the current value of y, beyond and above the information that is already included in the past values of y.

In testing for Granger non-causality, we proceed according to Toda & Yamamoto (1995) procedure and its step-by-step explanation provided by Giles (2011). Toda & Yamamoto (1995) showed how to estimate VAR in levels and test general restrictions on its parameters even if the variables in VAR are integrated or cointegrated of any order, and therefore developed a way of testing for Granger non-causality with non-stationary data. Stationarity is required to

[35]Selection criteria employed with the upper bound of 12 lags.

[36]For the small VAR, SIC suggests 3 lags and for the large VAR it suggests using only 1 lag, although in case of the large VAR, SIC could not be computed for higher lags than 10 due to the overparameterization.

[37]Given a linear model with i.i.d. observations.

[38]We use a term 'non-causality' because the null hypothesis is that one variable does not Granger-cause the other.

[39]Usually, the 5% significance level is taken as a threshold.

produce test statistics that converge to a convenient limiting distribution; however, if some of the variables are non-stationary, standard asymptotic theory is not valid and Wald test statistic computed to assess the null hypothesis above would not have the asymptotic chi-square distribution under the null (Lütkepohl (2007)). We decided to use this procedure because even after we transformed the data, there are some apparently non-stationary variables[40] that would make the test statistic used to evaluate the null of Granger non-causality not to follow any appropriate distribution, even asymptotically.

We choose to examine Granger causality among variables of the medium VAR that includes the variables[41] that are fundamental for predicting house prices (Gattini & Hiebert (2010)). The most important thing is to use the variables in (log-)levels, i.e., not to difference the data even if they are non-stationary. Therefore, we employ log-levels of real GDP, house price index, residential investment and CPI, and levels of mortgage and Wu and Xia rates.

Following Toda & Yamamoto (1995) and Giles (2011), our testing for Granger non-causality then proceeds as follows:

1. Determine the maximum order of integration among variables.

   Below we discuss the order of integration of each medium-VAR variable separately. Results from ADF and KPSS tests to which we below refer were already assessed in Table B.1 and in the corresponding discussion.

   Real GDP Clearly I(1); ADF and KPSS tests both indicate that it is non-stationary in log-levels, but it becomes stationary when we first-difference it.

   S&P/Case-Shiller house price index Appears to be I(2), because it exhibits non-stationary behavior in log-levels and even in log-differences as both ADF and KPSS tests agree, but when we difference it twice, it finally becomes stationary according to the tests.

   Residential investment Seems to be I(2), even though the ADF test for a series in log-levels rejects the null of a unit root at the 5% level, because KPSS strongly rejects stationarity. First differences are not stationary either, however, taking second log-differences finally results in a series that is marked stationary by both tests.

[40] This is of course caused because we refused to use 'extreme' transformations, like e.g. double-differencing the interest rates.

[41] List of variables included in the medium VAR can be found in Table 3.1.

CPI Clearly I(1) and the reasoning is similar as in the case of real GDP.

Mortgage average Also I(1) as ADF and KPSS tests both conclude that it is non-stationary in levels and stationary in first differences.

Wu and Xia rate Clearly non-stationary in levels, however, first-differencing induces a conflict in both tests: ADF test cannot reject the null of a unit root at the 5% level, while KPSS concludes that the series is level stationary. We could argue that because the power of ADF test is presumably low and its test statistic is close to the 5% critical value (test statistic is -2.8573 and 5% critical value is -2.88), rejection of the unit root null by ADF test could be assumed as well and the first-differenced series could be called stationary, resulting in Wu and Xia rate being integrated of order 1. Fortunately, it does not make any difference whether this series is actually I(1) or I(2)[42] since we are interested only in the *maximum* order of integration among the variables, which is indeed 2 because of the S&P/Case-Shiller house price index.

To summarize, we found out that the maximum order of integration among our variables is 2. We denote it by m, therefore, $m = 2$.

2. Set up the VAR in (log-)levels and determine the optimal lag length p using information criteria.

   Our VAR contains the following variables: *(y, hpi, rinv, cpi, ma, ir)*, where *y*, *hpi*, *rinv*, *cpi* are the log-levels of real GDP, house price index, residential investment and CPI, respectively, and *ma* and *ir* are the levels of mortgage average and Wu-Xia rate. We also included an intercept to each equation. As Giles (2011) emphasizes it is crucial to set up the VAR in (log-)levels and not to difference any variables, regardless of what we found in the previous step.

   Next, we need to determine the optimal lag length of our VAR in (log-)levels. For this purpose, information criteria (AIC, SIC, HQC) were computed for the lags up to 12. The final lag length was chosen to be 4 according to SIC which is also favored by Giles (2011). However, when we check the residuals for autocorrelation using the Ljung-Box Modified Portmanteau test for multivariate series, we find that there is no autocorrelation from $p = 7$ onwards, so we redefined the VAR lag length to $p = 7$. This VAR is

[42] Second differences are marked stationary by both tests.

also dynamically stable as all the eigenvalues of the companion coefficient matrix are inside the unit circle.

3. Estimate the VAR with selected number of lags p and add m additional lags of each variable into each equation. Then test for Granger non-causality using only the first p lags.

   We estimate VAR with variables *(y, hpi, rinv, cpi, ma, ir)* and 9 lags, because $p = 7$ is assumed to be the optimal lag length and $m = 2$ is the maximum order of integration among variables which was determined in step 1. Now we test for Granger non-causality as follows. For each pair of variables X and Y in the VAR, $X \neq Y$, we test that the first 7 lags of X in the equation for Y are jointly equal to zero using Wald test. The null hypothesis is that X does not Granger cause Y. The coefficients for the extra m lags are not included in the test because they are there only to fix up the asymptotics (Giles (2011)). Under the null hypothesis, the test statistic is asymptotically chi-square distributed. Rejection of the null at a sufficiently low significance level (usually 5%) can be considered as a firm evidence of Granger causality from X to Y.

Table 3.3 reveals the results of all pairwise Granger non-causality tests. We can see that there is a really strong evidence that real GDP Granger-causes all of the remaining variables, as the null hypothesis that the lagged coefficients of real GDP are jointly zero can be rejected in all equations at any possible significance level. The same situation occurs with residential investment which is not so surprising, given that it is a part of GDP. Developments in house prices Granger-cause real GDP and residential investment, as the test suggests. Moreover, past values of CPI help to predict the policy rate[43]. This also agrees with economic intuition, as monetary policy is believed to respond to CPI inflation and GDP that are both showed to Granger-cause the policy rate. Interestingly, mortgage average does not Granger-cause house prices as the null could not be rejected at the 5% level, even though there is a strong evidence that it Granger-causes residential investment. Importantly for our analysis, the null hypothesis that Wu and Xia rate does not Granger-cause house prices can be rejected even at the 1% level, implying that the monetary policy actions may affect the developments in house prices, even though we cannot assess the magnitude or the direction of such an effect from Granger causality analysis. For this, we have to use other tools, for example

[43]Measured by Wu and Xia rate.

y → hpi **0.0000**	**y → rinv** **0.0000**	**y → cpi** **0.0000**	**y → ma** **0.0000**	**y → ir** **0.0000**
hpi → y **0.0186**	**hpi → rinv** **0.0007**	hpi → cpi 0.0824	hpi → ma 0.3766	hpi → ir 0.4243
rinv → y **0.0000**	**rinv → hpi** **0.0000**	**rinv → cpi** **0.0000**	**rinv → ma** **0.0000**	**rinv → ir** **0.0000**
cpi → y 0.3013	cpi → hpi 0.4914	cpi → rinv 0.7639	cpi → ma 0.3680	**cpi → ir** **0.0002**
ma → y **0.0117**	ma → hpi 0.2141	**ma → rinv** **0.0000**	ma → cpi 0.3251	**ma → ir** **0.0021**
ir → y 0.2058	**ir → hpi** **0.0032**	**ir → rinv** **0.0021**	ir → cpi 0.1896	ir → ma 0.2329

Table 3.3: P-values from Granger non-causality tests. Variables: *y* - logarithm of real GDP, *hpi* - logarithm of house prices, *rinv* - logarithm of residential investment, *cpi* - logarithm of CPI, *ma* - mortgage rate in levels and *ir* - Wu-Xia rate in levels. Null hypothesis in each X → Y test is that X does not Granger-cause Y. Significant (at least at 5%) tests and their p-values are in bold.

the impulse response functions. However, the link between the policy rate and house prices is not bi-directional ("feedback") because the null that house prices do not Granger-cause the policy rate could not be rejected at any reasonable significance level. A significant bi-directional Granger causality can be found between house prices and residential investment and between mortgage rates and residential investment. Finally, house prices are also revealed as an important predictor for real GDP. However, results from Granger non-causality tests should be treated with caution because they are dependent on which variables appear in the VAR, seasonal adjustment of the data, presence of measurement errors, etc. Therefore, deriving any definite conclusions from those tests would be misleading.

Chapter 4

Methodology

In this chapter, we describe how the TVP-VAR model with stochastic volatility was estimated and how we extracted impulse responses and calculated forecast error variance decompositions.

4.1 Brief description of the model

The algorithm for estimating large TVP-VAR with stochastic volatility was developed by Koop & Korobilis (2013), and we follow their notation below. The model is the state-space model of the form:

$$y_t = Z_t \beta_t + \epsilon_t \tag{4.1}$$

$$\beta_t = \beta_{t-1} + u_t \tag{4.2}$$

where ϵ_t is i.i.d. $\mathcal{N}(0, \Sigma_t)$ and u_t is i.i.d. $\mathcal{N}(0, Q_t)$. ϵ_t and u_s are assumed to be independent of one another for all s and t. Equation 4.1 is called the observation equation because it links the unobserved states β_t to y_t, that is, for $t = 1, \ldots, T$, $M \times 1$ vector of observations on M time series variables. Equation 4.2 is the transition equation, which specifies the law of motion for the unobserved state variable. The matrix that links the unobserved states to y_t, $t = 1, \ldots, T$, is

$$Z_t = \begin{pmatrix} z_t' & 0 & \ldots & 0 \\ 0 & z_t' & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & z_t' \end{pmatrix}$$

$M \times K$ matrix with $K = M(1 + pM)$ and z_t being a vector[1] of an intercept and p lags of each of the M variables. β_t, $t = 1, \ldots, T$, is therefore a $K \times 1$ vector of time-varying VAR coefficients and we assume that those coefficients evolve as random walks.

TVP-VARs with stochastic volatility can be estimated by combining Gibbs sampling and Metropolis Hastings (MH) algorithm, whereas the latter is used for drawing stochastic volatilities from their posterior distribution. Briefly, the algorithm works as follows (for a detailed description, see Appendix). First, the researcher needs to select the specification for Σ_t and Q_t, the prior for the initial conditions (prior for β_0 and for the free elements of Σ_0 and Q_0, which depends on the nature of stochastic volatility assumed by the researcher) and the prior for any remaining parameters of the model (e.g., if there is a drift term in the transition equation). Next, β_t, $t = 1, \ldots, T$, is sampled from its conditional posterior, given Σ_t, Q_t and the remaining parameters of the model. In this step, usually a multi-move sampler proposed by Carter & Kohn (1994) is used. Conditional on β_t, Q_t and the remaining parameters of the model, Σ_t, $t = 1, \ldots, T$, is sampled from its conditional posterior. Then, conditional on β_t, Σ_t and the remaining parameters of the model, Q_t, $t = 1, \ldots, T$, is sampled from its conditional posterior. Sampling Σ_t and Q_t involves Metropolis Hastings algorithm if a non-linear law of motion for stochastic volatilities is assumed. Finally, conditional on β_t, Σ_t and Q_t, the remaining parameters of the model are sampled from their conditional posteriors.

However, the above approach is very time-demanding, especially for large models, for which the computational burden can easily become insurmountable. Therefore, Koop & Korobilis (2013) suggested to replace Σ_t and Q_t by estimates and then draw β from its posterior distribution, which in case of Carter and Kohn algorithm, amounts to drawing repeatedly from normal distributions with known mean and variance to get one draw of β_t, $t = 1, \ldots, T$. Replacing Σ_t and Q_t by estimates is achieved by using an Exponentially Weighted Moving Average (EWMA) estimator for Σ and forgetting factors for Q_t, both of which are explained below during the description of steps in which they enter the algorithm. This avoids using MCMC methods, and thus considerably reduces the computational burden. In the following we describe how to estimate β and select the best model in each time from those of different prior shrinkage parameters and different dimensions. The model features dynamic dimension selection and time-varying coefficients, therefore Koop & Korobilis (2013) call it TVP-VAR-DDS.

[1]By vector, we always refer to the column vector; therefore z_t' is a row vector.

4.2 Estimation of TVP-VAR-DDS

In this section we describe how we adjusted the algorithm of Koop & Korobilis (2013) to be able to draw β_t, $t = 1, \ldots, T$, from its posterior distribution, and subsequently to perform impulse response analysis and provide forecast error variance decompositions. We had to adjust the algorithm, because Koop and Korobilis focused on iterated forecasts of their model and its forecast performance as compared to other models, but our aim is not on forecasting. Individual steps of our algorithm are discussed below.

4.2.1 Kalman Filter with Forgetting Factors

Now consider we have one TVP-VAR model of the form 4.1 and 4.2 with the prior for the initial condition $\beta_0 \sim \mathcal{N}(b_0, P_0)$, where the form of b_0 and P_0 will be specified below, and the initial condition Σ_0 on Σ_t. We do not need to impose the initial condition on Q_t, because the algorithm uses forgetting factors to remove the need for estimating Q_t, as it soon becomes clear. Kalman filter is a recursive algorithm that provides the estimate of the state variable and its variance for each time period, given information up to that time period. The estimate of the state variable at time t given information up to time t is denoted $\beta_{t|t}$ and its variance $P_{t|t}$. To calculate those, the algorithm needs to be supplied with the initial conditions $\beta_{0|0}$ and $P_{0|0}$ and with values of all the parameters of the state-space model, which in our case reduce to Σ_t and Q_t, $t = 1, \ldots, T$.

Initial conditions $\beta_{0|0}$ and $P_{0|0}$

$\beta_{0|0}$ is set equal to the prior mean of β_0, b_0, while $P_{0|0}$ is set equal to its prior variance, P_0. Koop & Korobilis (2013) stressed the importance of choosing the prior for β_0 in the context of TVP-VARs, because in those models, the number of parameters is much higher than the number of observations, and therefore obtaining reasonable results hinges upon having an adequate prior. Therefore, they employed a Normal prior for β_0 that is similar to the Minnesota prior (see e.g., Doan *et al.* (1983)), and, if there is no time variation in the parameters, this prior will be the same as the Minnesota prior in a VAR with constant coefficients. We adopted similar prior for β_0, but we slightly changed its structure to make it compatible with our application. In this prior, prior mean $E(\beta_0)$ should be 0 for variables that are stationary and 1 for those that follow a random walk. Koop & Korobilis (2013) claim that they transformed all variables to stationarity, so they set $E(\beta_0) = 0$. We did not want to transform some variables by taking second log-differences even if they were obviously I(2) or to transform the interest rates that were mainly I(1) due to the easier interpretation of results, so we still have some variables that can be shown to follow a random walk (see Table after the transformation. For those variables, we set $E(\beta_0)$ to 1 for coefficients on their first own lags so as to shrink towards a random walk, and 0 otherwise. Namely, variables that clearly follow a random walk even after the transformation are[2]: S&P/Case-Shiller U.S. National Home Price Index, Civilian Unemployment Rate, Real Private Residential Fixed Investment, 10-Year Treasury Constant Maturity Rate, 30-Year Fixed Rate Mortgage Average and Wu-Xia Shadow Rate. There are also three variables with mixed evidence from the two complementary tests for stationarity, specifically Capacity Utilization: Manufacturing, Real Personal Consumption Expenditures and Mortgage Debt Outstanding. For those variables, ADF test rejects the null of non-stationarity at least at the 5% level, but KPSS test rejects the null of stationarity also at least at the 5% level. However, ADF test has low power, so rejection of the null can be considered as a fairly strong evidence of stationarity. Therefore, we set prior mean on first own lags for those variables equal to 0.95 so as to reflect a high degree of persistency which may produce conflicting test results, but still to express the belief that the series is stationary. It should also be noted that these are only our prior beliefs; a-posteriori, each series may follow a different process if there is enough information in the data to confirm it.

Minnesota priors typically assume that the prior covariance matrix is diagonal and Koop & Korobilis (2013) are no exception to this; therefore, our prior covariance matrix for β_0 will be diagonal. Let us denote $var(\beta_0) = P_0$ and $p_{0i}, i = 1, \ldots, K$, its diagonal elements. Then,

$$p_{0i} = \begin{cases} \frac{\gamma}{l^2} & \text{for coefficients on lag } l = 1, \ldots, p \\ \underline{\alpha} & \text{for intercepts} \end{cases} \tag{4.3}$$

This is a Minnesota-type prior which has one key hyperparameter - γ - that represents the tightness on the variance of the VAR coefficients on different lags. The lower the γ, the more centered are the VAR coefficients around their prior means because their variance is lower. This hyperparameter can also have the interpretation of the shrinkage hyperparameter; the lower the γ, the higher is

[2]Variables used only for a robustness check with the same property are: FHFA House Price Index and Krippner's SSR. For those variables, we also set the prior mean on their first own lag to 1.

the degree of shrinkage in which the VAR coefficients shrink towards their prior means. Large VARs and especially TVP-VARs usually require a high degree of shrinkage to be able to produce reasonable results due to the otherwise present overparameterization, and Minnesota-type priors can greatly reduce the overparameterization worries. Variance of the coefficients also decay with the lag to express the belief that more distant lags are less informative than recent lags, and therefore they are shrunk more to zero. The standard Minnesota prior has two shrinkage hyperparameters[3]: one for the coefficients on own lags and the other for coefficients on the lags of other variables, while it typically holds that the first one is higher than the latter one to reflect the fact that own lags are more important than the lags of other variables. Koop & Korobilis (2013) used only one shrinkage hyperparameter to make the computation easier and we follow their practice and set prior covariance matrix to 4.3. Prior means of intercepts are assumed to be noninformative, and thus we set $\underline{\alpha}$ to 10. The selection of γ will be discussed in section 4.2.2 after we describe the particular steps in the Kalman filtering procedure. However, first we focus on the estimation of Σ_t and Q_t which are needed for the Kalman filter.

Estimating Σ_t

To estimate Σ_t, Koop & Korobilis (2013) used an Exponentially Weighted Moving Average (EWMA) estimator, which takes a form:

$$\widehat{\Sigma}_t = \kappa\widehat{\Sigma}_{t-1} + (1-\kappa)\widetilde{\epsilon}_t\widetilde{\epsilon}_t' \tag{4.4}$$

where $\widetilde{\epsilon}_t = y_t - Z_t\beta_{t|t-1}$ is prediction error produced by the Kalman filter. This estimator requires to choose the decay factor κ, for which we followed the suggestion in Morgan & Reuters (1996) and set $\kappa = 0.96$, as Koop & Korobilis (2013) did. We also need the initial condition, Σ_0, that is established using the sample covariance matrix of y^τ, where τ is the size of the training sample which we set to the first 40 observations.

Estimating Q_t

Transition error covariance matrix Q_t is needed in Kalman filter recursions only for estimating the variance of the state variable at time t using information up to time $t-1$:

$$P_{t|t-1} = P_{t-1|t-1} + Q_t \tag{4.5}$$

[3]Three, if we consider also exogenous variables.

Therefore, if we follow the suggestion of Koop & Korobilis (2013) and replace this by

$$P_{t|t-1} = \frac{1}{\lambda} P_{t-1|t-1} \tag{4.6}$$

we do not have to estimate or simulate Q_t. λ is known as a forgetting factor and it holds that $0 < \lambda < 1$. Forgetting factors can reduce the computational burden associated with estimating TVP-VARs or other models with many parameters and, in the context of TVP-VARs, they were utilized by other authors as well (e.g., Dangl & Halling (2011))[4]. However, they serve only as an approximation to the exact MCMC methods, which in our case would involve drawing repeatedly β_t, Σ_t and Q_t from their conditional posteriors, but would not be computationally feasible for many-variable TVP-VAR that we employ. It follows from 4.5 and 4.6 that $Q_t = (\lambda^{-1} - 1)P_{t-1|t-1}$.

As noted by Koop & Korobilis (2013), observations j periods before time t get weight λ^j in the filtered estimate of β_t if we use forgetting factor as in equation 4.6. In many empirical applications using forgetting factors, λ is set to a number slightly lower than one, e.g., 0.99. This implies, for quarterly data as are those used by Koop & Korobilis (2013), that observations five years (20 quarters) in the past get approximately 80% (0.99^{20}) of the last observations' weight. According to Koop & Korobilis (2013), this implies a fairly stable model with gradual coefficient change and ensures that λ has properties of the Cogley & Sargent (2005) "business as usual" prior. Therefore, with our monthly data, we would need to set λ to 0.9963 to achieve approximately 80% relative weight for observations five years in the past if we were to use a fixed λ. However, Koop & Korobilis (2013) decided to estimate λ in a time-varying way and for this they used the updating equation of Park *et al.* (1991):

$$\lambda_t = \lambda_{min} + (1 - \lambda_{min})L^{f_t} \tag{4.7}$$

where $f_t = -NINT(\widetilde{\epsilon}'_{t-1}\widetilde{\epsilon}_{t-1})$ and $\widetilde{\epsilon}_t = y_t - Z_t\beta_{t|t-1}$ is again the prediction error produced by the Kalman filter. Function NINT returns the nearest integer; half-integers are rounded to the nearest higher integer in our setting. We choose values of λ_{min} and L to match those of Koop & Korobilis (2013), therefore $\lambda_{min} = 0.96$ and $L = 1.1$. This λ_t is used instead of λ in equation 4.6 which differentiates this

[4] For an in-depth discussion of forgetting factors see Jazwinski (1970) or Raftery *et al.* (2010).

modeling approach to forgetting factors from most of the existing literature that simply sets λ to a constant (Koop & Korobilis (2013)).

Now when we explained how to obtain the values of all the parameters used in the Kalman filter algorithm, we can describe its particular steps.

Kalman filter steps (for $t = 1, \ldots, T$):

- Set $\beta_{t|t-1} = \beta_{t-1|t-1}$. For $t = 1$, use $\beta_{0|0} = b_0$, i.e., the prior mean of β_0
- Calculate $\widetilde{\epsilon_t} = y_t - Z_t\beta_{t|t-1}$ (prediction error)
- For $t > 1$ estimate $\lambda_t = \lambda_{min} + (1 - \lambda_{min})L^{f_t}$, where $f_t = -NINT(\widetilde{\epsilon}'_{t-1}\widetilde{\epsilon}_{t-1})$
- Set $P_{t|t-1} = \frac{1}{\lambda_t}P_{t-1|t-1}$. For $t = 1$, set $P_{1|0} = P_{0|0} = P_0$, i.e., the prior variance of β_0
- Estimate $\widehat{\Sigma}_t = \kappa\widehat{\Sigma}_{t-1} + (1-\kappa)\widetilde{\epsilon_t}\widetilde{\epsilon_t}'$. For $t = 1$, set $\widehat{\Sigma}_1 = \kappa\Sigma_0$
- Calculate $\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1}Z'_t(\widehat{\Sigma}_t + Z_tP_{t|t-1}Z'_t)^{-1}\widetilde{\epsilon_t}$
- Calculate $P_{t|t} = P_{t|t-1} - P_{t|t-1}Z'_t(\widehat{\Sigma}_t + Z_tP_{t|t-1}Z'_t)^{-1}Z_tP_{t|t-1}$

This algorithm delivers $\beta_{t|t}$ and $P_{t|t}$ for $t = 1, \ldots, T$ and also the estimates of the measurement error covariance matrix $\widehat{\Sigma}_t$ and forgetting factor λ_t.

4.2.2 Dynamic Model Selection

Previous results apply to only one model. However, Koop & Korobilis (2013) decided to employ dynamic model averaging (DMA) and selection (DMS) methods developed by Raftery *et al.* (2010) in the context of time-varying parameter models. Those methods assume that there are $j = 1, \ldots, J$ competing models under consideration. For each competing model the researcher needs to calculate $\pi_{t|t-1,j}$, i.e., the probability that model j forecasts the best at time t given information up to time $t-1$ and $\pi_{t|t,j}$, which is the posterior model probability. Those probabilities are then used in doing dynamic model averaging or selection. Koop & Korobilis (2013) performs dynamic model selection that occurs if, in each time $t = 1, \ldots, T$, the model with the highest value of $\pi_{t|t,j}$ is selected. The calculation of $\pi_{t|t-1,j}$ and $\pi_{t|t,j}$ is handled using a fast recursive algorithm of Raftery *et al.* (2010).

The algorithm proceeds in two steps that are comparable with the Kalman filter's prediction and updating equations. After we specify the initial condition,

$\pi_{0|0,j}$ for $j = 1, \ldots, J$, the algorithm begins with the prediction equation:

$$\pi_{t|t-1,j} = \frac{\pi^{\alpha}_{t-1|t-1,j}}{\sum_{l=1}^{J} \pi^{\alpha}_{t-1|t-1,l}} \tag{4.8}$$

and continues with the updating equation:

$$\pi_{t|t,j} = \frac{\pi_{t|t-1,j} p_j(y_t|\widetilde{y}_{t-1})}{\sum_{l=1}^{J} \pi_{t|t-1,l} p_l(y_t|\widetilde{y}_{t-1})} \tag{4.9}$$

to evaluate $\pi_{t|t-1,j}$ for $t = 1, \ldots, T$. In equation 4.9, $p_j(y_t|\widetilde{y}_{t-1})$ is the predictive likelihood of model j - predictive density of this model evaluated at y_t given observations up to time $t-1$: $\widetilde{y}_{t-1} = (y_1', y_2', \ldots, y_{t-1}')'$. This predictive likelihood is available from the Kalman filter if we evaluate the predictive density $\mathcal{N}(Z_t^{(j)} \beta_{t|t-1}^{(j)}, \widehat{\Sigma}_t^{(j)} + Z_t^{(j)} P_{t|t-1}^{(j)} Z_t^{\prime(j)})$ at $y_t^{(j)}$, and it measures the forecast performance of model j at time t. In equation 4.8, α is the forgetting factor and controls how much weight the forecast performance of model j $i = 1, \ldots, t-1$ periods before time t receives in calculating $\pi_{t|t-1,j}$. Koop & Korobilis (2013) show that:

$$\pi_{t|t-1,j} \propto \prod_{i=1}^{t-1} [p_j(y_{t-i}|\widetilde{y}_{t-i-1})]^{\alpha^i}$$

where $p_j(y_{t-i}|\widetilde{y}_{t-i-1})$ is the predictive likelihood of model j i periods before time t and measures the forecast performance of model j in that point in time. As with the forgetting factor λ, if $\alpha = 0.99$ and for quarterly data, forecast performance five years ago receives approximately 80% (0.99^{20}) of the weight of the forecast performance at $t-1$. For monthly data we set $\alpha = 0.9963$ to obtain the same effect. Finally, initial conditions $\pi_{0|0,j}$, $j = 1, \ldots, J$, are specified as follows: $\pi_{0|0,j} = \frac{1}{J}$ to express the belief that all models are ex-ante equally likely.

The above algorithm is used to select one model in each time from the pool of J possible models. Following Koop & Korobilis (2013), we augment the model space in two ways:

First, we consider 3 TVP-VARs of different dimensions: small, four-variable VAR[5] with three key macroeconomic variables (real GDP, inflation and interest rate) and add house prices to it; medium, six-variable VAR with additional two

[5]From now on, whenever we use VAR we mean our specification of VAR, that is a VAR with time-varying parameters.

variables that are essential for determining the house-price movements (see Rahal (2016); or Gattini & Hiebert (2010)); and large, twenty-variable VAR containing all medium-VAR variables and capturing broad financial and macroeconomic conditions. List of variables included in each VAR is available in the Appendix. The only problem here is that the predictive densities $p_j(y_t|\widetilde{y}_{t-1})$ will not be comparable among TVP-VARs of different dimensions, because y_t has a different dimension for each of the small, medium and large TVP-VAR. Therefore, we decided that for each TVP-VAR model, we will calculate predictive density using only the variables of the small TVP-VAR. This means that we will always evaluate $p_j(y_t|\widetilde{y}_{t-1})$ using the vector y_t that includes real GDP, inflation, interest rate and house prices regardless of how many variables a given TVP-VAR has. Those variables are present in all TVP-VAR models under consideration, so it makes sense that we are interested in their joint predictive likelihood to determine which model is the best and when it is so.

Next, for a given TVP-VAR size (small, medium or large), we define 7 models based on the values of the prior shrinkage parameter γ. As explained before, this hyperparameter controls a degree of shrinkage of the VAR coefficients; see 4.3 and the associated discussion. In the TVP-VAR literature, γ is usually determined using training sample priors (see, e.g., Primiceri (2005)). This approach involves taking out the small subsample of the data that will then be discarded, usually from the beginning of the data set, and estimating hyperparameters on this subsample by, e.g., OLS. Instead of estimating γ from the training sample, we will choose one γ in each point in time among the seven pre-defined values of it: $\gamma \in [10^{-10}, 10^{-5}, 0.001, 0.005, 0.01, 0.05, 0.1]$. Different values of γ correspond to different priors and hence, in a Bayesian framework, to different models. Thus, in each time the algorithm estimates seven TVP-VAR models for each VAR size from step 1 (because we have seven possible values of γ), and then selects the value of γ for which the posterior model probability $\pi_{t|t,j}$ is maximized. This allows for switching between TVP-VAR models of different dimensions and for choosing the best shrinkage for each dimension in each time. Normally, higher shrinkage is needed in large VARs and this algorithm enables to choose γ from a very wide grid of values and to change a degree of shrinkage over time as compared to having a fixed γ.

To summarize, the full dynamic model selection algorithm proceeds as follows:

1. Set up the small, medium and large TVP-VAR. Medium TVP-VAR contains all variables of the small TVP-VAR plus some additional variables, and large

TVP-VAR contains all variables of the medium TVP-VAR plus additional variables[6].

2. For each TVP-VAR dimension from step 1, define 7 possible priors for β_0 based on the value of γ. Different values of γ correspond to different prior covariance matrices of β_0, and hence a model j, $j = 1, \ldots, 7$, will have covariance matrix $P_0^{(j)}$ with $\gamma^{(j)} \in [10^{-10}, 10^{-5}, 0.001, 0.005, 0.01, 0.05, 0.1]$ in its structure (see eq. 4.3).

3. Given a TVP-VAR size (small, medium or large), loop over $j = 1, \ldots, 7$ models determined by their prior for β_0. That is, for $t = 1, \ldots, T$ run Kalman filter recursions to obtain $p_j(y_t|\widetilde{y}_{t-1})$, and use it and the initial conditions $\pi_{0|0,j} = \frac{1}{7}$ to calculate $\pi_{t|t-1,j}$ and $\pi_{t|t,j}$. Perform this step for each TVP-VAR size. Therefore, at the end of this step, we will have for each TVP-VAR size $\pi_{t|t,j}$ for $t = 1, \ldots, T$ and $j = 1, \ldots, 7$.

4. For each $t = 1, \ldots, T$ and TVP-VAR dimension, select the model with the highest posterior model probability $\pi_{t|t,j}$ and denote its value of γ as $\gamma^{(j_{max})}$ and the associated $j \in [1, \ldots, 7]$ as j_{max}. This gives us the optimal value of the shrinkage coefficient for each time and TVP-VAR dimension.

5. Conditional on the optimal value of γ, $\gamma^{(j_{max})}$, compute the posterior model probabilities of the small, medium and large TVP-VAR: $\pi_{t|t,m}$ for $t = 1, \ldots, T$. Now $m = 1, 2, 3$ for the small, medium and large TVP-VAR, respectively, and we use $\pi_{t|t-1,j_{max}}$ and $p_{j_{max}}(y_t|\widetilde{y}_{t-1})$ to calculate $\pi_{t|t,m}$ according to the equation 4.9:

$$\pi_{t|t,m} = \frac{\pi_{t|t-1,j_{max}(m)} p_{j_{max}(m)}(y_t|\widetilde{y}_{t-1})}{\sum_{k=1}^{3} \pi_{t|t-1,j_{max}(k)} p_{j_{max}(k)}(y_t|\widetilde{y}_{t-1})}$$

where $\pi_{t|t-1,j_{max}(m)}$ is $\pi_{t|t-1}$ calculated using the optimal shrinkage parameter γ for a given TVP-VAR size m and similarly for $p_{j_{max}(m)}(y_t|\widetilde{y}_{t-1})$.

6. Now we evaluate which of the small, medium and large TVP-VAR, conditional on the optimal value of γ, is the best model at time $t = 1, \ldots, T$ in terms of posterior model probability. This is achieved by selecting $m \in [1, 2, 3]$ for each t with the highest $\pi_{t|t,m}$.

[6]For the list of variables used in the small, medium and large TVP-VAR consult the Appendix.

Therefore, for $t = 1, \dots, T$, dynamic model selection algorithm delivers the optimal dimension of TVP-VAR (small, medium or large) and the optimal value of the shrinkage coefficient γ. Those uniquely define the TVP-VAR model in each time for which we will draw from the posterior of β using the Carter and Kohn algorithm described in the subsequent section.

4.3 Carter and Kohn Algorithm

Up to this point, the modeling strategy follows Koop & Korobilis (2013). The aim of their paper is, however, on forecasting. Therefore, in the empirical part they used the last in-sample values of $\beta_{T|T}$ and $P_{T|T}$ obtained from the Kalman filter to simulate the path of β_{T+h}, where h is the forecast horizon[7]. Afterwards, they demonstrate the forecasting superiority of their model over the other frequently used time series models and possibly different modeling choices for their TVP-VAR, but they do not perform any kind of a structural analysis, which we however need because we are interested in obtaining impulse responses and forecast error variance decomposition (FEVD). We thus utilized their model to get filtered estimates of the states $\beta_{t|t}$ and their variance $P_{t|t}$, and mostly to be able to estimate even the large TVP-VAR. Their algorithm provides us with the estimates of the measurement error covariance matrix $\widehat{\Sigma}_t$ and the forgetting factors approximation of the transition error covariance matrix Q_t that make the estimation of the large TVP-VAR computationally feasible because they avoid the need of drawing from the conditional posteriors of Σ_t and Q_t. To perform structural analysis, we programmed the Carter and Kohn algorithm for this model to obtain draws from the posterior of β_t, $t = 1, \dots, T$, and used those draws to calculate impulse responses and FEVD.

Here we describe key equations of the Carter and Kohn algorithm applied to our model. In the following we assume that the parameters of the state-space model, Σ_t and Q_t, are known. For Σ_t we use its estimate $\widehat{\Sigma}_t$, and Q_t can be expressed from equations 4.5 and 4.6 as $Q_t = (\lambda_t^{-1} - 1)P_{t-1|t-1}$ given a time-varying nature of λ. Those are available for $t = 1, \dots, T$ from the algorithm of

[7]They employed the two strategies: 1) VAR coefficients are fixed out-of-sample, implying that $\widehat{\beta}_{T+j|T} \sim \mathcal{N}(\beta_{T|T}, P_{T|T})$ for $j = 1, \dots, h$, and 2) VAR coefficients are allowed to drift out-of-sample, for which $\widehat{\beta}_{T+j|T}$ are obtained by drawing recursively from $\mathcal{N}(\widehat{\beta}_{T+j-1|T}, P_{T|T})$ for $j = 1, \dots, h$, where for $j = 1$, $\widehat{\beta}_{T+j-1|T} = \beta_{T|T}$.

Koop & Korobilis (2013) for all TVP-VAR dimensions and values of the shrinkage parameter γ.

Let $\widetilde{\beta}_T$ denote the time series of β for $t = 1, \ldots, T$, i.e., $\widetilde{\beta}_T = (\beta_1', \beta_2', \ldots, \beta_T')'$. Similarly, $\widetilde{\Sigma}_T = (\Sigma_1', \Sigma_2', \ldots, \Sigma_T')'$, $\widetilde{Q}_T = (Q_1', Q_2', \ldots, Q_T')'$ and the observations through time T are denoted by $\widetilde{y}_T = (y_1', y_2', \ldots, y_T')'$. When using exact MCMC methods to draw from the joint posterior distribution of all unknown parameters and the state variable of the state-space model, Gibbs sampling algorithm is used to repeatedly draw from the conditional posterior of each parameter (or a group of parameters) and the state variable, given all other unknowns. In this algorithm, draws from $p(\widetilde{\beta}_T|\widetilde{\Sigma}_T, \widetilde{Q}_T, \widetilde{y}_T)$, i.e., the joint posterior of $\beta_1, \beta_2, \ldots, \beta_T$, can be obtained using the technique developed by Carter & Kohn (1994) – Carter and Kohn algorithm. They proposed a multi-move sampler for drawing from the conditional posterior of β, meaning that it samples a whole vector of β_t, $\forall t$, at one draw. Single-move algorithms that sample β_t for $t = 1, \ldots, T$ one at a time from $p(\beta_t|\widetilde{\Sigma}_T, \widetilde{Q}_T, \widetilde{y}_T, \beta_{-t})$, where $\beta_{-t} = (\beta_1', \ldots, \beta_{t-1}', \beta_{t+1}', \ldots, \beta_T')'$, are usually ineffective because the draws obtained from those algorithms tend to be highly correlated, which increases the number of draws that must be taken to ensure that the true posterior distribution is simulated accurately enough[8]. We decided to use the Carter and Kohn algorithm because it is arguably the most popular choice in the literature[9].

In our implementation of the Carter and Kohn algorithm, $\widetilde{\Sigma}_T$ and $\widetilde{Q}_T$ are not taken as random variables, because Σ_t is estimated by an EWMA estimator of the form 4.4 and Q_t is replaced using a forgetting factor λ (see 4.5 and 4.6). Therefore, we can express the posterior distribution of β as

$$p(\widetilde{\beta}_T|\widetilde{y}_T)$$

which is equal to the conditional posterior of β, $p(\widetilde{\beta}_T|\widetilde{\Sigma}_T, \widetilde{Q}_T, \widetilde{y}_T)$, provided that Σ_t and Q_t are known for $t = 1, \ldots, T$. As explained above, the treatment of Σ_t and Q_t in the model avoids the need for Gibbs sampling (and potentially Metropolis Hastings) algorithm, and thus provides a time-manageable way of estimating large TVP-VARs.

For a detailed discussion and derivation of all the equations in the Carter and Kohn algorithm see e.g. Kim & Nelson (1999). But the key step involves the

[8]For a summary and comparison of multi- and single-move algorithms, see Koop & Potter (2011).

[9]Other algorithms are also possible, e.g., Durbin & Koopman (2002) algorithm.

result that

$$p(\widetilde{\beta}_T|\widetilde{y}_T) = p(\beta_T|\widetilde{y}_T)\prod_{t=1}^{T-1} p(\beta_t|\beta_{t+1}, \widetilde{y}_T)$$

Therefore, a draw of $\widetilde{\beta}_T = (\beta_1', \beta_2', \ldots, \beta_T')'$ from its posterior distribution can be obtained by first drawing β_T from $p(\beta_T|\widetilde{y}_T)$, and then, for $t = T-1, \ldots, 1$ draw β_t from $p(\beta_t|\beta_{t+1}, \widetilde{y}_T)$. It can be shown[10] that

$$\beta_T|\widetilde{y}_T \sim \mathcal{N}(\beta_{T|T},\, P_{T|T})$$

and

$$\beta_t|\beta_{t+1}, \widetilde{y}_T \sim \mathcal{N}(\beta_{t|t,\beta_{t+1}},\, P_{t|t,\beta_{t+1}}) \text{ for } t = T-1, \ldots, 1$$

The mean and variance of $\beta_T|\widetilde{y}_T$ are the last Kalman filter estimates of the state variable and its variance, respectively, obtained from the Kalman filter. Mean and variance of $\beta_t|\beta_{t+1}, \widetilde{y}_T$ can also be computed using $\beta_{t|t}$ and $P_{t|t}$ from the Kalman filter. They are computed backwards from $t = T-1$ to 1 using the following recursions:

$$\beta_{t|t,\beta_{t+1}} = \beta_{t|t} + \lambda_{t+1}(\beta_{t+1} - \beta_{t|t}) \tag{4.10}$$

$$P_{t|t,\beta_{t+1}} = (1 - \lambda_{t+1})P_{t|t} \tag{4.11}$$

where $\beta_{t|t}$ and $P_{t|t}$ for $t = 1, \ldots, T-1$ are available from the Kalman filter, λ_{t+1} is obtained according to equation 4.7, and β_{t+1} is the draw of the state variable obtained in the previous step of this procedure. That is, for $t = T-1$, $\beta_{t+1} = \beta_T$ which was drawn from a normal distribution with mean $\beta_{T|T}$ and variance $P_{T|T}$; for $t = T-2$, $\beta_{t+1} = \beta_{T-1}$ which was drawn from a normal distribution with mean $\beta_{T-1|T-1,\beta_T}$ and variance $P_{T-1|T-1,\beta_T}$, etc. Equations 4.10 and 4.11 follow from the general Carter and Kohn recursions fitted to match our problem (for derivations, see the Appendix).

[10]See e.g. Kim & Nelson (1999).

4.4 Impulse Response Analysis

In this subsection we describe how to use draws of β_t, $t = 1, \ldots, T$, to calculate impulse responses. Since we are working with TVP-VARs, we would not have enough computer memory to store the draws of impulse responses for each t, therefore we decided to compute impulse responses only for selected time periods. Those time periods were chosen to include both stable periods where economic policy uncertainty is low, and those of high uncertainty that correspond to the major events in the recent financial crisis and subsequent recession. Periods of low uncertainty are chosen to be pre-crisis November 2006 and July 2007. High-uncertainty periods are represented by October 2008 (the month right after the collapse of Lehman Brothers) and August 2011, in which high uncertainty originating from the summer 2011 debt ceiling crisis, i.e., disputes in the Congress over raising of the debt ceiling that has been normally raised automatically, escalated. In addition to those, we also included December 2013 because it marks the turnover in quantitative easing (QE) policies of the Federal Reserve; on December 18, 2013 the Federal Open Market Committee announced its first QE tapering. Finally, we included the most recent period in our sample, April 2017, with economic policy uncertainty building up again which some observers attribute to the Donald Trump's election.

Economic policy uncertainty is measured by the U.S. Economic Policy Uncertainty Index which was constructed by Baker *et al.* (2015). The index is calculated based on occurrences of words related to economic uncertainty in articles in 10 leading U.S. newspapers.

Figure 4.1 displays the path of the index for the whole sample period and highlights the dates for which we calculated impulse responses. We can see that the period of maximum uncertainty corresponds to the debt-ceiling battle in August 2011 and the second highest marked peak to the aftermath of the Lehman Brother's collapse. On the other hand, November 2006 and July 2007 represent the periods of minimal uncertainty.

Next we show how to obtain impulse responses in the context of our model. The procedure is the same as with standard VARs, except now the coefficients are changing over time, so we will have different impulse response functions in each time, and the models[11] are changing over time, so in each time, we get the

[11] A model is determined by its dimension and the value of the shrinkage parameter γ; see subsection 4.2.2.

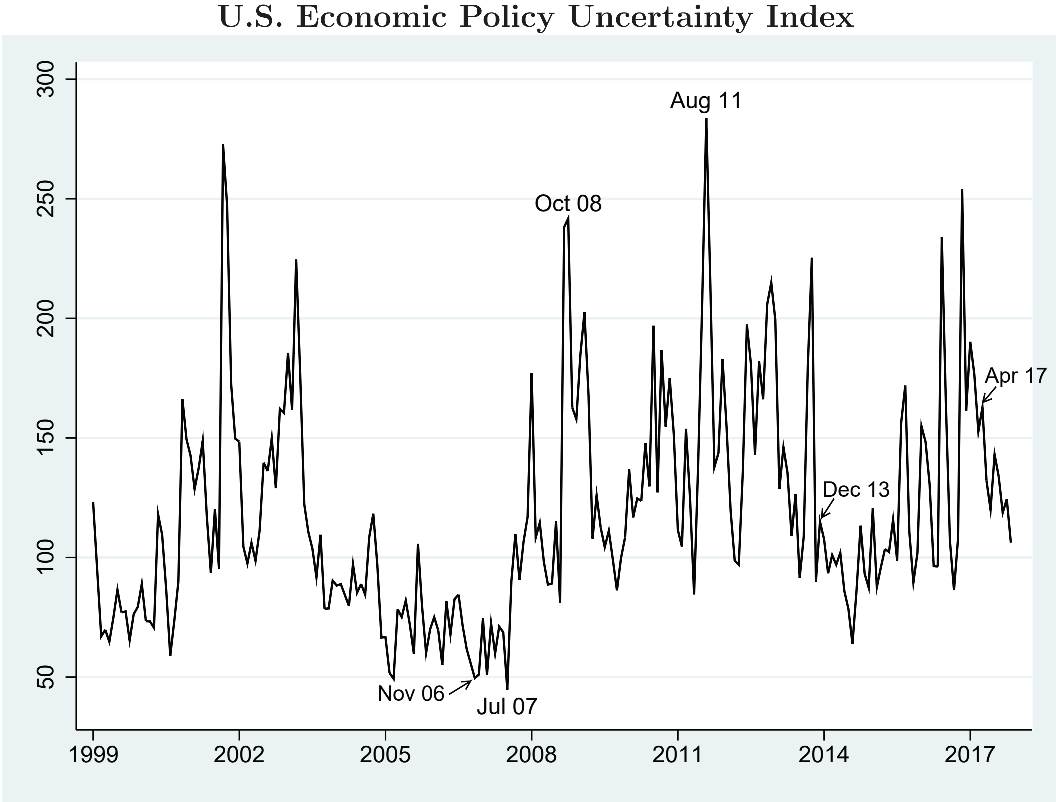

Figure 4.1: Monthly Economic Policy Uncertainty Index from January 1999 to November 2017. Highlighted periods correspond to those for which we obtain impulse responses and FEVD. *Source:* Data were downloaded from `policyuncertainty.com/us_monthly.html`

impulse responses of the "best" model, i.e. the one with the highest posterior model probability (see 4.2.2).

Consider impulse responses only for one particular time t, $t \in (1, \ldots, T)$. Suppose that the number of variables of the "best" model in time t is M, $M \in (4, 6, 20)$ depending on whether the "best" model in time t is the small, medium, or large TVP-VAR. Following Lütkepohl (2007) with his notation adjusted for time variation in the coefficients, we can write TVP-VAR as:

$$Y_t = \boldsymbol{\mu}_t + \mathbf{A}_t Y_{t-1} + E_t \tag{4.12}$$

where

$$Y_t := \underset{(Mp\times 1)}{\begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \end{bmatrix}}, \qquad \boldsymbol{\mu}_t := \underset{(Mp\times 1)}{\begin{bmatrix} \mu_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}},$$

$$\mathbf{A}_t := \underbrace{\begin{bmatrix} A_{1t} & A_{2t} & \cdots & A_{p-1t} & A_{pt} \\ I_M & 0 & \cdots & 0 & 0 \\ 0 & I_M & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_M & 0 \end{bmatrix}}_{(Mp\times Mp)}, \quad E_t := \underbrace{\begin{bmatrix} \epsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{(Mp\times 1)}.$$

A_{it} for $i = 1, \ldots, p$ are $M \times M$ matrices of coefficients on the $i-th$ lag; in the first row, they include the coefficients on the $i-th$ lag for all M variables in the first equation, in the second row, they include the coefficients on the $i-th$ lag for all M variables in the second equation, etc. Matrices $\mathbf{A}_t$ and $\boldsymbol{\mu}_t$ can be completely determined from our vector of coefficients β_t obtained from the Carter and Kohn algorithm.

Provided that the process Y_t is stable[12], 4.12 has a moving-average (MA) representation (without deterministic terms as they are irrelevant for the impulse response analysis):

$$Y_t = \sum_{i=0}^{\infty} \mathbf{A}_t^i E_{t-i}$$

If we premultiply this by $J := [I_M : 0 : \cdots : 0]$, which is $(M \times Mp)$ matrix, we get the MA representation of y_t

$$y_t = \sum_{i=0}^{\infty} J\mathbf{A}_t^i J' J E_{t-i}$$

Following Lütkepohl (2007), this can be rewritten as

$$y_t = \sum_{i=0}^{\infty} \Phi_{it} \epsilon_{t-i} \tag{4.13}$$

if we define $\Phi_{it} := J\mathbf{A}_t^i J'$, because $E_t = J'JE_t$ and $JE_t = \epsilon_t$.

It can be shown[13] that the MA coefficients Φ_{it} contain the impulse responses to the innovations in ϵ_t. However, this representation of impulse responses assumes that the shocks in y_t variables are independent, which may not be true. Therefore, we decided to use orthogonalized impulse responses which, as the name

[12] In 4.12, Y_t is stable if all eigenvalues of $\mathbf{A}_t$ have modulus less than one.
[13] See Lütkepohl (2007) on page 52.

suggests, are responses to orthogonal (i.e. independent) shocks. Those responses are obtained if we manipulate the MA representation 4.13 as

$$y_t = \sum_{i=0}^{\infty} \Phi_{it} P_t P_t^{-1} \epsilon_{t-i} = \sum_{i=0}^{\infty} \Theta_{it} w_{t-i} \tag{4.14}$$

where $\Theta_{it} := \Phi_{it} P_t$ and $w_t = P_t^{-1} \epsilon_t$. P_t is a lower triangular matrix obtained from Cholesky decomposition of Σ_t, i.e. the covariance matrix of TVP-VAR errors: $\Sigma_t = P_t P_t'$. It follows that w_t is white noise with the covariance matrix $\Sigma_{wt} = P_t^{-1} \Sigma_t (P_t^{-1})' = P_t^{-1} P_t P_t' (P_t^{-1})' = I_M$, therefore impulse responses Θ_{it} will be responses to orthogonal shocks. The difference with TVP-VARs with stochastic volatility is that P_t is not time-invariant because Σ_t is changing over time, and Φ_{it} are not time-invariant because the VAR coefficients are changing. The elements of the matrix Θ_{it} represent the responses to unit innovations in w_t. Hence, the $jk-th$ element of Θ_{it}, $\theta_{jk,it}$, can be interpreted as the response of variable j to a unit innovation in variable k that occurred i periods ago. Moreover, the impulse responses can be computed even if the MA representation 4.14 does not exist (Hashimzade & Thornton (2013)), which occurs e.g. in case if VAR is not stable in which case the shocks might permanently affect the variables of the system.

The matrix of contemporaneous impulse responses is $\Theta_{0t} = \Phi_{0t} P_t = P_t$, because $\Phi_{0t} = J\mathbf{A}_t^0 J' = I_M$. It follows from the lower triangular nature of P_t that the first variable in the system can contemporaneously respond only to its own shocks, the second variable can contemporaneously respond only to the shocks in the first variable and its own shocks, ..., k-th variable can contemporaneously respond only to shocks in variables $1, \ldots, k$. This is a restrictive assumption because, in the empirical part, we could not have e.g. asset prices contemporaneously responding to monetary policy shocks and at the same time monetary policy immediately responding to asset prices, but some studies (e.g., Bjø rnland & Jacobsen (2013)) found that this interdependence is vital for revealing the role of asset prices in the monetary-policy transmission mechanism.

We estimated the impulse responses Θ_{it} by replacing Σ_t by its estimate $\widehat{\Sigma}_t$ from eq. 4.4 and using the draws of β_t to construct Φ_{it}. For each time period t, $t \in$ (November 2006, July 2007, October 2008, August 2011, December 2013, April 2017), we took the "best" model in that time period which is uniquely identified by TVP-VAR dimension and the shrinkage coefficient γ[14], and the draws of β_t and estimates $\widehat{\Sigma}_t$ used to construct the impulse responses are for that model. Because

[14]See section 4.2.2

the draws of β_t are produced by Carter and Kohn algorithm, we get the whole vector of β_t, $t = 1, \ldots, T$, at one draw but we use only the coefficients in time periods in which the impulse responses are calculated. Moreover, it follows that the dimension of the matrix that contains impulse responses for a particular time t and impulse response horizon i, Θ_{it}, can differ among t based on the TVP-VAR dimension of the winning model in period t.

4.5 Forecast Error Variance Decomposition

Given the impulse response estimates for t, $t \in$ (November 2006, July 2007, October 2008, August 2011, December 2013, April 2017), FEVD at time t can be calculated as follows[15]. Let $\omega_{jk,ht}$ denote the proportion of the h-step forecast error variance of variable j that can be explained by innovations in variable k (Lütkepohl (2007)), given the time period t. It holds that

$$\omega_{jk,ht} = \frac{\sum\limits_{i=0}^{h-1} \theta_{jk,it}^2}{\sum\limits_{i=0}^{h-1} \sum\limits_{m=1}^{M} \theta_{jm,it}^2} \tag{4.15}$$

where $\theta_{jk,it}$ is the jk-th element of the impulse response matrix Θ_{it}, and thus represents the response of variable j to a shock in variable k that occurred i periods ago, and M is the number of variables in the TVP-VAR which depends on the dimension of the winning model in time t. Note that if we sum up $\omega_{jk,ht}$ for $k = 1, \ldots, M$, we get one, i.e., the forecast error variance of variable j can be fully explained by innovations in variables of the system (including the innovations in variable j itself). FEVD in time t are calculated for each matrix of impulse response coefficients Θ_{it}, and therefore for each draw of β_t.

[15] In TVP-VARs, FEVD is time-varying because VAR coefficients (and therefore impulse responses) are time-varying. Therefore the time periods in which we calculate FEVD are the same as those in which we provide impulse responses.

Chapter 5

Results

This chapter presents the results of our analysis whose main aim is to assess the link between house prices and the stance of monetary policy. First, we describe the main characteristics of the employed model and support them with some empirical evidence. Then, we provide impulse responses and forecast error variance decompositions for several chosen time periods. Finally, results of the robustness checks are revealed.

5.1 TVP-VAR with dynamic dimension selection

In this section, estimation results on some important aspects of using a time-varying parameter VAR whose dimension can change over time, are presented. First, we briefly summarize the main characteristics of this model[1 2]. The model follows Koop & Korobilis (2013) in that it features both time-varying VAR coefficients and time-varying covariance matrices[3], while the dimension of a model can also change over time based on the past predictive likelihoods of different sized models. Time variation in VAR coefficients is controlled by a forgetting factor λ_t which is estimated in each point in time and the degree of switching between TVP-VAR models of different dimensions is controlled by another forgetting factor, α. Basically, we select among 3 TVP-VARs: small VAR with 4 variables, medium with 6 variables, and large VAR that contains 20 variables[4]. Each time, posterior model probabilities are computed for each model and the

[1] Koop & Korobilis (2013) named such a model 'TVP-VAR-DDS' where 'DDS' stands for 'dynamic dimension selection'.

[2] For a detailed description of the estimation procedure see Chapter 4.

[3] Specifically, covariance matrix of VAR errors and covariance matrix of VAR coefficients.

[4] See Table 3.1 for variables included in each model.

one with the highest probability is selected. Posterior model probabilities are determined based on the past predictive likelihood of the models, and α controls the weight of those past likelihoods. The key difference and a novelty of Koop & Korobilis (2013) approach is that covariance matrices are not sampled from their posterior distributions, but they are estimated. Measurement error covariance matrix is estimated by an EWMA estimator, and covariance matrix that controls the volatility of VAR coefficients is approximated by the use of the above mentioned forgetting factor λ_t.

Besides the selection of one TVP-VAR dimension in each point in time, Koop & Korobilis (2013) decided to augment the model further by choosing among 7 predefined values for the prior shrinkage coefficient γ. This coefficient controls a degree by which VAR coefficients are pushed to zero so that the model with a large number of parameters can be estimated without the fear of overparameterization. Shrinkage is very important in TVP-VARs because the number of parameters is much larger than with standard VAR models, and thus usually a high shrinkage is needed for obtaining reasonable results. Each time, one value of γ for each of the small, medium and large TVP-VAR is selected based again on the predictive likelihood of the models with different γ. Therefore, each time the model with the highest posterior model probability[5] can be fully characterized by its dimension and shrinkage coefficient γ. All parameters that enter the model are summarized in D.1 along with their brief description.

Figure 5.1 plots the selected values of the prior shrinkage parameter, γ, for each TVP-VAR dimension. As expected, lower values of γ are associated with TVP-VAR of the largest dimension, implying that VAR coefficients are more centered around their prior means of zero (lower γ means lower variance and thus higher shrinkage, see equation 4.3). Interestingly, with medium and small TVP-VARs, a necessary degree of shrinkage exhibits a large drop shortly after 2008 and then continues to decline[6] which suggests that the (prior) variance of VAR coefficients needs to be larger from 2008 onwards that could be attributed to increased volatility following a financial crisis. Minimum shrinkage is attained by both medium and small TVP-VARs at the end of the sample, while the large TVP-VAR does not go above $\gamma = 0.001$ during the sample period.

[5] "winning" model

[6] Rising γ implies a declining shrinkage.

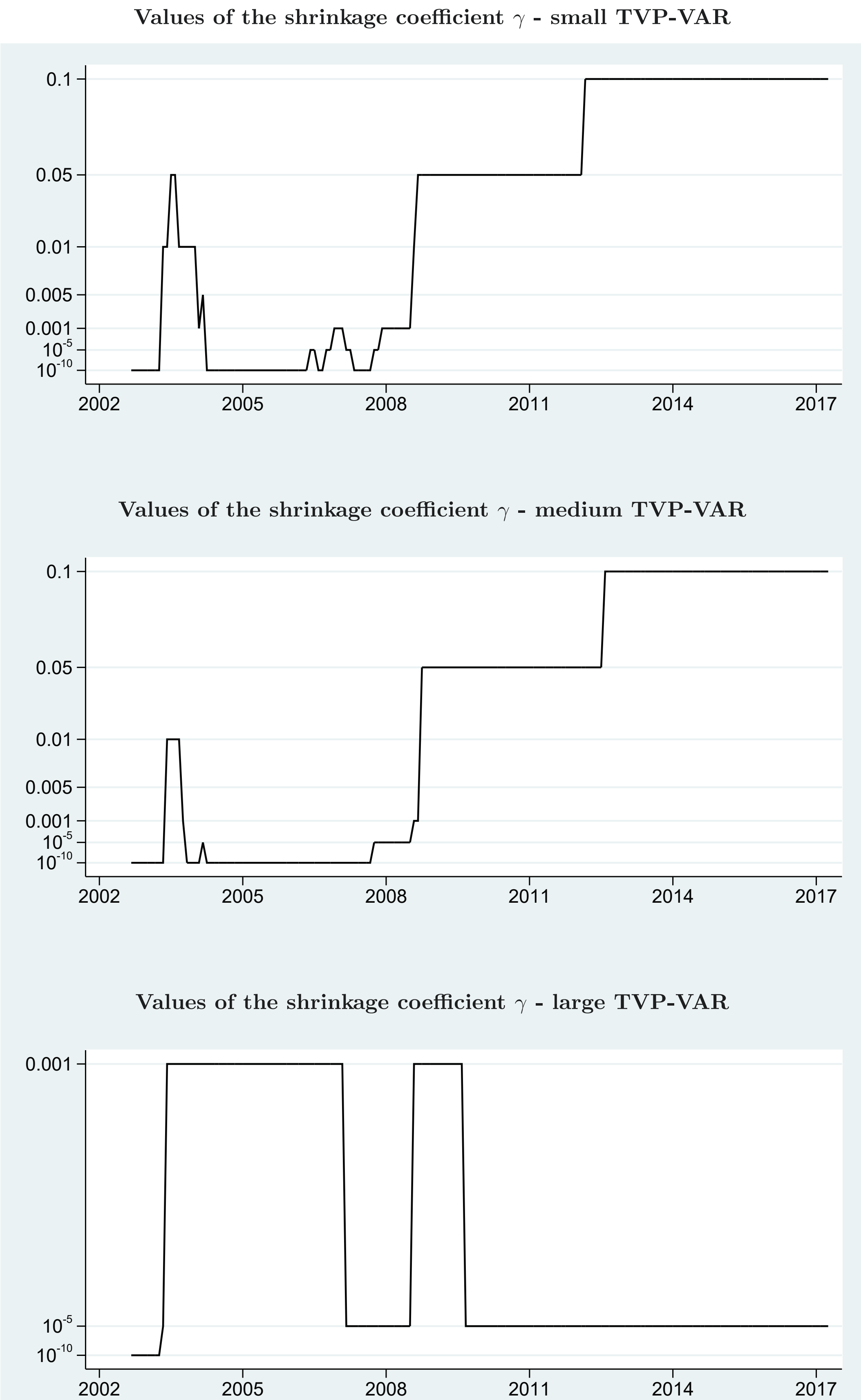

Figure 5.1: Values of the optimal prior shrinkage parameter γ for each TVP-VAR dimension and time period.

Values of λ_t that controls the degree of time-variation in VAR coefficients are displayed in Figure 5.2 in case of the small TVP-VAR[7]. Following Koop & Korobilis (2013), λ_t is allowed to vary over the interval from 0.96 to 1 which should induce a gradual change in the coefficients. Higher values of λ_t correspond to lower changes in VAR coefficients (see equation 4.6). We can see that in the beginning and at the end of the sample, coefficients are changing less than in the middle of the sample. The highest changes are occurring between 2008 and 2014 and can be associated with unstable periods of high policy uncertainty (see Figure 4.1).

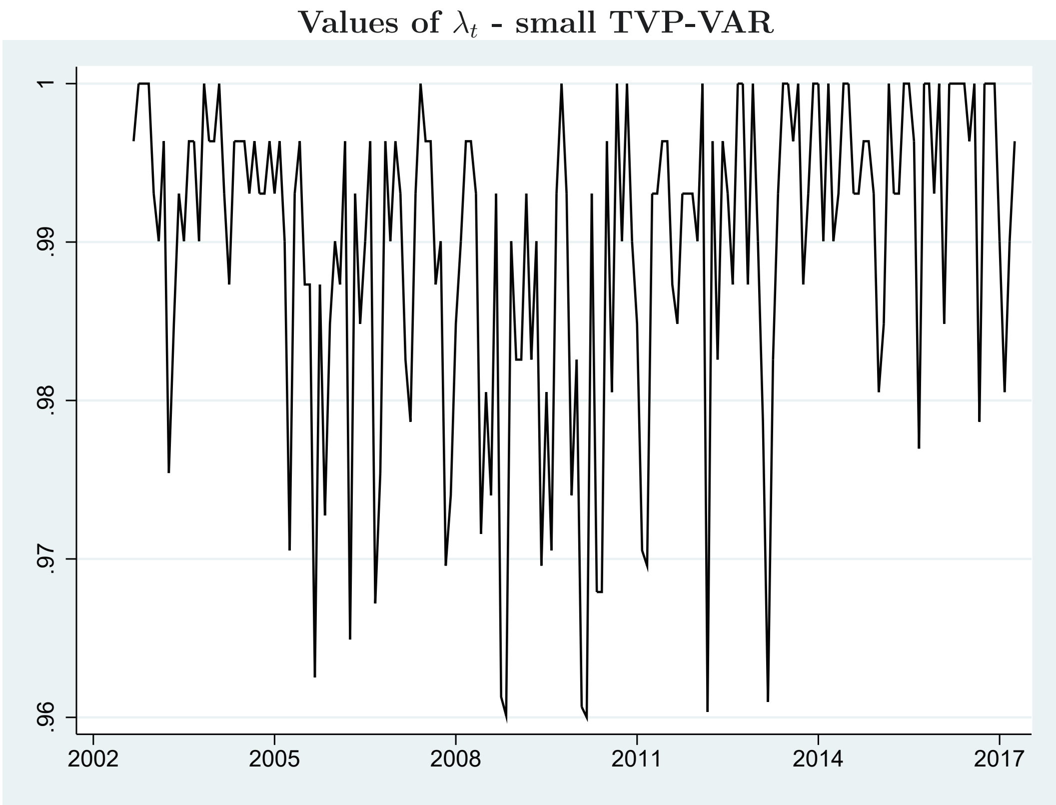

Figure 5.2: Estimated λ_t for the small TVP-VAR with the optimal shrinkage parameter γ in each time.

To select between small, medium and large TVP-VAR for which we already determined the optimal shrinkage γ[8], posterior model probabilities must be evaluated according to equation 4.9. Those probabilities are plotted for each TVP-VAR size (with the optimal shrinkage coefficient) in Figure 5.3. It follows immediately from the figure that TVP-VAR dimension changes over time (mostly between the

[7]Values of λ_t for medium and large TVP-VARs exhibit similar patterns.
[8]See Figure 5.1.

small and the large model). We can also conclude that bigger VARs perform better in stable times prior to 2008, but from 2008 onwards, following a surge in small model's posterior probability, small and medium models are preferred.

Time-varying dynamic dimension selection probabilities

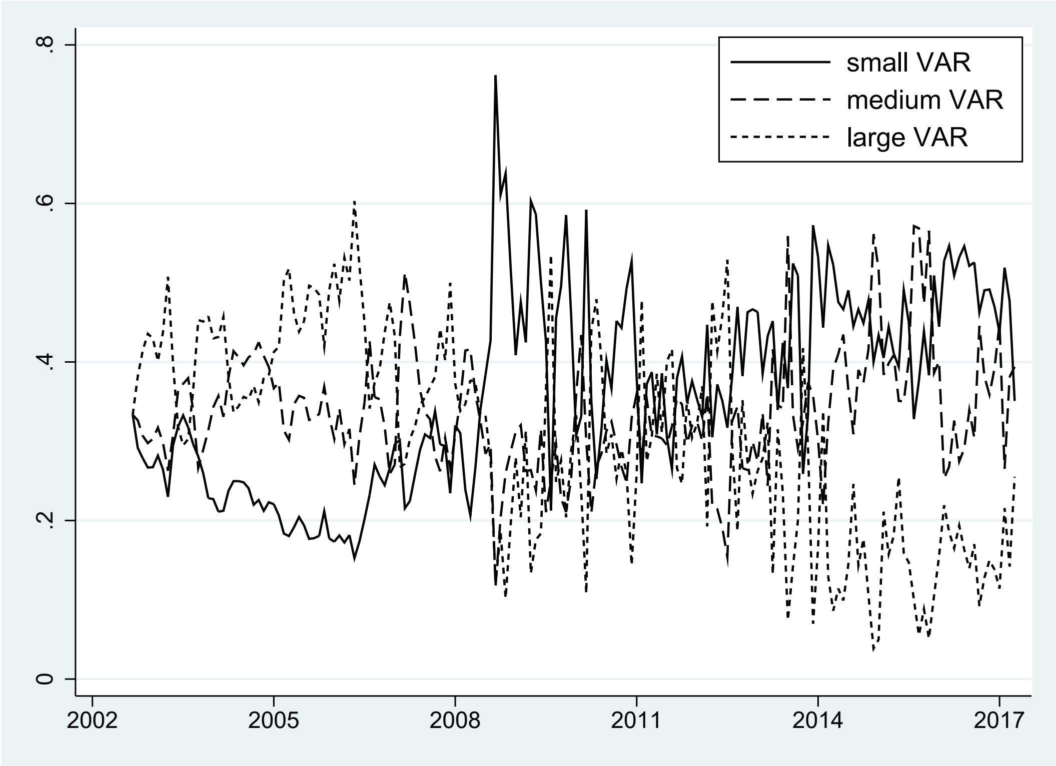

Figure 5.3: Dynamic dimension selection probabilities of the small, medium and large TVP-VARs. Each time, the optimal value of the shrinkage parameter γ was chosen for each TVP-VAR and the probability plotted is for that γ.

Due to the time-varying nature of the model, we present empirical evidence only for selected time periods. Those time periods were chosen to correspond with remarkable events that influenced the path of macroeconomic variables[9]. Based on the selected model and its shrinkage coefficient, we use 5 different TVP-VAR models to study the behavior of house prices after a monetary policy shock. Those time periods and the respective "winning" models are presented in Table 5.1.

[9]The selection of time periods is discussed in Section 4.4.

Time period	TVP-VAR dimension	Shrinkage γ
November 2006	Large	0.001
July 2007	Large	10^{-5}
October 2008	Small	0.05
August 2011	Large	10^{-5}
December 2013	Small	0.1
April 2017	Medium	0.1

Table 5.1: Models with the highest posterior model probability in each time period.

5.2 Impulse responses, FEVD

Here, we present impulse responses of house prices to a negative, one percentage point shock in Wu and Xia shadow rate. House prices are measured by S&P/Case-Shiller U.S. National house price index. Since we have a different VAR coefficients in each time, we would have a different set of impulse responses for each month in our sample which would be difficult to store given that we need to take a certain number of draws from their posterior distribution for each time. Therefore, impulse responses are computed only for 6 time periods. Number of draws from the posterior distribution of coefficients is set to 10 000[10]. We do not need to discard any draws, as it is recommended when using Gibbs sampling, because we employed the Carter and Kohn algorithm to draw from the posterior of VAR coefficients, which was possible since we do not need to draw covariance matrices[11]. Another modeling choice involves the ordering of variables which matters since we are using a Cholesky decomposition to identify a monetary policy shock, and therefore a position of the policy rate[12] among other variables determines which variables will be contemporaneously affected by the shock. We ordered the policy rate last to allow for an immediate response of monetary policy to shocks in other variables. This is a standard assumption in macroeconomic VAR literature. It follows that according to this ordering, house prices react with a lag to monetary policy shocks. This is not always (but arguably predominantly) assumed in the literature as some papers stress the importance of allowing for simultaneous reaction of house prices and monetary policy (e.g., Bjørnland and Jacobsen, 2013). That is, house prices should be allowed to respond contemporaneously to mone-

[10] Although, setting it to 1000 does not influence the results.
[11] See Chapter 4.
[12] Wu and Xia rate or Krippner's SSR in the robustness checks.

tary policy shocks and, at the same time, monetary policy should not be restricted from immediately responding to house price shocks. This can be accomplished by employing various long-run or sign restrictions, i.e., a different identification scheme. In robustness checks[13], we provide impulse responses with house prices ordered below the policy rate which allows for a contemporaneous reaction of house prices to a monetary policy shock.

Figures 5.4 plot the impulse responses of S&P/Case-Shiller house price index to an expansionary monetary policy shock. Impulse responses were rescaled to correspond to a minus one percentage point shock in Wu and Xia rate. The graph shows posterior median impulse responses along with their probability bands represented by 16^{th} and 84^{th} percentiles. Impulse response horizon (on the x-axis) is in months. For November 2006, July 2007 and August 2011, the impulse responses display exploding behavior, even though the median impulse response stays slightly above zero. This is caused by the fact that in those time periods, the winning model used to generate the impulse responses is the large VAR with the value of shrinkage coefficient 10^{-5} or 0.001. Unfortunately, large VAR was not able to produce stable draws under several different specifications unless, as will soon become clear, we would increase the value of the shrinkage coefficient γ. However, higher γ translates into higher variance of VAR coefficients, and thus less shrinkage to zero. Therefore, the coefficients (not just at higher lags, see equation 4.3) become more important and the computational burden so immense that it is not computationally feasible to draw from the posterior of coefficients. In the remaining time periods, impulse responses were obtained from a small VAR for October 2008 and December 2013, and from a medium VAR for the most recent period in our sample, i.e., April 2017. All those models feature a low degree of shrinkage as is apparent from Table 5.1.

Median responses of house prices generated by small and medium TVP-VARs are positive, around 4% after approximately 3 years, but the effect is insignificant in December 2013 which is a period of QE tapering. In this period, the confidence bands are wider which could be a signal of higher uncertainty. However, Figure 4.1 reveals that even though there is a local peak in policy uncertainty in December 2013, this uncertainty was even larger in April 2017 and much higher in October 2008 for which the impulse responses are significant[14]. Still, economic policy

[13]See Section 5.3

[14]Impulse responses in October 2008 become significant after 6 months and in April 2017 after 8 months.

uncertainty index, as it is designed[15], puts more weight on events that are arguably not so important for the developments in the main macroeconomic variables[16], for example, it creates as much as uncertainty following the Trump's inauguration in the beginning of 2017 as it was during the financial crisis, even though the economic conditions were remarkably better in 2017. Therefore, the index may not fully capture the uncertainty about the future movements of monetary policy and some kind of monetary policy uncertainty index could be more relevant in the context of our model.

[15]See section 4.4.
[16]Except for the widely-recognized critical events such as the collapse of Lehman Brothers.

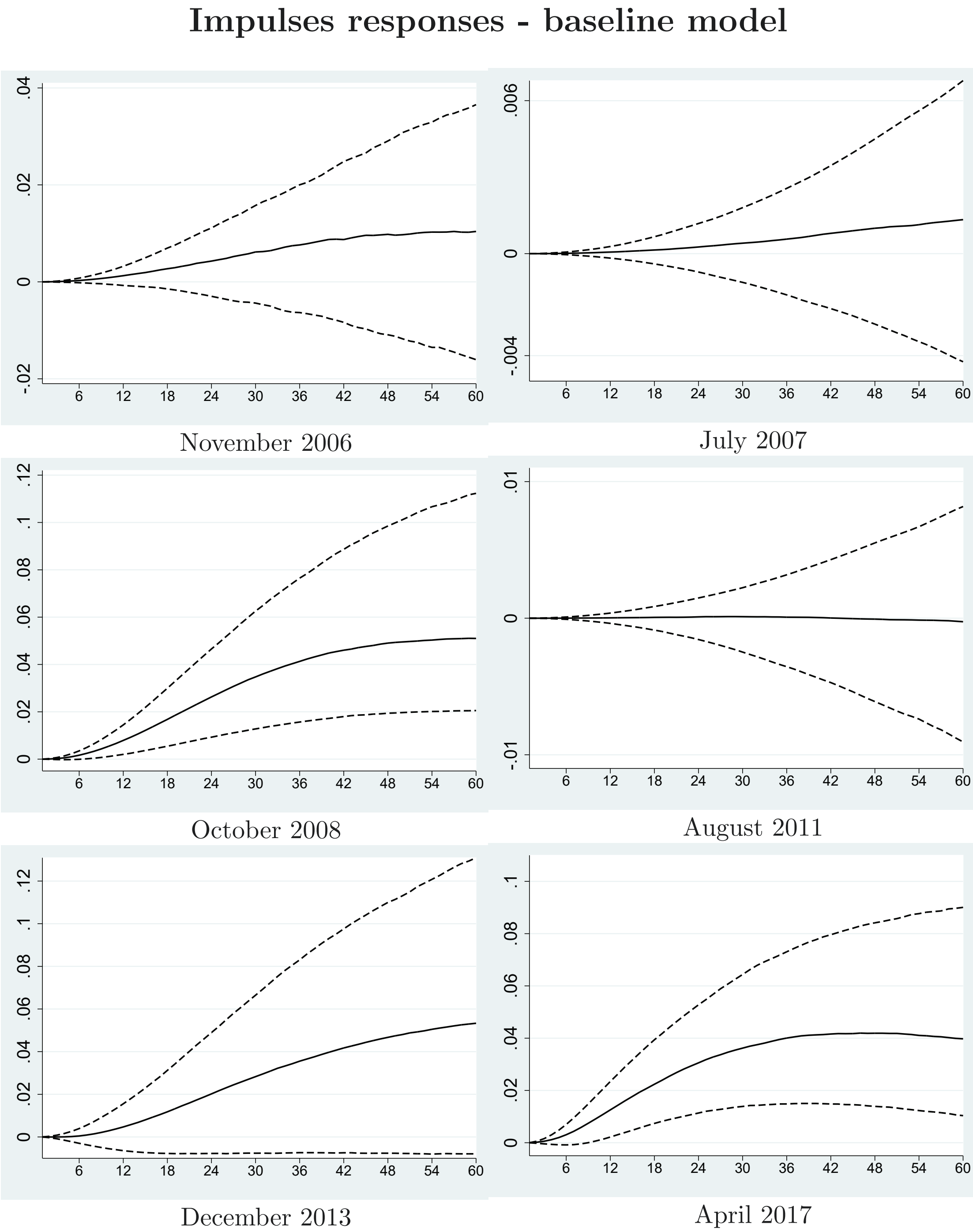

Figure 5.4: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Wu-Xia rate. In November 2006, July 2007 and August 2011, impulse responses were generated by large TVP-VAR. In October 2008 and December 2013, impulse responses are from the small TVP-VAR, and in April 2017, they were generated by medium TVP-VAR.

To extend our discussion about the importance of the shrinkage coefficient γ,

Figure 5.5 depicts the impulse responses of house prices to the same negative 100-basis-point shock in Wu-Xia rate generated by medium VAR for the two extreme choices of γ. Impulse responses were plotted for November 2006 and July 2007. Those are the periods in which the large VAR is the winning model that, however, produces strange impulse responses. Therefore, we decided to use a medium VAR in those periods[17] and experiment with a different values of γ. The left-hand-side panel shows impulse responses when γ is set to its minimum, $\gamma = 10^{-10}$, implying a maximal degree of shrinkage in VAR coefficients, whereas the right-hand side sets γ to its maximum, $\gamma = 0.1$. Values of γ near the minimum are typical for a large VAR due to the presence of many coefficients.

The figure makes it apparent that choosing the maximum degree of shrinkage produces very strange results, even with a medium VAR, similar to those produced by a large VAR. This may be because γ is simply too small: prior variance of coefficients in equation 4.3 is then very low, making the coefficients to be tightly centered around their prior means of zero, and if there is not enough information in the data to outweigh it, those coefficients will be essentially zero. That could result in essentially zero impulse responses, as can be seen in the LHS panel of the figure, even though the confidence bands are widening. This may be one explanation for the behavior of impulse responses from a large VAR captured in Figure 5.4. However, we were not able to produce the impulse responses from a large VAR with high values of γ because it would require a huge computational power. Therefore, we replace a large VAR in periods when it is the winning model by a medium VAR with the highest possible value of γ and provide the impulse responses for that model.

Figure 5.6 shows the corresponding impulse responses. In October 2008, December 2013 and April 2017, the models are the same as the winning models in those periods. However, for the other periods, large VAR was replaced by a medium VAR with $\gamma = 0.1$. Responses of house prices in stable periods of November 2006 and July 2007 are significantly greater than zero right after the expansionary monetary policy shock. They are also gradually increasing till they reach the maximum of around 2% (for the median impulse response) about two and a half years from the shock. Responses become significant after some initial period for all dates, except for December 2013 which was already discussed above.

[17] And in August 2011 which is another period in which large VAR was originally selected. Results from August 2011 are not presented here because they are qualitatively similar and will be assessed in the following analysis for the maximum value of γ.

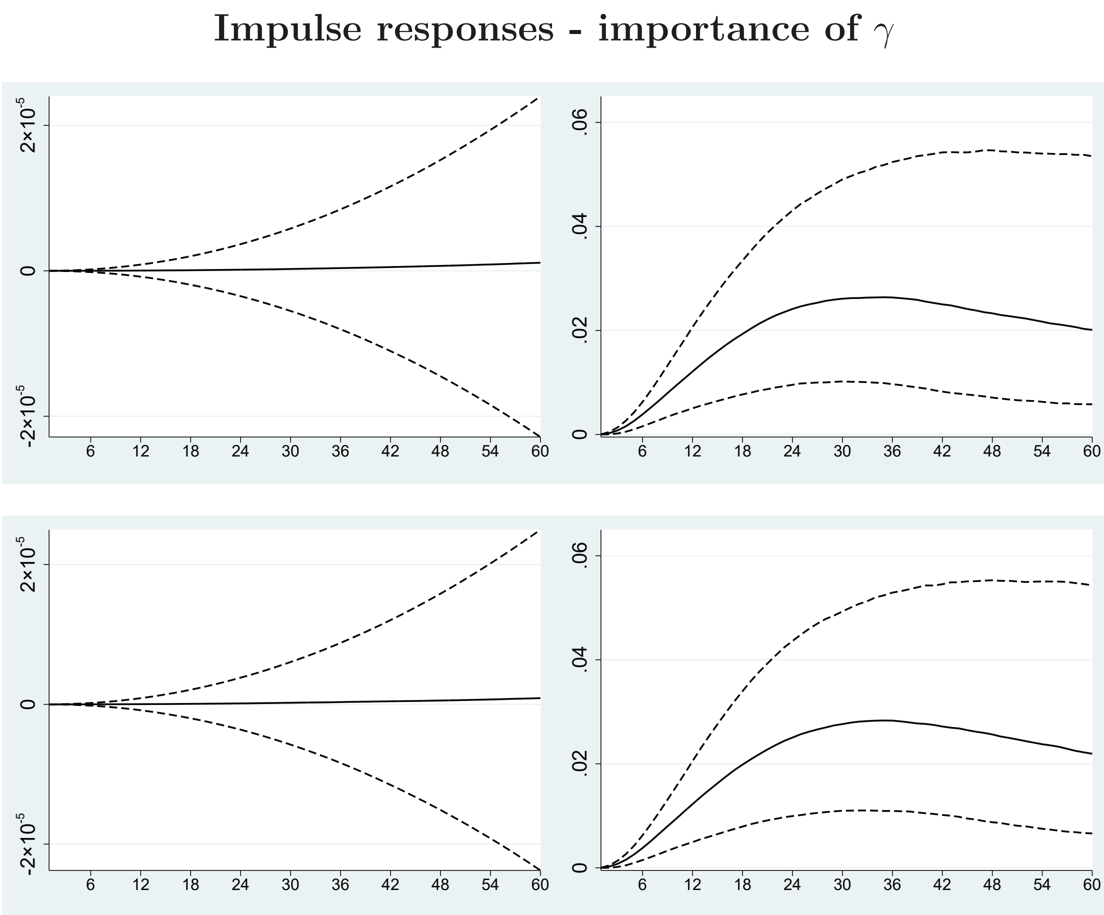

Figure 5.5: LHS panel shows the impulse responses of house prices to a negative 100-basis-point shock in Wu-Xia rate from medium TVP-VAR with shrinkage coefficient $\gamma = 10^{-10}$. RHS panel displays the corresponding impulse responses, but now the medium TVP-VAR has shrinkage coefficient $\gamma = 0.1$. First row: impulse responses in November 2006, second row: impulse responses in July 2007.

For October 2008, August 2011 and April 2017, median IRFs are stronger, around 4% after approximately 3 years. In all figures, the response of house prices seem to be quite persistent which has been also found in the literature (e.g., Bjørnland and Jacobsen, 2013). A delayed and persistent response of house prices is usually being explained by lengthy construction processes that makes property prices different from other assets (Eickmeier & Hofmann (2010)).

The results from November 2006 and July 2007 are comparable with Smets & Jarociński (2008) who also used S&P/Case-Shiller house price index and found a mean response of -0.5% after two and a half years following a positive 25 basis-point shock in the federal funds rate[18]. However, they used a mixture of zero and sign restrictions to identify the monetary policy shock, but they also restrict house prices from responding contemporaneously to monetary policy shocks. Results from October 2008, August 2011 and April 2017 can be qualitatively related to those of Eickmeier & Hofmann (2010), who estimated a FAVAR model on the U.S. data and obtained a median impulse response of S&P/Case-Shiller house price index of -5% after approximately four years following a contractionary monetary policy shock in the form of a 100-basis-point increase in the federal funds rate. The response was also found to be significant from the beginning and persistent.

Next, we present the forecast error variance decomposition of house prices, i.e., the contribution of different variables to the forecast error variance in house prices. Figures 5.7, 5.7 and 5.9 plot the results for three time periods: October 2008, December 2013 and April 2017. Those periods were chosen because these are the three periods from our results evaluating times for which small or medium TVP-VARs were selected as 'winning' models[19]. In all graphs, mean forecast error variance decompositions obtained from 10 000 draws of VAR coefficients are displayed.

The results slightly differ among the medium and small models. For small models, innovations in real GDP explain most of the variation in house prices, while for medium VAR, the contribution is divided between real GDP, residential investment and house prices. Wu and Xia rate is becoming more and more important in explaining house prices and it accounts for almost 4.3% of the variation in house prices for the longest forecast horizon given a medium VAR. Therefore, the effect of monetary policy shocks on house prices is delayed.

Tables D.2, D.3 and D.4 reveal the complete results of variance decompositions for each variable in the corresponding VAR. We focus on the medium VAR. House price shocks are there estimated to explain around 16% of the variation in real GDP in the long run and they also account for 23% of the variation in residential investment. They are also found to explain roughly 22% of the variation in Wu and Xia rate after five years, which is slightly less than the 30% found by Bjørnland and Jacobsen (2013) after the same time period with the effective federal funds rate in place of Wu and Xia rate.

[18]A negative 100-basis-point shock would therefore result in a response of 2% after the same period.

[19]See Table 5.1.

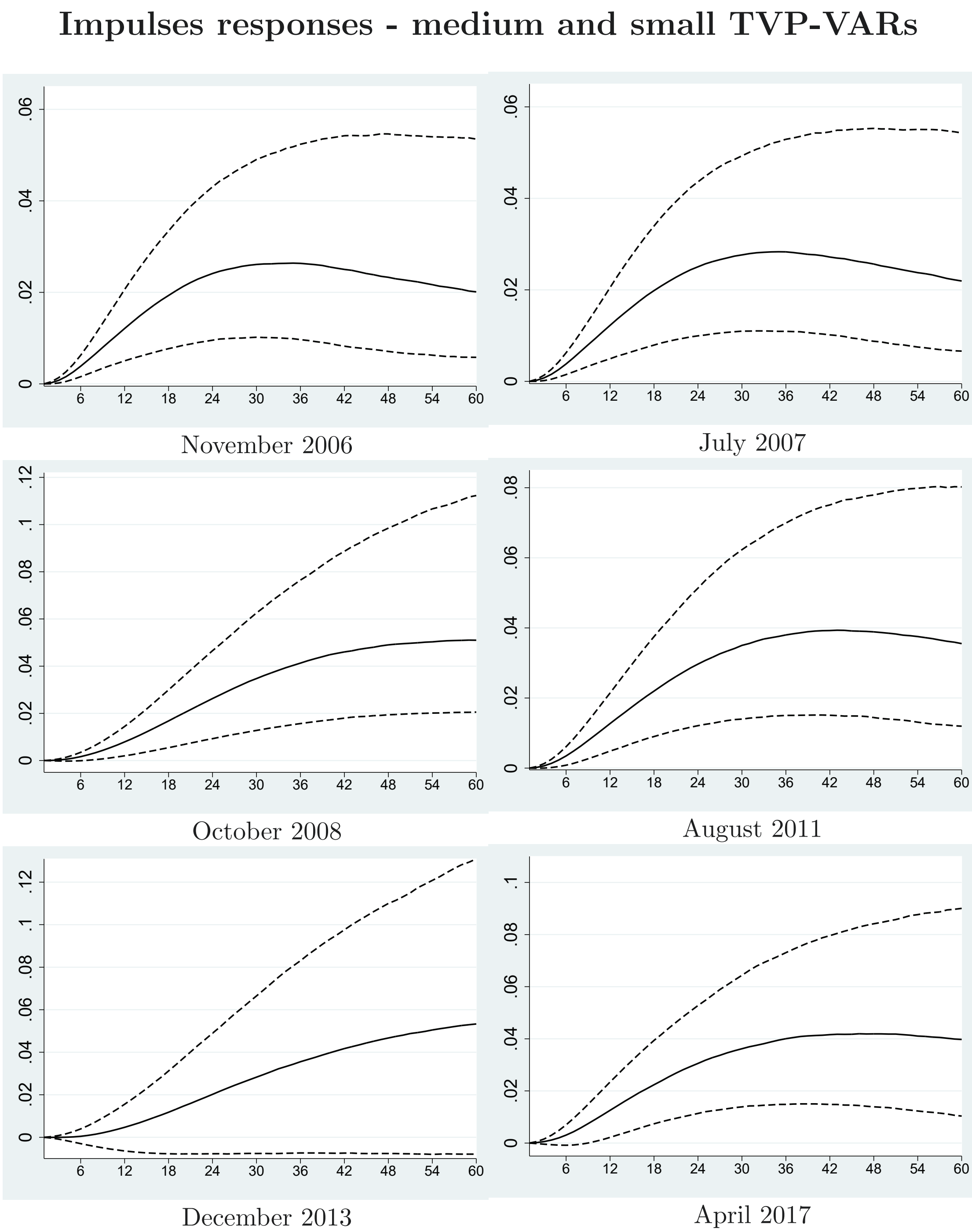

Figure 5.6: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Wu-Xia shadow rate. Responses of the large TVP-VAR were replaced by responses from the medium TVP-VAR with shrinkage coefficient $\gamma = 0.1$.

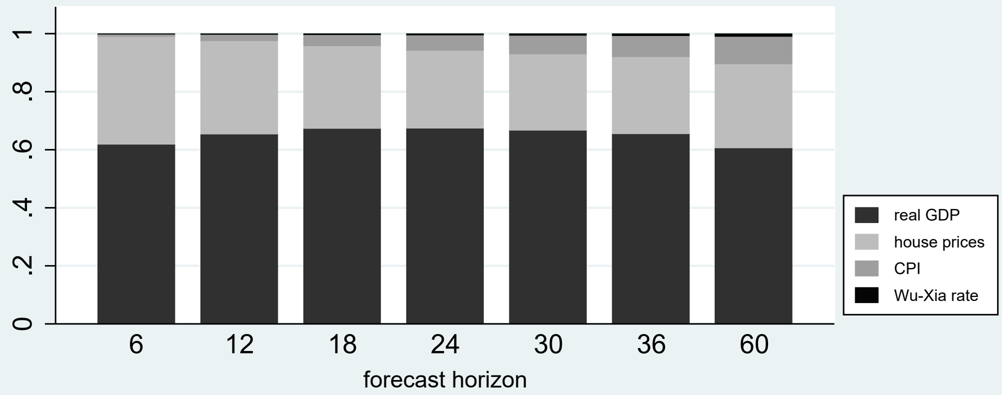

Figure 5.7: Forecast error variance decomposition of S&P/Case-Shiller house price index based on impulse response coefficients in October 2008. In this period, small TVP-VAR with $\gamma = 0.05$ is chosen as a model with highest posterior model probability, and therefore the figure shows the proportion of forecast error variance of house prices accounted for by innovations in 4 variables.

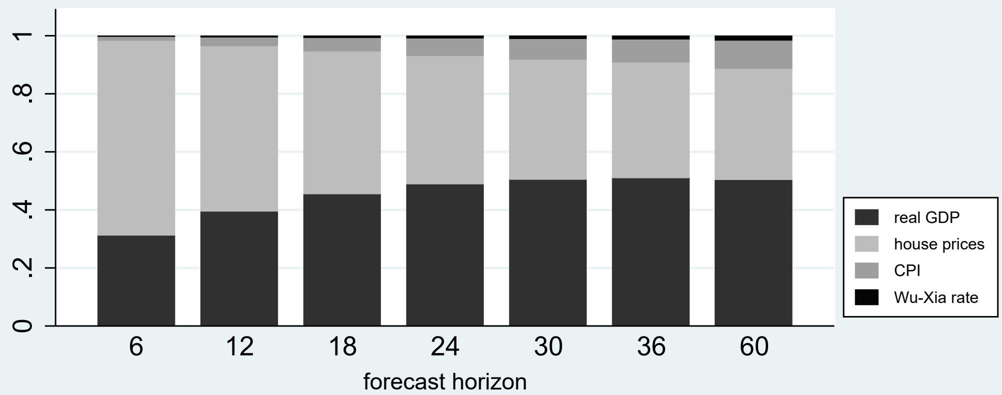

Figure 5.8: Forecast error variance decomposition of S&P/Case-Shiller house price index based on impulse response coefficients in December 2013. In this period, small TVP-VAR with $\gamma = 0.1$ is chosen as a model with highest posterior model probability, and therefore the figure shows the proportion of forecast error variance of house prices accounted for by innovations in 4 variables.

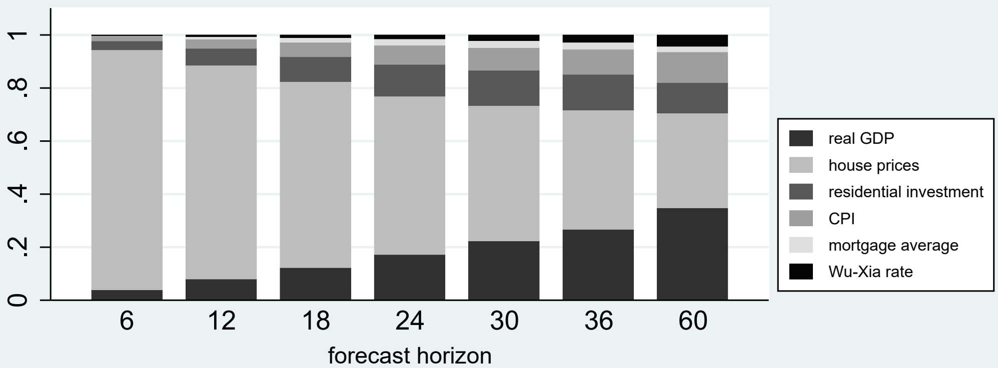

Figure 5.9: Forecast error variance decomposition of S&P/Case-Shiller house price index based on impulse response coefficients in April 2017. In this period, medium TVP-VAR with $\gamma = 0.1$ is chosen as a model with highest posterior model probability, and therefore the figure shows the proportion of forecast error variance of house prices accounted for by innovations in 6 variables.

5.3 Robustness Checks

Several robustness checks were performed, particularly for:

- Krippner's Shadow Short Rate instead of Wu and Xia shadow rate
- Industrial production index instead of real GDP that was converted to monthly figures using cubic spline interpolation in the baseline model
- Macroeconomic Advisers' monthly estimates of real GDP instead of real GDP
- FHFA house price index instead of S&P/Case-Shiller house price index
- Different ordering of variables, mainly, switching the order of house prices and policy rate so as to allow for the immediate effect of monetary policy shocks on house prices

Robustness checks are provided for the same time periods as the results for the baseline model and they use only small or medium VARs. Therefore, results from robustness checks are comparable to those in Figure 5.6. All robustness checks have a baseline (medium or small) model as a starting point and just replace one particular variable or change the ordering according to the list above. We used 10 000 draws from the posterior of VAR coefficients to obtain median impulse responses and their 16^{th} and 84^{th} percentiles. Monetary policy shock is approximated by an unexpected decline in the policy rate (Wu-Xia or Krippner's shadow rate) of 100 basis points.

Results from the first robustness check are plotted in Figure D.1. Overall, the impulse responses tell the same story as those from the baseline model (Figure 5.6). However, there are a few dissimilarities. In August 2011, a period of the highest policy uncertainty[20], and in April 2017, median impulse responses for Krippner's SSR are significantly different from zero only for around 1 year after the shock, while those obtained from Wu-Xia rate shock are significant in August 2011 for all impulse response horizons and become significant in April 2017 after approximately half a year following the shock. This may be caused by the differences in methodology used to construct both shadow rates that make the Krippner's rate more volatile and the changes in Wu-Xia rate more gradual (see

[20]See Figure 4.1.

Figure 3.2). To conclude which shadow rate should be preferred would require further analysis, however, in most applications involving some kind of a shadow rate, Wu-Xia rate is used. Nevertheless, Krippner (2015) forcefully argues that Wu-Xia rate is not robust as compared to his Shadow Short Rate, and therefore should be avoided. Forecast error variance decomposition (in Table D.6) is comparable to the one from the baseline model, except that now residential investment explains more variation in house price.

To extend the present analysis, we further compare responses to Wu-Xia rate shock and the shock to Krippner's SSR, but now using residential investment and mortgage average as the variables whose responses are examined. Figures 5.10 and 5.11 display the impulse responses of residential investment (in levels) and mortgage average (in percentage points) to a 100-basis-point negative shock in Wu-Xia and Krippner's shadow rate, respectively. Impulse responses are presented for the two periods in which the economic policy uncertainty index attains its minimum and maximum over the whole sample; November 2006 is the period with minimal uncertainty and August 2011 marks the other extreme.

Higher uncertainty is visible in the RHS panel of Figure 5.11 as it translates into much wider confidence bands. In case of Wu-Xia rate, confidence bands are also wider in August 2011, but the pattern is not so apparent as with Krippner's SSR. As expected, residential investment increases after an expansionary monetary policy shock and mortgage rate declines, though neither of responses is significant except for the response of residential investment to Wu-Xia rate shock in stable period, which becomes significant after approximately one year.

Robustness checks using industrial production index or Macroeconomic Advisers' real GDP estimates (Figures D.2 and D.3) yield practically similar impulse responses as the baseline model, therefore we do not need to describe them further. This result is expected, as the monthly estimates of real GDP are designed to match the quarterly path of real GDP (which was converted to monthly figures using cubic spline interpolation for the baseline model), and industrial production index is frequently used in the literature as a measure of real activity. However, there is one difference concerning forecast error variance decompositions of those robustness checks and that of the baseline model. Particularly, the role of real GDP in explaining forecast error variance in house prices in those robustness checks is much lower and replaced by residential investment.

Next, we assess the robustness of the results to changes in the house price index. For this purpose, we used FHFA house price index instead of S&P/Case-Shiller HPI. The main difference between those two indices is that the first uses

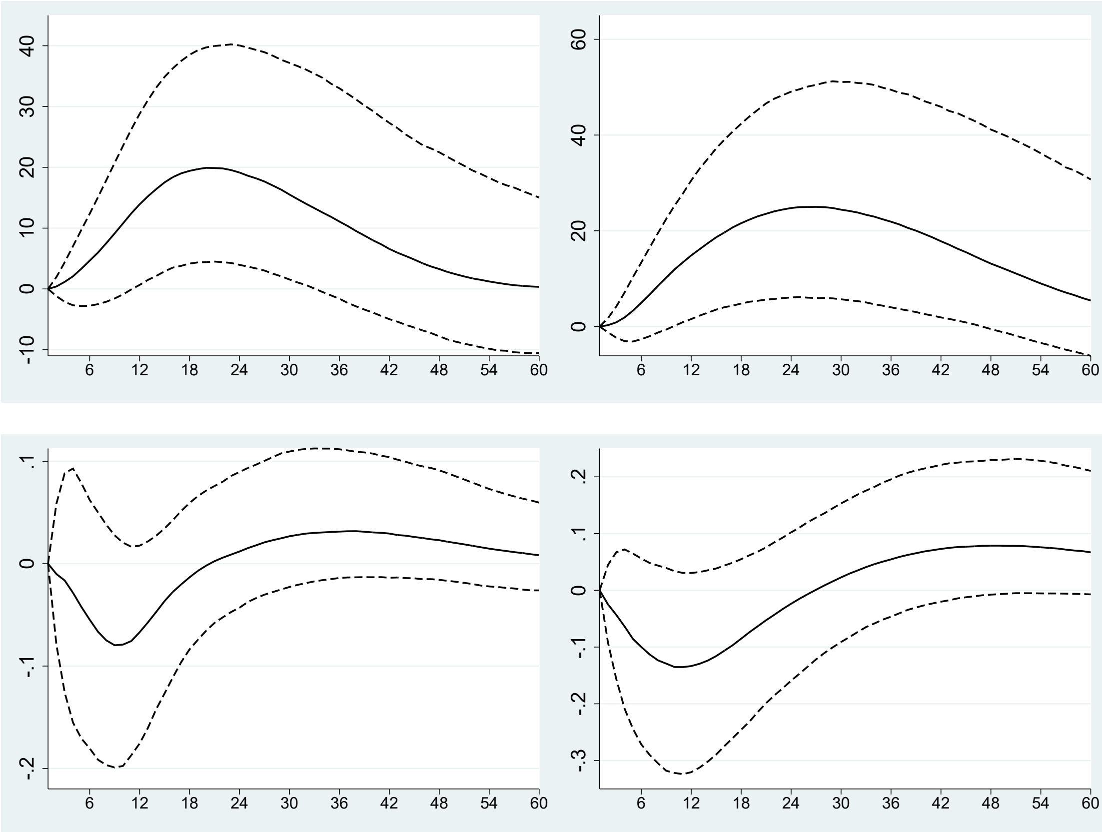

Figure 5.10: Impulse responses of residential investment (first row) and mortgage average (second row) to a negative 100-basis-point shock in Wu-Xia shadow rate. Impulse responses in the left-hand-side panel are from November 2006 and those in the right-hand-side panel are from August 2011.

only the mortgages purchased or securitized by Fannie Mae or Freddie Mac, while the latter includes also non-agency financed homes by sub-prime mortgages. Impulse responses of FHFA HPI to a 100-basis-point unexpected decrease in the policy rate are available in Figure D.4. The responses are rather similar to those from the baseline model, with the exception of December 2013, for which the response becomes significant after approximately one year and a half. This partly contradicts the findings of Eickmeier & Hofmann (2010) who obtained stronger responses of S&P/Case-Shiller HPI to a monetary policy shock. Results from FEVD are similar except that now residential investment and CPI gains more importance.

Lastly, we examine the robustness to changes in the ordering of variables. The sensitivity of the results with respect to a different ordering of variables should be examined when using a Cholesky decomposition to identify a monetary policy

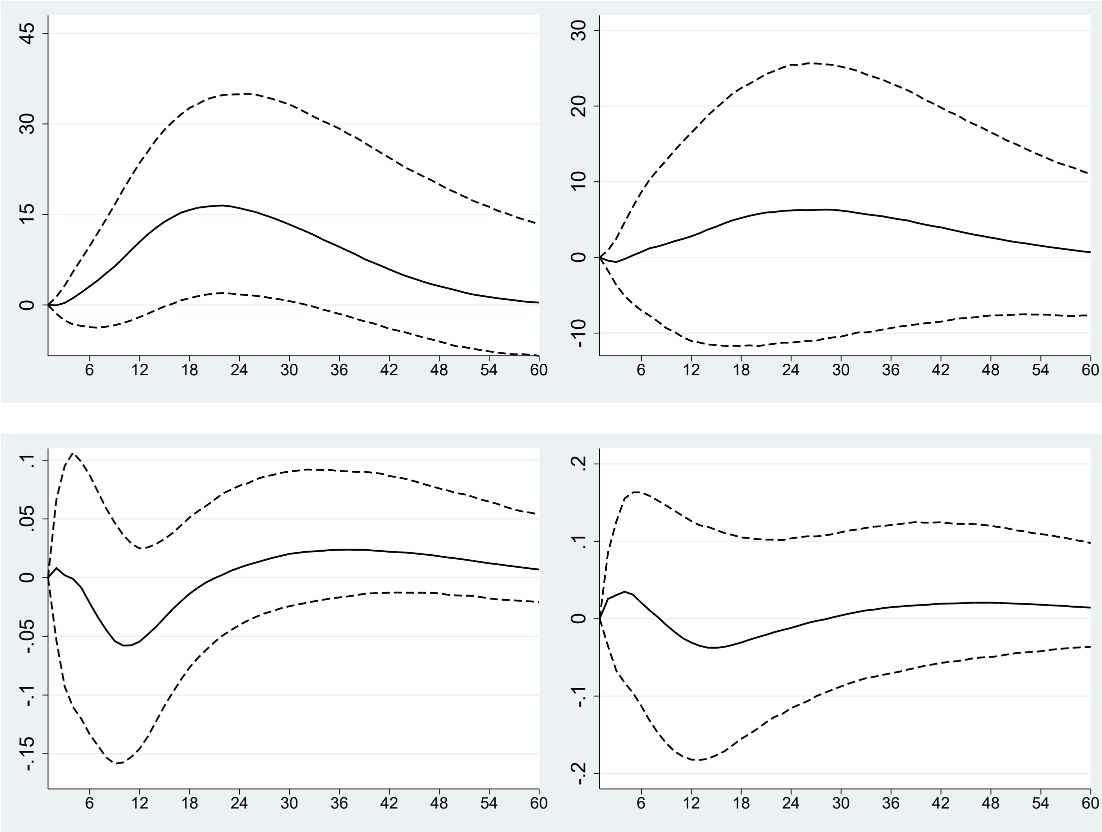

Figure 5.11: Impulse responses of residential investment (first row) and mortgage average (second row) to a negative 100-basis-point shock in Krippner's Shadow Short Rate. Impulse responses in the left-hand-side panel are from November 2006 and those in the right-hand-side panel are from August 2011.

shock. We choose to order the asset prices (and therefore the house price index) after the policy rate so that it can react contemporaneously to monetary policy shocks. This ordering is compatible with Bernanke *et al.* (2004) who divide the variables into fast- and slow-moving, and order the fast-moving variables after the policy rate. Hence, the ordering is now: real GDP, residential investment, CPI, policy rate, mortgage average and house prices for medium VAR and the same for small VAR provided that it uses only the four of the above variables (does not contain residential investment and mortgage average). With this ordering, monetary policy is restricted from contemporaneously responding to the shocks in house prices and mortgage rates.

Impulse responses for this ordering are depicted in Figure D.5. These impulses responses differ in two aspects from the baseline model. First, for October 2008, confidence bands are larger, and therefore the response becomes significant at higher horizons than in a baseline model, and the response in August 2011 is considerably lower, but still significant. Second, the initial response of house prices is negative for all examined time periods. The first result is reasonable since the uncertainty in October 2008 and August 2011 was very high, while the second result is the consequence of allowing for a contemporaneous response of house prices to a monetary policy shock. In the baseline model, those responses are restricted to zero which is the usual assumption of macroeconomic VAR models that employ house prices. Therefore, we can conclude that the ordering of house prices after the policy rate changes the initial response of house prices and slightly changes the results for highly unstable periods, but the preferred ordering is that of a baseline model because it does not produce negative immediate responses of house prices to an expansionary monetary policy shock.

Chapter 6

Conclusion

This thesis estimates a TVP-VAR model with stochastic volatility and dynamic dimension selection in order to assess the link between house prices and monetary policy. The model is from Koop & Korobilis (2013) and features changing a TVP-VAR dimension among small TVP-VAR with 4 variables, medium TVP-VAR containing 6 variables, and large TVP-VAR with 20 variables. We specified all models to be housing-oriented. Moreover, the model allows for the estimation of prior shrinkage hyperparameter in a time-varying manner. As expected, the necessary degree of shrinkage increases with the number of parameters in the model.

We extended the model by providing a way to perform impulse response analysis and forecast error variance decomposition. As the model is time-varying in many aspects, impulse responses and FEVD are also changing over time. Therefore, impulse responses of house prices, residential investment and mortgage average to a monetary policy shock are presented for several time periods. Those periods include both "stable" times with supreme economic conditions and the times of high uncertainty. The impulse responses are generally more invariant to different model specifications in stable times and they are also always significant during those times. Moreover, the behavior of impulse responses seem to be highly dependent on the value of the shrinkage hyperparameter, and we show that stable responses can be generated only from models with lower shrinkage, i.e., those that allow for more variation around the prior means of their coefficients.

Overall, the results indicate that there is a connection between monetary policy and the housing market, even though it appears to be less significant in periods of high uncertainty. House prices positively respond to an expansionary monetary policy shock approximated by the shock in Wu and Xia rate and the response is significant except for December 2013, which is a period of QE tapering. The responses are sluggish and persistent, with the peak median response of 2-4% after approximately 3 years following a 100-basis-point decrease in the policy rate. This corresponds to findings in the literature (e.g., Eickmeier & Hofmann (2010)). Responses of residential investment and mortgage average have the expected direction, although they are found to be insignificant for most of the examined periods, except for the most stable period of November 2006 where the response of residential investment is positive and becomes significant after approximately one year.

Several robustness checks were performed to assess the stability of the results. The most prominent ones include changing the measure of monetary policy from Wu and Xia rate to Krippner's Shadow Short Rate and changing the order of variables in order to allow for a contemporaneous response of house prices to a monetary policy shock. Main results are not qualitatively affected, even though there is one period in which the responses of house prices to Krippner's SSR are insignificant, while for Wu and Xia rate they were mostly significant. This period is the one with the highest uncertainty - August 2011 - in which the disputes in the Congress over raising of the debt ceiling escalated. Different ordering affects the immediate response of house prices to a monetary policy shock that is no longer restricted to zero and becomes slightly negative. Also, the confidence bands become rather wider following this ordering, and therefore, we concluded that house prices should be preferably ordered before the policy rate to allow for contemporaneous response of monetary policy to developments in house prices. The preferred ordering also implicitly restricts house prices from responding immediately to a monetary policy shock.

Possible extensions for future work are threefold. First, the impulse responses could be estimated based on an identification scheme that allows for a simultaneity between monetary policy and house prices. Therefore, house prices could respond contemporaneously to monetary policy shocks, while monetary policy could react immediately to house price shocks. This would require using sign or long-term restrictions. Second, one could use this model to examine the response of house prices to a monetary policy shock in periods when the policy rate is below the rate suggested by Taylor rule and immediately after the recession, and check whether the response is stronger. The idea would be that the central bank does not know whether it should increase the rates yet, but we get a better picture ex-post from the data that were not previously available. Lastly, we could repeat the analysis using the data for the Czech Republic and compare the results.

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Appendix A

Description of variables

Variables used in the empirical part and their transformation codes (Tcode), frequencies in which the data were downloaded, data sources and the original units. Transformation codes are as follows: if y_{it} is the original (untransformed) variable i at time t and $\widetilde{y}_{it}$ is the corresponding transformed variable; 1 - no transformation ($\widetilde{y}_{it} := y_{it}$), 2 - first difference ($\widetilde{y}_{it} := y_{it} - y_{it-1}$), 4 - logarithm ($\widetilde{y}_{it} := \log y_{it}$) and 5 - first difference of logarithm ($\widetilde{y}_{it} := \log y_{it} - \log y_{it-1}$).

Variable	Tcode	Frequency	Source	Units	SA
Real GDP	5	Quarterly	FRED	Billions of 2009 USD	✓
S&P/Case-Shiller U.S. National HPI	5	Monthly	FRED	Index Jan 2000=100	✓
Capacity Utilization: Manufacturing	1	Monthly	FRED	Percent	✓
Civilian Unemployment Rate	1	Monthly	FRED	Percent	✓
Real Disposable Personal Income: Per Capita	5	Monthly	FRED	2009 USD	✓
Real Personal Consumption Expenditures	5	Monthly	FRED	Billions of 2009 USD	✓
Real Private Residential Fixed Investment	1	Quarterly	FRED	Billions of 2009 USD	✓
CPI: All Items	5	Monthly	FRED	Index 1982-1984=100	✓
Mortgage Debt Outstanding	5	Quarterly	FRED	Millions of USD	
Housing Starts: Total	5	Monthly	FRED	Thousands of Units	✓
Total Reserves of Depository Institutions	5	Monthly	FRED	Billions of USD	
M2 Money Stock	5	Monthly	FRED	Billions of USD	✓
10-Year Treasury Constant Maturity Rate	1	Monthly	FRED	Percent	
30-Year Fixed Rate Mortgage Average	1	Weekly	FRED	Percent	
Spot Crude Oil Price: WTI	5	Monthly	FRED	USD per Barrel	
S&P 500 Index	5	Monthly	Yahoo Finance	Index	
Producer Price Index for All Commodities	5	Monthly	FRED	Index 1982=100	
Real Broad Effective Exchange Rate	5	Monthly	FRED	Index 2010=100	
CBOE Volatility Index: VIX	5	Daily	FRED	Index, NSA	
Wu-Xia Shadow Rate	1	Monthly	Wu's personal website	Percent	
Industrial Production Index^{RC}	5	Monthly	FRED	Index 2012=100	✓
Macroeconomic Advisers' Real GDP Index^{RC}	5	Monthly	Macroeconomic Advisers	Billions of 2009 USD	✓
FHFA House Price Index^{RC}	5	Monthly	FHFA	Index Jan 1991=100	✓
Krippner's SSR^{RC}	1	Monthly	RBNZ	Percent	

Table A.1: Variables and their transformation codes, available frequencies, data sources and original units. FRED denotes the Federal Reserve Bank of St. Louis database (`fred.stlouisfed.org`), data from Yahoo Finance are available at `finance.yahoo.com`, Cynthia Wu's personal website is at `sites.google.com/site/jingcynthiawu/`, Macroeconomic Advisers at `macroadvisers.com`, FHFA stands for the Federal Housing Finance Agency (`www.fhfa.gov`) and RBNZ for the Reserve Bank of New Zealand (`www.rbnz.govt.nz`). SA denotes seasonally adjusted data.

Appendix B

Results from ADF and KPSS tests

Variable	**ADF test**		**KPSS test**	
	Constant	Constant+trend	Constant	Constant+trend
Real GDP		-2.7458		0.53373^{***}
S&P/Case-Shiller U.S. National HPI		-2.7211		0.81887^{***}
Capacity Utilization: Manufacturing	-3.7769^{***}		0.53908^{**}	
Civilian Unemployment Rate	-2.2231		1.6708^{***}	
Real Disposable Personal Income: Per Capita		-2.5254		0.66962^{***}
Real Personal Consumption Expenditures		-1.8597		0.66809^{***}
Real Private Residential Fixed Investment		-3.6178^{**}		0.62257^{***}
CPI: All Items		-1.7654		0.78962^{***}
Mortgage Debt Outstanding	-2.5914^{*}	-2.506	1.1891^{***}	1.0462^{***}
Housing Starts: Total	-1.1456	-1.271	3.1951^{***}	0.65145^{***}
Total Reserves of Depository Institutions		-2.1347		1.0109^{***}
M2 Money Stock		-0.3847		1.2055^{***}
10-Year Treasury Constant Maturity Rate	-1.3076	-2.8114	4.7055^{***}	0.17914^{**}
30-Year Fixed Rate Mortgage Average	-1.1883	-2.3385	4.7951^{***}	0.21991^{***}
Spot Crude Oil Price: WTI	-1.9414	-1.6737	2.8102^{***}	0.73161^{***}
S&P 500 Index		-1.3242		0.87968^{***}
Producer Price Index for All Commodities		-1.4426		0.69251^{***}
Real Broad Effective Exchange Rate	-1.478		2.9895^{***}	
CBOE Volatility Index: VIX	-2.7826^{*}		0.63069^{**}	
Wu-Xia Shadow Rate	-2.324	-2.6363	3.4732^{***}	0.29798^{***}
Industrial Production Index^{RC}		-3.6739^{**}		0.27732^{***}
Macroeconomic Advisers' Real GDP Index^{RC}		-1.95		0.53948^{***}
FHFA House Price Index^{RC}		-2.9406		0.78647^{***}
Krippner's SSR^{RC}	-1.7099	-1.4786	3.1088^{***}	0.37572^{***}

Table B.1: Test statistics from ADF and KPSS tests for variables in levels. Superscript RC denotes variables used for a robustness check. Columns Constant and Constant+trend indicate which deterministic terms are included in the regressions. Null hypothesis: ADF test: Variable has a unit root, KPSS test: Series is level (Constant) or trend (Constant+trend) stationary. Critical values for test statistics - Constant: ADF test: -3.46 (1%) -2.88 (5%) -2.57 (10%), KPSS test: 0.739 (1%) 0.463 (5%) 0.347 (10%); Constant+trend: ADF test: -3.99 (1%) -3.43 (5%) -3.13 (10%), KPSS test: 0.216 (1%) 0.146 (5%) 0.119 (10%). The asterisks indicate that the null hypothesis can be rejected at the 10% (*), 5% (**) and 1% (***) significance levels.

Variable	ADF test	KPSS test
Real GDP	-4.2644^{***}	0.29269
S&P/Case-Shiller U.S. National HPI	-1.9802	1.0364^{***}
Capacity Utilization: Manufacturing	-3.7769^{***}	0.53908^{**}
Civilian Unemployment Rate	-2.2231	1.6708^{***}
Real Disposable Personal Income: Per Capita	-6.2982^{***}	0.11106
Real Personal Consumption Expenditures	-3.2482^{**}	0.66284^{**}
Real Private Residential Fixed Investment	-2.8508^{*}	2.454^{***}
CPI: All Items	-5.6101^{***}	0.32024
Mortgage Debt Outstanding	-3.1327^{**}	0.7499^{***}
Housing Starts: Total	-4.8678^{***}	0.24555
Total Reserves of Depository Institutions	-5.2214^{***}	0.16307
M2 Money Stock	-5.7601^{***}	0.16083
10-Year Treasury Constant Maturity Rate	-1.3076	4.7055^{***}
30-Year Fixed Rate Mortgage Average	-1.1883	4.7951^{***}
Spot Crude Oil Price: WTI	-6.1341^{***}	0.25996
S&P 500 Index	-5.1123^{***}	0.23703
Producer Price Index for All Commodities	-5.3967^{***}	0.19354
Real Broad Effective Exchange Rate	-5.7281^{***}	0.24455
CBOE Volatility Index: VIX	-7.3573^{***}	0.026794
Wu-Xia Shadow Rate	-2.324	3.4732^{***}
Industrial Production Index$^{\mathrm{RC}}$	-3.8986^{***}	0.10068
Macroeconomic Advisers' Real GDP Index$^{\mathrm{RC}}$	-4.3024^{***}	0.24279
FHFA House Price Index$^{\mathrm{RC}}$	-1.5066	0.95728^{***}
Krippner's SSR$^{\mathrm{RC}}$	-1.7099	3.1088^{***}

Table B.2: Test statistics from ADF and KPSS tests for variables used in the empirical part. Superscript RC denotes variables used for a robustness check. ADF and KPSS tests include intercept and not trend. Null hypothesis: ADF test: Variable has a unit root, KPSS test: Series is level stationary. Critical values for test statistics: ADF test: -3.46 (1%) -2.88 (5%) -2.57 (10%), KPSS test: 0.739 (1%) 0.463 (5%) 0.347 (10%). The asterisks indicate that the null hypothesis can be rejected at the 10% (*), 5% (**) and 1% (***) significance levels.

Appendix C

Carter and Kohn algorithm

For the general state-space model of the form:

$$y_t = Z_t\beta_t + \epsilon_t, \ \ var(\epsilon_t) = \Sigma_t \tag{C.1}$$

$$\beta_t = \mu + F\beta_{t-1} + u_t, \ \ var(u_t) = Q_t \tag{C.2}$$

$\beta_{t|t,\beta_{t+1}}$ and $P_{t|t,\beta_{t+1}}$ can be calculated using

$$\beta_{t|t,\beta_{t+1}} = \beta_{t|t} + P_{t|t}F'(FP_{t|t}F' + Q_{t+1})^{-1}(\beta_{t+1} - \mu - F\beta_{t|t}) \tag{C.3}$$

$$P_{t|t,\beta_{t+1}} = P_{t|t} - P_{t|t}F'(FP_{t|t}F' + Q_{t+1})^{-1}FP_{t|t} \tag{C.4}$$

where the meaning of $\beta_{t|t}$, $P_{t|t}$ and β_{t+1} is the same as in 4.10 and 4.11. Equations 4.10 and 4.11 can be obtained from C.3 and C.4 if we realize that in our model $F = I_K$, $Q_{t+1} = (\frac{1}{\lambda_{t+1}} - 1)P_{t|t}$, and $\mu = 0$.

Appendix D

Additional results, robustness checks

Parameter	Value	Description
α	0.9963	forgetting factor for DDS probabilities
λ_{min}	0.96	minimal forgetting factor for the time variation in VAR coefficients
L	1.1	weight of the prediction error in the estimation of λ_t
κ	0.96	decay factor for the measurement error volatility
$\underline{\alpha}$	10	prior variance of intercepts
γ	$[10^{-10}, 10^{-5}, 0.001, 0.005, 0.01, 0.05, 0.1]$	prior shrinkage coefficient
τ	40	size of the training sample
p	4	VAR lag length

Table D.1: Summary of parameters employed in TVP-VAR with dynamic dimension selection (DDS).

forecast error in	forecast horizon h	proportion of forecast error variance h periods ahead accounted for by innovations in			
		real GDP	house prices	CPI	Wu-Xia rate
real GDP	6	0.968	0.02	0.01	0.002
	12	0.928	0.048	0.019	0.004
	18	0.883	0.08	0.031	0.005
	24	0.836	0.113	0.044	0.006
	30	0.792	0.144	0.057	0.008
	36	0.753	0.171	0.067	0.008
	60	0.644	0.250	0.095	0.011
house prices	6	0.619	0.372	0.008	0.002
	12	0.654	0.322	0.022	0.003
	18	0.673	0.285	0.038	0.004
	24	0.675	0.267	0.053	0.005
	30	0.667	0.263	0.064	0.006
	36	0.655	0.265	0.073	0.007
	60	0.606	0.290	0.094	0.010
CPI	6	0.280	0.162	0.556	0.003
	12	0.289	0.133	0.574	0.004
	18	0.280	0.160	0.554	0.007
	24	0.278	0.208	0.505	0.009
	30	0.287	0.252	0.451	0.010
	36	0.303	0.284	0.402	0.011
	60	0.373	0.333	0.282	0.012
Wu-Xia rate	6	0.279	0.528	0.040	0.153
	12	0.226	0.510	0.134	0.130
	18	0.292	0.385	0.239	0.084
	24	0.358	0.300	0.286	0.056
	30	0.394	0.269	0.294	0.043
	36	0.406	0.273	0.285	0.036
	60	0.382	0.341	0.248	0.030

Table D.2: Mean forecast error variance decomposition in October 2008 for small TVP-VAR with shrinkage parameter $\gamma = 0.05$. Means were computed using 10 000 draws from the posterior distribution of parameters and rounded to three decimal places, therefore the numbers in each row do not have to sum up to unity.

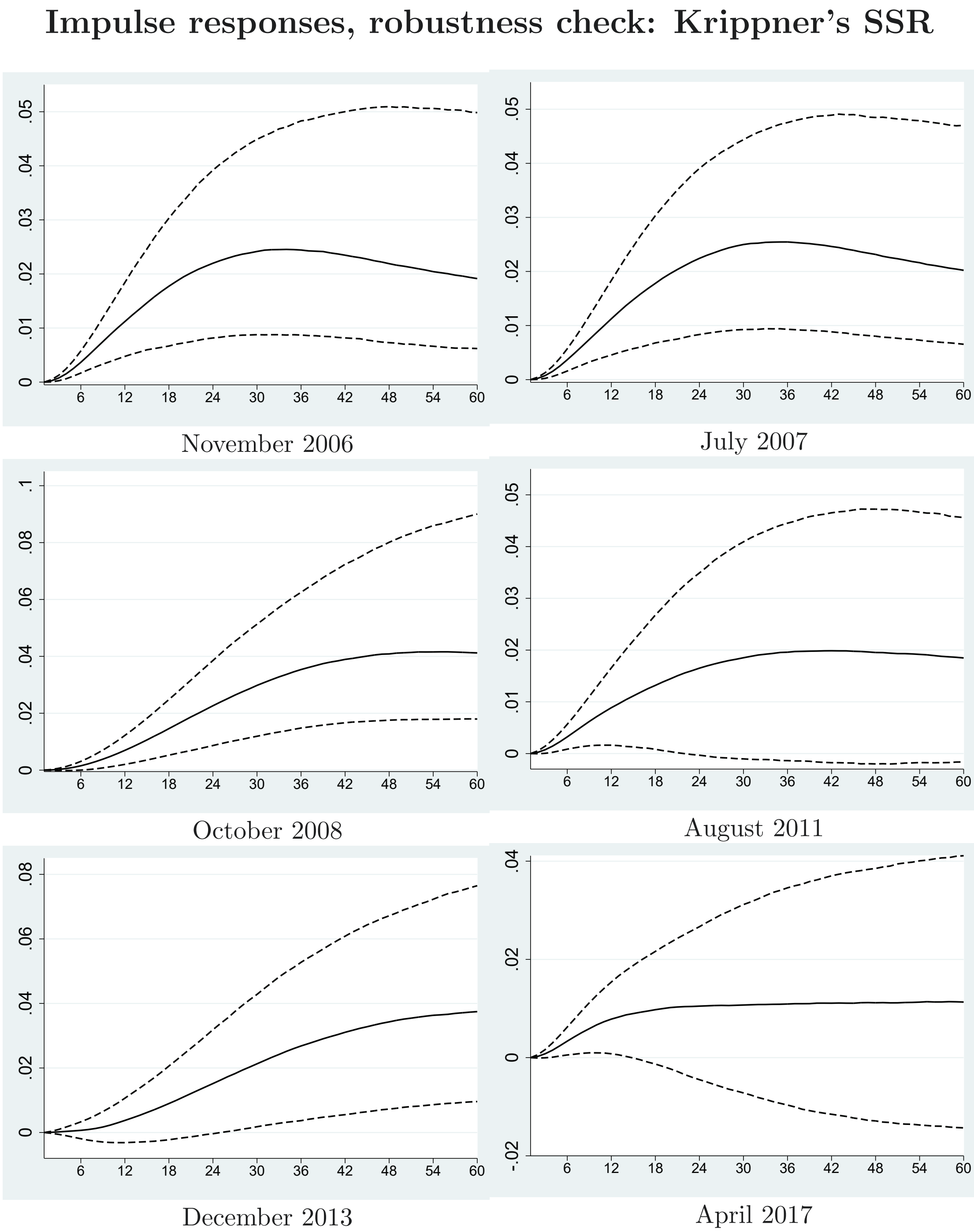

Figure D.1: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Krippner's SSR. For November 2006, July 2007, August 2011 and April 2017, impulse responses were generated from the medium TVP-VAR with $\gamma = 0.1$. In October 2008 and December 2013, small TVP-VAR with $\gamma = 0.05$ and $\gamma = 0.1$, respectively, was selected.

Impulse responses, robustness check: Industrial production index

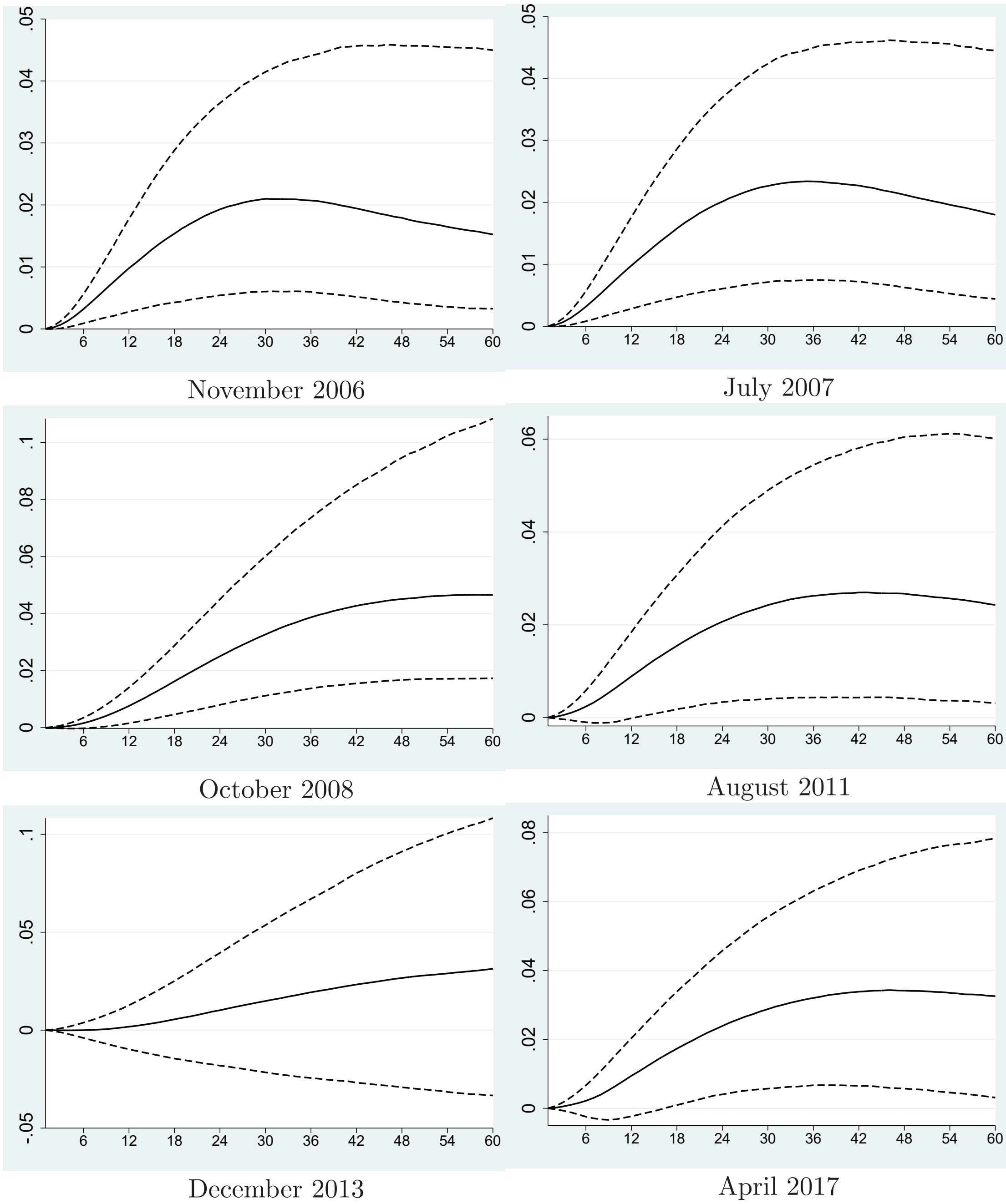

Figure D.2: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Wu-Xia shadow rate. In the baseline model, real GDP was replaced by industrial production index. For November 2006, July 2007, August 2011 and April 2017, impulse responses were generated from the medium TVP-VAR with $\gamma = 0.1$. In October 2008 and December 2013, small TVP-VAR with $\gamma = 0.05$ and $\gamma = 0.1$, respectively, was selected.

Impulse responses, robustness check: Macroeconomic Advisers' monthly GDP

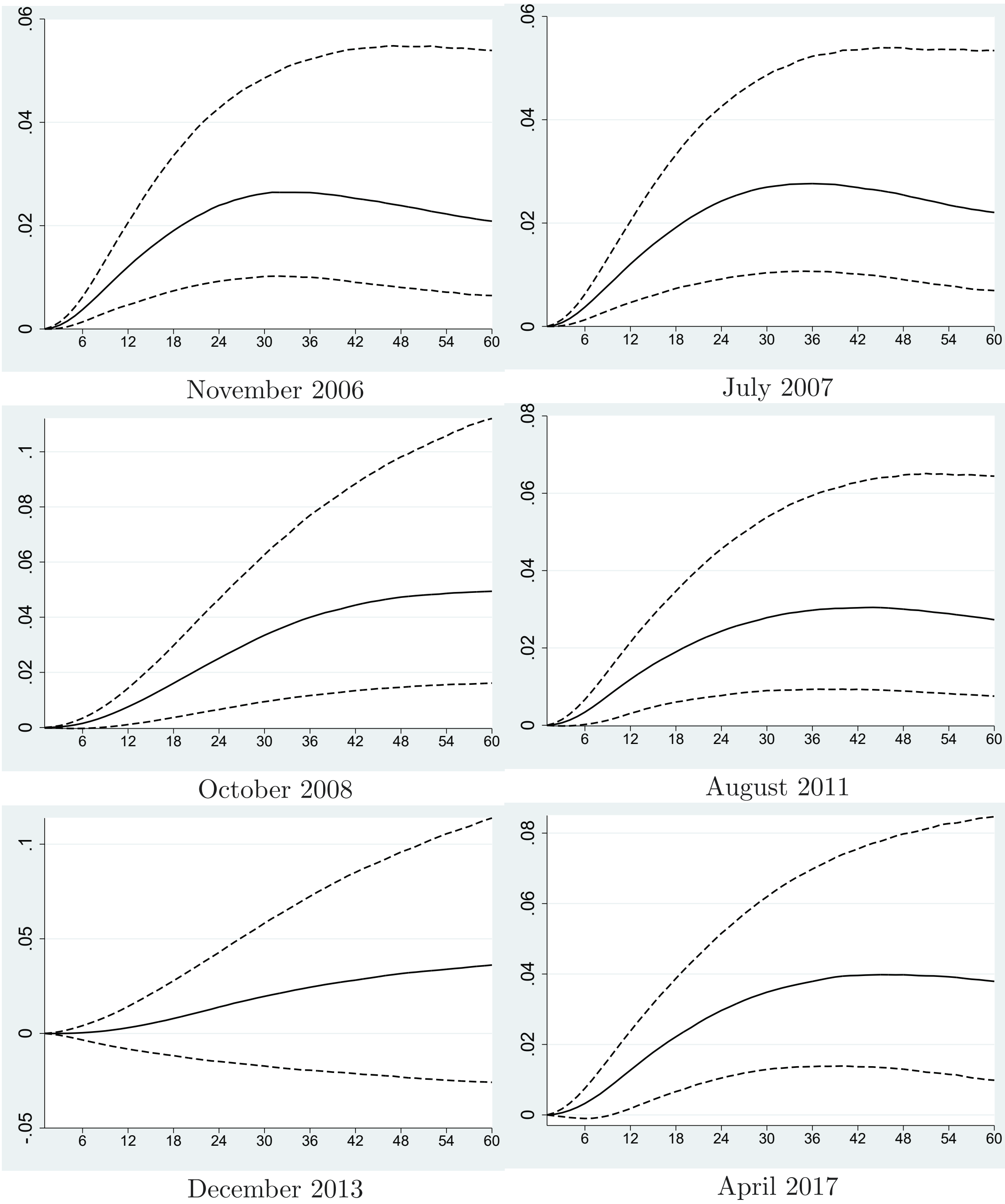

Figure D.3: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Wu-Xia shadow rate. In the baseline model, real GDP was replaced by Macroeconomic Advisers' monthly real GDP estimates. For November 2006, July 2007, August 2011 and April 2017, impulse responses were generated from the medium TVP-VAR with $\gamma = 0.1$. In October 2008 and December 2013, small TVP-VAR with $\gamma = 0.05$ and $\gamma = 0.1$, respectively, was selected.

Impulse responses, robustness check: FHFA house price index

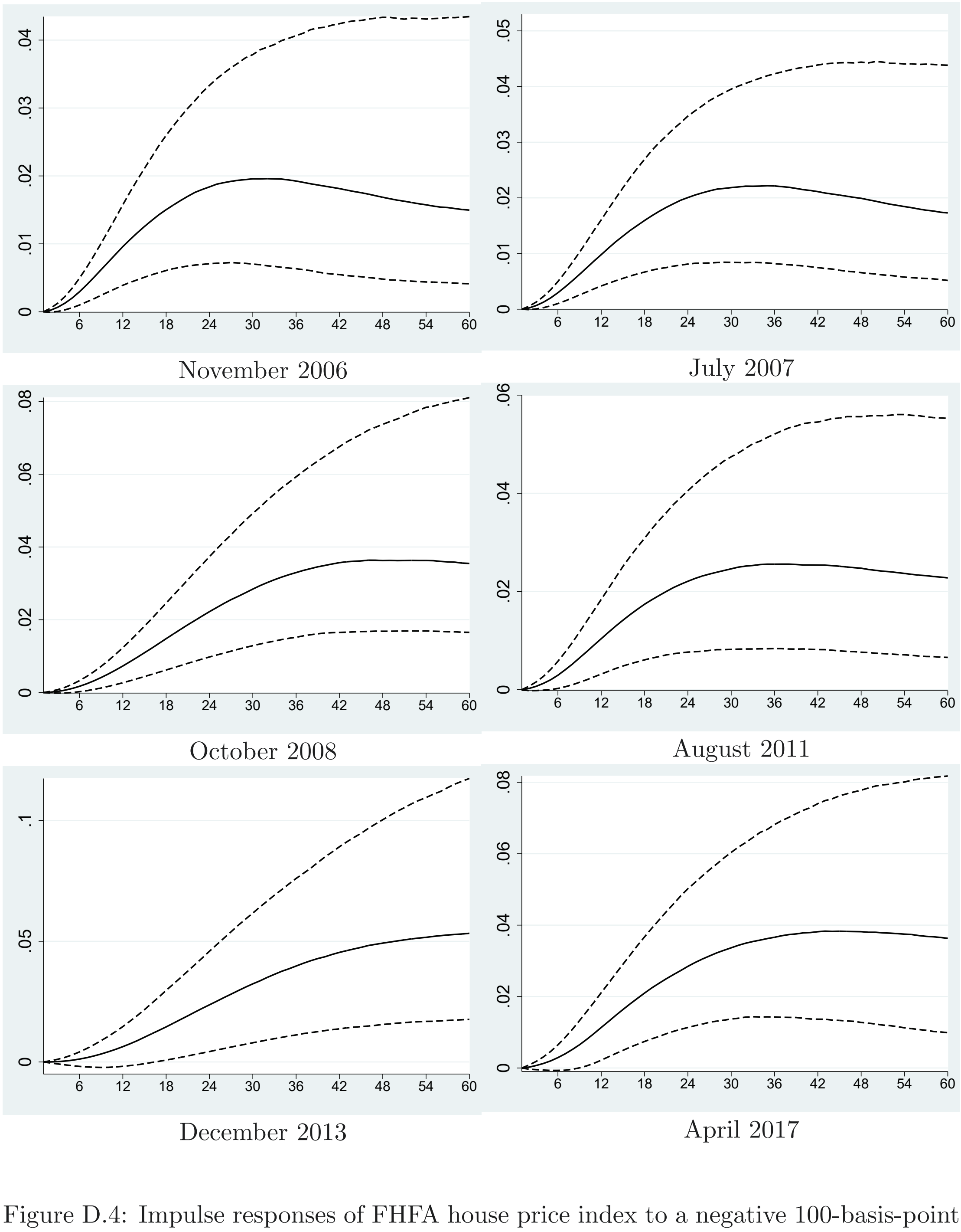

Figure D.4: Impulse responses of FHFA house price index to a negative 100-basis-point shock in Wu-Xia shadow rate. For November 2006, July 2007, August 2011 and April 2017, impulse responses were generated from the medium TVP-VAR with $\gamma = 0.1$. In October 2008 and December 2013, small TVP-VAR with $\gamma = 0.05$ and $\gamma = 0.1$, respectively, was selected.

Impulse responses, robustness check: different ordering of variables

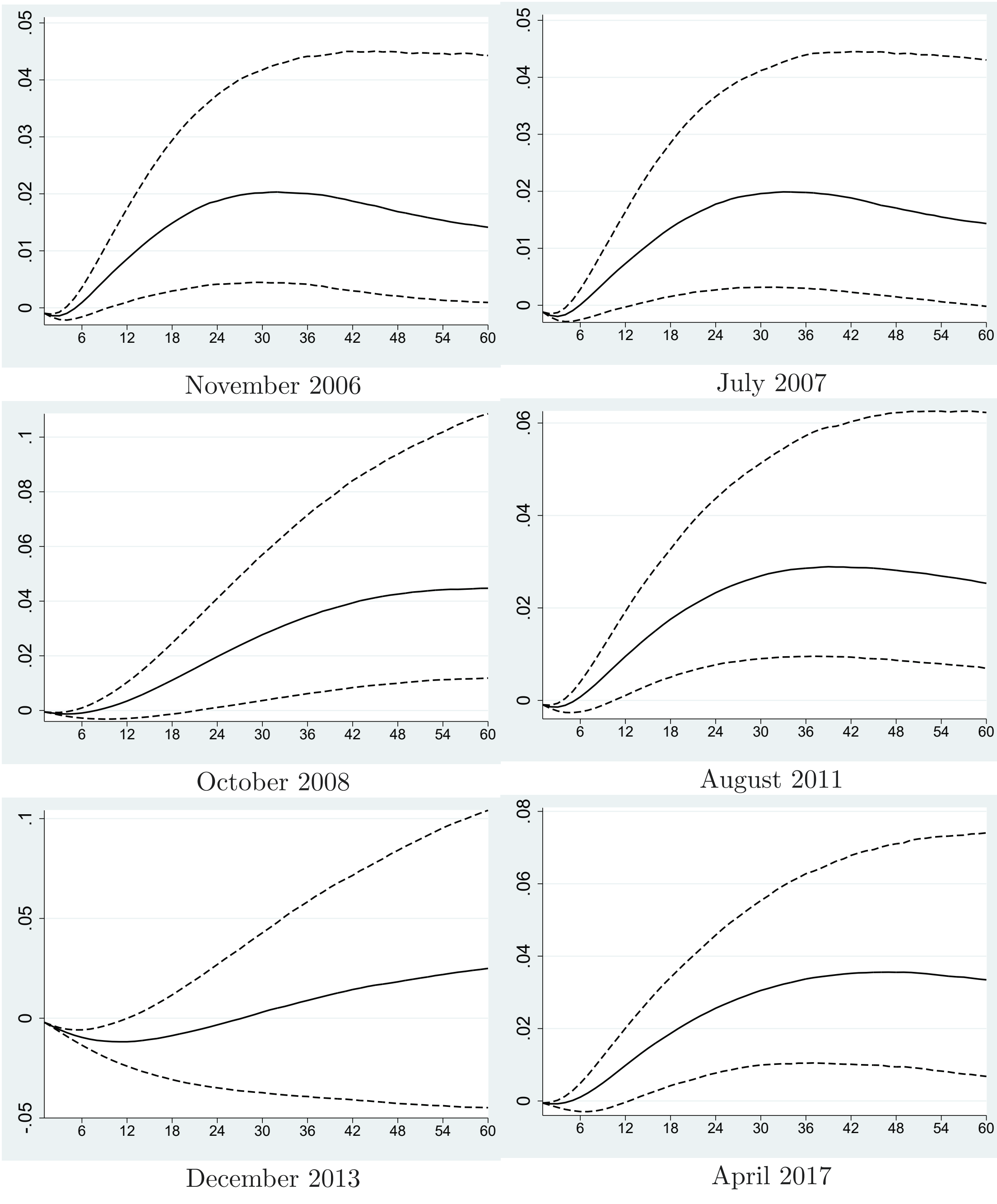

Figure D.5: Impulse responses of S&P/Case-Shiller house price index to a negative 100-basis-point shock in Wu-Xia shadow rate. S&P/Case-Shiller house price index is now ordered *after* Wu-Xia rate to allow for immediate effects of monetary policy shocks on house prices. For November 2006, July 2007, August 2011 and April 2017, impulse responses were generated from the medium TVP-VAR with $\gamma = 0.1$. In October 2008 and December 2013, small TVP-VAR with $\gamma = 0.05$ and $\gamma = 0.1$, respectively, was selected.

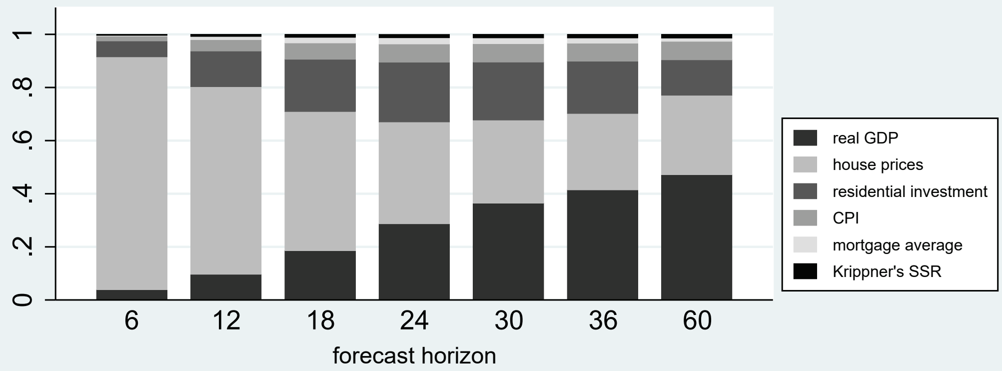

Figure D.6: FEVD of house prices, robustness check: Krippner's SSR instead of Wu and Xia rate.

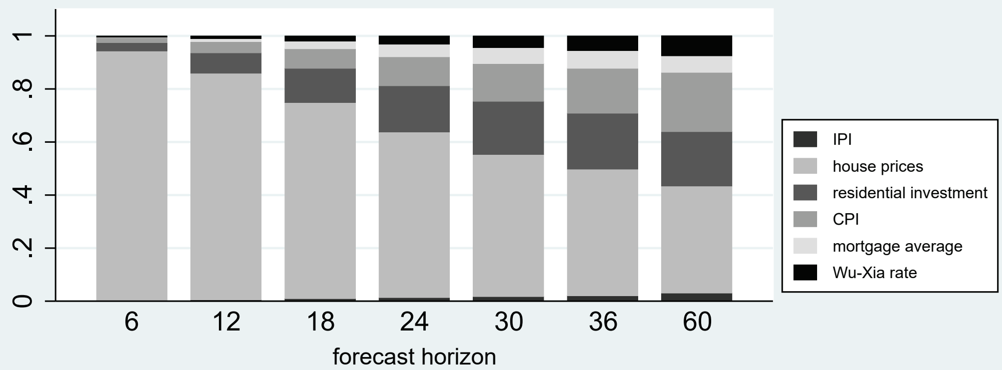

Figure D.7: FEVD of house prices, robustness check: Industrial production index (IPI) instead of real GDP.

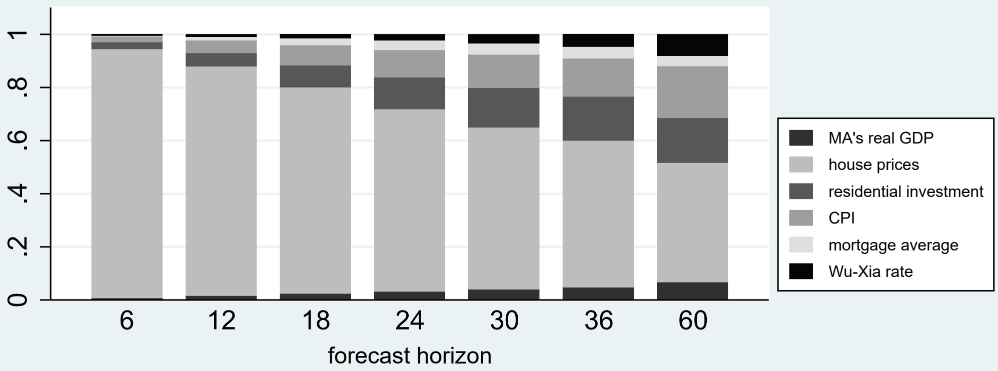

Figure D.8: FEVD of house prices, robustness check: Monthly estimates of GDP produced by Macroeconomic Advisors instead of real GDP.

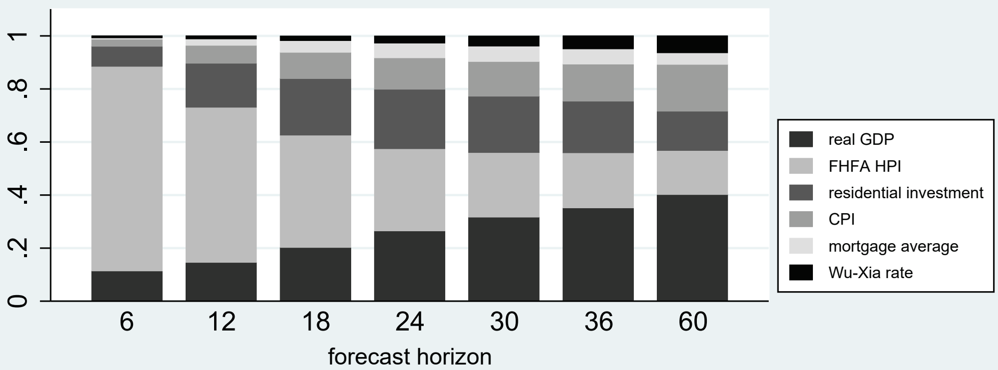

Figure D.9: FEVD of house prices, robustness check: FHFA house price index instead of S&P/Case-Shiller HPI.

forecast error in	forecast horizon h	proportion of forecast error variance h periods ahead accounted for by innovations in			
		real GDP	house prices	CPI	Wu-Xia rate
real GDP	6	0.974	0.015	0.008	0.002
	12	0.934	0.044	0.017	0.005
	18	0.891	0.074	0.028	0.007
	24	0.851	0.102	0.039	0.008
	30	0.816	0.127	0.048	0.010
	36	0.786	0.148	0.055	0.011
	60	0.708	0.203	0.075	0.014
house prices	6	0.312	0.671	0.014	0.003
	12	0.395	0.570	0.030	0.005
	18	0.455	0.492	0.046	0.007
	24	0.489	0.442	0.060	0.008
	30	0.505	0.414	0.071	0.010
	36	0.510	0.399	0.079	0.012
	60	0.504	0.383	0.097	0.016
CPI	6	0.330	0.022	0.645	0.004
	12	0.408	0.065	0.522	0.005
	18	0.449	0.114	0.431	0.007
	24	0.480	0.153	0.357	0.009
	30	0.508	0.181	0.301	0.011
	36	0.531	0.197	0.260	0.011
	60	0.576	0.227	0.185	0.013
Wu-Xia rate	6	0.127	0.097	0.274	0.503
	12	0.276	0.114	0.394	0.216
	18	0.346	0.142	0.401	0.111
	24	0.370	0.178	0.382	0.071
	30	0.369	0.218	0.357	0.055
	36	0.361	0.255	0.334	0.049
	60	0.353	0.322	0.282	0.043

Table D.3: Mean forecast error variance decomposition in December 2013 for small TVP-VAR with shrinkage parameter $\gamma = 0.1$. Means were computed using 10 000 draws from the posterior distribution of parameters and rounded to three decimal places, therefore the numbers in each row do not have to sum up to unity.

forecast error in	forecast horizon h	proportion of forecast error variance h periods ahead accounted for by innovations in					
		real GDP	house prices	residential investment	CPI	mortgage rate	Wu-Xia rate
real GDP	6	0.941	0.032	0.014	0.009	0.001	0.003
	12	0.886	0.055	0.024	0.022	0.006	0.007
	18	0.819	0.087	0.031	0.042	0.011	0.011
	24	0.763	0.110	0.036	0.063	0.014	0.014
	30	0.723	0.124	0.042	0.080	0.015	0.016
	36	0.695	0.135	0.046	0.091	0.016	0.018
	60	0.643	0.161	0.053	0.106	0.015	0.023
house prices	6	0.039	0.905	0.033	0.017	0.002	0.003
	12	0.080	0.806	0.063	0.035	0.010	0.006
	18	0.123	0.701	0.094	0.054	0.018	0.010
	24	0.172	0.597	0.120	0.072	0.024	0.015
	30	0.223	0.511	0.133	0.086	0.026	0.021
	36	0.267	0.450	0.134	0.095	0.026	0.028
	60	0.348	0.357	0.115	0.116	0.021	0.043
residential investment	6	0.033	0.068	0.870	0.017	0.007	0.005
	12	0.087	0.102	0.678	0.088	0.023	0.022
	18	0.143	0.126	0.545	0.120	0.026	0.041
	24	0.183	0.152	0.452	0.131	0.025	0.057
	30	0.210	0.176	0.386	0.136	0.023	0.069
	36	0.228	0.195	0.340	0.140	0.021	0.075
	60	0.285	0.232	0.249	0.148	0.018	0.069
CPI	6	0.154	0.103	0.041	0.694	0.002	0.005
	12	0.277	0.106	0.066	0.538	0.006	0.007
	18	0.354	0.112	0.071	0.446	0.009	0.009
	24	0.414	0.120	0.069	0.376	0.010	0.011
	30	0.459	0.129	0.067	0.323	0.010	0.012
	36	0.493	0.135	0.064	0.284	0.011	0.013
	60	0.554	0.150	0.061	0.209	0.011	0.014
mortgage rate	6	0.521	0.091	0.172	0.099	0.111	0.006
	12	0.516	0.232	0.109	0.075	0.061	0.006
	18	0.520	0.275	0.088	0.069	0.041	0.007
	24	0.518	0.287	0.085	0.069	0.033	0.008
	30	0.510	0.288	0.091	0.074	0.029	0.009
	36	0.500	0.287	0.098	0.079	0.026	0.010
	60	0.474	0.281	0.108	0.097	0.022	0.018
Wu-Xia rate	6	0.103	0.204	0.111	0.154	0.005	0.423
	12	0.174	0.198	0.151	0.237	0.008	0.233
	18	0.219	0.186	0.161	0.273	0.009	0.152
	24	0.242	0.181	0.167	0.286	0.010	0.114
	30	0.255	0.183	0.170	0.284	0.011	0.097
	36	0.265	0.191	0.169	0.275	0.011	0.090
	60	0.293	0.224	0.150	0.241	0.010	0.081

Table D.4: Mean forecast error variance decomposition in April 2017 for medium TVP-VAR with shrinkage parameter $\gamma = 0.1$. Means were computed using 10 000 draws from the posterior distribution of parameters and rounded to three decimal places, therefore the numbers in each row do not have to sum up to unity.