Příloha 1: Kód pro výpočet PIG/PIE z kříže (“cross”)

```
input = (‘k:\images\’);
flr = dir([input, '*.png']);

img = imread([input, flr(1).name]);
      % read a grayscale (black-white) image "img"

for alpha = 0.1:0.1:4                                                   %
for each value of "alpha" from interval 0.1 to 4 with step 0.1

    if alpha == 1                                                       % if
"alpha" equals 1,
        alpha = 0.99
            % supply value 1 (for Shannon entropy) by value 0.99
    end
      % end

    PIG = zeros(size(img, 1), size(img, 2));
            % create zero matrix "PIG" of the same size as studied image
"img"

    for i = 1:size(img, 1)
      % for each pixel in each row "i"
        for j = 1:size(img, 2)
                 % for each pixel in each column "j"
            [y] = imhist(horzcat(img(i,:), img(:,j)'));
                             % create intensity histogram "y" from the
whole row and the whole around pixel at the position img(i,j)
            y(double(img(i,j))+1) = y(double(img(i,j))+1)-1;
                             % remove an excess point with same intensity
like point img(i,j) from the intensity histogram "y"
            p = y/sum(y);
                 % recalculate intensity histogram to a probabilistic
histogram "p"
            y(double(img(i,j))+1) = y(double(img(i,j))+1)-1;
                             % remove studied point of the same intensity
like point img(i,j) from the intensity histogram "y"
            p2 = y/sum(y);
                 % recalculate intensity histogram "y" to a probabilistic
histogram "p2"
            PIG(i,j) = log2(sum(p.^alpha)./(sum(p2.^alpha)))/(1-alpha);
                                   % calculate Point Information Gain from
probabilistic histograms "p" and "p2", save the resulted value at the
position PIG(i,j)
        end
      % end
    end
      % end

    PIE = sum(sum(PIG));
      % calculate Point Information Gain Entropy "PIE" as a sum of all
values in the matrix "PIG" for a particalar "alpha"

    imgPIG = double(img.*0);
      % create zero matrix imgPIG of the same size as studied image "img"
```

```
    imgPIG = uint8(round((PIG-min(min(PIG)))/(max(max(PIG)) -
min(min(PIG)))*255));        % rescale PIG values (in double precision
floating point format) to 8-bit unsigned integers (8-bit intensity image,
variable "imgPIG"): the minimal and maximal PIG value in the PIG matrix
is intensity 0 and 255, respectively
    imwrite(imgPIG, [input, 'Cross\', num2str(k), '_', num2str(alpha),
'.png']);        % save the resulted entropic image "imgPIG" for
particular "alpha"
end                                                                % end
```

Příloha 2: Kód pro výpočet PIG/ PIE z celku obrázku („whole“)

```
input = ('i:\images\');
flr = dir([input, '*.png']);

img = imread([input, flr(i).name]);
            % read a grayscale (or black-white) image "img"
[y] = imhist(img);
      % create intensity histogram "y" of the image "img"
y = y';
      % transpose matrix "y"
p = y/sum(y);
      % recalculate the intensity histogram "y" to a probabilistic
histogram "p"
PIG = [];                                                             %
create an empty vector "PIG"


for alpha = 0.1:0.1:4
      % for "alpha" from 0.1 to 4 with the step of 0.1

    if alpha == 1
      % if "alpha" equals 1 (for Shannon entropy),
        alpha = 0.99;
                  % supply value 1  by value 0.99
    end
            % end

    for i = 1:length(y)
                  % for each intensity (bin) in intensity histogram "y"
        Y = y;
                  % create intensity histogram "Y", the same as intensity
histogram "y"
        if Y(i) ~= 0
                  % if the bin in the intensity histogram "Y" at the
position "i" is not occupied (not valid in the case of a black-white
image),
            Y(i) = Y(i)-1;
                        % remove a point at the position "i" in the
intensity histogram "Y"

        end
            % end
        p2 = Y/(sum(Y));
                  % recalculate the intensity histogram "Y" without the
examined point to a probabilistic histogram "p2"
        PIG(i) = log2(sum(p.^alpha)./(sum(p2.^alpha)))/(1-alpha);
                              % calculate Point Information Gain "PIG" for
particular "alpha" and intensity "i" from probabilistic histograms "p"
and "p2"
    end
            % end

    PIE = sum(PIG.*y);
            % calculate Point Information Gain Entropy "PIE" as a sum of
all products of PIG values with their particular numbers in the intensity
```

```
histogram "y" (element multiplication of the vector "PIG" with the vector
"y")

    imgPIG = double(img.*0);
            % create an empty matrix "imgPIG" of the same size as the
image "img"

    for i = 1:numel(img)
            % for each pixel "i" in the image "img"
       imgPIG(i) = PIG(img(i)+1);
                  % write the particular value "PIG" at the position of
the pixel "i" in the image "img" into the matrix "imgPIG"
    end
            % end

    imgPIG = uint8((imgPIG-min(min(imgPIG)))/(max(max(imgPIG)) -
min(min(imgPIG)))*255);            % rescale PIG values (in double
precision floating point format) to 8-bit unsigned integers (8-bit
intensity image, variable "imgPIG"): the minimal and maximal PIG value in
the PIG matrix is intensity 0 and 255, respectively
    imwrite(imgPIG, [input, 'img\whole.png']);
                        % save the resulted entropic image "imgPIG" for
particular "alpha"
end                                                                     %
end
```

Příloha 3: Text článku

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Study of human visual perception with the usage of information entropy analysis of patterns

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Abstract:

We introduce a new research methodology based on the Partial Least Squares Regression (PLSR) statistical method to examine human perception of shapes and patterns in a set of images. The matrix of predictors was formed by a computed spectra of dependency of a new information-entropic variable – Point Information Gain Entropy – on a dimensionless coeficient α. It distinguishes between the semantic, syntactic, and total information brought by each image. The matrix of responses was gained from a sample of the human population. The matrix is a set of classification vectors assigning the identity number of the classification group to each image for each human evaluator. For each evaluator, we analyzed weighted regression coefficients (outputs of the PLSR model), which give weights of the coefficients α for classification of images into groups. Testing the method on a group of 86 evaluators with black-and-white drawings by František Kupka shows relatively uniform classification, which clearly relates to the computed semantic information (meaning) of the image.

Keywords: *Pattern recognition; Visual complexity; Image analysis; Rényi entropy; Sensometrics;*

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1. Introduction

Order is a necessary condition for anything the human mind is to understand. Arrangements such as the layout of a city or building, a set of tools, a display of merchandise, the verbal exposition of facts or ideas, or a painting or piece of music are called orderly when an observer or listener can grasp their overall structure and the ramification of the structure in some detail. Order makes it possible to focus on what is alike and what is different, what belongs together and what is segregated. When nothing superfluous is included and nothing indispensable left out, one can understand the interrelation of the whole and its parts, as well as the hierarchic scale of importance and power by which some structural features are dominant, others subordinate. (…) In many instances, order is apprehended first of all by the senses. The observer perceives an organized structure in the shapes and colors or sounds facing him. But it is hard, perhaps impossible, to find examples in which the order of a given object or event is limited to what is directly apparent in perception. Rather, the perceivable order tends to be manifested and understood as a reflection of an underlying order, whether physical, social, or cognitive.

(Arnheim, 1971)

Vision is the most informative and complex sense among higher organisms such as humans. People receive 90% of information about their environment through vision. Sensation, perception, and thought exist only as subjective realities within the inner world of an individual. To emulate this nonmaterial world, we must investigate an objective physical system that generates such subjective realities – the human brain *(Yarichin, 2008)*.

Observations made by the mind and soul have been accompanied by mathematical calculations and formal scientific methods since the time of early philosophers *(Zielinski, 2008)*. The questioning about human perception of reality reaches back to the most ancient of philosophers, such as Empedocles or Democritus to Plato and Aristotle. An epistemology of perception was established at the turn of the 17th and 18th centuries (Descartes, Reid, Locke, Hume, Berkeley). Throughout the 20th century a lot of elaborated theories (*e.g.*, philosophy of phenomenology) and new scientific disciplines connecting philosophy, art, and science (Whitehead, Bergson, Arnheim) were established. Many theories of art were often developed by artists themselves (Cézanne, Kandinsky, Kupka) including the pattern language of the architect Christopher Alexander *(Beck & Cunningham, 1987; Alexander et al., 1977)*. The development of theories of perception was accompanied by the development of psychology – mainly cognitive psychology, behaviorism, gestaltism, psychoanalysis, experimental psychology – and neuroscience.

Since the 1960´s *(Youguo et al., 2008)* there has been a strong trend of studying information transmission in humans mediated by human sensory perception and (re)cognition using exact (mathematical and algorithmical) methods. This approach has been made possible by the extensive development of computer hardware as well as programming tools, which enable algorithm development and implementation for a variety of tasks. In this way, it is possible to test virtually any statement. This provides tests not only of the aspect under study, but also of the statement itself, namely its self-consistency. This approach is epistemologically anchored in systems theory, cybernetics, complexity, and information theory. It is implemented with multivariate analysis,and statistical approaches to analysis of phenomena resulting from nonlinear dynamics *(e.g., Fouda et al., 2014; Zhang & Xiao, 2014; Shatheesh et al., 2014)*, namely various types of information entropy and multifractal analysis *(e.g., El-Sayad et al., 2013; Abry et al., 2013)*.

Nevertheless, models and algorithms of perception and mind are still far from being complete, chiefly due to the vast complexity of the processes underlying brain functioning. The necessity to create new recognition algorithms which resemble human perception, is still unfulfilled *(Scheirer et al., 2014)*.

In this article, we aim to analyze the perception of a piece of visual art as a reflection of a Lyapunov stable object − the human brain in a given dynamic state − interacting with a perceived image of the object. (a discussion of Lyapunov stability in relation to living structures is given by *Zhyrova et al., 2015* in this volume.) We introduce a new method whose novelty lies in the usage of the information entropy approach − Point Information Gain $PIG_{\alpha,x,y}$ and Point Information Gain Entropy PIE_{α} (*Štys et al., 2011a; Štys et al., 2011b*) − which enables us to measure the contribution of an element (pixel) to the semantic and syntactic information of the studied images − a set of treated black-and-white drawings by a Czech 20th century artist František Kupka. With the use of sensometric methods, the goal is to find the most proper weight of individual PIE_{α} values of the PIE_{α} spectral dependency on the Rényi coefficients α (dimensionless coefficients in the $PIG_{\alpha,x,y}/PIE_{\alpha}$ calculations) which would map the automated analysis onto the human perception of shapes and patterns in the images in the given sample of human population as well as possible.

2. Method

2.1. Description and processing of digitized pictures

For the classification by human volunteers, 35 digitized reproductions of the series *Four stories of black and white* (about 1925, released in 1926) by the Czech artist František Kupka (1871−1957) were obtained from the Kampa Museum of Prague (CZ).

These original reproductions were then transformed into black-and-white images (Figure 1) and normalized to the same resolution of 1452 × 1985 px^2 *via* computation of global image threshold using Otsu's method in Matlab® software (Mathworks, USA).

The information of each black-and-white digital picture was calculated in Matlab® using modified Rényi information entropy. Firstly, a unique value was calculated for each pixel of each image − Point Information Gain ($PIG_{\alpha,x,y}$) − describing change of information when the pixel at any given position is omitted *(Štys et al., 2011a; Štys et al., 2011b)* as

$$PIG_{\alpha,x,y} = \frac{1}{1-\alpha}\log_2\frac{\sum_{i=1}^{n} p_{i,x,y}^{\alpha}}{\sum_{i=1}^{n} p_{i}^{\alpha}},$$

where the dimensionless coefficient α characterizes the distribution of probabilities of occurrence of given intensities (39 values of α = {0.1, 0.2, 0.3,…, 0.9, 1.1, 1.2,…, 4.0} were used). The logarithmic terms with probabilities $p_{i,x,y}$ and p_i were calculated from the probability density function without and with the examined point at coordinates (x,y), *respectively*. Further, i was the examined digital level and n was the number of digital levels (n = 2 for a black-and-white image).

Consequently, Point Information Gain Entropy (PIE_{α}), a cumulative value unique for each image sized $s \times r$ px^2 and characterizing information carried by all studied pixels was calculated as

$$PIE_{\alpha} = \sum_{x=1}^{s} \sum_{y=1}^{r} PIG_{\alpha,x,y}.$$

The $PIG_{\alpha,x,y}/PIE_{\alpha}$ values were calculated in two ways:

1. by the Whole method in which we created statistics of the occurrence of intensities from the whole image and obtained semantic information of an image ($PIE_{\alpha,\mathrm{Wh}}$) and
2. by the Cross method in which the intensity occurrence was calculated from the intensities of a cross whose shanks intersect at the examined point giving syntactic information of an image ($PIE_{\alpha,\mathrm{Cr}}$).

The total information of each picture of the studied series at a given α parameter was evaluated as a vector $PIE_{\alpha,\mathrm{Tot}} = [PIE_{\alpha,\mathrm{Cr}}, PIE_{\alpha,\mathrm{Wh}}]$.

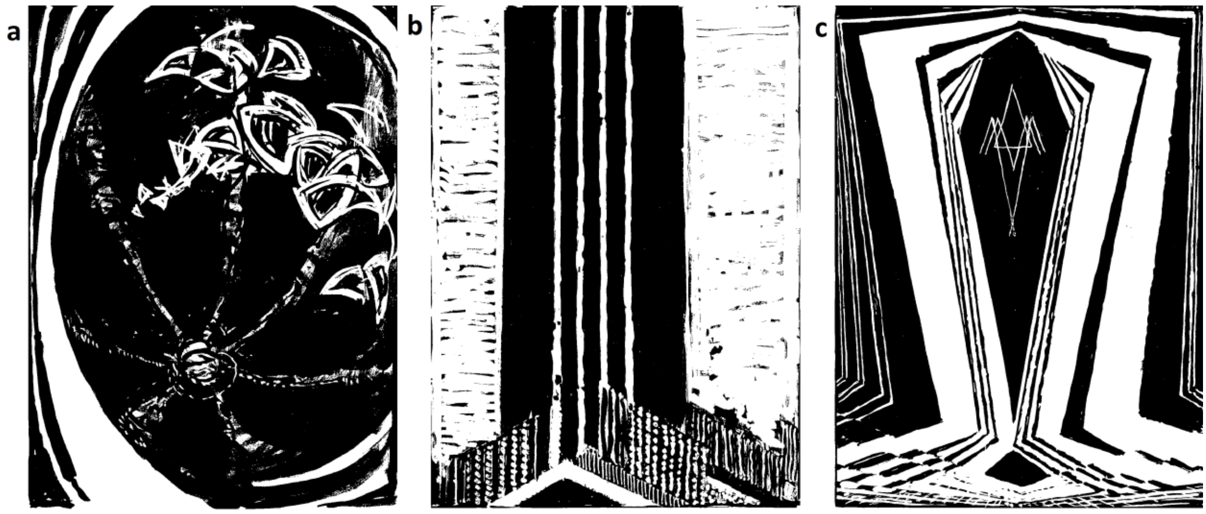

Figure 1: Examples of treated black-and-white pictures by František Kupka used for the experiment on human perception of shapes and patterns. a – Picture 6 with the highest average value of $PIE_{\alpha,\mathrm{Wh}} = (10.700 \pm 9.8705) \times 10^{-8}$ bit and $PIE_{\alpha,\mathrm{Cr}} = (-16.302 \pm 12.139)$ bit, b – Picture 26 with the lowest average value of $PIE_{\alpha,\mathrm{Cr}} = (-242.2 \pm 89.63)$ bit, c – Picture 27 with the lowest average value of $PIE_{\alpha,\mathrm{Wh}} = (5.0698 \pm 2.867) \times 10^{-7}$ bit. The average value of PIE_{α} was calculated for spectrum of α = {0.1, 0.2, …, 0.9, 1.1, 1.2,…, 4.0}.

2.2. Human evaluators

The group of human volunteers (evaluators) who classified 35 pictures arbitrarily into 5 groups consisted of 86 members: 40 males (46.51%, Evaluators 1 – 46) and 46 females (53.49%, Evaluators 47 – 86). To achieve the highest possible correlation of classification vectors between evaluators, the matrix of classification vectors was remapped *via* evaluation of cosine similarity and percentage of overlapping for each vector between each pair of evaluators. The groups of vectors were labeled in ascending order, *i.e.*, the group with Picture 1 corresponding to the majority of evaluators was labeled No. 1.

2.3. Multidimensional data processing

The matrix of $PIE_\alpha = f(\alpha$, Pic. No.) consists of predictors (**X**-matrix), whereas the correlated vectors of classification obtained from human volunteers were responses (**Y**-matrix) of the Partial Least Square Regression (PLSR; NIPALS algorithm, cross validation, 32 factors for $PIE_{\alpha,\mathrm{Wh}}$ and $PIE_{\alpha,\mathrm{Tot}}$, and 25 factors for $PIE_{\alpha,\mathrm{Cr}}$) using Unscrambler X® software (CAMO, Norway). The weighted regression coefficients β_w obtained from the PLSR were normalized to the 0 – 1 range to show the weight of PIE_α, which corresponded to each coefficient α for human classification of the pictures.

3. Results and discussion

3.1. Design of experiment on human visual perception

The domain of our research extends to several areas (Figure 2a). It is mainly the examination of human perception, particularly the perception of shapes and, more specifically, the perception of shapes in pieces of visual art. Drawings, which we use as a sample in this study, are just one of the ways how one can test the orientation of people in the visual complexity of their environment and how one can explain how they find patterns. However, we do not examine the perception of art works separately from the rest of the human processing of visual information and application to computer science and machine learning.

Clustering is an important topic in pattern recognition. Since only the structure of the data dictates the grouping (in unsupervised learning), information theory provides an obvious criterion to establish the clustering rule *(Gokcay & Principe, 2002)*. Our goal is to find such information-entropic parameters, which reflect the set-up of the human mind. For this purpose, we applied the method of the modified Rényi entropy for analysis of an image set and a group of human volunteers.

Figure 2b illustrates the design of the experiment for analysis of human perception by the classification of images (in detail in Section 2). The experiment consisted of two main parts. The first one is constituted by subjective clustering (Figure 2b). Volunteers (experiment participants) were asked to group cards of the images into groups according to percieved similarity without providing any other instruction and independent of number of pictures in each group. Decrease of variability in the matrix of classification vectors was achieved *via* the evaluation of the percentage of overlap for each pair of classification subvectors (groups of classification for each individual).

The second part of the experiment was an objective characterization of the same digitized images (Characterization in Figure 2b). The Point Information Gain Entropy (PIE_α) method of calculation was chosen as being suitable for the characterization. This method is derived from the Rényi entropy and was primarily developed for analysis of inner structures of microscopic images of living cells as self-organizing multifractal objects *(Štys et al., 2011a;*

Štys et al., 2011b). However, this procedure captures the information in other digitized structured experimental objects such as patterns in the examined test images.

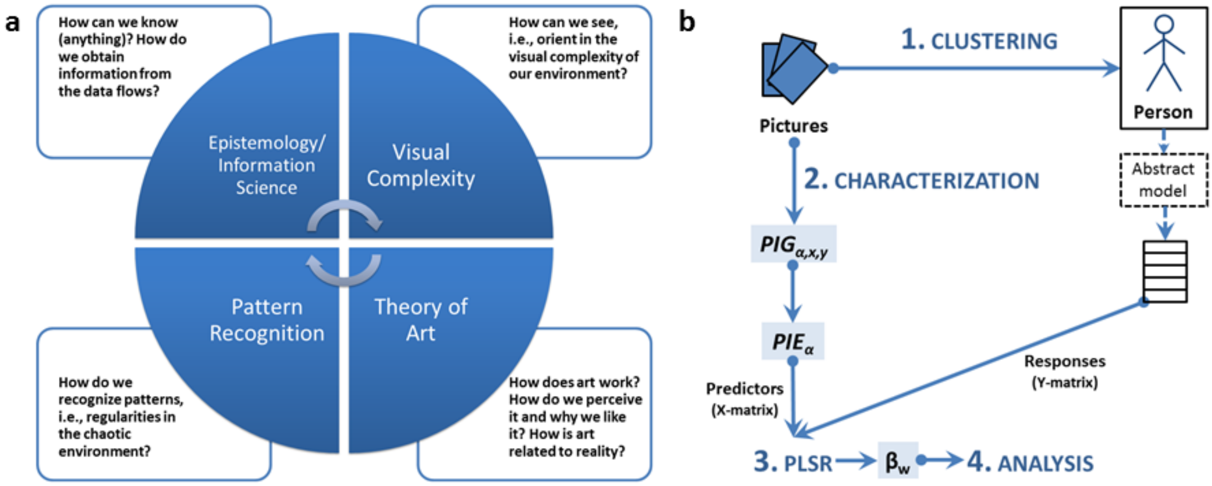

Figure 2: Scheme of (a) background and (b) design of the experiment on human perception of the shapes in images.

In the first step of the calculation, we calculated Point Information Gain ($PIG_{\alpha,x,y}$), a unique value for each pixel in the image at a particular value of the Rényi coefficient α, which describes the change of information (entropy) after omitting this pixel. In the formula for calculation of $PIG_{\alpha,x,y}$, each pixel is explained by its probability of the occurrence of its intensity in an image histogram. Low values of coefficient α highlight the pixels with low frequency of occurrence in an image, whereas higher α values draw uniform areas. To cover as many deformed distributions of intensities as possible, we decided to use 39 values of the dimensionless coefficient $\alpha = \{0.1, 0.2, \ldots, 0.9, 1.1, 1.2, \ldots, 4.0\}$.

As the next step, we calculated the Point Information Gain Entropy (PIE_{α}), a unique value specifying the information in an image at given parameter α, as the sum of all $PIG_{\alpha,x,y}$ values in the image.

The suitability of this methodological approach also lies in its ability to distinguish semantic and syntactic information perception in tested images. Semantic information deals with the overall meaning of the picture, whereas syntactic information reflects the structure (composition) of the picture. In our case, the semantic information is defined as change of distribution in an intensity histogram (explained as a frequency of occurrence of intensities) obtained from whole image after removing an examined point (abbrev. $PIE_{\alpha,\mathrm{Wh}}$). Syntactic information was calculated similarly, however, a histogram is formed by pixels, which create the cross with the examined (omitted) point at the intersection of its shanks (abbrev. $PIE_{\alpha,\mathrm{Cr}}$). The total information contents in each picture were measured as a joint vector of $PIE_{\alpha,\mathrm{Cr}}$ and $PIE_{\alpha,\mathrm{Wh}}$ at a given parameter α (abbrev. $PIE_{\alpha,\mathrm{Tot}}$). The definition of the syntactic element was in this case chosen arbitrarily and should be a subject to discussion and, perhaps, experimental testing.

We believe that the $PIG_{\alpha,x,y}/PIE_{\alpha}$ method is also able to describe the complexity of human perception of patterns. We assume that the human perception of shapes and patterns has been formed to select salient features of the observed reality, whereas irrelevant features have been naturally suppressed. Thus, this fact manifests itself during the classification of pictures into groups according to the similarity of their shapes and patterns. We expected to find that grouping pictures according to their depicted shapes (patterns) would be predicted by spectrum of PIE_α values for each image.

Therefore, a method of multivariate analysis ordinarily used in sensometrics *(e.g., Sundberg, 2000)* – the Partial Least Squares Regression (PLSR) – was applied to find an appropriate spectrum of weights of coefficients α controlling the grouping of tested images by each human evaluator. These weights are explained as weighted regression coefficients β_w, outputs from the PLSR. The resulted positive values of β_w directly influences the grouping, whereas the negative ones negatively relate to the output of the PLSR model of human perception.

As seen in Figure 2, the inputs to the PLSR are two matrices. The first one is a predictor matrix (**X**) containing vectors of PIE_α values as a function of parameter α for each image, while the second one is a response matrix (**Y**) formed by classification vectors obtained from human evaluators. The usage of the $PIE_{\alpha,\mathrm{Cr}}$, $PIE_{\alpha,\mathrm{Wh}}$, and $PIE_{\alpha,\mathrm{Tot}}$ matrix, *respectively*, as an **X**-matrix allows us to extract the semantics and syntax perceived by each human evaluator.

As test images, we intentionally decided to use 35 digital reproductions by the early 20th century Czech avant-garde painter František Kupka (Figure 1). According to *Kupka (1999)*, an artist who wants to create a cohesive art work should not imitate nature, but rather “form as nature”, to capture the natural dynamics of nature without imitating it. Work by Kupka is also made similar to shapes on computer models of nonlinear systems and processes and shows features of fractal geometry *(Anděl et al., 2000)*. These fractal features make art work by František Kupka suitable for analysis by the $PIG_{\alpha,x,y}/PIE_\alpha$ method. It is also important for our work that Kupka also dealt with the theme of vision, the complexity of visual perception, and looking at the physiology of vision as vital inspiration. In another words, the philosophy of Kupka´s creativity was similar to that with which the PIE_α was developed.

In order to ensure the classification of the reproductions only according to the drawn shapes, the black-and-white and identically resoluted forms of the reproductions were prepared. Nevertheless, providing the usage of, *e.g.*, a set of color images, the described method is also suitable for the examination of the human perception of colors.

3.2. Application and verification of experiment on human visual perception

According to the methodology described in Section 3.1, the cards of the treated black-and-white reproductions by František Kupka were classified by an 86-member team of evaluators (in detail in Section 3.1) into 5 groups. In this section, we shall set aside some individual and unique features of the examined population and try to identify some general properties of human vision.

The correlated vectors of this classification are depicted in Figure 3. In general, the set of classification vectors is divided into two main parts. The first (upper) one is more homogenous and contains mostly groups 3 – 5 with pictures No. 21–35 illustrating sharp rectangular and triangular shapes as seen in Figure 1b,c. For six evaluators (No. 12, 24, 30, 36, 39, and 50), this subset of the pictures forms only one single undivided group. The second part of the diagram is formed by pictures No. 1 – 20 with round shapes (*e.g.*, Figure

1a), mostly classified in groups 1–3. This set of pictures is seen as a one group by evaluator No. 19.

In Figure 3, 34 identical clusters (groups) of pictures are observable. Some combinations are identical for more than two evaluators. Even pictures No. 26 – 29 and 34 were identically classified into the same group labeled No. 5 by up to 15 evaluators (No. 16, 19, 23, 35, 44, 49, 59, 60, 63, 66, 68, 75, 78, 84, and 86). The recurrent classifications into at least two identical groups of pictures is also relatively frequent (17 different combinations). The uniformity for as many as two evaluators are emphasized: Evaluators 19, 38, 63, and 68 (groups 4 and 5), Evaluators 16, 23, 49, 66, 75, 78, and 86 (groups 4 and 5), Evaluators 19, 59, and 84 (groups 3 and 5), Evaluators 59, 75, and 83 (groups 2 and 5), Evaluators 4, 27, 34, 81, and 83 (groups 2 and 5), Evaluators 32, 38, 59, 67, 74, and 83 (groups 3 and 4), Evaluators 34, 62, 75, and 81 (groups 2 and 4), and Evaluators 59, 83, and 84 (groups 2 and 3). Thus, Evaluator 59 shows agreement in three groups with both No. 83 and No. 81. Classification groups 2, 4, and 5 are also identical for Evaluators 34 and 81. Evaluators 38 and 83 have groups 3–5 identical, too. Agreement in the classification of 31 pictures makes them the most similar pair.

Pictures 5, 12, 9, and 26 were being assigned to another picture with difficulties by evaluators No. 1, 72, 36, 49, 31, 40, 62, 85, 41, 43, and 52. In this case, they form separated one- or two-element groups. Despite the representation of round shapes, many evaluators classified pictures 5, 12, and 9 into groups 1 or 2. No clear relation has been found between the values of $PIE_{\alpha,\mathrm{Wh}}$ and $PIE_{\alpha,\mathrm{Cr}}$ of the pictures and their classification into groups.

In general, according to the results in Figure 3, we claim that the classification of the pictures is very uniform among our volunteers.

As mentioned in Section 3.1, we decided to use the PLSR method to explain the variability in classification according to semantic ($PIE_{\alpha,\mathrm{Wh}}$), syntactic ($PIE_{\alpha,\mathrm{Cr}}$, both in Figure 4), and total ($PIE_{\alpha,\mathrm{Tot}}$ in Figure 5) information measured in the pictures. This method also enables us to find a general spectrum of the weighted regression coefficients β_{w} *vs.* Rényi coefficients α, which control (influence) grouping of the pictures *via* human visual perception.

For semantic information calculated from $PIE_{\alpha,\mathrm{Wh}}$, the significant values of spectra of the coefficients β_{w} as a function of the Rényi coefficients α are situated at α in the range of 1.8–3.4. The majority (62) of the evaluators has the minimal value of β_{w} at α in the range of 2.3–2.5. The occurrence of the local maxima at α equal to 1.8, 2.0, 2.2, 2.5, 2.7, 2.8, 3.1, and 3.3 is the most frequent (47 evaluators). A combination of the minimal values of β_{w} coefficients at α = 2.4 with maximal values at α of 2.7 and 3.3, is the most frequent (11 evaluators for each α parameter). For some evaluators, a relatively strong region of β_{w} coefficients at $\alpha \leq 0.5$ is observable as well.

The weighted regression coefficients β_{w} calculated from the values of $PIE_{\alpha,\mathrm{Cr}}$ as an input matrix to the PLSR, are more uniform. They have a lower variability in comparison to the values calculated from $PIE_{\alpha,\mathrm{Wh}}$. The significant part of β_{w} coefficients occurs at α in the range of 0.7–1.6. One strong zone of local maxima can be seen at α = 3.7. Most of the evaluators have minimal and maximal values of β_{w} at α of 1.5 and 1.6, *respectively*.

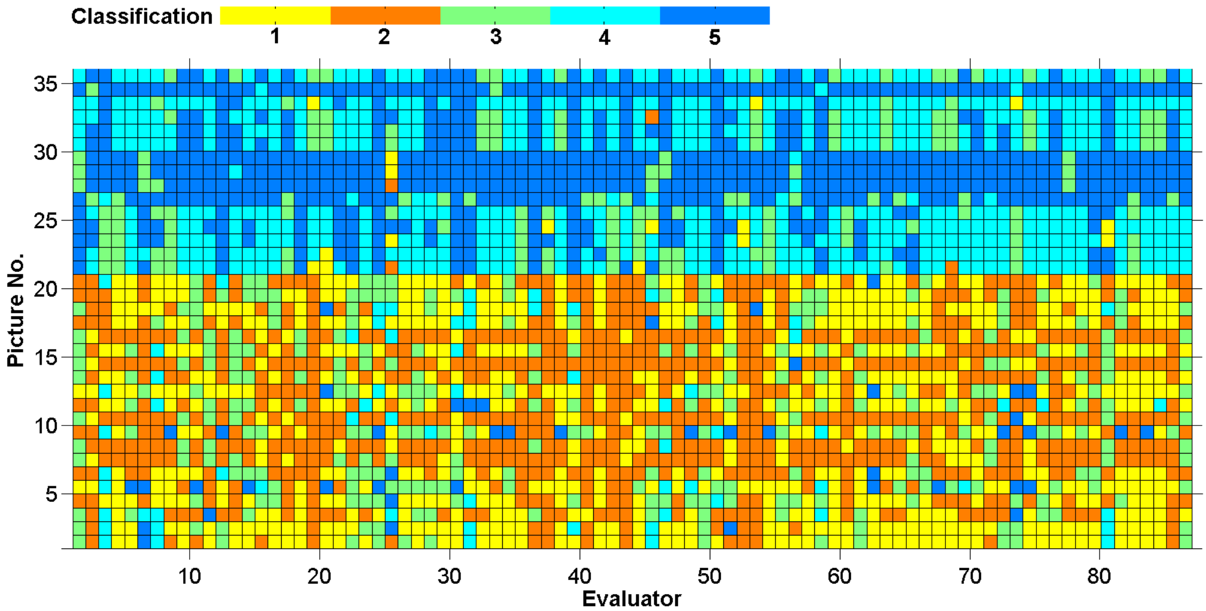

Figure 3: Vectors of classification of 35 black-and-white pictures into 5 color-coded groups obtained from 86 human evaluators.

The model of the total information correlates to the previous ones. The β_w coefficients significant for understanding human perception lie in the semantic part of graph in the range of α from 1.8 to 3.1. For a substantial number of the evaluators, the calculated β_w coefficients reach the minimal and maximal values at α in the range of 2.4–2.5 and 2.7–2.8, respectively. Many evaluators classified the pictures according to α of 2.1 (23 evaluators), 2.5 (10 evaluators), 2.2 (10 evaluators), 3.0 (17 evaluators). In agreement with Figure 4, the significant zone of β_w coefficients for syntactic information lies at $\alpha \leq 1.6$ with one stronger region at $\alpha \geq 3.0$.

Similar to the experiment performed by *Reed (1972)*, we managed to describe mathematically (to model) some general features of human perception based on application of the $PIG_{\alpha,x,y}/PIE_{\alpha}$ method. As seen in Figures 4 and 5, people perceive mainly semantics (meaning) of the tested pictures. Syntactic information is secondary for the process of classification. During classification, they were guided by (they perceive) mainly large areas and did not concentrate on details in the drawings so much. In our future research, we will search for such a grouping method and metrics that reflect this model provided by the human brain as best as possible. Such an algorithm would serve as a suitable tool used in, *e.g.*, cryptography *(e.g., Fouda et al., 2014)* or quality assessment of art works and the visual preferences of people *(e.g., Cunningham et al., 2007)*.

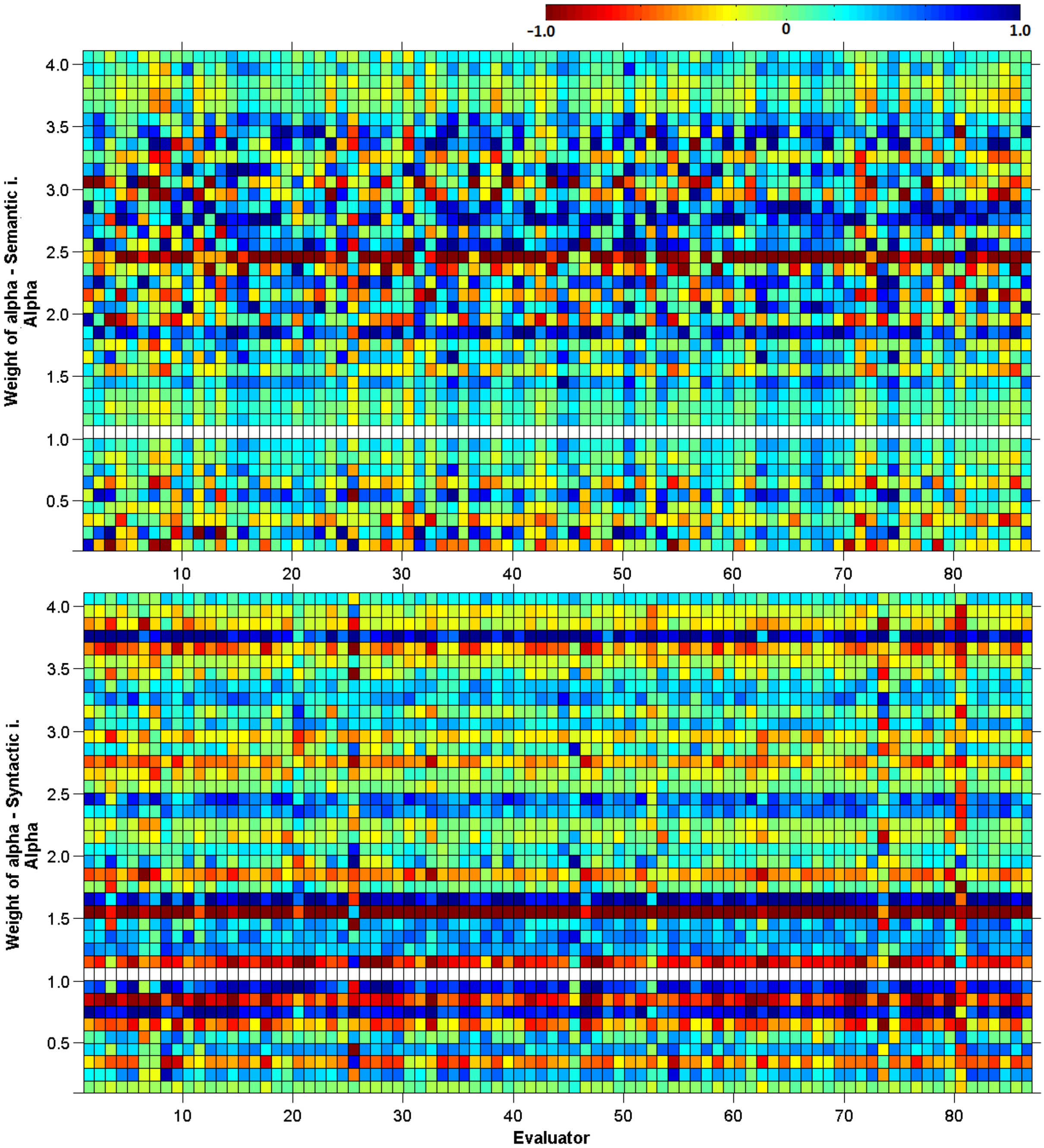

Figure 4: Normalized weighted regression coefficients βw obtained by PLSR and describing the weight of the Rényi coefficient α for the classification of 35 black-and-white pictures into 5 groups according to $PIE_{\alpha,\mathrm{Wh}}$ (upper panel) and $PIE_{\alpha,\mathrm{Cr}}$ (lower panel). Red – coefficients βw with minimal (or negative) relation to model, blue – coefficients βw with a positive relation to the model.

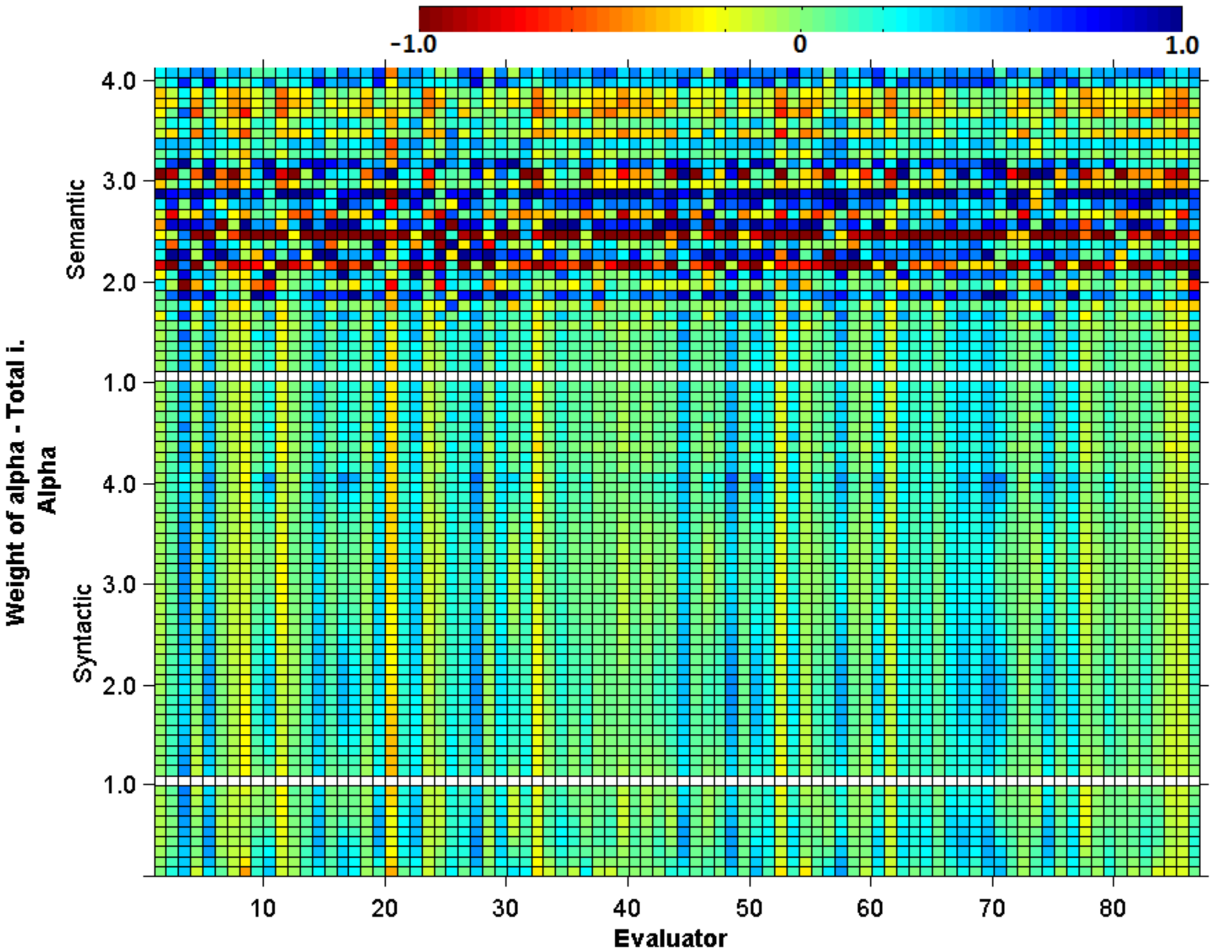

Figure 5: Normalized weighted regression coefficients *β*w obtained by PLSR and describing the weight of the Rényi coefficient *β*w for the classification of 35 black-and-white pictures into 5 groups according to $PIE_{\alpha,\mathrm{Tot}}$. Red – coefficients *β*w with minimal (or negative) relation to model, blue – coefficients *β*w with a positive relation to model.

4. Conclusions

This work is a contribution to the objective analysis of philosophical and epistemological questions, such as whether there are any general patterns and paradigms for/of human (re)cognition. We describe an exact mathematical methodology which connects the information entropy approach and multivariate data analysis, with the potential to study them. Verifying the method on a group of human evaluators with the use of drawings by the Czech artist František Kupka shows relatively uniform classification, related mainly to a computer-based semantic kind of information – meaning of the drawings. The syntactic information, reflecting the structure (composition), is secondary, however, there is also a clear similarity between groups of people in the weight of this parameter.

We believe that application of the research results in development of artificial intelligence and encryption systems, as well as in consumer research from the point of view computational aesthetics is possible.

5. Supplementary data

Supplementary data are available at *(ftp)* and contains:

1. Folder „Kupka_Pic“ with
   a. 35 original scans of sketches of *Four stories of black and white* series by František Kupka (subfolder „orig“),
   b. treated scans of the series into black-and-white version using *transformImg.m* Matlab® script (subfolder „bw“),
   c. and an 8bit resolution of computed $PIG_{\alpha,x,y,\mathrm{Wh}}$ (saved also in „PIG_Whole.xlsx“) and $PIG_{\alpha,x,y,\mathrm{Cr}}$ values for each picture (subfolders „cross“ and „whole“). For each Rényi coefficients α, white pixels are perceived in pictures with the highest probability by evaluators, whereas perception of black pixels is quite suppressed. In case of $PIG_{\alpha,x,y,\mathrm{Wh}}$, the more illustrative are graphical dependences of raw $PIG_{\alpha,x,y,\mathrm{Wh}}$ values on coefficient α *(*see *Graph1* for Picture 1*)*.
2. Folder „matlab_script“ containing
   a. implementation of algorithms for calculation of $PIG_{\alpha,x,y,\mathrm{Wh}}/PIE_{\alpha,\mathrm{Wh}}$ (*whole.m*) and $PIG_{\alpha,x,y,\mathrm{Cr}}/PIE_{\alpha,\mathrm{Cr}}$ (*cross.m*) into Matlab®.
   b. txt-file containing *transformImg.m* Matlab® algorithm for treatment of scans,
   c. *remap.m* Matlab® script (in txt-file) useful for remapping of original classification vectors obtained from human volunteers (Figure 3).
3. All PLSR models in „PLS.unsb“ file (Unscrambler®).
4. For each image, dependences $PIE_{\alpha,\mathrm{Wh}}$ and $PIE_{\alpha,\mathrm{Cr}}$ *vs.* α („Char_Pictures.xlsx“).
5. Tables of original vectors of classification obtained from human volunteers (List „orig_vectors), identical groups of classification among pairs of human volunteers (List „identical groups“), and remapping groups of classification according to similarity (Lists „remapping_1st step“ and „remapping last step“; all in „Class_Vectors.txt“ file).

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Příloha 4: Vstupní obrazová informace
(František Kupka: studie k cyklu Čtyři příběhy černé a bílé, 1924---25)