Posudek oponenta disertační práce Anny Horské
"Cut Elimination and Consistency Proofs"
předkládané v roce 2017 na Katedře logiky

1   Overview of the thesis

The thesis investigates Gentzen's cut elimination procedure. In Part 1, the
author focuses on Gentzen's original proof of consistency of PA. This is partly a
historical venture (simpler and stronger proofs have since been developed), but
it is supplied with careful mathematical analysis. In Part 2, the author compares
efficiency of different cut-elimination procedures in Propositional Calculus.

2   Brief evaluation

The thesis demonstrates Anna Horská's good understanding of Mathematical
Logic, as well as her ability to meet standards of scientific writing.

3   Detailed evaluation

The thesis is quite carefully written. I discovered almost no typos or errors – a
remarkable fact for a text of this length. The author expresses herself clearly.
Sections 1.1 - 1.4 are especially enjoyable to read: they are motivated by a clear
a goal and the goal is achieved in a coherent way. The subject at hand involves
many technical details, and the author does a good job in handling them.

In terms of the content, the thesis contains some new insights into cut elim-
ination. As far as I know, Theorem 4 from Part 2 is a new result. In textbooks,
cut-elimination usually follows the same strategy; Theorem 4 states that all
strategies give the same outcome (at least in Propositional Calculus). I find
this interesting. From Part 1, I learned that Gentzen's original proof does not
give what I thought it did – instead of transfinite induction up to ϵ_0, it implicitly
uses a higher ordinal.

The thesis has some imperfections too. Most importantly, the author chooses
formal correctness at the cost of legibility. Many of the proofs and definitions
are very tedious to read. The candidate should realize that a good mathematical
proof should be correct, but also accessible to the reader. For example, the proof

of Theorem 4 consists of nineteen pages of case distinctions. Similarly, Section 1.5 contains formulas that stretch over a whole page. This should be avoided if possible. (Again, the subject matter of Sec 1.5 is not an easy one.) Furthermore, one might want to be more economical with lemmas and definitions. Do we really need all the Lemmas 19-32?

I also wish the thesis had included a section placing Gentzen's result into a broader mathematical/historical context. Why is Gentzen's theorem important? And what actually *is* Gentzen's theorem? Another remarkable fact for a text of this length is the shortness of its bibliography.[1]

4   Questions for discussion

1. What we today call "Gentzen's consistency theorem" talks about transfinite induction up to ϵ_0 (as opposed to $\Phi_\omega(0)$). Is this a later improvement of Gentzen himself, or is it due to someone else? Did Gentzen even think of the question in this framework?

2. Theorem 3. Gentzen's theorem is sometimes stated in Primitive Recursive Arithmetic + Transfinite Induction. Hence, why do we need $I\Pi_3$ in Theorem 3 (rather than $I\Sigma_1$)?

3. Definition 39. Consider even more general strategy: pick any cut and eliminate it. Is it obvious that in some cases, the procedure does not terminate?

4. What goes wrong with Theorem 4 in predicate calculus?

5   Conclusion

Předložená disertační práce splňuje požadavky kladené na disertační práci, a proto ji doporučuji k obhajobě a předběžně ji klasifikuji jako prospěla.

5.10. 2017   Pavel Hrubeš

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[1]I suspect Section 1.5 could be greatly improved, had the author taken some inspiration from additional sources related to the subject.