Abstract The thesis consists of two parts. The first one deals with Gentzen's consistency proof of 1935, especially with the impact of his cut elimination strategy on the complexity of the proof. Our analysis of Gentzen's cut elimination strategy, which eliminates uppermost cuts regardless of their complexity, yields that, in his proof, Gentzen implicitly applies transfinite induction up to $\Phi_\omega(0)$ where Φ_ω is the ω-th Veblen function. This is an upper bound and $\Phi_\omega(0)$ represents an upper bound on heights of cut-free infinitary derivations that Gentzen constructs for sequents derivable in Peano arithmetic (PA). We currently do not know whether this is a lower bound too. The first part also contains a formalization of Gentzen's proof of 1935. Based on the formalization, we see that the transfinite induction mentioned above is the only principle used in the proof that exceeds PA.

The second part compares the performance of Gentzen's and Tait's cut elimination strategy in classical propositional logic. Tait's strategy reduces the cut-rank of the derivation. Since the propositional logic does not use inference rules with eigenvariables, we managed to organize the cut elimination in the way that both strategies yield identical cut-free derivations in classical propositional logic.