This thesis is about graph-indexed random walks, Lipschitz mappings and graph homomorphisms. It discusses connections between these notions, surveys the existing results, and shows new results.

Graph homomorphism is an adjacency-preserving mapping between two graphs. Our main objects of study are graph homomorphisms to an infinite path. We are interested in two parameters: maximum range and average range. The average range of a graph is the expected size of the image of a uniformly picked random homomorphism to an infinite path. We obtain formulas for several graph classes and investigate main conjectures on this parameter. For maximum range parameter we show a general formula and an algorithm to compute it for general graphs. Besides that, we study the problem of extending a prescribed partial graph homomorphism to a full graph homomorphism. We show that this problem is polynomial in some cases.