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Doctoral Thesis of Martin Kalousek

Dear Prof. Kratochvíl,

this is my report on the Doctoral Thesis of Martin Kalousek on the

„Analysis of flows on non-Newtonian fluids“.

In his thesis Mr. Kalousek studies the homogenization of systems with non-linear structure. These systems are strongly motivated by the flow of non-Newtonian fluids, where the fluid responds in a non-linear way to the shear-rate. The thesis consists of three parts, where Mr. Kalousek studies different aspects of this homogenization process.

In the first chapter Mr. Kalousek studies the homogenization of a non-Newtonian fluid. The problem is to pass from a microscopic model, that is valid for small cells that are repeated periodically over a large domain, to a macroscopic model. This reduction step is necessary to reduce the complexity of the problem to something that will later be accessible by numerical methods. In this first chapter Mr. Kalousek concentrates on the case of the homogenization of the non-linear Stokes problem. The extra stress is already a non-linear function of the shear rate. Mr. Kalousek considers here the very general case that the growth is given by an Orlicz function that satisfies minimal requirements (which still satisfy ellipticity and uniform convexity of the problem). For some special cases a corresponding theory has been developed in [6] by Bourgeat and Mikeić. However, Mr. Kalousek generalizes this work in the sense that he also allows more general Orlicz functions φ, which are singular (in the sense of $\varphi'(0) = 0$) at zero. Moreover, he includes the instationary case in his analysis. It should be pointed out that there is no convection in this part of the thesis, so there is no restriction of the Orlicz function from below to ensure compactness.

In the second chapter of his thesis Mr. Kalousek studies the homogenization of a fluid with a far more complicated behavior. In this situation the Orlicz function that describes the dependency of the extra stress from the shear rate may also depend on other given parameters. In particular, one can treat the dependence as an Orlicz function that depends additionally on the space variable. Moreover, the full non-linear system is treated that includes the convective term. In this situation the convective term has to be treated as a compact perturbation. However, depending on the technique very different growth conditions from below are necessary for the Orlicz function. Mr. Kalousek is able to use the most advanced technique here that allows for the smallest possible known growth condition from below. It is merely needed that the convective term is well defined in a distributional sense. Nevertheless, he is able to pass to the homogenized system. This method is based on the Lipschitz truncation technique. Unfortunately, this methods requires strong tools from harmonic analysis for the underlying spaces. For this reason the growth of the Orlicz function,

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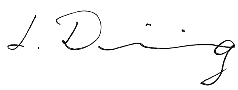
which is dependent on the spatial variable, needs to behave for large values like a fixed Orlicz function, which is independent of the spatial variable. It will be a very difficult step to overcome this simplification, in particular in this situation, where the lower growth of the Orlicz function should be as small as possible. This chapter is from a technical point of view the most sophisticated one and shows that Mr. Kalousek is able to use very advanced techniques.

I have only one small technical remarks of minor importance for this chapter: In Lemma 2.2.5 the Lipschitz truncation from [7] is generalized to the context of zero boundary values. However, the technique from [7] has been further developed already in this direction by Diening, Kreuzer and Süli (Finite element approximation of steady flows of incompressible fluids with implicit power-law-like rheology. SIAM Journal on Numerical Analysis, 51(2), 984-1015 (2013)). Moreover, the truncation in that paper is stable in all L^q and $W^{1,q}$, which would allow to simplify a few steps for example in Lemma 2.3.7.

In his third chapter Mr. Kalousek concentrates on the elliptic situation, where the growth condition is given by very general space dependent Orlicz functions. The system considered is without pressure and in particular without a convective term, which certainly removes all the difficulties that appear in the second chapter. Instead the difficulty in this chapter arises, first, from the the space dependency of the Orlicz function and, second, from the lack of super-linear growth (avoiding the ∇_2-condition). Since the pressure and the convective term is avoided the affect of the spatial dependence of the Orlicz function is however not very significant to the theory. Certainly, it is necessary to use Musielak-Orlicz spaces instead of Orlicz spaces, but there is no hard difficulty in this step. The technical harder part is to avoid the lack of superlinearity. Although I am not so sure on the necessity of this generalization in view of the application, it is still a technical difficult step. Mr. Kalousek shows in his thesis that he is also able to cope with this mathematically challenging situation.

The thesis of Mr. Kalousek is very well written. It contains many novel results, which show the authors great ability for creative scientific work. The results in the three chapters require the use of many quite sophisticated techniques, which show that he is certainly an expert in this area. It was a pleasure to read his work.

Sincerely,



Prof. Dr. Lars Diening