Report on the doctoral thesis "Analysis of flows of non-Newtian fluids"

by Mgr. Martin Kalousek
for the study program mathematics at the Department of Analysis, Charles University

The author presents in his thesis three scientific articles in three chapters. All chapters treat homogenization problems of incompressible non-Newtonian fluids with non-constant viscosity. On the one hand the models under considerations are physically important and mathematically interesting objects. Due to the fact that the viscosity depends in every instant on the particular shear rate even the respective Stokes problems are essentially non-linear. One famous example that is included in all the theory presented here are the so called power law fluids first studied mathematically by O. Ladizenskaya and J.-L. Lions in the late 60s. On the other hand homogenization is an important tool to describe macroscopical flow behavior as limits of microscopical approximations. Mathematically this idea goes back to L. Tartar in the late 70s, but has ever since been used in a plethora of kinetic modeling. This thesis connects these to approaches in a physically relevant manner.

The first chapter has as its content his single author publication in Nonlinear Anal. (2016). Here the author considers limits of fluids which are homogeneous mixtures of a non-Newtonian fluid with tiny particles. The author shows that a limit system (letting the size of the particles going to zero) of the respective stationary and instationary Stokes system exists; moreover, that it can be approached as a weak converging sequence of a homogenization procedure. The main technical highlight is the development of the structural background for general Orlicz spaces.

The second chapter consider stationary electrorheological fluids. These are fluids that are modeled in the setting of variable exponents, since the viscosity dependence on the shear rate changes locally with respect to a given electric field. It has a physical meaning to assume that the electric field oscillates rapidly. Therefore, a homogenization with respect to the oscillatory behavior is natural and should give some physical insight. Indeed, the limit stress tensor is not of (variable) power law any more but a generalized Orlicz function. The authors extend the results of V. V. Zhikov (Complex Var. Elliptic. Eqn. 2011) to a more general class of exponents. However, the restrictions on the viscosity are of such kind that the functional analytical background stays in the framework of classical Lebesgue spaces.

Chapter three is yet another homogenization result. It has the most general assumptions with respect to growth conditions. However it considers elliptic systems, which are in some respects simpler to treat, then systems of Stokes or Navier Stokes type. Nevertheless, the result is remarkable since it extends the many known literature of homogenization of elliptic systems considerably; indeed, the assumptions on the operator are so general that the mere existence of solutions to the non homogenized problems have been established only recently. The assumptions include various strictly convex (but not uniformly convex) minimizers/ operators that may have a log-Hölder continuous dependence on the spacial variable.

The thesis is written satisfying international scientific standards. Besides some inaccuracies (especially in Chap. 3) all proofs seem to be complete. The results are of good mathematical quality and do have the potential for future applications. The author of the thesis has proven to be capable to develop and formulate mathematically interesting and challenging results. Therefore, I can fullheartedly recommend to accept his application for a PhD.

Dr. Sebastian Schwarzacher 9.5.2017