

Review of the doctoral thesis of Mgr. Martin Kalousek

Homogenization of flows of non-Newtonian fluids and strongly nonlinear elliptic systems

The thesis consists of three results

- M. Kalousek: *Homogenization of incompressible generalized Stokes flows through a porous medium*. *Nonlinear Anal.* 136:1-32 (2016).
- M. Bulíček, M. Kalousek, P. Kaplický: *Homogenization of an incompressible stationary flow of an electrorheological fluid*. *Annali di Matematica* (2016). doi:10.1007/s10231-016-0612-5.
- M. Bulíček, P. Gwiazda, M. Kalousek, A. Świerczewska-Gwiazda: *Homogenization of nonlinear elliptic systems in nonreflexive Musielak-Orlicz spaces*, submitted.

The results deal with homogenization of evolutionary and stationary system of PDE's. They are connected by the structure of the elliptic term and by the basic applied method which is the two-scale convergence. In the first result an evolutionary and stationary variant of the generalized Stokes problem is considered. The growth of the elliptic term is given by an Orlicz function and the small parameter $\epsilon > 0$ describes size of periodic holes in the domain. The second result deals with a different variant of the Stokes problem. The elliptic term has a variable growth and the parameter describes speed of its oscillations. Finally the third result is concerned with elliptic nonlinear problem with a rather general growth given by an N function that may depend also on space variable. Moreover it is not assumed that Δ_2 and ∇_2 conditions are satisfied. Again ϵ describes speed of oscillations in the space variable.

I think that the results in the thesis are interesting, important, and develop the understanding of homogenization of flows of generalized Newtonian fluids and of a nonlinear elliptic problems. They nicely demonstrate how Martin mathematically developed. In the first article he applied as a main tool the two scale convergence, while in the second he combined it with the method of approximation of Sobolev functions by Lipschitz functions and Div-Curl lemma. In the last also further tool about nonreflexive Banach spaces, modular convergence, Young measures were added.

The formal layout of the thesis is good, it is typeset in L^AT_EX.

The collaboration with Martin Kalousek was pleasant. Even though some problems that he was asked to solve appeared not solvable, he did not hesitate and found a new fields where he could proceed further. At the final stage of his Ph.D. studies he worked completely independently.

I am sure that I can recommend to accept the thesis of Martin Kalousek for the doctoral degree.

In Prague, May 17, 2017

doc. Mgr. Petr Kaplický, Ph.D.
supervisor of the thesis