The aim of this work is to investigate the influence of perturbation by point mass on the caustic structure of the Navarro-Frenk-White model using the inverse ray shooting method. We specifically focus on the description of metamorphoses between different caustic topologies when changing the relative mass and position of the point. It turns out that in the combined model of discrete and continuous matter there appear some types of metamorphoses, such as elliptical umbilic, lips and probably also hyperbolic umbilic, that do not exist in purely discrete models. The main, and somewhat surprising, result of the work is the finding that even at the relative mass of the point 10^{-4}-10^{-3} the perturbation is strong enough to cause changes in the caustic structure which are in size comparable to the original caustics.