Report on the Bachelor thesis
'Characterization of functions vanishing at the boundary' by Hana Turčinová

Let Ω be an open subset of $\mathbb{R}^n$, let $p \in (1, \infty)$, denote by d the distance function given by $d(x) = \text{dist}\,(x, \partial\Omega)$ and let $W^{1,p}(\Omega)$ be the familiar Sobolev space; $W_0^{1,p}(\Omega)$ will stand for the closure of $C_0^\infty(\Omega)$ in $W^{1,p}(\Omega)$. It is well known that if $\partial\Omega$ has a mild regularity property, then $u \in W_0^{1,p}(\Omega)$ if and only if $u \in W^{1,p}(\Omega$ and $u/d \in L^p(\Omega)$. The usefulness of this classical characterisation led to various attempts to weaken the condition $u/d \in L^p(\Omega)$: first it was shown that it is enough to require that u/d should belong to the weak L^p space $L^{p,\infty}(\Omega)$; and then, very recently, that $u/d \in L^1(\Omega)$ will do. The thesis takes this process of discovery a step further in the special case when $n = 1$ and Ω is an interval (a, b), showing that $u \in W_0^{1,p}(I)$ if and only if $u' \in L^p(I)$ and u/d belongs to the Lorentz space $L^{1,p}(I)$. By means of a counterexample it is shown that it is not sufficient to require that u/d should belong to $L^{1,\infty}(I)$.

The style of writing is effective and to the point: the necessary definitions and basic results are given in just the right amount of detail, and the proofs of the new material given in reassuring but not overwhelming detail. I have a most favourable opinion of the thesis, which displays mastery of the material together with considerable originality and ingenuity: the main result is new and of definite interest.

I find the thesis to be well above the standard expected of a Bachelor thesis and strongly recommend its acceptance.

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