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Bachelor thesis

2017 **Samuel Maroš Kožuch**

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Using the log-periodic power-law model to detect bubbles in stock market

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Abstract

Stock market crashes were considered as an chaotic even for a long time. However, more than a decade ago a specific behavior was observed, which accompanied most of the crashes: an accelerating growth of price and log-periodic oscillations. The log-periodic power law was found to have an ability to capture the behavior prior to crash and even predict the most probable time of the crash. The log-periodic power law requires a complicated fitting method to find the estimated values of its seven parameters. In the thesis, an alternative simpler fitting method is proposed, which is equally likely to find the true estimates of parameters, thus generating an equally good fit of log-periodic power law. Furthermore, four stock indices are fitted to log-periodic power law and examined for possible log-periodic oscillations in different time periods, including a very recent period of 2017. In all of the analyzed indices, a log-periodic oscillations could be observed. One index, analyzed in past period, was fitted to log-periodic power law, which was able to capture the oscillations and predict the critical time of crash. In the rest of the selected stocks, which were analyzed in a recent period, the critical time was estimated with varying results.

Abstrakt

Pády akciových trhov boli dlho považované za nepredvídateľné. Pred viac ako desaťročím, bolo spozorované špecifické správanie, ktoré sprevádza väčšinu pádov: zrýchlujúci sa rast cien a log-periodické oscilácie. Log-periodické mocninné pravidlo bolo navrhnuté ako spôsob, ako zachytiť tieto oscilácie. Navyše, toto pravidlo je schopné odhadnúť najpravdepodobnejší čas pádu trhu. Mocninné pravidlo vyžaduje zložitú metódu fittingu, pre určenie odhadovaných hodnôt jeho siedmych parametrov. V práci je navrhnutá alternatívna metóda fitting-u, ktorá zjednodušila tento proces, a napriek tomu dokázala veľmi dobre estimovať hodnoty parametrov. Táto metóda tým pádom produkuje rovnako dobrý fit log-periodického mocninného pravidla. Okrem iného, štyri významné akciové indexy, v rôznych i nedávných časových obdobiach, boli analyzované pomocou log-periodického mocninného pravidla. Vo všetkých indexoch boli nájdené log-periodcké oscilácie. U jedného indexu, ktorý boli analyzovaný v dávnejšej dobe, log-periodické mocninné pravidlo dokázalo zachytiť oscilácie a predikovať predpokladaný čas pádu. U ostatných indexov, ktoré boli analyzované v nedávnej minulosti, bol tiež odhadnutý kritický čas s rôznymi výsledkami.

Keywords

log-periodic power law, stock market, prediction of crashes, market bubbles, econophysics

Klíčová slova

log-periodické mocninné pravidlo, akciový trh, predikcia pádov trhu, trhová bublina, ekonofyzika

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

I grant a permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, 17 May 2017

Signature

Acknowledgment

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Bachelor Thesis Proposal

Author	Samuel Maroš Kožuch
Supervisor	doc. PhDr. Ladislav Krištoufek, Ph.D.
Proposed topic	Using the log-periodic power-law model to detect bubbles in stock market

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Preliminary scope of work

Throughout the history, many people asked a question how, or whether there even is a way to predict stock market crashes. These huge markets contain a lot of invested money in them, thus any plummet of prices would cause a great amount of investment to be lost, throwing the economy into recession. Thus, the ability to predict these unfavorable events would be more than welcome. However, as many economists believe, these events are mostly unpredictable, happen randomly and can cause great damage. However, in the past years, many econometricians tried to answer again the question related to predictability. In fact, some even constructed models, based on stock market data, that could in fact predict these crises. In my thesis, I would like to focus on these models, study their accuracy and even simulate the stock market to uncover how these model function.

In 2001, Sornette et al. proposed a log-periodic equation, which should be able to predict these downfalls. Since then, many researches were conducted to determine, whether this model is accurate and I believe, that further research of this topic will provide a better understanding of the topic.

In my thesis, I aim to test the utility of the models, compare them with the real life situations and thus try to find, which of the models is the best under what conditions. I will try to find, whether these models follow real life situations and try to propose changes to models based on my observations. I believe, that this will contribute greatly to the already well defined models.

Therefore my overall contribution shall be connecting together these models and trying to apply them to real life situations.

Methodology

I will conduct a regression analysis of time series to determine the overall efficiency of the models. I will mainly work with a very complex power law

$$y = A + B\left(\frac{t_c - t}{t_c}\right)^{-m} \times \left[1 + C\cos\left(\omega \log\left(\frac{t_c - t}{t_c}\right) + \phi +\right)\right] \quad \text{for } t < t_c,$$

where t_c is the crash-time and other variables are free parameters. According to Sornette, the period of oscillations converges to the rupture point at $t = t_c$. Another model, the so called hazard rate, calculates the probability of the future crash and is defined by the hazard function

$$h(t) \approx \beta_0 (t_c - t)^{-\alpha} + \beta_1 (t_c - t)^{-\alpha} \cos\left(\omega \log(t_c - t) + \phi\right),$$

which has to be greater than 0, since it is a probability function. As t_c approaches t, the hazard rate approaches 1. Of course, there exist many more proposals, which will be later discussed in the thesis. My analysis will be conducted on time series data obtained from stock markets across the industries.

Proposed Outline

1. Introduction to market crashes, past occurrences and why a prediction method would be welcome
2. Literature review
3. Introduction to proposed models
4. Data - What type of data will be used?
5. Methodology
6. Empirical results
7. Discussion
8. Conclusion

Bctibliography

1. Sornette, D. (2003). Why stock markets crash: Critical events in complex financial systems. Princeton, NJ: Princeton University Press.

2. Graf v. Bothmer, Hans-Christian, and Christian Meister. "Predicting Critical Crashes? A New Restriction for the Free Variables." Predicting Critical Crashes? A New Restriction for the Free Variables. Physica A: Statistical Mechanics and Its Applications, Volume 320, P. 539-547., n.d.

3. Sornette, Didier, and Anders Johansen. "ArXiv.org Cond-mat ArXiv:cond-mat/0004263." The Nasdaq Crash of April 2000: Yet Another Example of Log-periodicity in a Speculative Bubble Ending in a Crash. Institute of Geophysics and Planetary Physics, University of California, n.d.

4. Grech, D. "Can One Make Any Crash Prediction in Finance Using the Local Hurst Exponent Idea?" Can One Make Any Crash Prediction in Finance Using the Local Hurst Exponent Idea? Institute of Theoretical Physics, University of Wroclaw, n.d.

5. Jiang, Zhi-Qiang, Wei-Xing Zhou, Didier Sornette, Ryan Woodard, Ken Bastiaensen, and Peter Cauwels. "Bubble Diagnosis and Prediction of the 2005-2007 and 2008-2009 Chinese Stock Market Bubbles." Bubble Diagnosis and Prediction of the 2005-2007 and 2008-2009 Chinese Stock Market Bubbles. N.p., n.d.

Contents

Introduction

Stock market has been an area of great focus in recent years primarily due to increasing importance of stock market for risk management and portfolio allocation. Individuals, companies, governments and investment groups participate in the stock market on daily basis, either directly or indirectly, further enhancing the importance of stock market. With a high participation and dependency rate, a sudden plunge in prices may have a disastrous effect on the market. Depending on a magnitude of a plunge, a market may not be able to fully recover to its original state for multiple years. A prediction of expected huge market drops, crashes, is therefore more than needed.

Crash happens when a sudden and substantial drop in prices occurs. Explained in terms of economic theory, the price of an asset in a stock market is driven by the equilibrium, the point where supply and demand meet. The equilibrium, however, is influenced by many exogenous variables, which may be the cause of crash. In this case, an exogenous crash occurs, as many participants put a plethora of sell orders at the same time, causing a distortion of a stock market, based on a piece of news. Usually, the news have almost immediate effect on the price that can last for a number of days or even weeks. A notable example of such a market 'crash' are the events, which occurred on the 11^{th} of September 2001, when an attack on the United States took place. Even though the markets remained closed for six days, the effect of attacks was felt anyway. On the first day of trading, the market fell 684 points, which is to this day the biggest single day loss. However, a nature of exogenous shocks is short-lived. One month after the attack, all of the major indices regained their pre-attack value and the confidence of the market increased furthermore. Opposed to exogenous shocks, endogenous shocks do not need any outside influence to be triggered. A common source of endogenous crash is a speculative bubble, i.e. over-valuation of an asset takes place after a sharp increase in prices due to a positive feedback. The bubble is often a result of a group thinking among individuals, when a majority takes the same action without any central direction.

Endogenous crashes are caused by bursting of a bubble. Bubble occurs, when market price of a stock is driven up above its true fundamental value. It is endogenous types of crashes that are the most dangerous. Not only is their destructive power for the economy of a vast nature, these crashes tend to last for a longer period of time thus amplifying the magnitude of economic disaster.

Due to the disastrous consequences of crashes a numerous methods were proposed as a warning sign of a incoming crash. Some are based on the comparisons of gross-domestic product of the country to stock indices with a goal of determining the overvaluation of the market, others use market returns to estimate market's behavior. In general, the accuracy of all methods varies, as there are many variables affecting the results and their likelihood. Predictions of the some methods may sometimes result in false positives, which is a case of predicting the crash, which does not happen in the end, thus raising a false alarm.

The Johansen-Ledoit-Sornette model was originally proposed in 1996 and was put to spotlight by Sornette's book *'Why Stock Markets Crash?'* in 2003. The three physicists, after which the model is named, propose an idea, that bubbles are characterized by faster-than-exponential growth of price rather than an exponential increase of price (Sornette et al., 2013). The model uses the log-periodic power law function with 3 linear and 4 non-linear parameters to detect log-periodic oscillations of price, which are regarded as a leading indicator of emerging crash. Log-periodic oscillations are oscillations that are periodic, but with respect to the logarithm of the independent variable of time, rather than the variable itself (Chang and Feigenbaum, 2008).

In the thesis, I would like to construct the Johansen-Ledoit-Sornette model and try to check for log-periodicity in the current market. The main focus of the thesis will be leading stock indices of American, Argentinian and Japanese market. The thesis is organized as follows. In the first section of literature review, a field of econophysics and its history will be introduced. Furthermore, a summary of already published research papers on this topic will be done. The model's assumptions will be stated in this part as well. In the second section of dataset description, the price indices, which were chosen to be analyzed, their short record and importance to their respective market will be introduced. In the methodology section, the basic notion behind the model will be introduced, the herding behavior of market agents, which might be a reason for a crash to happen, will be analyzed and the fitting procedure used in the paper will be discussed. In the fourth section, a separate case studies for all of the indicies will be included along with results of detecting log-periodicity results. Moreover, possible issues with the fit will be talked about in the section. In the final part, the thesis and its results will be summarized and possible future actions will be proposed along with recommended further research.

Literature review

A notion of applying tools from physics to a field of economy is relatively new. By the mid 1980s, a huge amount of financial data was already available and a lot of phenomena occurring in the market remained unexplained. A standard approach dealing with homogeneous agents and equilibrium situations was not able to explain most of the phenomena, since these occurrences did not comply with the homogeneity condition. A field of econophysics emerged as a new field that was able to tackle complex economic problems by applying tools of physics to economics.

The term 'econophysics' was first used by H. Eugene Stanley to describe a huge amount of papers written by physicists about distinct financial market problems. Officially, it first appeared in a conference on statistical physics in Calcutta in 1995 (Sharma, 2012).

Although, a field of econophysics is recent compared to other scientific fields, a relationship between economy and physics was recorded before. In fact, through 19^{th} century, physics was a key factor in developing economic theory. Moreover, many of the leading neoclassical economists were originally physicists, who decided to use their training in founding the economic field. One of the honorable mentions is Irving Fischer, who was one of the founders of neoclassical theory in economics and an apprentice of Jan Tinbergen, who became the first Nobel laureate in economics in 1969, for the development of applied dynamic models for the analysis of economic processes.

A revolution of the field began in 1980s, with adoption of electronic trading a huge amount of financial data became available. Application of concepts, such as power-law distributions, scaling and unpredictable time series, which are regularly used in physics, helped physicists achieve remarkable results in economic field. At the time, it was recognized that unpredictable time series and stochastic processes are not equivalent (Savoiu and Iorga-Siman, 2008). Indeed, a great effort was put into studying of chaos theory, originally proposed by Henri Poincaré in 1880s, specifically whether the evolution of asset prices in financial markets might be due to underlying nonlinear dynamics of a finite number of variables.

In 1996, a behavior of major stock indices in the United States during the crash of October 1987 was analyzed, when some of the indicies experiences a decline in prices by 30 percent. In the paper by Sornette, a common cusp-like behavior of price evolution growth is observed (Sornette et al., 1996). The behavior could be fitted to a pure power law, expressed by

$$F(t) = A + B(t_c - t)^m, \tag{1}$$

where t_c denotes a critical point, or a point in time with the highest probability of crash occurring. A power law, which describes a relationship, when a change in one variable is a power of another, was before found to be a supporting element of Kepler's laws of planetary motion (Russell, 1964).

In 2001, Sornette found a more specific behavior of endogenous crashes, that is log-periodic oscillations. A new log-periodic power law, a simple power law with a log-periodic extensions, was proposed in following form

$$F_{lp}(t) = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \log((t_c - t) + \phi)), \tag{2}$$

as a solution to detecting log-periodic oscillations (Sornette et al., 2001). The formula is dependent on 3 linear parameters A, B and C and 4 non-linear parameters m, t_c, ω and ϕ. A parameter t_c, as well as in the case of simple power law, is the time, when the crash is most likely to occur. The aftershock patterns were identified as well, being the mirror opposite to the pre-shock patterns, having decelerating price and decreasing log-periodic oscillations.

The behavior of anti-bubbles was defined as well, with the main idea being inverted. The price decrease is therefore followed by a price rebound (Sornette, 2003a). The Johansen-Ledoit-Sornette (JLS) model was extended to study the negative bubbles using the mechanism originally developed to detect earthquakes (Yan et al., 2012).

In 2013, Brée criticized the model for its slopiness and complexity, which could lead to a potential over-fitting. He based his conclusions on multiple attempts to fit stock indices from different markets and across different periods, that produced inconsistent results with relatively large errors (Brée et al., 2013).

A proposal was made to use Levenberg-Marquart algorithm in fitting the JLS model to empirical data in 2010. Although the proposal did not have negative impact on the quality of the model, it made the model even more complex to fit to the data and was marked as not suitable, provided that correct methodology is used (Brée and Joseph, 2010).

Multiple time periods and their respective lengths were fitted to the log-periodic power law producing results with differing quality. It was found, the best way to achieve a good fit of a time period is to fit it to various period lengths and use the fits with the best goodness-of-fit measure. Regarding the price transformations, the same method is recommended as used in the time period lengths (Sornette et al., 2013).

Due to abundance of parameters in the expression and the strong structure of the equation, calibrating the model was exposed to many difficulties. The optimization happens in a quasi-periodic structure with multiple minima where Taboo search or other algorithms have to used to find the global extreme. Despite this, the correct solution is not guaranteed to be found. The transformation, which was proposed to reduce the number of non-linear parameters from 4 to 3 was proposed, significantly decreasing the complexity of calibration procedure as well as increasing efficiency. Furthermore, the stability of the model is improved as it is characterized by smooth properties of its cost function with a single minimum present (Filimonov and Sornette, 2013). To further improve efficiency and time consumption, additional transformation is made to slave the remaining 3 non-linear parameters as variables of t_c, or the critical point, when the bubble may burst or a new regime may occur. This, in return, limits the search of global minimum to a three-dimensional space, therefore no further algorithms are needed to find the extreme, rather the search can reliably rely to local search algorithms.

Log-periodic power law was successfully used to evaluate whether log-periodic oscillation are present in the Indian market, which saw multiple market crashes prior to the research (Sarda et al., 2010). Leading Indian indices could be fitted to a log-periodic power law throughout multiple periods from 1997 to 2009, thus proving the crashes in the market follow log-periodic power law.

A more complex analysis of a single stock index was conducted by Chang and Feigenbaum. The duo was able to fit the log-periodic power law to the two largest drawdowns of Standard & Poor's 500 index since 1950. The evidence of log-periodic oscillation was found prior to the drawdowns. Furthermore, an extension of the model was introduced, to detect regime-switching. However, even after repeated attempts to fit the log-periodic power law extension, a stable estimates of parameters could not be obtained (Chang and Feigenbaum, 2008).

The log-periodic power law model's performance was further exploited by Korzeniowski and Kuropka, which used various periods of Dow Jones Industrial Average and Polish stock market index WIG20 to fit the Johansen-Ledoit-Sornette model. During the selected periods, both of the indices experienced large drawdowns, which needed to be examined for possible log-periodic oscillations. Three results with a relative prediction error below 5% and three results with the error above 15% were produced (Korzeniowski and Kuropka, 2013).

Twenty-two significant bubbles in Latin American and Asian markets were analyzed for log-periodic behavior by Johansen in 2001. The log-periodic power law had the descriptive power to capture all of the bubbles and the log-periodic behavior. Moreover, three of the bubbles in the Japanese market were found to be followed by anti-bubbles, a behavior opposite to a creation of speculative bubble (Johansen and Sornette, 2001).

In my thesis, I would like to perform an analysis of the major stock market indices using the model proposed by Sornette, Johansen and Ledoit, test the model's performance, exploit the fitting method used by the log-periodic power law, discuss the possible issues of fitting the model and test for a presence of log-periodicity in current stock market.

Dataset description

In the paper, a selection of stock indices will be used to evaluate the model's performance. The selection was based on importance of the index to the market, number of previous crashes that affected the index and the amount of previous researches conducted on the index.

For the construction and evaluation of the JLS model, following indices were chosen: Dow Jones Industrial Average (DJIA), Standard and Poor's 500, Mercado de Valores Buenos Aires (MERV) and Nikkei 225 (N225). Each of the chosen indicies is a leading index in its respective market and is commonly used to evaluate performance of the national market.

To successfully use the log-periodic power law model to evaluate the presence of log-periodic oscillations, daily prices are more than sufficient. Any transformation of prices may be fitted to the log-periodic power law, including log-prices (Sornette, 2003b).

The time periods and their lengths differ from stock to stock. The lengths and respective time scopes were chosen, so that the assumption of faster-than-exponential growth is satisfied and the log-periodic power law can be successfully fitted.

In the next subsections, each of the stock indices will be looked at from a financial perspective, its development over time will be assessed, together with its importance to the related market.

Dow Jones Industrial Average

The Dow Jones Industrial Average (DJIA) was first published in 1885. Named after its founder Charles Dow and statistician Edward Jones, the index includes thirty publicly owned companies and shows how the companies traded during an ordinary trading day (Sullivan and Sheffrin, 2003). Originally, companies included in the DJIA were involved in heavy industry, such as oil mining industry, car industry, etc. Currently, the name is considered to be of archaic meaning, as a limited number of companies are engaged in traditional heavy industry.

DJIA is one of the most known price indices and is argued to be a good representation of the United States stock market. This sparks a lot of criticism with special attention to the calculation of DJIA. In the past, DJIA was calculated as a sum of stock prices divided by the count of stocks included in the calculation - simple arithmetic average. Currently, DJIA is calculated as

$$DJIA = \frac{\sum_i p_i}{d}, \tag{3}$$

where p_i is the price of the i-th stock price and d is the Dow Jones divisor (Beneish and Gardner, 1995). The divisor is used to maintain the the historical continuity which is threatened by stock splits, spinoffs and changes in a list of companies included in Dow Jones calculation, that occured 51 times.

At the time of its publication, DJIA reached a value of $40. The value has been growing constantly and in March 2017, the DJIA broke a record value of 21000. With recent rapid growth observed in DJIA, a question arises whether the bubble is present.

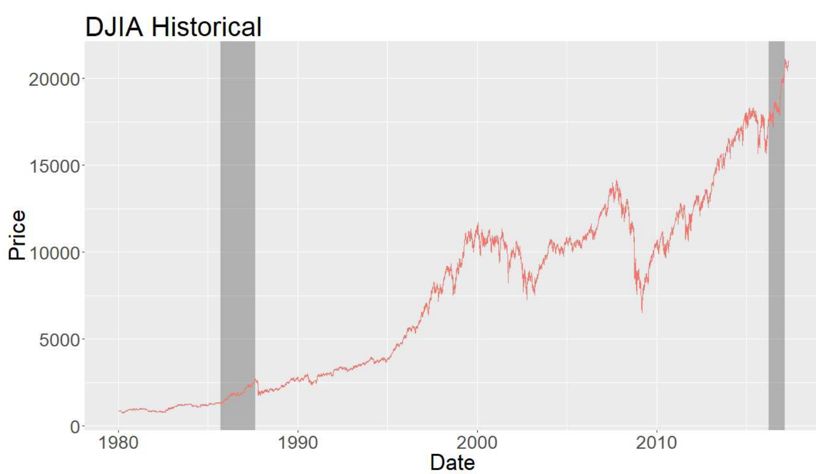

Figure 1: Historical development of DJIA from 1980 to 2017

In the thesis, two periods of DJIA were selected for further analysis, marked by grey areas in the figure presented above. Firstly, a period before Black Monday, September 1985 - August 1987, will be evaluated and fitted to the log-periodic power law. The primary purpose of this period is not to reveal presence of the bubble, but rather to use the period as a comparison period of the fitting method used in the thesis and those used in other researches with the same topic. The period prior to Black Monday was fitted in multiple published research papers, hence a good source of fitted parameters is present for comparison.

The second period includes all the trading days between April 2016 to March 2017, one of the fastest growing periods of the index in the history. The overall price of Dow Jones gained approximately 3600 points, rising from 17500 to 21100. One of the main factors behind the growth are considered US presidential elections accompanied by positive feedback regarding to business development in the United States.

Standard and Poor's 500

The Standard and Poor's 500 (SP500) is together with DJIA regarded as a leading indicator of US equities and reflection of performance of largest companies in the market. Unlike DJIA, however, SP500 consists of 500 companies, thus representing a larger sample of market. Moreover, the 500 companies are selected by a committee of economists based on market cap, liquidity and industrial grouping.

The index is calculated as a weighted average with weight being assigned by the size of the market cap of each company. Similarly to DJIA, SP500 uses the divisor to keep the price continuous in case of spin-offs, splits or change in basket. Starting in 2005, only publicly traded companies can be part of the basket (Podobnik et al., 2009).

Unlike DJIA, SP500 did not see a significant development in price during past 20 years. Although the trend of both series is the same, the scale differs, with SP500's price being significantly less than that of DJIA. The maximum value of SP500 is merely 2400 USD.

In the thesis, the period ranging from January 2013 to February 2017 will be in scope, the first period of continuous growth since the crash of 2008. Although, it may be thought that the development of SP500 and DJIA is the

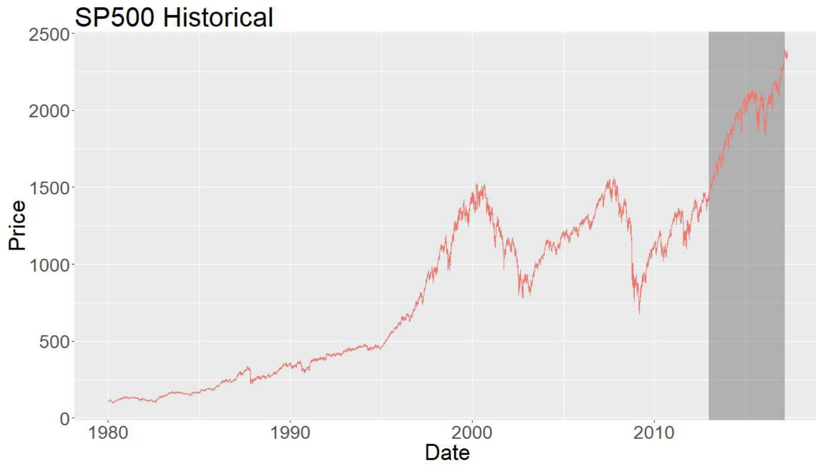

Figure 2: Historical development of SP500 from 1980 to 2017

same, that is not the case. In the SP500 index, less oscillations are present thus the results of fitting the log-periodic power law to the two indices will differ, presumably the critical time t_c will not be closely matched.

Mercado de Valores Buenos Aires

The Mercado de Valores (MERVAL) index is the most important index of Buenos Aires stock exchange. Similarly to DJIA, it is a basket weighted index calculated as the market value of a selection of stocks based on the market share, number of transactions and price. The index is revised every 3 months taking into account transactions, which took place during past year. As of 2016, MERVAL includes 23 companies, mainly invested in heavy industries such as oil mining and steel production.

The MERVAL index had an extraordinary development in past 15 years. During 2001, Argentina suffered from massive economic crisis, which ultimately led to an end of peso peg against the US dollar. Despite the crisis, the Argentinian stock market experienced a boom. MERVAL more than doubled during the crisis period and continued to accelerate even after the crisis ended. As the main factor of the counter intuitive behavior of MERVAL during the 2001 period is considered a shift of investments from the United States to Argentina, ultimately leading to a positive growth since then (Melvin, 2003).

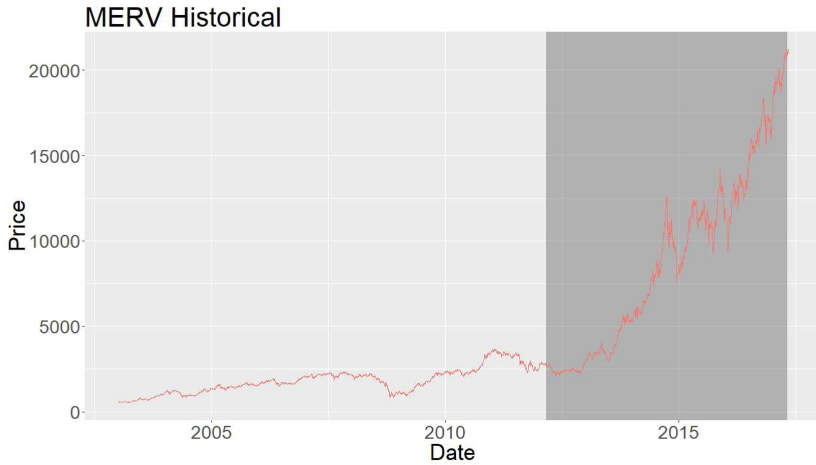

Figure 3: Historical development of MERVAL from 1996 to 2017

The fitting of the log-periodic power law to MERVAL data will be focused on a greyed-out period ranging from March 2012 to May 2017, a period when MERVAL, similarly to DJIA, experienced a huge boom, rising from approximately 2100 to 21100 USD. The growth of MERVAL is accompanied by plethora positive news, such as upgrading Argentinian market to an Emerging Market.

Nikkei 225

Nikkei 225 (N225) is the premier index of Tokyo Stock Exchange. N225 is not only one of the oldest indices in Japanese stock market, it is one of the most quoted as well, commonly used as a representation of Japanese stock market. Many financial products interlinked with N225 were created as a result of popularity of N225.

The Nikkei 225 includes upper 225 domestic common stocks of Tokyo Stock Exchange. The companies included in the index are revised once a year in October based on the annual reviews. The main aspects affecting the selection are liquidity and sector balance. At the moment, 6 sectors comprised of 36 industrial classifications are included in the basket.

The index calculation starts with setting a presumed par value for each member of the basket. The price of a stock is then divided by the par value to obtain the adjusted price of a stock. Adjusted prices are summed and divided by the divisor. Divisor's mission is in this case the same as in the case of Dow Jones, to sustain continuation of the price of the index (Greenwood, 2008).

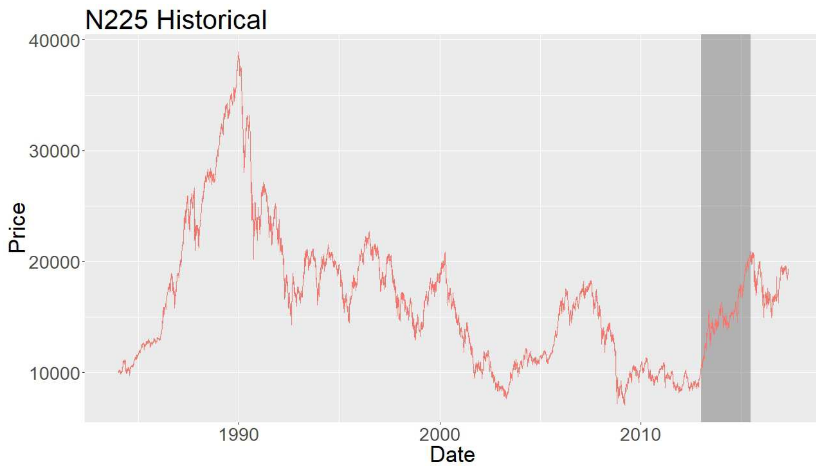

Figure 4: Historical development of Nikkei 225 from 1980 to 2017

The development of Nikkei 225 was quite volatile. At one point in time, the index reached as high as 38900. A swift upturn was followed by a Japanese asset price bubble burst in 1989, that led the index to fall significantly. The Nikkei never again reached its pre-1989 value but is still being considered as one of the most significant indices.

In the thesis, the period from January 2013 to June 2015 will be analyzed, a period during which the index grew significantly only to fall again in July 2015.

Methodology

In this section, an underlying mechanisms behind the JLS model will be introduced as well as fitting of the equation to the data and choice of best fitted parameters will be discussed. In the beginning, a building stone of JLS model will be introduced - the drawdowns, followed by the derivation of a model and an introduction to human and price dynamics. After that, a restrictions and recommendation of parameters will be discussed. In the end a brief overview of a fitting process will be introduced and a method of finding the best fit will be mentioned.

Drawdowns

In majority, the price movements of financial assets are based on the returns, which can be expressed in terms of hours, days, weeks or even years. The basic definition of return is a difference between market price in a scoped period. The problem with returns is their impossibility to capture the real dynamics of the market. In the case of crash, a market price goes through a substantial drop in value with large negative returns being present as a consequence. However, returns are fixed in time due to their non-changing time units and are assumed to be time-independent, thus making the capture of real market dynamics somewhat worse, when compared to other methods.

To imagine the problem, let's assume a market price drops by $X\%$ within a period of three days. Looking at the daily returns, a successive losses of $Y\%$ would be recorded. However, these records are considered to be independent and no focus is put on the successiveness of the losses.

The price dynamics in the JLS model is defined by drawdowns, a continuous decrease in close price over a period of consecutive days. A drawdown is terminated by the next increase in the price. Mathematically, the dynamics can be defined as the change in price from the local maximum to the next local minimum (Jacobsson, 2009). Drawdowns can easily capture the dynamics of the price as both the magnitude of the shift and its duration are taken into consideration. The changes are not considered to be autonomous, thus this measure captures both time dependence of price returns and the memory of the market. Their distribution thus captures the way successive drops can influence each other and construct in this way a persistent process (Sornette et al., 2001).

As with the returns, large drawdowns are extreme events in the market associated with tremendous price decreases. A complex analysis of the numerous drawdowns being present in the market lead to approximation of distribution of these events. The majority of drawdowns could be parametrized and fitted with the stretched exponential function, that is

$$f(x) = \begin{cases} z\left(\frac{x^{z-1}}{\lambda^z}\right) exp\{-\left(\frac{x}{\lambda}\right)^z\} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0, \end{cases} \tag{4}$$

where z is the shape parameter and λ is the scale parameter. A vast majority of 98 percent of drawdowns could be extrapolated by the stretched exponential function with a good result. The rest of the events were not following the distributions and could not be estimated well. These drawdowns are deemed as outliers and are rooted as crashes in the JLS model (Sornette and Johansen, 1997).

Derivation of the JLS model

The JLS model works with a term critical point, which is a point in time with the highest probability of crash (Sornette, 2003b). When the critical point is reached, three scenarios in the regime of growth may happen:

- the regime switches smoothly
- the market crashes
- the regime turns into anti-bubble.

The model does not only focus on the price dynamics but tries to implement expectation of human behavior as well. In the next sections, I will introduce the background logic of the model.

Feedback dynamics

The market price is fluctuating such that it satisfies supply and demand, thus ultimately reaching the equilibrium. In an equilibrium, an equal amount of buyers and sellers is present. A more alternative approach of defining market status is through terms of order and disorder (Johansen et al., 2000).

A market supply and demand is driven by agents, which in any situation have three options: to buy, to sell or to wait. Agents will choose the action to maximize their utility and meet their risk preferences.

In normal situations, although sounding contradictory, a market is in disorder. The same number of agents willing to buy and sell are present in the market and the price is driven by the 'invisible hand' of the market. Order occurs, when the balance of agents is distorted. A disagreements between the agents are diminished and the market becomes synchronized. Agents with lack of information will tend to stay rational, maximize their utility and hence follow the agents which seem to have more information. The resulting behavior creates a feedback, either positive or negative, which creates bubbles in the stock market. A positive feedback influences the present situation in the market based on the past events, sending out the positive signals. Finally, the price is driven up in the process known as self-reinforcing loops and the whole market is driven out of equilibrium. A speculative bubble, which is created as a result of behavior of the agents, lures people into thinking irrationally and not considering possible risks.

This type of behavior is known in the economics as the Bandwagon effect, when individual agents believe, that they are better off by copying the behavior of others rather than obeying their beliefs. This sort of crowd behavior is present in everyday life and is performed by individuals who lack the information and decide to follow the crowd as the opinion that crowd is always right prevails. As agents are organized in various social, economic or ethnic groups, more people will decide to join the crowds, creating further imbalance in the market. Under these conditions, the market becomes extremely unstable and vulnerable to any exogenous shock, which may trigger the crash.

To include the effect of herding into the equation a thorough look is needed into a behavior of a group. At any moment there are finitely many agents present in the market denoted by $i = 1, \ldots, I$. A subset of the agents $N(i)$ can be chosen such that each member of the subset is in some way connected to the agent i. Since there are finitely many agents present in the market, the group of interconnected agents $N(i)$ has to be finite as well and consists of $j = 1, \ldots, n$ agents.

In the group $N(i)$, each member can chose to take two possible actions: buying or selling. The selling of the stock would result in a price decrease, while buying would affect the price positively (thus, selling = -1, buying = $+1$). The movement of the price can thus be expressed by the formula

$$\Delta p = \sum_{i=1}^{I} s_i, \tag{5}$$

where s_i is the decision of the i-th agent to buy or sell. If the sum is negative, the best strategy is to sell, since the price is dropping. The opposite scenario applies to the case, when sum is positive. If the sum is zero, the market is filled with equal number of buyers and sellers, thus everybody's needs are met, equilibrium is present and no change to the price is necessary. However, the equation (5) is not known to a single trader. The best strategy for a single trader is to imitate the behavior of his neighborhood in a hope, that it represents the whole population well (Jacobsson, 2009). To evaluate, the agent has to determine prior distribution of probabilities for each trader, that is buying or selling. The optimal strategy for the i-th agent based on imitation of his network is given by

$$s_i(t+1) = sign\left(K \sum_{j=1}^{n} s_j(t) + \sigma\epsilon_i\right), \tag{6}$$

where variable K represents the relative imbalance of buyers and sellers and is inversely proportional to the market depth. Thus, in layman terms, K serves as an indicator of strength of imitation of the individual agent to his network. ϵ is an error with a standard normal distribution acting as a compensation for the agent's inaccuracy of judging the network as a good representation of population. Parameter σ represents the tendency towards distinct behavior.

Macroscopic modeling

The theory behind a single network of agents has to be somehow combined to not only represent one network, but the whole set to represent the behavior of the market. A critical value K_c exists, which is determined by the characteristics of the system of imitation. During periods when K is less than K_c, disorder is present in the market. Agents are not agreeing on each other's preferences (Johansen et al., 2000).

As K approaches the critical value K_c, agents start to agree with each other and order starts to appear as more agents imitate each other. Any piece of information might trigger huge collective behavior, eventually resulting in a rally or a crash. Logically, as the K gets closer to the critical value, the probability of the crash increases, although crash can happen at any value of K though being unlikely. In physics, this relationship is captured by the critical phenomenon represented by a simple power law

$$\chi \approx A\left(K_c - K\right)^{-\gamma}, \tag{7}$$

where A is a positive constant and γ is the critical exponent. These two parameters are the most important parameters of the simple power law.

In the JLS model, the simple power law is adjusted to reflect the fact, that highest probability of crash is at critical time t_c (Johansen et al., 2000). The hazard rate is dependent on time and is expressed by

$$h(t) = \frac{B}{(t_c - t)^{\alpha}}, \tag{8}$$

where the exponent α is specified as $\alpha = (\epsilon - 1)^{-1}$, with ϵ representing the number of traders in one network. The ϵ is bounded by the interval $\langle 2, \infty)$, as every agent must be connected to at least one other agent to be a part of a network. The time variable t specifies in what time period is the hazard rate calculated, while B is a positive constant.

By definition, the exponent will always be smaller than one restricting the price to be finite. It also important to stress, that his form of hazard rate can only be used up to the critical point t_c and not once, after it happens, e.g. $t < t_c$. Given the conditions, there exists a probability that the speculative bubble will not end smoothly or change into another regime expressed by the equation

$$\int_{t_0}^{t_c} h(t)dt > 0. \tag{9}$$

Price dynamics

In financial markets, there are many factors that have an effect on agents' decisions. Worth to mention are interest rates, dividends, risk aversion, information asymmetry and market clearing condition. However, these factors would be hard to implement into the model and thus the JLS model was build with a blind eye to these conditions. Instead, JLS model builds on an assumption of rational expectation (Sornette, 2003b).

The model of rational expectation assumes that every agent in the efficient market tries to maximize their wellbeing by using all of the available information. The rational expectation hypothesis then states, that the aggregated predictions of the future price will be the expected value conditioned of the information revealed up to time t, which will be the price of the financial asset at time t (Blanchard, 1979). The assumption can be captured by

$$E\left(p(t')|t\right) = p(t) \qquad \forall t' > t. \tag{10}$$

The solution to the equation (10) when no noise is present in the market is $p(t) = p(t_0) = 0$, with t_0 signifying the initial time interval. The equation (10) says that the observed price p_0 is captured as a linear combination of fundamental value p^* and the bubble component p, therefore:

$$p_0 = p^* + p. \tag{11}$$

The fundamental value in the market can be specified by any variation model for example geometric random walk. The bubble price component is independent of the fundamental value. The JLS model builds on this relationship by adding the log-periodic power law structure (Sornette et al., 2013), which is used to detect presence of bubble.

The power law structure assumes that the dynamics of the change in price can be expressed by a simple stochastic differential equation with a drift and jump:

$$\frac{dp}{p} = \mu(t)dt + \sigma dW - \kappa dj, \tag{12}$$

where p is the stock market bubble price, $\mu(t)$ represents the trend in the market price, dW is the increment of Wiener process, a continuous-time stochastic process (Wiener and Masani, 1957) with zero mean and unit variance. The σ in this case, is derived from equation (6) and captures the same tendency towards distinct behavior of an agent. The jump in the equation is portrayed by the term dj, which can have two possible values: $dj = 0$, before the crash happened, and $dj = 1$, after the crash occurred. The jump acts as a dummy variable for κ, the loss amplitude in price after the crash happened. κ is bound to be in interval $(0, 1)$ and signifies a fixed percentage by which the price drops (Brée and Joseph, 2013).

The dynamics of the jump is captured by the hazard rate, which forecasts the probability, that the crash will happen at any time t', which belongs to a positive dt neighborhood of variable t, conditional on the fact, that it has not yet happened ($t' \in (t, t + dt)$). Under these conditions, the mean of dj is defined as:

$$E\left(dj|t\right) = 1 \times h(t)dt + 0 \times (1 - h(t)dt) \tag{13}$$

$$= h(t)dt. \tag{14}$$

The hazard rate in this situation is similar to hazard rate seen in equation (8) but with an expansion of aforementioned log-periodic power law structure. The JLS model assumes, that the aggregate effect of noise traders can be captured by the following dynamics of crash hazard rate:

$$h(t) = B'(t_c - t)^{m-1} + C'(t_c - t)^{m-1} \cos(\omega \log(t_c - t) - \phi'). \qquad (15)$$

The cosine part of equation (15) accounts for hierarchical cascades of hastening panic arising in the market (Sornette and Johansen, 1997) resulting from preexisting hierarchy in noise trader sizes (Zhou et al., 2005). The extension also captures the positive feedbacks from agents, which drives the price above its true value.

By using the knowledge gained from equation (14) and taking the expectation of (12), the drift is set as:

$$\mu(t)dt = E\left(\frac{dp}{p}\right) = \kappa h(t)dt. \qquad (16)$$

Plugging equations (15) into equation (16), then integrating the resulting formula yields the log-periodic power law used in the JLS model:

$$\log E(p(t)) = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \log((t_c - t) + \phi)), \qquad (17)$$

where A, B and C are linear parameters. Both B and C are dependent on values of other non-linear parameters and are calculated as:

$$B = -\frac{\kappa B'}{m} \qquad C = -\frac{\kappa C'}{\sqrt[2]{m^2 + \omega^2}}. \qquad (18)$$

As was stressed before, this form of JLS model does not specify what happens after the critical time t_c is reached and thus can be used only up to that time. The time t_c is an estimation and its precise value is not known. The observed time of crash can be expressed by:

$$t_c^{real} = t_c^{estimated} + \epsilon, \qquad (19)$$

with ϵ being the residual error term distributed to some distribution (Sornette et al., 2013).

Characteristics of the JLS model

Since the log-periodic power law used in the JLS model is a relatively complex equation, it needs to meet a number of criteria to function properly. The criteria are imposed on bubble characteristic, parameters and even price evolution.

Firstly, the JLS model can only detect endogenous crashes in the market. Specifically, it is for detecting bubbles, which arise as a result of positive feedback mechanism of which the most common mechanisms present in the financial markets are herding and imitation of agents. Exogenous crashes show a very chaotic behavior as a result of big influence of new in the financial markets. In a bubble analysis, it was identified that approximately one third of all financial crashes are exogenous, thus unpredictable by the JLS model (Johansen et al., 2010). Still, the log-periodic power law is able to describe about two-thirds of the crashes.

Secondly, the JLS model, contrary to other models, uses faster-than-exponential growth of the price during the bubble. During the bubble, the prices accelerate rapidly above their fundamental value. However, most of the financial and econometric models are not able to detect a bubble, since the fundamental value in these models is poorly constrained and they are unable to distinguish an exponential growth of the fundamental price from exponential growth of the bubble price (Lux and Sornette, 2002). The JLS model is able to recognize apart fundamental value from the bubble, since it is based on the process, which is considered to be intrinsically transient due to positive feedback mechanisms that create an unsustainable regime (Sornette et al., 2013).

The JLS model uses seven parameters to describe a possible bubble. A wide variety of data can be fitted to the log-periodic power law equation, such as normalized prices, close prices, log-prices. It is recommended though to use the log-prices (Sornette et al., 2013), since the initial logic behind the model is based on them.

The most important parameters in the model are exponent of growth m, parameter of oscillations ω and estimated time of crash t_c. Exponent m is the exponent which affects the faster-than-exponential growth and has to be in a region of $(0, 1)$. If the m would be negative, the unrealistic divergence in price could be observed in a model. If the parameter is greater than one, the signs of decelerating price could be observed. Generally, in case of speculative bubbles m is expected to fall into region (Sornette, 2003b):

$$m = 0.33 \pm 0.18. \tag{20}$$

ω takes control of the oscillations, which can be observed in the financial market prior to the crash. The higher the value of ω, the higher frequency oscillations reach. The recommended values lay between 5 and 15, but observations suggest (Sornette et al., 2013), it is expected to be bounded by:

$$\omega = 6.36 \pm 1.56. \tag{21}$$

The fitting process

Due to complexity of the log-periodic power law, it is not an easy task to fit the equation to the data. The parameters shall be estimated in such way, that the residual sum of squares is minimized between the data and each day predicted by the model, thus a solution to the optimization problem is defined by:

$$\min F(t_c, m, \omega, \phi) = \sum_{i}^{N} \left(y(t_i) - \hat{y}(t_i)\right)^2, \tag{22}$$

where $y(t_i)$ and $\hat{y}(t_i)$ is the log-price or normalized price of the financial asset and the fitted price obtained by the JLS model, respectively. Variables A, B and C are left out of the optimization problems, as they do not carry any structural information and are only shift parameters. It can be easily proved, that:

$$\min F(A, B, C, t_c, m, \omega, \phi) = \min F(t_c, m, \omega, \phi). \tag{23}$$

In order to find the optimal solution to the problem, a Taboo search is recommended (Sornette et al., 2001). Firstly, a table for all possible values of t_c and ω is constructed and fitted values of m and ω are chosen as the estimates with lowest error. Only values, where m satisfies $m \in (0, 1)$ are chosen and a Simplex search is performed to find the best fit. A proposed method can be lengthy.

In the thesis, a following approach will be used. To optimize the fitting procedure a non-linear least squares estimation will be used. The algorithm searches for the optimal solution to the problem by using a relative-offset convergence criterion that compares the numerical imprecision at the current parameter estimates to the sum of squared residuals. The non-linear least squares algorithm needs starting values in whose ϵ neighborhood a search for the best fit is performed. The starting values will be provided in a form of combination grid of all possible values of parameters.

The second stage of finding the best fit will be indifferent to aforementioned process, hence only fits, for which m does obtain suitable values will be included in further analysis. Afterwards, only the best fit will be selected. However, due to large number of starting values and inaccuracy of estimation, there exists a moderate chance of several vectors of estimators having the same performance. In this highly likely situation, all of the fits, which have best goodness-of-fit measure, will be selected and a breeding mechanism will be performed. In particular, out of all of the fits a median values for all estimators will be taken and the fitting procedure will be performed again. The resulting fitted parameters should have the lowest sum of squared residuals and thus showing the best performance.

Results

In the following section, the results of the fitting process will be analyzed. At first, a comparison of the results of the thesis to the results obtained by other research papers will be made. After that, each of the stocks with their respective periods will be analyzed.

Dow Jones Industrial Average (1985 - 1987)

By using the fitting method described in the previous section, the results were pretty consistent with results of other researches. Compared to (Korzeniowski and Kuropka, 2013), the differences in the main variables affecting the log-periodic power law m and ω are negligible. The predicted times, in both papers, are defined as decimal parts of a year. One day is defined as

$$\frac{1}{365} \approx 0.0027. \tag{24}$$

This number is then multiplied by a serial number of a given day and added to a serial number of a year, e.g. 100$^{\text{th}}$ day of year 1987 equals to $1987 + 100 \times 0.0027 \approx 1987.27$.

Fit	A	B	C	t_c	m	ω	ϕ	R^2
Thesis	3852.51	-1812.90	-102.97	1987.98	0.36	8.29	5.32	0.9868
Research	3808.17	-1768.21	-102.74	1987.98	0.37	8.27	0.53	0.9868

Table 1: The thesis' results for DJIA compared to (Korzeniowski and Kuropka, 2013)

The goodness-of-fit measure is the same as well as the predicted time of the crash. The difference in other parameters does not affect the results greatly as the parameters are only considered to shift the fit and carry no structural information.

It is notable to mention, that the goodness-of-fit measure is extremely high. However, this is result is expected as the the log-periodic power law tries to model prices, a process in which high goodness-of-fit measures are normal.

The visualization of the fit can be observed below. As was mentioned before, the log-periodic power law can only be used up to the point of crash, which is clearly distinguishable in the graph, as the fitted log-periodic power law (blue line) starts to deviate from the historical value of Dow Jones Industrial Average. The JLS model is unable to capture the dynamics after the crash and thus it is not expected to follow the price after the critical time t_c is reached. The maximum value of LPPL is reached at time t_c, or the predicted time of crash (green line) and its value is given by A.

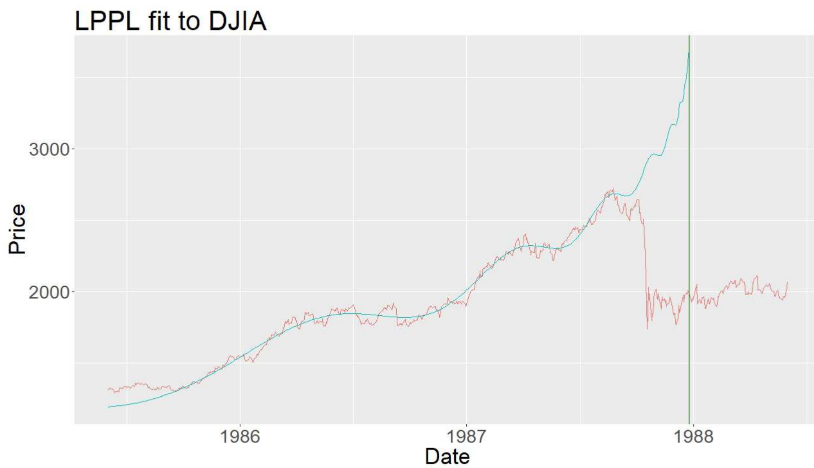

Figure 5: Fitted LPPL to DJIA in pre-1987 time period

Dow Jones Industrial Average (2016 - 2017)

Although, in present time Dow Jones Industrial Average follows the log-periodic patterns, the fit was somewhat worse. Despite that, the fit is good enough to assume, that DJIA does follow the log-periodic power law. The estimates obtained by fitting the model to data are provided below.

Fit	A	B	C	t_c	m	ω	ϕ	R^2
DJIA_2016	25054.15	-7780.60	408.43	2017.21	0.22	5.22	14.27	0.9252

Table 2: Estimates of LPPL for DJIA in the period from 2016 to 2017

The parameter of exponential growth m is well between the boundaries, determined by Sornette, that were defined by the most significant crashes. The ω, which signifies the frequency of oscillations also lays in the region recognized to be dangerous.

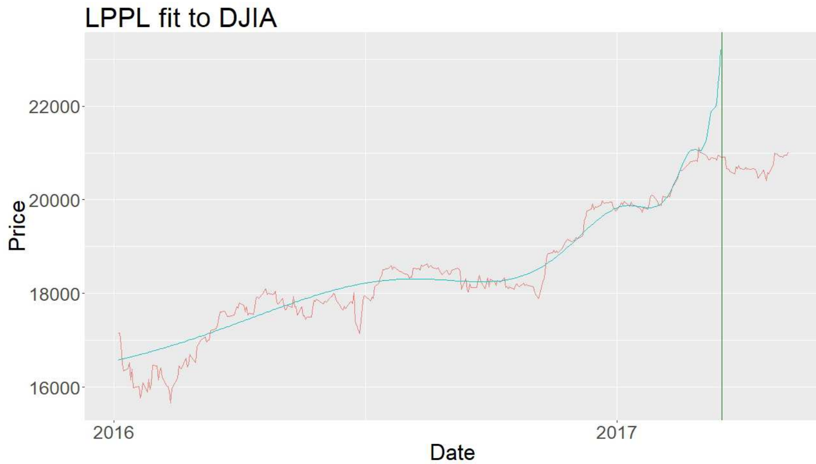

Figure 6: Fitted LPPL to DJIA to current time period

As can be observed from the graph, the fitted equation starts to deviate from the price just before the peak is reached. The power law seems to be wrong in two estimated things: the estimated price at the critical time and the critical time itself. The price, determined by A, is estimated at 22319.66, which would mean, that DJIA would have to reach the new highest price. To satisfy the estimated critical time, this act would have to happen in the mid 2017 at last, so the deviation from the critical time is not as high.

The possible explanation behind the worse fit of the present DJIA data is the unexpected drop in November 2016, which was influenced by negative news regarding to US Presidential elections. The dip in price caused the index to stop following the log-oscillations temporarily only to start the patterns again shortly after. Unfortunately, the period after the US Presidential elections did not contain enough trading days (observations) to successfully fit the JLS model.

Standard & Poor's 500 (2013-2017)

Since the Standard & Poor's 500 index seemed to be less volatile, the expectation for better fit of the log-periodic power law was in place. Indeed, the goodness-of-fit measure is greater in comparison to the DJIA in the present period, although the difference in lengths of periods has to be taken into consideration. The table with estimates is shown in Table 3, below.

Fit	A	B	C	t_c	m	ω	ϕ	R^2
SP500_2013	2509.53	-220.44	45.20	2018.37	0.80	6.06	5.19	0.9495

Table 3: Estimates of LPPL for SP500 in the period from 2013 to 2017

The thing that stands out to SP500's counterpart, DJIA, is the estimated critical time. While the estimate Dow's estimate was in the first third of 2017, in case of SP500, the estimated critical point is more than a year later. The cause of such a 'delay' is easily recognizable. Although the variable of oscillations ω falls withing range of most crashes, the other important variable, m, does not, which is the main factor producing the results with nearly a year long delay. The higher the value of m, the less accelerating the stock is. In this case, the acceleration of the stock is not sufficient for the log-periodic power law to determine the critical point in the nearby future. The behavior can be observed in a graph below, where the log-periodic power law assumes, that oscillations will continues as it dips below the actual price of the stock index.

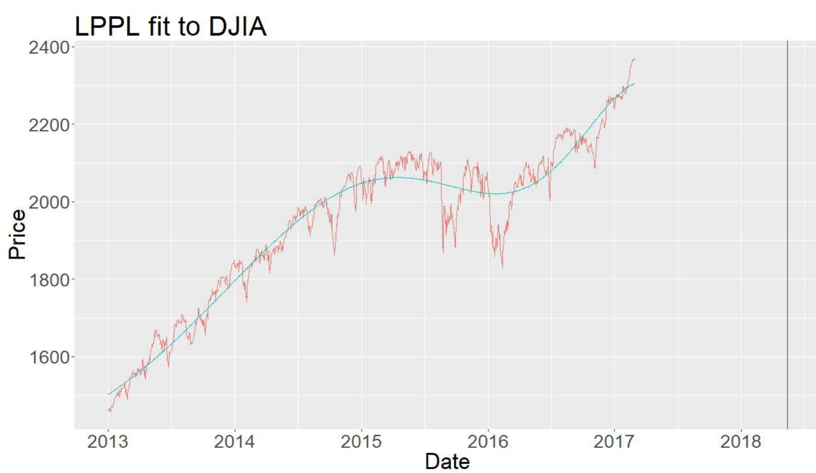

Figure 7: Fit of log-periodic power law to recent SP500 prices

Another possible explanation behind the difference in prediction of critical point might be the difference in time period, when smaller volatilities are ignored. The neck-to-neck comparison within the same time period would be needed to compare the real results of the DJIA and SP500. The main problem of the comparison would be the aforementioned presidential election. The SP500 suffered in a less rapid fashion over a greater period of time in comparison to DJIA. This small, but significant change seems to throw off the log-periodic power law, which is no longer able calculate sensible estimates.

Mercado de Valores Buenos Aires (2012-2017)

Mercado de Valores Buenos Aires is the index that is market with a constant and very fast growth since 2009. The increasing oscillations could be observed in the index's price, thus implying log-periodicity. In fact, the goodness-of-fit of log-periodic power law to MERV and the calibration example of DJIA in 1987, was only different by 0.01 in favor of Dow Jones, which is the exhibition piece of log-periodicity. Below are the estimated parameters of the power law.

Fit	A	B	C	t_c	m	ω	ϕ	R^2
MERV_2012	23799.56	-9914.01	813.91	2017.34	0.46	5.21	14.80	0.9787

Table 4: Estimates of LPPL for MERV in the period from 2013 to 2017

As was mentioned, the quality of fit is very good, one of the best in the thesis. Both crucial parameters m and ω are within norms and do imply log-periodicity. Below, a fitted JLS model is shown.

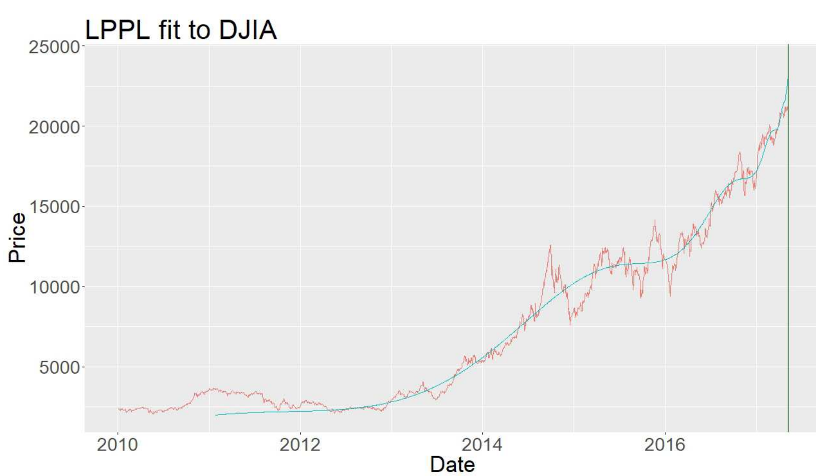

Figure 8: A log-periodic behavior observed in MERV

According to the log-periodic power law, the predicted most critical point is in May 2017 with a predicted price of 23797.01. The price of the index was at 21212.03 as of May 1st 2017. The fast growing nature of the Mercado de Valores, especially in recent years, may indicate that at this rate, the predicted crash price is nearing and following in a not so distant future will be the bubble burst of the index.

Although MERV was following log-periodicity very well, there is no guarantee that the theorem behind the prediction is correct. The fast growing nature of the Mercado de Valores index seems to baffle the log-periodic power law. If any other period, with the same starting date but ending date prior to May 2017 (with exception of periods of drawdowns) would be fitted to the log-periodic power law, there are non-negligible chances that it would predict the critical point not far after the period's end, due to fast growing nature of the index.

Nikkei 225 (2013-2015)

Unfortunately, the behavior of Nikkei 225 during writing of this thesis did not represent any kind of log-periodic behavior. Nevertheless, a look into not so distant past reveals a recent log-oscillations in the Japanese stock index. Although this behavior is not as clean as it was with the previous object of interest, the LPPL could be fitted anyways. The parameter estimates are provided below.

Fit	A	B	C	t_c	m	ω	ϕ	R^2
N225_2013	21537.58	-6003.76	798.03	2015.47	0.52	5.45	3.51	0.9220

Table 5: Estimates of LPPL for N225 in the period from 2013 to 2015

The goodness-of-fit is mediocre. In later stages of the period, the log-periodic power law seems to copy well the price of the index. It is the earlier stages that are the Achilles' heel of the LPPL. Big volatile disruptions are not copyable by the JLS model thus the goodness-of-fit suffers a lot during these stages.

Essential parameters m and ω are again in their relative norms. The predicted time falls somewhere to June 2015, an estimate that is extraordinarily accurate to the time at which bubble burst occurred in early July 2015 in the Japanese market. Although the magnitude of the 'crash' was relatively small, it was predictable by the log-periodic power law.

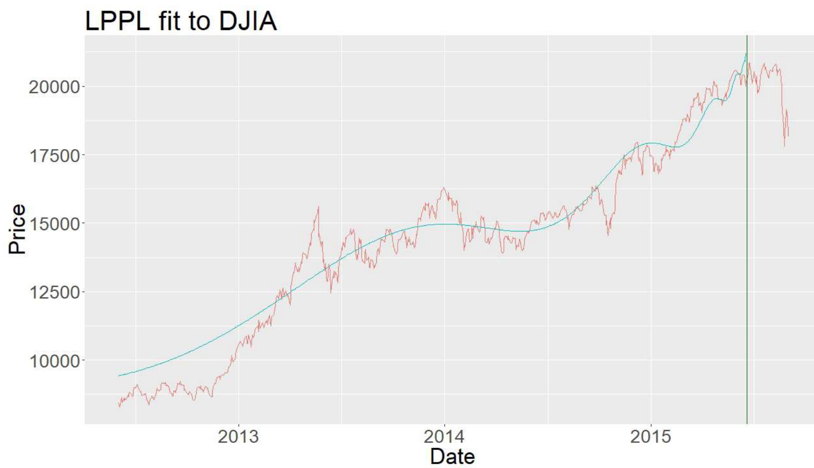

Figure 9: The fit of LPPL to N225 data with huge disruptions at the beginning

Unfortunately, the disruptions at the beginning of the observed period could not be skipped, due to being part of the oscillation.

Strengths and weaknesses of the JLS model

During the not so short usage of the log-periodic power law to detect bubbles in the stock market, a few of its good prospects and some of its unexpected drawbacks were noticed. These will be summarized in the following section.

Probably, the most appreciated feature of the log-periodic power law is the variety of data it can be fitted to. The estimation can rely on the hourly data, daily data, weekly data and even monthly data provided their abundance and low variance. Also, any transformation of the prices is acceptable as well. Preferred transformation are log-transformations. In the thesis, a standard non-transformed price was used as the better results were achieved when compared to the log-transformations.

Furthermore, the parameters in the equation are easy to interpret. All of them have clearly defined meaning and purpose and their ranges are well specified. In case that any parameter is outside its range, it usually takes seconds to identify why it is so. In a number of situations, no other analysis than a visual look is needed to detect a possible problem. Maybe one of the main reasons of this ease in problem solving is very well understandable human and price dynamics background, on which the JLS model builds its predictions.

Last, but probably one of the most important strengths of the JLS model, is the prediction ability. This has been tested post crashes in multiple other researches and also prior to the crashes. Bubbles were successfully identified in recent bubble bursts in Indian stock market, and many others. Under certain conditions, the log-periodic power law can certainly be considered a strong predictor.

On the other hand, the log-periodic power law comes with a number of weaknesses as well. One of the main disadvantages of the log-periodic power law is the complicated fitting process of the parameters. Since the parameters have wide range of possible values (especially the linear parameters), and the fitting process is very sensitive to starting values, a huge combination matrix must be created, so the best possible fit may be found. Even then though, the chances are that the best fit will not be found due to optimization taking place in seven dimensional space.

Moreover, the log-periodic power law is extremely sensitive to even smallest deviations in price, which may greatly affect the estimates of parameters. To eliminate the effect of the deviations and achieve the best fit possible, multiple time scopes have to be tested and analyzed to determine the chance for the best fit. Combined with the fitting process, this search for the best fit may take days.

Another, rather underwhelming feature of the log-periodic power law, is its inability to predict magnitude of the crash. As could be seen in part Nikkei 225, the predicted bubble burst was followed by rather small and short drop in prices. With LPPL, even though the crash is predicted, the magnitude and duration of the crash remains unknown.

Conclusion

In this thesis, an analysis of bubbles present in the market was conducted. A compromise fitting process was proposed. All of the fits with exponential component above the value of 1 or below the value of 0 were not included in the testing. After this, only the best fits were taken into consideration.

The calibration performed on the Dow Jones Industrial Average in the period from 1985 to 1987 showed equal results to other researches using different fitting methods. The fitting method then proceeded onto the four selected stock indices, 3 of which were tested in very recent period.

The log-periodicity was present in all of the stocks with recent time period in scope, although two, DJIA and SP500, had rather unconvincing results. On these indices, it is recommended to follow up with an analysis on future prices, as these two showed strong potential for possible log-periodic behavior. In contrast, MERV had a very good fit to the log-periodic power law, although the predicted time of crash remains uncertain due to doubts concerning the critical point prediction in fast growing stocks.

The other stock, Nikkei 225, which was tested in a period prior to its 2015 drop, could also be fitted to the LPPL albeit the magnitude of the follow up crash was very small.

The log-periodic power law was found to have strong prediction capabilities with advantages of being easily interpretable and understandable. Its main drawbacks can be found in complicated fitting procedure, which was simplified in the thesis but still took much of computational capacity to find the best fit, and inability to determine the magnitude of the crash, predicted by the log-periodic power law.

References

Beneish, M. D. and Gardner, J. C. (1995). Information costs and liquidity effects from changes in the dow jones industrial average list. *Journal of Financial and Quantitative Analysis*, 30(01):135–157.

Blanchard, O. J. (1979). Speculative bubbles, crashes and rational expectations. *Economics letters*, 3(4):387–389.

Brée, D. S., Challet, D., and Peirano, P. P. (2013). Prediction accuracy and sloppiness of log-periodic functions. *Quantitative Finance*, 13(2):275–280.

Brée, D. S. and Joseph, N. L. (2010). Fitting the log periodic power law to financial crashes: a critical analysis. *International Review of Financial Analysis*.

Brée, D. S. and Joseph, N. L. (2013). Testing for financial crashes using the log periodic power law model. *International review of financial analysis*, 30:287–297.

Chang, G. and Feigenbaum, J. (2008). Detecting log-periodicity in a regime-switching model of stock returns. *Quantitative Finance*, 8(7):723–738.

Filimonov, V. and Sornette, D. (2013). A stable and robust calibration scheme of the log-periodic power law model. *Physica A: Statistical Mechanics and its Applications*, 392(17):3698–3707.

Greenwood, R. (2008). Excess comovement of stock returns: Evidence from cross-sectional variation in nikkei 225 weights. *Review of Financial Studies*, 21(3):1153–1186.

Jacobsson, E. (2009). How to predict crashes in financial markets with the log-periodic power law. *Master diss., Department of Mathematical Statistics, Stockholm University.*

Johansen, A., Ledoit, O., and Sornette, D. (2000). Crashes as critical points. *International Journal of Theoretical and Applied Finance*, 3(02):219–255.

Johansen, A. and Sornette, D. (2001). Bubbles and anti-bubbles in latin-american, asian and western stock markets: An empirical study. *International Journal of Theoretical and Applied Finance*, 4(06):853–920.

Johansen, A., Sornette, D., et al. (2010). Shocks, crashes and bubbles in financial markets. *Brussels Economic Review*, 53(2):201–253.

Korzeniowski, P. and Kuropka, I. (2013). Forecasting the critical points of stock markets indices using log-periodic power law. *Ekonometria*, 1:100–110.

Lux, T. and Sornette, D. (2002). On rational bubbles and fat tails. *Journal of Money, Credit, and Banking*, 34(3):589–610.

Melvin, M. (2003). A stock market boom during a financial crisis?: Adrs and capital outflows in argentina. *Economics Letters*, 81(1):129–136.

Podobnik, B., Horvatic, D., Petersen, A. M., and Stanley, H. E. (2009). Cross-correlations between volume change and price change. *Proceedings of the National Academy of Sciences*, 106(52):22079–22084.

Russell, J. L. (1964). Kepler's laws of planetary motion: 1609–1666. *The British Journal for the History of Science*, 2(01):1–24.

Sarda, V., Karmarkar, Y., Lakhotia, N., and Sen, P. (2010). What can the log-periodic power law tell about stock market crash in india? *Applied Economics Journal*, 17(2):45–54.

Savoiu, G. and Iorga-Siman, I. (2008). Some relevant econophysics' moments of history, definitions, methods, models and new trends. *Romanian Economic and Business Review*, 3(3):29.

Sharma, B. (2012). A brief review of econophysics. *Journal of Pure Applied and Industrial Physics Vol*, 2(3A):286–402.

Sornette, D. (2003a). Critical market crashes. *Physics Reports*, 378(1):1–98.

Sornette, D. (2003b). *Why stock markets crash critical events in complex financial systems*. Princeton Univ. Press, Princeton, NJ.

Sornette, D. and Johansen, A. (1997). Large financial crashes. *Physica A: Statistical Mechanics and its Applications*, 245(3-4):411–422.

Sornette, D., Johansen, A., and Bouchaud, J.-P. (1996). Stock market crashes, precursors and replicas. *Journal of Physics*.

Sornette, D., Johansen, A., et al. (2001). Significance of log-periodic precursors to financial crashes. *Quantitative Finance*, 1(4):452–471.

Sornette, D., Woodard, R., Yan, W., and Zhou, W.-X. (2013). Clarifications to questions and criticisms on the johansen–ledoit–sornette financial bubble model. *Physica A: Statistical Mechanics and its Applications*, 392(19):4417–4428.

Sullivan, A. and Sheffrin, S. M. (2003). Economics: Principles in action. upper saddle river, new jersey 07458: Pearson prentice hall.

Wiener, N. and Masani, P. (1957). The prediction theory of multivariate stochastic processes. *Acta Mathematica*, 98(1):111–150.

Yan, W., Rebib, R., Woodard, R., and Sornette, D. (2012). Detection of crashes and rebounds in major equity markets. *International Journal of Portfolio Analysis and Management*, 1(1):59–79.

Zhou, W.-X., Sornette, D., Hill, R. A., and Dunbar, R. I. (2005). Discrete hierarchical organization of social group sizes. *Proceedings of the Royal Society of London B: Biological Sciences*, 272(1561):439–444.

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