

This bachelor thesis is an interesting application of the standard methods of Markov chain theory and martingale theory. According to the strong law of large numbers, all betting strategies in roulette are doomed to end up in an eventual loss and thus the only difference between the various strategies is in studying statistics such as the distribution of the maximum of the betting account, the distribution of the last exit time from zero or the number of visits to zero. Most people would find somewhat counterintuitive that the distribution of the maximum has the largest expected value for a single number strategy when the bets are not diversified in contrast to other strategies such as the red-black strategy. Similar result is true for the distribution of the last exit from zero, the last exit happens statistically later for undiversified bets. The thesis documents that the last exit time from zero has the mean somewhere at 46,700 and 95% quantile somewhere at 128,135.

The main contribution of the thesis is a creative application of the standard methods of Markov chain theory and martingale theory. The student studies two major strategies: the basic red-black strategy and the strategy of betting on a single number. The statistics of the distribution of the maximum of the betting account, the distribution of the last exit time from zero and the number of visits to zero are studied. While the red-black strategy can be fully computed analytically, the single number strategy needs to use numerical techniques to determine the distribution of the maximum and the distribution of the last exit time. The distribution of the maximum of the betting account requires solving a polynomial equation of the 36th degree. All results are illustrated by the Monte-Carlo simulation. The distribution of the last exit time is analytical, but computational software such as Mathematica cannot easily handle computation of the expected value. Thus the student uses Stirling's approximation to get a formula that happens to be analytically tractable.

In conclusion, this is a solid bachelor thesis and meets the usual expectations on such work. There are only minor issues: typo on page 22 (median of the last exit time of the single bet strategy). It would be also interesting to show analytically that $P(M=0)=1/36$ for a single number strategy, but this seems highly nontrivial.