

Posudek vedoucího pro diplomovou práci Tobiáše Hudce

Jan M. Swart

4. srpna 2016

Autor:	Tobiáš Hudec
Typ práce:	diplomová
Název práce:	Absorption cascades of one-dimensional diffusions / Kaskády absorpce jednorozměrných difuzí
Jméno vedoucího:	Jan M. Swart

Způsob pracování

Pán Hudec během práce ukázal, že je schopný samostatně pracovat na matematickém problému. S mou pomocí našel relevantní literaturu, z které se potom sám potřebné znalosti naučil. Rozumí nejen samotným výsledkům ale vidí i její souvislost a cíl a je schopný je samostatně aplikovat. Při psaní důkazů dbá na detail ale při tom nestrácí přehled. Rozdělí práci a samostatně formuluje potřebná lemata.

K samotné práci

An intertwining relation between two Markov processes is an algebraic relation between two Markov semigroups with possibly different state spaces and a probability kernel from one state space to the other. For processes with countable state space, J.A. Fill has shown that such an intertwining relation implies that the two Markov processes can be coupled such that at any time, the probability kernel describes the conditional law of one process given the other. This has been applied by P. Diaconis and L. Miclo to couple a birth-and-death process on $\{0, 1, \dots, n\}$ for which n is a trap to a pure death process on the same space such that both processes arrive at n at the same time. J.M. Swart has found another such intertwining, with a

kernel that goes in the opposite direction compared to the one of Diaconis and Miclo.

The aim of the diplom thesis was to generalize the latter result to the case where the birth-and-death process is replaced by a Wright-Fisher diffusion on $[-1, 1]$ and the second process is an explosive pure birth process on $\mathbb{N} \cup \{\infty\}$, in such a way that the diffusion gets absorbed at the boundary of the interval $[-1, 1]$ at the same time as the pure birth process explodes. In his thesis, T. Hudec uses the theory of Feller processes (in particular, the Hille-Yosida theorem for Feller semigroups) to show that the corresponding intertwining relation between the semigroups of a Wright-Fisher diffusion and a pure birth process indeed exists (Theorem 3.7).

He then investigates whether there also exists a coupling of both processes in the spirit of the work of J.A. Fill. Here, his results are less conclusive. Instead of looking at the Wright-Fisher diffusion on $[-1, 1]$, for technical reasons, it turns out to be easier to look at the Wright-Fisher diffusion on $[0, 1]$ with reflection at 0 and absorption at 1. It is shown that this process can be coupled in an appropriate way to a pure birth process on $\{0, 1, \dots, n\}$ with $n < \infty$, where the case $n = \infty$ is left unsolved, again for technical reasons.

The exposition of the results is clear. The organization of the thesis is good and the style of writing is pleasant. There are several new results. Even though the methods closely follow those of earlier papers, the extension of these methods to the continuum setting is nontrivial.

All (small) errors and typos that I knew of have been corrected in the final version. The thesis is written in English which is generally quite good (and certainly better than my Czech above).

Závěr

Práci doporučuji uznat jakou diplomovou.