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BACHELOR THESIS

Do crypto-currencies form a new asset class?

Author: **Samuel Mayr**

Supervisor: **PhDr. Ladislav Krištoufek Ph.D.**

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, July 30, 2015

Signature

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Abstract

This paper examines statistical properties of crypto-currencies' price variations in comparison with statistical properties of price variations in common financial markets. Price data of Bitcoin, ripple and Litecoin have been directly compared with price data of euro currency and stock index S&P500. Additionally, and compared with set of stylized facts of asset returns. The properties in scope of this work include an autocorrelation of day-to-day returns, a shape of return distributions, a volatility clustering, a leverage effect and a volume/volatility correlation. To answer the question of this thesis, we have tried to find unique differences in the way prices of crypto-currencies behave. After every point of the data analysis has been checked, we have concluded that the only major difference is in the shape and the significance of autocorrelation in day-to-day returns. While crypto-currencies seem to autocorrelate, there has been no such a cross-autocorrelation found in the benchmark values. Therefore, we argue that it is the most distinctive sign of crypto-currencies and the reason for crypto-currencies to be regarded as separate asset class.

JEL Classification	C12, C32, G12, G23
Keywords	crypto-currency, Bitcoin, ripple, Litecoin, stylized fact, asset return, autocorrelation, leverage effect, volatility clustering

Author's e-mail	mayr.samuel@gmail.com
Supervisor's e-mail	ladislav.kristoufek@fsv.cuni.cz

Abstrakt

Tato práce zkoumá statistické vlastnosti cenových variací kryptoměn, ve srovnání se statistickými vlastnostmi kolísání cen na běžných finančních trzích. Data o změnách cen kryptoměn Bitcoin, ripple a Litecoin byla přímo srovnávána se změnami cen evropské měny euro a akciového indexu S&P500. Zároveň byla data srovnávána se sadou stylizovaných faktů výnosů finančních aktiv. Vlastnosti zkoumané v této práci jsou: autokorelace denních výnosů, tvar rozdělení výnosů, shlukování volatility, pákový efekt a korelace objemu a volatility. K tomu, aby jsme mohli odpovědět na otázku této práce, jsme se snažili najít unikátní rozdíly v chování výnosů kryptoměn. Poté, co byl zkontrolován každý bod této analýzy, jsme dospěli k závěru, že jediný zásadní rozdíl je ve tvaru a významnosti autokorelace denních výnosů. Zatímco výsledky analýzy ukazují, že kryptoměny autokorelují, ostatní finanční aktiva tuto vlastnost obecně nevykazují. Závěrem tedy konstatujeme, že autokorelace jako nejvýraznější statistická odlišnost denních výnosů kryptoměn je dostatečným důvodem považovat kryptoměny za samostatnou třídu aktiv.

Klasifikace JEL	C12, C32, G12, G23
Klíčová slova	krypto-měna, Bitcoin, ripple, Litecoin, stylizovaný fakt, návratnost aktiv, autokorelace, pákový efekt, shlukování volatility

E-mail autora	`mayr.samuel@gmail.com`
E-mail vedoucího práce	`ladislav.kristoufek@fsv.cuni.cz`

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Acronyms

ACD Autoregressive Coditional Duration

ARMA Autoregressive Moving Average

BTC Bitcoin

CHF franc

EGARCH Exponential Generalized Autoregressive Conditional Heteroskedasticity

EUR euro

GBP pound sterling

JPY Japanese yen

LTC Litecoin

S&P500 Standard & Poor's 500

USD United States dollar

VAR Vector Autoregression

XRP ripple

Bachelor Thesis Proposal

Author	Samuel Mayr
Supervisor	PhDr. Ladislav Krištoufek Ph.D.
Proposed topic	Do crypto-currencies form a new asset class?

Preliminary scope of work In this work, I would like to analyse phenomena of last couple of years - crypto-currencies, also known as virtual currencies. To simplify the whole analysis a little bit, I will primarily focus on first broadly known and by far the biggest crypto-currency currently used called Bitcoin. Obviously, there are many others but it is unnecessary to study each of them separately as they operate similarly and their market capitalization is much lower, meaning that their significance in real economy is much lower as well.

On the example of Bitcoin currency I will try to spot and describe differences between virtual currencies generally and other known asset classes. For that purpose, I will examine its properties from statistical point of view using econometric modelling of financial time series and I will use stylized facts of stocks, bonds and other common asset classes to compare them with price and return data properties of the Bitcoin currency.

This research should provide us with an answer to the question, whether crypto-currencies are or are not a new asset class itself and how could the observed differences be possibly used in an investment portfolio.

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Core literature

1. Cont, R. (2001): "Empirical properties of asset returns: stylized facts and statistical issues." *Quantitative Finance* **1(2)**: pp. 223–236.
2. BriĂ¨re, M. , K. Oosterlinck, & A. Szafarz (2013): "Virtual currency, tangible return: portfolio diversification with bitcoins." *Journal of Asset Management*
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Introduction

There are 178 different circulating currencies all over the world[1]. Some of them are more valuable and some of them are less. Some of them are used more than the others and some of them rarely ever change hands. But still, they all share similar characteristics. They are backed by credible institution — a state, which ensures the value of those currencies and at the same time creates artificial demand for the currency itself by accepting it as means of payment. Alternative currencies might be backed by another institution such as a company, but there is still an assurance of value provided by this entity (the company ensures users that it will accept the currency for predetermined number of days). Widely used currencies also tend to be tangible, which increases the trust even more. That is why it came as a shock when news about success of a new crypto-currency named Bitcoin (BTC) started to emerge in early 2013.

Crypto-currency is a specific type of currency, which uses cryptography to control its supply, its security, and functioning of its transaction processes. Those currencies are mostly internet based and rarely exist in other form than intangible. That allows for nearly instant transactions between any two places in the world (under condition of internet access). Although there are hundreds of such currencies, BTC is the most used one and also the first one to be known among wide public[2][3]. It has been created in 2008 by an individual or a group of people going by the name of Satoshi Nakamoto[4]. Originally, it was meant as a response to common internet payments, which force buyers to tell more information about themselves just to cover the risk of fraud and have relatively high transaction costs which make small casual transactions unattractive.

On August 15, 2008 encryption patent[5] application number 20100042841

[1] http://www.currency-iso.org/en/home/tables/table-a1.http

[2] http://www.forbes.com/forbes/2011/0509/technology-psilocybin-bitcoins-gavin-andresen-crypto-currency.html

[3] http://cryptocoincharts.info/

[4] https://bitcoin.org/bitcoin.pdf

[5] http://www.google.com/patents/US20100042841

has been filed and created basis on which BTC was going to be later built on. Three days later internet domain bitcoin.org has been registered. And finally, on October 31, 2008 famous paper named Bitcoin: A Peer-to-Peer Electronic Cash System has been published (anonymously, under the Satoshi Nakamoto pseudonym). In this work, Nakamoto (2008) solved notorious double-spending problem which haunts digital currencies (issue of possible multiplication of a unit of currency), described the cryptographic process behind the transactions and questions of privacy, and allowed the whole system to start working in January 2009. First transaction has been made on January 12, 2009. First real-world transaction then on 22 May, 2010[6]. At that time the value of one BTC was not even 0.01$. One year later it was around 8$ and on November 27, 2013 the currency broke 1000$ per BTC. At the time of writing this thesis (July 2015), the price is at around 200 – 300$ per BTC. (Prices vary between the inividual exchanges, as there is no official exchange rate.)

When BTC was originally created in 2008, it was nothing in terms of value compared to its boom days of post 2013 period. People were cautious and name of the currency was spreading slowly. And actually for a good reason: BTC is not backed by any organization, state, nor firm. Its value is based on the trust of its users and its supply is constrained solely by process called mining and rules which it entails; built in the way that it reflects behaviour of rare assets, so that the rate of supply converges to zero. The fact that it had been (and still is) mostly used on deep web — parts of internet which are not widely accessible (not accessible through standard search engines nor standard internet browsers) and have very bad reputation, mostly for trade with narcotics, weapons and other illegal activities, does not help either.

Controversially, it was its security flaws and high volatility which brought attention to the currency. There were many articles involving BTC circulating during year 2013 and as name was mentioned more and more often the value of BTC grew. Obviously, people who wanted to try BTC were not scared by possible failure and even some of big e-commerce companies started to accept the crypto-currency. High volatility linked to the relationship between the currency's value and user's trust lured in investors who saw a possibility of high returns in short term and new way of differentiating their portfolios.

Other crypto-currencies followed the trend that BTC started, and some of them were able to grow quite big themselves. The biggest ones by market value and therefore those which are subject of this thesis, next to the BTC are

[6] https://bitcointalk.org/index.php?topic=137.msg1195#msg1195

ripple (XRP) and Litecoin (LTC). XRP was created mainly to compete with BTC with its super-fast transactions. Transaction on XRP typically takes only couple of seconds[7], which is compared to BTC transactions that take in average 5 – 10 minutes[8], rather big difference. Main disadvantage is, of course, still relatively low market capitalization compared to BTC. LTC, third and last of the observed currencies, has been created in 2011[9] and is very similar in the cryptographic mechanisms behind the currency to BTC. It also enables faster transactions than BTC, but it is even further from being as used as BTC is. (Market capitalization[10] as of May 8, 2015: BTC $3,356,134,992; XRP $236,144,980; LTC $56,282,044)

From all of the above comes the main question of this thesis. If BTC, and crypto-currencies generally seem so different, are statistical properties of their returns also different from regularly used fiat currencies? Are they different from other financial assets? And could they therefore form a whole new asset class? Even if none of the above is true, there still might be some unusual characteristics in the way the returns occur. One of the ways that these questions can be answered is by observing the asset's behaviour and using statistical procedures to compare observed behaviour with stylized facts of other investment assets (Cont 2001). This method covers quite broad spectrum of different statistical views on the shape of the distribution and, to our knowledge, no article that would already use this approach has been written yet.

Cont (2001) argues that even though it is not very obvious at the first glance, return data of various financial assets (stocks, commodities, indexes) share similar properties. Therefore crypto-currencies will not be any different from statistical point of view, or at least the associate returns, than regular currencies and investment assets in case that those properties hold even for them. In this thesis, we perform a statistical analysis parallel to the one of Cont (2001) to show whether BTC shares these properties with other assets or whether it differs significantly.

This thesis is structured as follow: Chapter 1 informs on research which has already been done in subject of this thesis. Chapter 2 describes the process of data collection and presents collected data and the reasoning behind the data selection. Chapter 3 presents the methodology used, process of analysis and its results. This chapter is also divided into eight subsections — stylized facts, ab-

[7]https://ripple.com/knowledge-center/understanding-ripple/
[8]https://blockchain.info/charts/avg-confirmation-time
[9]https://bitcointalk.org/index.php?topic=47417.0
[10]http://coinmarketcap.com/

sence of autocorrelation, heavy tails, gain/loss asymmetry, aggregational gaussianity, volatility clustering, slow decay of autocorrelation in absolute returns, leverage effect, volume/volatility correlation. Finally, Chapter 4 recapitulates the major findings of this thesis. Graphs and tables that are not included in the text, can be found in Appendix A and Appendix B.

Chapter 1

Literature review

There are many papers on the BTC currency, as the topic is rather popular, but only a few works which would examine behaviour of crypto-currencies generally from the statistical point of view and the differences between them and other broadly used financial assets as fiat currencies or stocks.

One of such is Wilson-Nunn & Zenil (2014). The authors argue that BTC shares similarities with stock and precious metal markets and that LTC has similar, even though not exactly the same and more currency-like, characteristics as BTC. They draw such a conclusion after they observe mean, standard distribution, kurtosis, skewness and generally the shape of distribution functions of various financial assets, and after they apply more advanced information-theoretic, algorithmic and fractal measures (Shannon's entropy, compressibility, algorithmic probability. theory of roughness). This research has been initiated by decision of Internal Revenue Service to consider BTC a property rather than currency for tax purposes, and authors show that BTC indeed behaves more like a property. However, according to this paper, with such complex patterns of behavior, and displaying signs of both property and currency, BTC could be classified as a hybrid instrument.

Very interesting approach showed, in their work, Cocco *et al.* (2014). They created an artificial agent-based market for BTC which simulated the real one. The model is quite complex and covers various trading strategies and Pareto-law wealth distribution among the virtual participants. It entails two kinds of actors — Random traders and Chartists (speculators), who participate on trade with different approaches and strategies using crypto-currency, which is modeled in the way that reflects real world BTC supply, and fiat currencies. It also accounts for an increase in the number of traders in the artificial society.

On this quite complicated model they were able to observe BTC's absolute returns, their autocorrelation and cumulative distribution function and they reproduced characteristics, which were very similar to that of real markets: power-law behavior in tails of complementary cumulative distribution function, low autocorrelation and volatility clustering.

Kristoufek (2013) studies relationship between BTC prices and number of Google respectively Wikipedia search queries. Google and Wikipedia search queries were chosen are used as a measure of interest in BTC and as expected, paper describes high correlation between the queries and BTC prices. Additionally, it finds out that the correlation goes both directions — it is not only the search trends that influence price of BTC but also the price itself influences number of queries involving the currency (aka bidirectional relationship). Which was expected, due to nature of BTC. The fact that there is no state or any institution that could investors watch to predict movements in price means that the price will be influenced mostly by speculators trying to catch a wave of profit. Moreover the number of searches, which denotes interest in the currency, drags the price further from the trend. Meaning that in case that price is below the trend, higher number of search queries causes price to dip even lower and in case that the price is above the trend, it will grow even more with higher number of visits on BTC Wikipedia or number of times BTC is searched for via Google search engine. In another work of his, Kristoufek (2015) performs a wavelet coherence analysis to determine which of the most often claimed drivers of its price are really behind the BTC's price movements. The most important finding of this work is that BTC currency exhibits properties of both speculative and standard financial assets and therefore its price is also, mainly in the long term, determined by trade usage, supply and price level. It also takes a look at a belief that the United States' BTC market is influenced by Chinese BTC market. However, it does not find any hard evidence that would support such a claim.

Other articles which should be mentioned here are Valstad & Vagstad (2014), and Baek & Elbeck (2015). Authors of the first publication have observed intraday volatility and have constructed intraday exchange risk measurement (Intraday Value at Risk based on Monte Carlo simulation and log-Autoregressive Coditional Duration (ACD)-Autoregressive Moving Average (ARMA)-Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model) of BTC, euro (EUR) and gold using ultra-high-frequency data. The results have shown that BTC is by far the riskiest of those three in an intraday horizon and has Intraday Value at Risk around 10 times higher than the other two assets.

Second publication also observes BTC's volatility and its returns. Authors do so in order to find out whether BTC is more suited to be a long term investment or short term speculation and their question is motivated by lack of studies that would work with BTC as with an investment vehicle. They have chosen Standard & Poor's 500 (S&P500) index as a benchmark, to find out how BTC price data fare in comparison with stock market data. Finally, the work concludes that BTC's volatility is for most part internally driven (volatility is created by individual decisions of the trade participants and not by any macroeconomic fundamentals) and it is way too volatile and therefore not suitable to be used as an investment asset. It is 26 times more volatile than S&P500 stock market index. Nevertheless, authors of both above mentioned papers do not discourage potential BTC users and mention possible drop in the crypto-currency's volatility in case of future rise in number of BTC users.

Works that study BTC and other similar crypto-currencies from broader perspective are more frequent. Yermeck (2013), in his working paper, takes the issue from basics and describes the fact that BTC does not even fulfill the main functions of currency. For an asset to be considered a money, it has to, according to the traditional economic definition, serve as medium of exchange, store of value, and unit of account[1]. Main problems, as stated in the paper, lie in the second and third functions. That is because of the currency's high volatility and *virtually zero* correlation of BTC's prices and other big currencies' (namely EUR, Japanese yen (JPY), franc (CHF) and pound sterling (GBP)) exchange rates with United States dollar (USD). Additionally, it does not correlate with exchange rate between USD and gold either. He also tries to come up with some ideas that could help BTC to become a real currency — introduction of consumer protection, and stronger connection to current banking and payment systems.

To come up with ideas how to improve the currency and to warn before possible gaps in the design was an idea behind a work of Barber *et al.* (2012). The broad study that has been created deals with the currency's structural problems — deflationary characteristics and *history-revision* attack; as well as it deals with possibility of theft/loss of BTC and technical scalability of the currency. Finally, they describe BTC's anonymity problems, as the currency still *exposes their users to a weak form of linkability* (Barber *et al.* 2012) and

[1]according to the traditional economic definition

therefore is not ideal in cases where the user does not want to be associated with the receiver or the transaction itself. Anonymity in case of BTC is a huge issue and is a topic of many articles. The most important work on anonymity is probably that of Reid & Harrigan (2012), observing structure of the currency's networks and structure of transaction history. By performing such investigation they were able to find the specific gaps in the currency's anonymity. They also mention that anonymity is not a main focus of the currency's design and they warn before misinformation among BTC's user that often think about BTC as strongly anonymous currency.

Chapter 2

Data

Downloading data is not a big problem for BTC, as the currency has every single transaction recorded, exchanges' rates are mostly public and there are many people which are keen on completing those data. Slightly more problematic is gathering data for lesser known crypto-currencies than BTC. Those data are much harder to come by and the few sources which gather them usually do not provide day to day prices for longer periods than one year, or do not provide their datasets at all. But for consistency reasons it is always better to use one source for the currencies' prices. Finally, after searching for the right source of the data, website called Coinplorer was the best for the purposes of this thesis, as it is possible to download .json data for historical exchange rates of all BTC, XRP and LTC to USD and it is possibly the only source which covers these three currencies in the desirable format and time range.

BTC data is originally from webpage Bitstamp[1]; XRP exchange rate is computed using daily XRP to EUR exchange rate from The Rock Trading Ltd[2] and then recomputed for USD prices using data from Open Exchange Rates[3]. LTC data is generated similarly, using Bitstamp for BTC/USD exchange rate and BTC-E[4] for BTC/LTC exchange rate.

Logarithmic day-to-day returns on each of the aforementioned currencies have been then derived by deducting logarithmic exchange rate from any given day from logarithmic exchange rate measured on the day after. That can be rewritten as:

$$x(t) = lnS(t)$$

[1]https://www.bitstamp.net/

[2]https://www.therocktrading.com/

[3]https://openexchangerates.org/

[4]https://btc-e.com/

where S is a function of value in USD at time t. The returns R then have form of:

$$R(t) = x(t + 1) - x(t)$$

For BTC, we will use only data between June 29, 2011, which is date of BitPay launch and is just before the top of first BTC price peak that has been coming a week later on July 8 2011, and March 1, 2015. March 1 was chosen so the dataset is as large and relevant as possible, relatively to the date of writing this thesis. This way, the sample has 1,339 values and is large enough for conclusions to be reliable. Three missing values from November 29, 2013 (peak day), December 1, 2013 and January 15, 2015 do not play significant role in the analysis, as the omitted values are relatively low to the sample volume. Additionally, those values were not taken into consideration when returns were computed and day-to-day return was not computed if value from any of the two days in question was missing. Therefore from 1,339 BTC/USD exchange rate values, we obtain 1,336 day-to-day return values. This approach has been used for all the data involved and also for computation of returns over more than two days (e.g. four day return has not been computed if value for first or fourth day in the sequence was missing).

The rest of datasets will not be as big, due to lower overall accessibility of data relating to crypto-currencies other than BTC. As beginning of the observed period has been used September 26, 2013 when Ripple payment protocol became open-source and free to use. Dataset ends on March 1, 2015 as in case of BTC currency. This way dataset of 469 daily XRP prices and 452 day-to-day returns is generated. Quite unlucky is that there is 53 days for which the values are missing, that is approximately 10.15% of the original period. Although it should not have a high impact due to the way we handle and adjust our return data for missing values, it is still important to take this fact into a consideration when conclusions are based specifically on XRP data.

LTC data have been also restricted. In this case by November 1, 2013 and March 1, 2015. March 1, 2015 has been chosen for consistency and relevancy purposes, and November 2013 was a month when LTC has started to be more widely used and recognized due to boom in popularity of BTC. There are 470 daily price values in the dataset and values for total of 16 days are missing (3.29%) — subsequently, dataset of 458 logarithmic day-to-day values has been generated.

For parts of the thesis, where comparison with EUR/USD exchange rate or S&P500 index has been needed, return data were gathered and modeled in similar fashion as for the crypto-currencies (finance.yahoo.com[5] and coinplorer.com[6] respectively have been used as source for the data). Time range has been adjusted so the time range over which the data are gathered is the same as for BTC (i.e. June 29, 2011 – March 1, 2015). There are 67 missing values in dataset of 1,342 return values (4.99%) for EUR/USD exchange rate. S&P500 index data are the least complete, with 370 missing values out of total 1,292, due to missing values for weekends. However, it should not affect the analysis, as return values are strictly day-to-day and are not heavily influenced by missing index price values.

There was also a need for trade volumes for the crypto-currencies due to the nature of the last stylized fact — correlation of volume and volatility. Those data has been gathered from website coinmarketcap.com which gathers statistics for approximately 560 crypto-currencies. And although the time range over which the data has been stored is not the best (reason why this website has not been used for the exchange rates) it is sufficiently large to give a decent picture. Only data after November 1, 2013 and before February 16, 2015 has been used for all the currencies in question, because of the limited time range and need for consistency and comparability. Volume time series for each of the crypto-currencies has been obtained for one out of every three days. Although, this approach made volume data not as frequent, it is still able to pair the data with return data and obtain datasets large enough to take the correlations as an appropriate reflection of the real state of things. This way it was possible to collect 181 observations for BTC, 174 observations for XRP and 192 observations for LTC. All volume data have been then converted to USD values for the purpose of comparability. The data in this form were obviously not stationary (Dickey-Fuller test does not reject presence of a unit root) and for that matter, volume data has been transformed by de-trending the time series using natural logarithm and differences. And even though, the data for trading volume has been collected irregularly — for one out of two or three days, it should not change much in the analysis, as the focus is more on the great scale of things. Thus, trading volume has been transformed by taking natural logarithms and then taking differences in trading volume since the last measurement. That ultimately creates changes in logarithmic volumes over two or three day periods.

[5]https://finance.yahoo.com/q/hp?s=%5EGSPC+Historical+Prices
[6]https://coinplorer.com/Charts?fromCurrency=EUR&toCurrency=USD

Chapter 3

Methodology & Analysis of Results

3.1 Stylized Facts

Our aim is to generally compare the statistical behavior of various crypto-currencies with other financial assets. For that purpose, we have chosen an approach based on work of Cont (2001), who has created a list of stylized empirical facts and so far has been the most succesful in creating such a list based on empirical observations of financial markets. The presented stylized facts, serve as a uniform set of characteristics, which hold *across a wide range of instruments, markets and time periods* (Cont 2001) and therefore they have been taken as a status quo and expectation for the way in which financial assets generally behave. From here, we have been trying to find any discrepancies between the stylized facts as described by Cont (2001) and the data obtained for BTC, XRP and LTC crypto-currencies. That has helped us to identify the regions in which crypto-currencies differ. Originally, Cont (2001) has described eleven of such stylized facts, but for simplicity only eight of them have been used in this thesis, due to the more complicated nature of the three. Those that are in scope of this thesis are:

- **Absence of autocorrelation** — tendency of asset returns to exhibit insignificant (linear) autocorrelations.
- **Heavy tails** — asset returns' distribution tails display a power-law or Pareto-like shapes. Heavy tails are also defined by Asmussen (2003) as distribution tails heavier than those of an exponential distribution.
- **Gain/loss asymmetry** — extremely low prices and extremely low index values are more frequent than extremely high values.

- **Aggregational Gaussianity** — with increase in time scale, shape of return distribution looks more and more as normal distribution.
- **Volatility clustering** — unusually high returns and unusually low returns tend to clump together and contrast with regions of moderately high/low returns (volatility measures has positive autocorrelation).
- **Slow decay of autocorrelation in absolute returns** — the autocorrelation function of absolute returns as a function of time lag decays slowly to zero.
- **Leverage effect** — measures of volatility tend to be negatively correlated with returns.
- **Volume/volatility correlation** — volatility is correlated with trading volume.

To check for the differences or similarities between the data and stylized empirical facts, we have used multiple statistical approaches, which are described in each of the section. For comparison, the same approach has been applied on data on EUR/USD exchange rate and S&P500 index. Stata 13 has been use for all the computations in this thesis.

3.2 Absence of Autocorrelations

According to Cont (2001), returns of various investment assets usually do not exhibit autocorrelation (serial correlation); meaning, that the returns do not correlate with their past values. And although Cont (2001) argues that in case of very small time scales (within minutes) there might be an autocorrelation due to occurrence of price and time micro-structure effects, it is not in scope of this work due to the data availability issues. Therefore, the first question of this thesis is whether or not crypto-currencies' returns correlate with past returns and what do their autocorrelation functions look like.

To find this out, it is necessary to compute a correlation of different time lags on the initial value of returns. We have decided to compute the correlation at seven lags to capture possible weekly influences. One of the suitable tools is correlogram as described by Box *et al.* (2008), who present us with values of autocorrelation function (auto-covariance function at given time over auto-covariance function at origin of the time series) at different lags, partial autocorrelations, respective portmanteau Q statistics and p-values. If data exhibit autocorrelation then we can observe significant statistical relationship between different time lags shown by appropriate constant being statistically different from zero. Q test statistics and their p-values are also used in Ljung-Box Q test, also known as white-noise test, which is another way to detect absence of autocorrelation.

In case of BTC, it is quite straightforward and obvious from Figure 3.1 and Table A.1 which is placed in Appendix A. For the first lag, the returns are strongly autocorrelated with constant 0.1198. Following lags are not significantly different from zero. Also Ljung-Box Q test's low p-value, which is 0.0000 for all lags except for the second where it is 0.0001, confirms the observed behaviour of the autocorrelation function and rejects the null hypothesis of *no autocorrelation* on 0.01 significance level. From the above comes the conclusion that in case of BTC autocorrelation is present and therefore, BTC's autocorrelation does not fulfil first of R. Cont's stylized facts, which is a significant sign of BTC's difference.

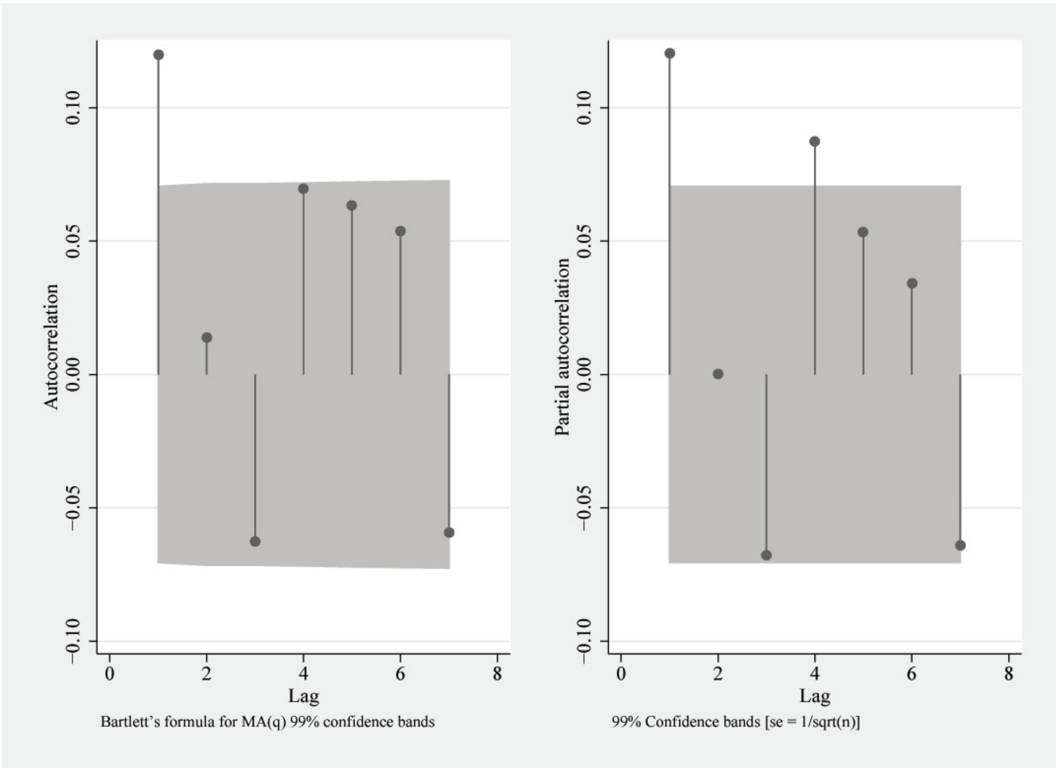

Figure 3.1: Autocorrelation and Partial autocorrelation of BTC

One would expect the other crypto-currencies to exhibit generally similar properties to BTC. That is the case only partially. Even though there is again high influence of first lag, the autocorrelation functions' coefficients are essentially zero for the rest of the lags — Figure 3.2, Table A.2. The first one is the only lag for which the autocorrelation function spikes out of 0.99 confidence bands.

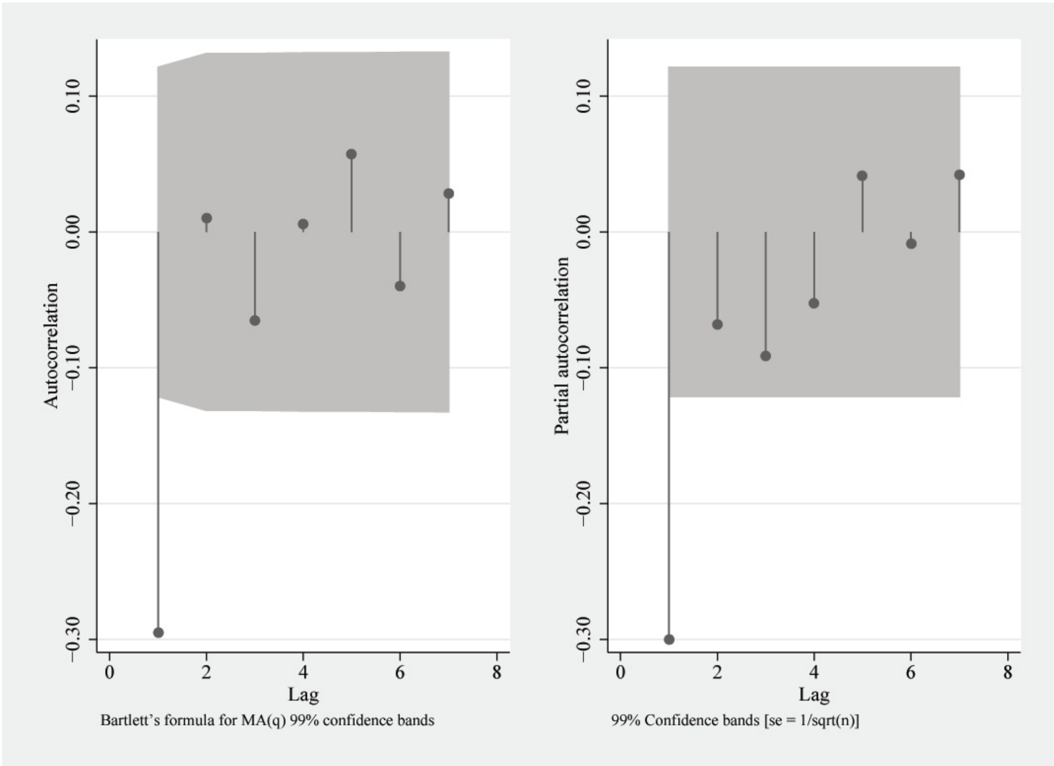

Figure 3.2: Autocorrelation and Partial autocorrelation for XRP

In case of XRP (documented on Figure 3.2 and in Table A.2) the coefficient is -0.2950 and therefore XRP exhibits negative autocorrelation (while BTC's autocorrelation is positive); meaning that, positive returns generally do not tend to be followed by gains, rather they are followed by losses. As in the previous cases, the autocorrelation functions is not significantly different from zero at higher lags.

In case of LTC it is a similar story. Again, there is only one spike in the autocorrelation function, which would significantly differ from zero while using 0.99 confidence band. And as for XRP, the spike is negative — see Table 3.1and Figure 3.3. Correlation for the rest of LTC's lags is, again, insignificant.

The Ljung-Box Q test rejects *no autocorrelation* on 0.01 significance level, mainly due to the effect of first lag, for all three currencies. Although, we can reject the stylized fact that describes absence of autocorrelation, we can argue

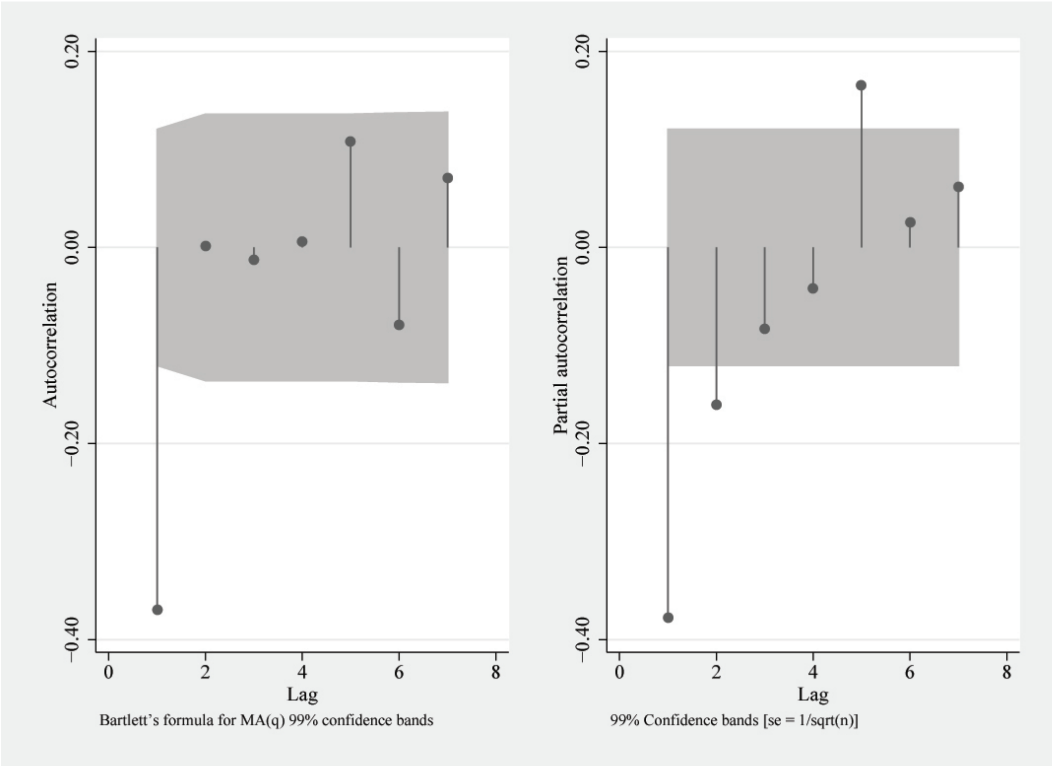

Figure 3.3: Autocorrelation and Partial autocorrelation for LTC

that the autocorrelations are decaying to zero and the effect is caused mainly by the correlation of two successive returns.

Table 3.1: Correlogram for LTC

Lag	**AC**	**PAC**	Q	Prob > Q
1	-0.3696	-0.3776	62.974	0.0000
2	0.0019	-0.1606	62.976	0.0000
3	-0.0130	-0.0825	63.055	0.0000
4	0.0060	-0.0419	63.071	0.0000
5	0.1076	0.1653	68.46	0.0000
6	-0.0786	0.0259	71.34	0.0000
7	0.0712	0.0615	73.709	0.0000

Now when we see how the correlations do behave, it is relevant to be able to determine whether it is an anomaly in a world of finance. To better see what is really happening here, and to have an idea of how much correlated the returns really are, it is necessary to also look at other financial time series.

Firstly, the same process is applied on EUR/USD exchange rate returns

(Figure 3.4). What is seen, is that the biggest difference is actually in the way the first lag correlation behaves. In case of crypto-currencies, first lag is hugely correlated; first lag autocorrelation coefficient for BTC is 0.1198, it is -0.2950 for XRP and -0.3696 for LTC.

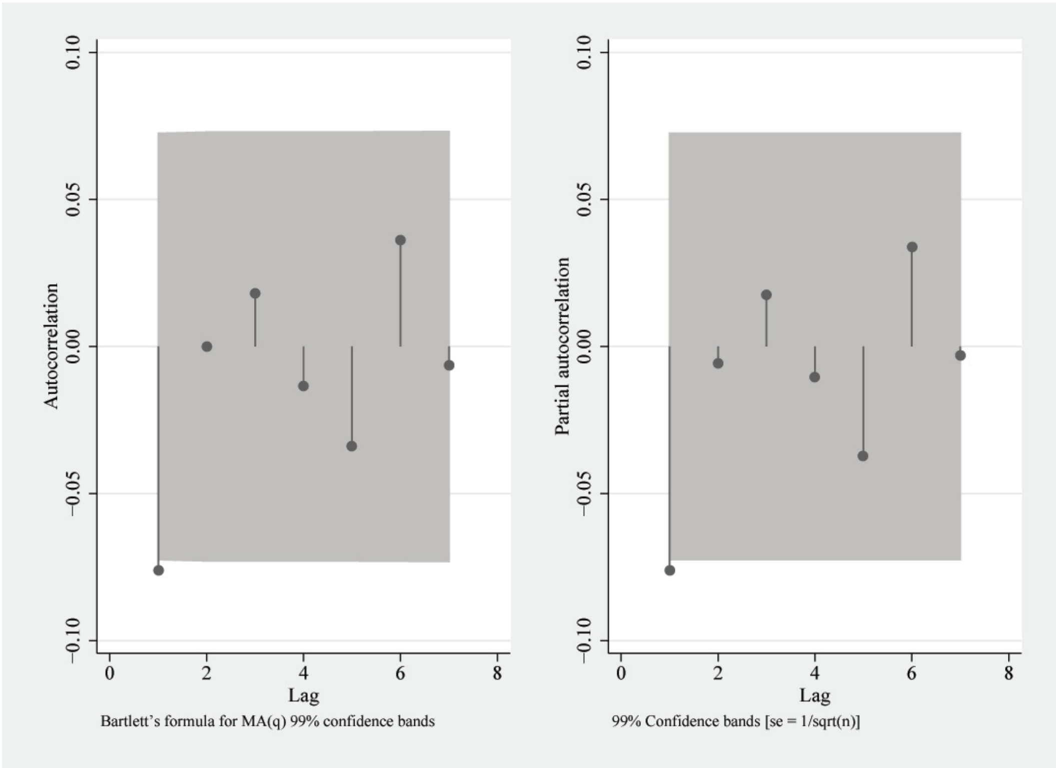

Figure 3.4: Autocorrelation and Partial autocorrelation for EUR

We can conclude, that the correlation of first lag is present in EUR/USD example too, but the correlation is much lower with coefficient of -0.0760. When we take in account all 7 lags, p-values for Ljung-Box Q test do not even allow us to reject *no autocorrelation* hypothesis on 0.1 significance level (p-value of 0.1330). This is better seen in detailed Table A.3.

Secondly, the same is done with S&P500 index data (Figure 3.5). What is interesting is the observation that S&P500 index returns do not seem to exhibit no autocorrelation. And even though first lag autocorrelation is statistically indifferent from zero and the correlations are lower than for EUR/USD rate, there are other lags which do quite significantly differ — especially fifth and seventh lags. Additionally, Ljung-Box Q test rejects its null hypothesis for S&P500, rejecting absence of autocorrelation for the stock index (p-value of 0.0005). Detailed results can again be seen in Appendix A, Table A.4.

All in all, according to the gathered data, crypto-currencies generally exhibit

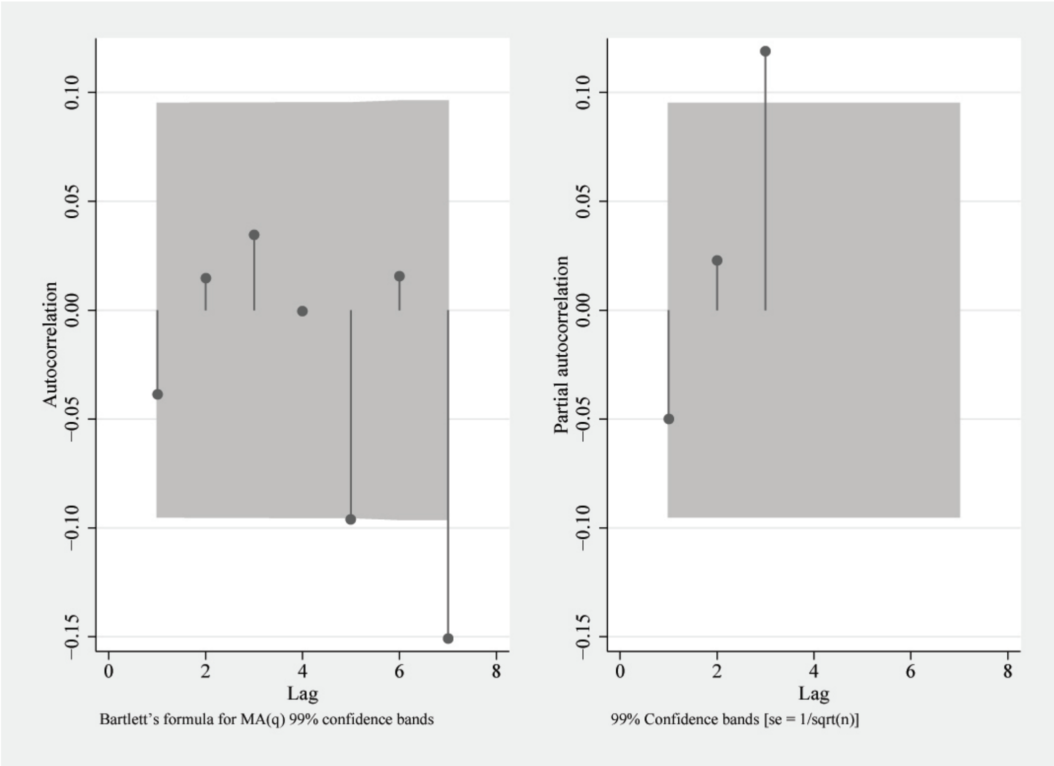

Figure 3.5: Autocorrelation and Partial autocorrelation for S&P500

higher level of correlation among lags of their returns than both S&P500 and the EUR. Having said that, the difference is the most significant in case of the first lag, which is so strong it practically dismiss any chance the autocorrelation being absent. Another important conclusion is the fact that BTC is the only of the observed assets that has positive significant correlation between its returns and returns on the last trading day. Characteristic that indicates that returns of a given sign are mostly followed by returns of the same sign. This indicates that returns are usually followed by returns and losses are more likely to be followed by more losses, creating clusters of returns with the same sign.

3.3 Heavy Tails

Next stylized fact to take care of is that of heavy tails. Financial time series, according to Cont (2001), exhibit heavy tails. The probability of outliers, or the probability of very extreme values is generally higher than it would be under the normal distribution. Although those very extreme values are appearing more often it does not necessarily mean that the distribution will have higher variance. The stylized fact also specifies that the tails of common financial series are similar to those of power-law distribution and that their tail index is usually between 2 and 5.

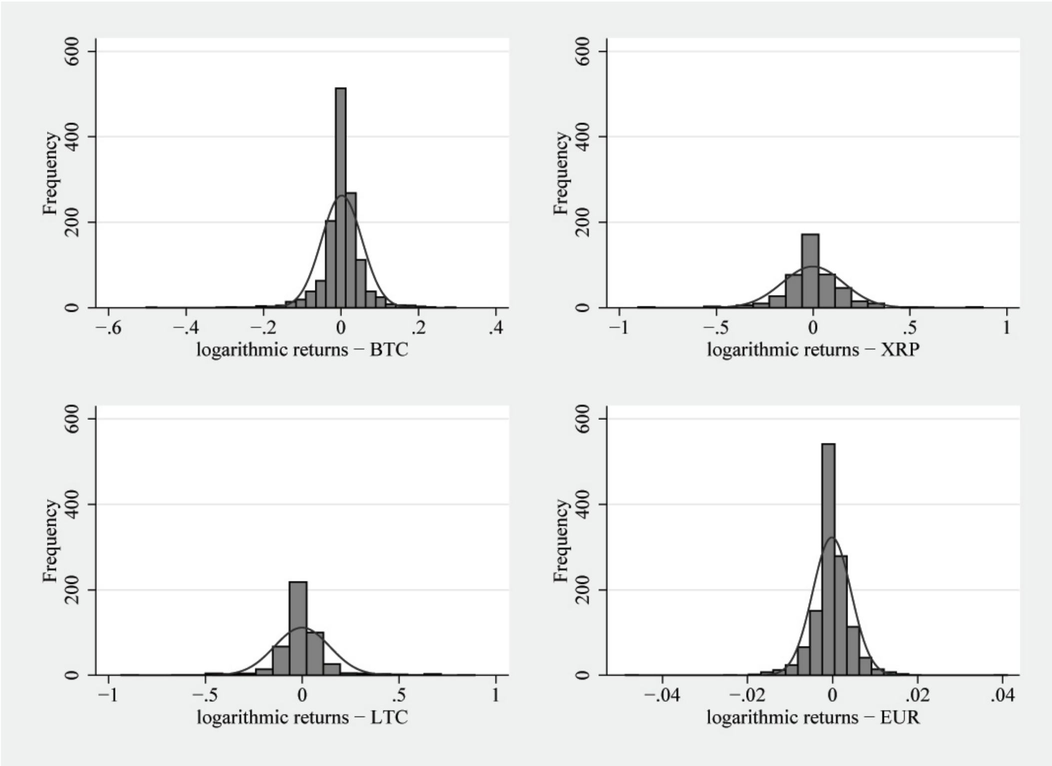

Figure 3.6: Histograms for BTC, XRP, LTC and EUR currencies

From the distribution of BTC (Figure 3.6) it seems, that its tails are indeed heavier than if BTC's returns were normally distributed.

To support such a claim, the data have been used for computation of kurtosis for each of the currencies and their Hill's tail index estimators (Hill 1975). Estimator for right tails, which has been used, can be rewritten as:

$$\xi^{Hill}_{(k(n),n)} = \frac{1}{k(n)} \sum_{i=n-k(n)+1}^{n} \ln(X_{(i,n)}) - \ln(X_{(n-k(n)+1,n)}),$$

where n denotes number of variables in the original sample and $k(n)$ is number of tail variables, which are to be used in the computation of the tail index. Unfortunately the right choice of $k(n)$ is very complicated and incorrect values might reproduce faulty tail indices. Nevertheless, for $k(n) = 65$, which has been chosen, based on graphical representation of the distribution, BTC has Hill's tail index of 2.5103011. For XRP the Hill's tail index has been computed as 2.8704092 using $k(n) = 15$. The same value of $k(n)$ has been also used in case of LTC, due to the similar sample sizes. Hill's tail index for LTC came up as 1.8396171 ($k(n) = 15$) and 1.3712198($k(n) = 35$), depending what value of $k(n)$ has been used (first choice was based on the sample size, second choice on the shape of the distribution). Those indicate that LTC has the heaviest tails of the crypto-currencies and that generally all of them have quite heavy tails, compared to estimate of Cont (2001), who estimates tail estimator to be usually in-between 2 and 5. Mind that $k(n)$ might have been chosen incorrectly.

Moreover, the highest kurtosis has been also found observing behaviour of LTC which exhibits kurtosis of 15.18995, while normal distribution has kurtosis equal to 3. Kurtosis of BTC currency is not far off as it seems to be also around 15; it is 15.06433 with the above described data. Kurtosis of XRP, even though lower than the others, was also very high at 11.20429. Although, kurtosis of the EUR/USD rate data is little bit higher at 20.45168, kurtosis of S&P500 is much lower at 7.01612. Measured kurtosis is still high enough to support the initial hypothesis of crypto-currencies' distribution being heavy tailed. Normal distribution has kurtosis of 3.

The kurtosis has been computed using formula:

$$g_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2)^2},$$

where n is a number of values in the observed sample.

It is also possible to use quantile-normal plot comparing distribution of BTC vis-à-vis normal distribution — Figure 3.7. It graphically shows, that the actual distribution of crypto-currencies' day to day returns is most distinctively different from normal distribution in the tail areas. The same applies for the other two currencies as can be seen on their respective quantile-normal plots on Figure B.2.

Alternative would be normal probability plot but it focuses more on the peak and shoulders of the distribution and therefore is less suited[1]. From these three tests, it is possible to conclude that the crypto-currencies do exhibit heavy tails and that their shape is quite similar to that of S&P500, which can be seen in Figure B.1, and possibly other stock indices.

[1] http://www.stata.com/manuals13/rdiagnosticplots.pdf

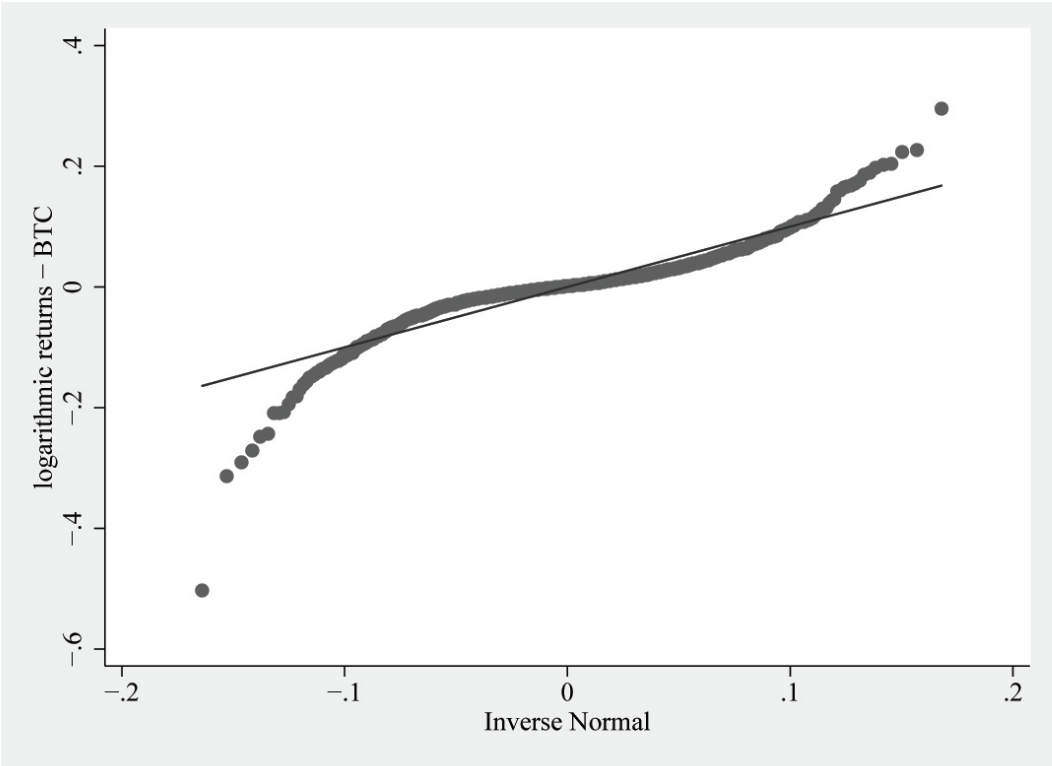

Figure 3.7: Quantile-normal plot for BTC

3.4 Gain/Loss Asymmetry

According to our hypothesis, one observes higher number of downward moving extremes in financial time series of market returns than is the number of extremes going in the opposite direction. It would mean that the above discussed tails would also be asymmetric and therefore shapes and density of right and left tails would differ. That is quite important characteristics as it is directly linked to computation of Value-at-Risk (a measure of how risky a given investment is) that is widely used in determination of an investment strategy. Taken how often crypto-currencies are used only for investment purposes, this is one of the more useful stylized facts. In context of this stylized fact, Cont (2001) notes that gain/loss asymmetry generally does not hold for currencies. That is due to the tendency to explain value of any given currency in terms of an another currency. In case of regular fiat currencies, participants on foreign exchange market are usually able to buy any of those two given currencies using the other one. Decrease in price (negative return) of EUR denoted in USD is then accompanied by increase in price (and positive return) of USD denoted in EUR. Every movement on foreign exchange market is then composed from those two contradictory moves and therefore, the stylized fact can not hold true for currencies generally.

Nevertheless, crypto-currencies are not intertwined as tightly with one another and we can take them as a separate group. They are rarely than not traded for other crypto-currencies and the largest exchanges operate on USD basis. Meaning that decrease in value of BTC in terms of USD increases value of USD in terms of BTC, but it does not directly influence value of LTC.

Although in case of BTC the difference is not as evident as in case of S&P500 index, basic symmetry plot shows that downward movements in return function are much more probable to happen. Even though the trend decreases as the observed data time frame is shortened (i.e. if we take into an account only data from last year), the most extreme values are still more often below the median than above it.

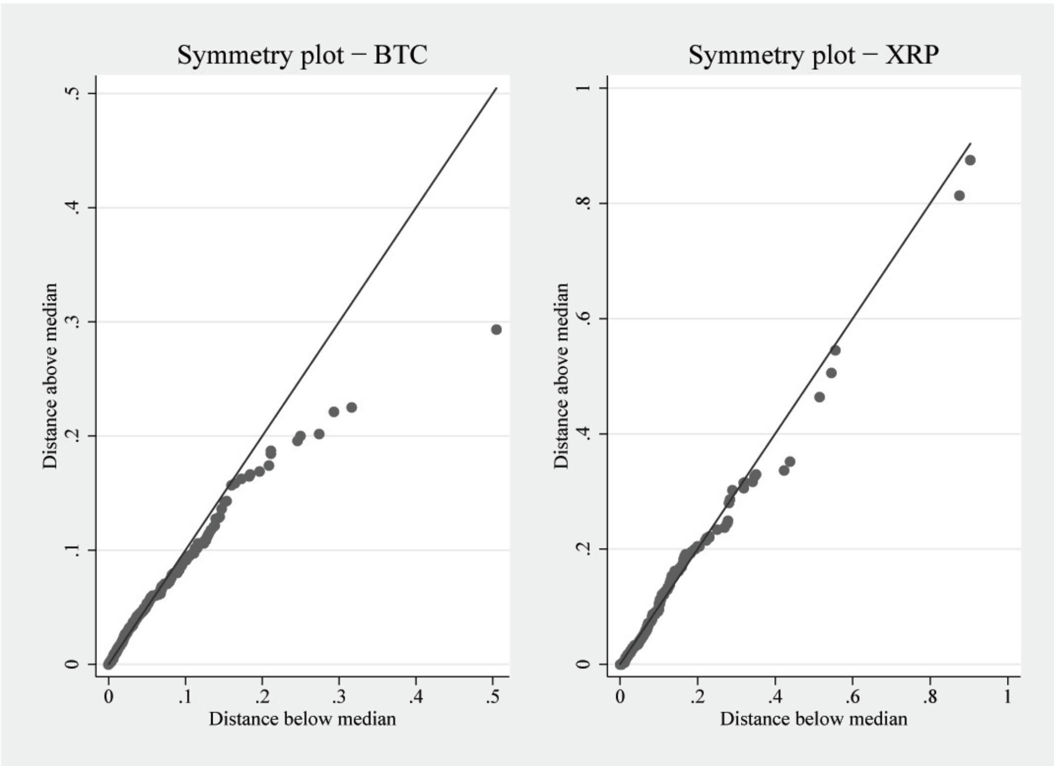

Figure 3.8: Symmetry plot for BTC and XRP

For XRP currency, similar behaviour can be observed on. Data of EUR/USD exchange rate also show, albeit very slight, asymmetry in a direction of loss. Symmetry plots of those four assets can be seen on Figure 3.8 and Figure B.3. On the other hand, gain/loss asymmetry in case of LTC — Figure B.4 seems to be the other way around. According to the dataset used, LTC currency exhibits gain/loss return asymmetry in a favor of gain.

Although this observation could mean that, unlike other financial assets, which do tend to have significant negative returns more often than positive significant positive returns (and data from the benchmark time series used in this thesis do not differ), returns on LTC behave in an opposite way, after taking into a consideration only second half of available data the above mentioned observation did not hold true any more. Conclusion is that gain/loss asymmetry held for LTC in favour of gain in the period of last two months of year 2013 and first half of 2014, but it does not seem to hold for this crypto-currency universally. Finally, we can conclude that negative movements in prices of crypto-currencies are generally more frequent and that crypto-currencies are in this stylized fact more similar to S&P500 index and possibly to other stock indices.

3.5 Aggregational Gaussianity

Generally, financial time series exhibit a property of aggregational gaussianity, which is defined as a tendency of their returns' distribution to resemble normal distribution with increase of time over which are the returns computed — τ. Formula for computation of the returns can be therefore rewritten in context of this section to:

$$R(t) = x(t+\tau) - x(t)$$

where

$$x(t+\tau) = \ln S(t+\tau)$$

.

Due to large number of observations, aggregational gaussianity of crypto-currencies' returns is the most effectively tested on BTC data. After adjusting the data by creating 2-day, 4-day, 8-day, 16-day and 32-day returns, it is possible to show how much distribution functions changes with increase in time scale. What one finds out, by simple graphical observation of data, is that the main difference is quite significant decrease in kurtosis with increase in τ. The problematic part in observing these changes on graphs is rather big difference in number of values.

For more concrete evidence, it is possible to use test of normality introduced by Shapiro & Wilk (1965) and adjusted for higher number of observations

(Royston 1992a). Another option, which also has been used is skewness and kurtosis test for normality in sense of Jarque & Bera (1987), introduced by D'Agostino *et al.* (1990) and its respective adjustment by Royston (1992b). In Shapiro-Wilk test, the null hypothesis is that the sample is taken from normally distributed population, therefore if p-value coming from Shapiro-Wilk test's z score is less than pre-selected alpha level, the null hypothesis can be rejected and it is safe to assume that the sample is not taken from normally distributed population (Shapiro & Wilk 1965). In Jarque-Bera test, the null hypothesis is based on value of skewness being equal to zero and value of kurtosis being equal to three, this the test checks whether those two values are in accordance with normal distribution (Jarque & Bera 1987).

Until now it was safe to omit missing values on the basis of their relatively low impact. Unfortunately, it is not possible to omit them during Shapiro-Wilk and Jarque-Bera tests, due to the fact, that value of sample used gets lower and that omitting them would lead to time spans in individual tests being inconsistent, as omitted values would create larger *jumps* between daily rates. To mitigate the error, returns has been computed only at those periods for which we have price data available for both first day and last day of the period. Obviously, it will make our dataset smaller, but completely omitting the values would lead to distribution tails being heavier than they really are and that could possibly influence the analysis much more.

Using this test, we are able to compute W — Shapiro-Wilk test statistics, V — transformation of the aforementioned test statistics, which is equal to one for normal distribution, and z — standard score, which allows us to compute the p-values. The result is that generally, p-values are increasing with increase in time scale. As time over which the returns are computed increases, the return values get closer to normal distribution. Nevertheless, in the case of BTC, p-values are still rather small — 0.00009 and 0.00016 respectively over time intervals of eight and sixteen days, but increase to 0.02049 for period of thirty two days — Table 3.2. That implies that the distributions is getting closer to normal, as one cannot reject the null hypothesis of thirty two day BTC returns being out of normal distribution while using the 0.01 significance level.

We get similar outcomes for the skewness-kurtosis test (Table 3.3), which is not able to reject the null hypothesis on 0.01 significance level for thirty two day periods either. Similarly, it is also rejecting the null hypothesis for all the other time periods.

Table 3.2: Shapiro-Wilk Test for BTC

Variable	Obs	W	V	z	Prob $> z$
two-day log. return	667	0.86811	57.568	9.867	0.00000
four-day log. return	334	0.90562	22.120	7.306	0.00000
eight-day log. return	166	0.95906	5.197	3.756	0.00009
sixteen-day log. return	83	0.92744	5.133	3.591	0.00016
thirty-two-day log. return	41	0.93454	2.637	2.044	0.02049

Table 3.3: Skewness-Kurtosis Test for BTC

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj χ^2 (2)	Prob $> \chi^2$
two-day log. return	667	0.0000	0.0000	.	0.0000
four-day log. return	334	0.0003	0.0000	50.49	0.0000
eight-day log. return	166	0.0006	0.0005	19.25	0.0001
sixteen-day log. return	83	0.0000	0.0001	24.43	0.0000
thirty-two-day log. return	41	0.0113	0.0976	7.97	0.0186

Fortunately, both tests work well even for a smaller sample sizes and therefore, it is safe to use them and the same two to and thirty two day periods on the other crypto-currencies as well. Shapiro-Wilk test (results in Table 3.4) on eight day returns of XRP currency comes up with a p-value of 0.05186 while the test on thirty two day returns comes up with a p-value of 0.06701. Compared to one day or the two day returns which results in a p-values of 0.0000, it is safe to say that XRP returns do begin to resemble a normal distribution with increase in the variable τ.

Table 3.4: Shapiro-Wilk Test for XRP

Variable	Obs	W	V	z	Prob $> z$
two-day log. return	227	0.93609	10.649	5.477	0.00000
four-day log. return	110	0.93569	5.751	3.901	0.00005
eight-day log. return	56	0.95852	2.134	1.627	0.05186
sixteen-day log. return	27	0.90719	2.728	2.062	0.01961
thirty-two-day log. return	11	0.86509	2.184	1.498	0.06701

Additionally, the skewness-kurtosis test confirms those results and cannot reject the null hypothesis of skewness/kurtosis being normal-like for both sixteen and thirty two day returns on the 0.01 significance level. Detailed results from the test are summed up in Table 3.5.

Table 3.5: Skewness-Kurtosis Test for XRP

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj χ^2 (2)	Prob > χ^2
two-day log. return	227	0.0137	0.0000	26.32	0.0000
four-day log. return	110	0.0012	0.0006	17.60	0.0002
eight-day log. return	56	0.0196	0.0202	9.22	0.0099
sixteen-day log. return	27	0.0103	0.0681	8.40	0.0150
thirty-two-day log. return	11	0.1033	0.6958	3.38	0.1844

The Shapiro-Wilk test of sixteen day returns on the LTC currency comes up with a p-value of 0.00241 and on thirty two day returns with a p-value of 0.0002, shown in Table 3.6. Therefore, there is a very weak resemblance of a normal distribution.

Table 3.6: Shapiro-Wilk Test for LTC

Variable	Obs	W	V	z	Prob > z
two-day log. return	230	0.87331	21.354	7.093	0.00000
four-day log. return	112	0.86542	12.218	5.587	0.00000
eight-day log. return	54	0.85435	7.279	4.253	0.00001
sixteen-day log. return	27	0.86582	3.945	2.819	0.00241
thirty-two-day log. return	12	0.51396	8.121	4.081	0.00002

The skewness-kurtosis test shows similar results — Table 3.7 — with even lower p-values indicating that the LTC's skewness and kurtosis are far from values which characterize a normal distribution. Therefore, it is possible, to reject the null hypothesis of normality on a 0.01 significance level for all of the observed values of τ, and LTC is the only of the three crypto-currencies that does not seem to converge to a normal distribution with an increase in time over which we have measured its returns (τ). In case that it converges, then we can say that it converges only very slowly.

Table 3.7: Skewness-Kurtosis Test for LTC

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	adj χ^2 (2)	Prob > χ^2
two-day log. return	230	0.0000	0.0000	46.75	0.0000
four-day log. return	112	0.0002	0.0000	24.91	0.0000
eight-day log. return	54	0.0014	0.0002	18.31	0.0001
sixteen-day log. return	27	0.0016	0.0056	13.49	0.0012
thirty-two-day log. return	12	0.0000	0.0002	19.91	0.0000

However, both tests on LTC did not show any movement towards normal distribution, there might have been possibly influence of the chosen data. Tests performed on XRP and BTC did show some changes for higher time frame returns and therefore crypto-currencies generally might exhibit tendency to approach normal distribution as variable τ increases. Yet, they do so rather slowly.

Because the above described results alone do not show enough about crypto-currencies comparison vis-à-vis returns on other financial assets, the same tests have been applied on EUR/USD exchange rate returns and S&P500 index return data. For the EUR currency, Shapiro-Wilk test shows p-value of 0.00018 for four day returns and 0.95080 for eight day returns (Table 3.8). Then there is a drop in p-value for sixteen day returns to 0.11413 and to 0.05908 in case of the thirty-two day returns. Nevertheless, values for all measured returns with $\tau \geq 8$ are still above 0.01 and therefore, fore them, we cannot reject the null hypothesis - sample being from normally distributed population.

Table 3.8: Shapiro-Wilk test for EUR

Variable	**Obs**	W	V	z	Prob > z
two-day log. return	633	0.96336	15.254	6.619	0.00000
four-day log. return	317	0.97964	4.556	3.569	0.00018
eight-day log. return	157	0.99601	0.483	-1.653	0.95080
sixteen-day log. return	78	0.97420	1.734	1.205	0.114131
thirty-two-day log. return	38	0.94458	2.106	1.563	0.05908

Again, similar story shows also the skewness-kurtosis test summarized in Table 3.9. Altogether, it is obvious that those p-values are generally higher than in case of crypto-currencies and that distribution of returns on EUR currency (denominated in USD) converge to normality faster than distributions on returns of the three crypto-currencies.

Table 3.9: Skewness-Kurtosis Test for EUR

Variable	**Obs**	**Pr(Skewness)**	**Pr(Kurtosis)**	**adj χ^2 (2)**	Prob > χ^2
two-day log. return	633	0.0000	0.0000	57.66	0.0000
four-day log. return	317	0.0011	0.0039	16.07	0.0003
eight-day log. return	157	0.4940	0.3379	1.41	0.4951
sixteen-day log. return	78	0.1640	0.0860	4.86	0.0880
thirty-two-day log. return	38	0.2803	0.0322	5.50	0.0640

Also for the S&P500 index the aggregational gaussianity shows up quite nicely and p-values increase from 0.00000 for four-day returns, through 0.00197 for eight-day returns up to 0.43149 for returns over sixteen day periods. Results for both tests can be seen in Table 3.10 and Table A.5 respectively.

Table 3.10: Shapiro-Wilk Test for S&P500

Variable	**Obs**	W	V	z	Prob $> z$
two-day log. return	276	0.94506	10.871	5.579	0.00000
four-day log. return	139	0.92474	8.205	4.753	0.00000
eight-day log. return	92	0.95211	3.689	2.882	0.00197
sixteen-day log. return	35	0.96957	1.086	0.173	0.43149
thirty-two-day log. return	17	0.88704	2.386	1.734	0.04142

Finally, we can see that both XRP and LTC seem to converge to normal distribution at much lower rate than the typical financial time series, while BTC's returns converge at approximately the same rate as those of S&P500.

3.6 Volatility Clustering

Volatility clustering is a tendency of extreme values (both, above and below mean) to group up. In our case creating periods of time with extremely high and extremely low returns per day on one side and then creating periods of time with mild returns moving just relatively slightly around the mean. This phenomena has been described by Mandelbrot (1963) (on an example of cotton prices) so that *large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes.*

In first part of this thesis we have described how crypto-currencies, compared to other financial assets, do exhibit autocorrelation. To show and measure volatility clustering, it is possible to follow more or less the same steps, with the the only difference being their returns modified into absolute or square returns. This approach will measure autocorrelation of absolute and square returns on the financial assets and therefore show how absolute values of past returns influence absolute values of returns in the current period. We have also, in this and the following section, broadened the number of observed lags to thirty — so the maximum difference in correlated days is approximately one month instead of a week. That way we are able to better see actual shape of autocorrelation functions and possible longer-term trends.

In his works Cont (2001; 2007) strongly argues in favor of appearance of this phenomenon in most financial time series and this time, to support his claim, we have begun by observing correlation of EUR and S&P500 data first.

When EUR data are transformed into absolute values and correlogram is created, it is already clear that there is a strong and positive correlation among different lags. Estimated autocorrelations leave 99% confidence bands in thirteen out of thirty observed lags and Ljung-Box Q test's p-values are at 0.0000 for all the lags, as can be seen in Table A.6. It is also possible to see weekly trend in autocorrelation (Figure 3.9), where the highest correlation is between absolute returns, which are seven, fourteen, twenty one or twenty eight (multiples of seven) lags from each other. Little bit difficult is explanation of the negative autocorrelation, which should theoretically not happen, while there are no negative absolute returns. This could be explained by data from Saturdays on which the absolute returns are generally zero, as foreign exchange market is closed. If we delete those days from our dataset we obtain slightly different version of the original results — Figure 3.10. In this way, we do not see as much of negative autocorrelation any more, but the weekly trend is still present.

When squared values were observed instead of absolute values the estimates came out quite bit differently (Table A.7), although with the same conclusion. Autocorrelation functions are now positive and significant only at first lag and there is no weekly trend. Additionally, the autocorrelation is so high (coefficient estimate of 0.40004 and 0.3945 after the adjustment) that even though other lags are not significantly different from zero, it still indicates relatively high amount of volatility clustering present in the data only due to the influence of this first lag.

Also for S&P500 the volatility clustering is quite clearly present. In the case of absolute returns, partial autocorrelation function is significantly different from zero (using 0.01 significance level) and positive for thirteen lags — Table A.10. Even here, it is possible to see a trend of higher correlation between returns which are approximately one week apart, but the influence is not as strong as in case of EUR.

Similar picture can be reproduced using the squared values, where autocorrelation function is positive and significantly different from zero for eleven out of thirty (Table A.11). Both autocorrelation functions are also positive and mostly significant — Figure 3.11.

When we take absolute values and squared values of returns on BTC and apply the same process (results in Table A.12 and Table A.13) — creating

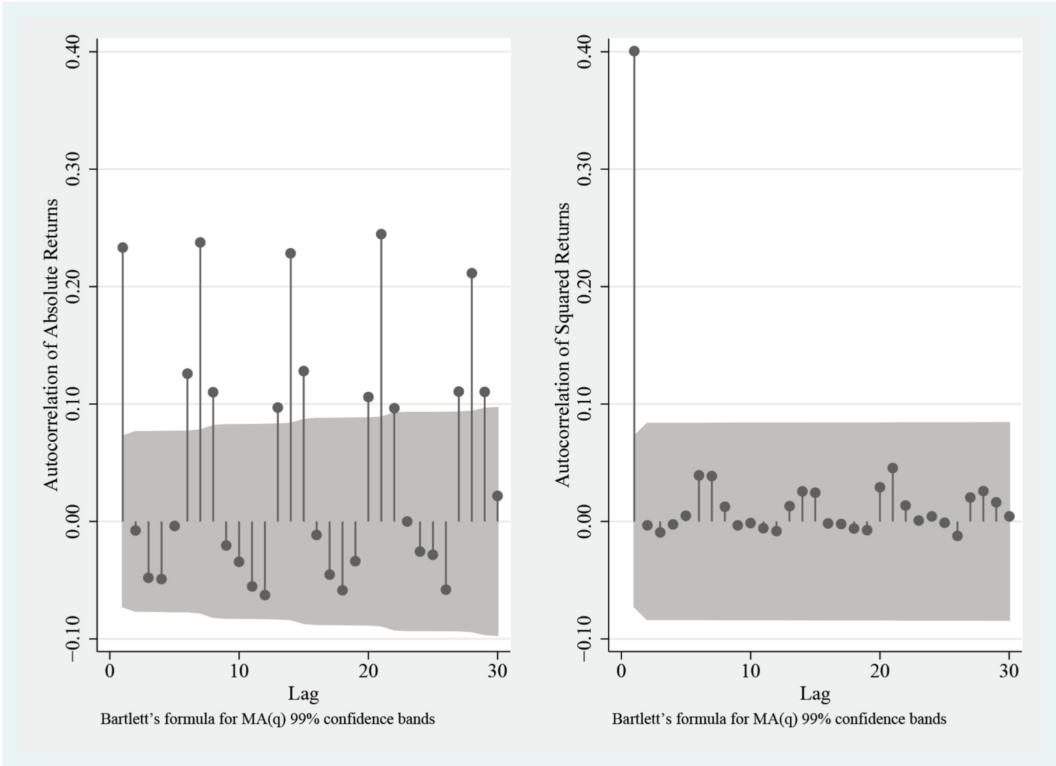

Figure 3.9: Autocorrelations of Absolute and Squared Returns for EUR

correlogram with thirty lags — we see that the correlation is even stronger. Autocorrelation function of the absolute values statistically differs from zero in all but six lags and autocorrelation function of the squared values statistically differs in ten lags, while the correlation coefficients are generally higher. From the correlograms it seems that BTC exhibit even higher volatility clustering on its returns then do EUR or S&P500 index. Both autocorrelation functions of BTC are displayed in Figure B.5.

In the case of the XRP, the autocorrelation graphs are not as dramatic and in both of them (for absolute and squared values) the most decisive is the first lag (estimates of 0.3607 and 0.4221) (Figure B.7). Estimates of autocorrelation at other lags are not statistically different from zero. This description holds for both correlograms which are presented in Table A.14 and Table A.15.

The last of our crypto-currencies — LTC, does not differ much in autocorrelation properties of its absolute and squared returns from XRP and the shape of their autocorrelation functions is very similar. The main factor stays the first lag as the estimates came out at 0.4216 and 0.4614 for absolute and squared values respectively. After that, the autocorrelation functions decrease and stay

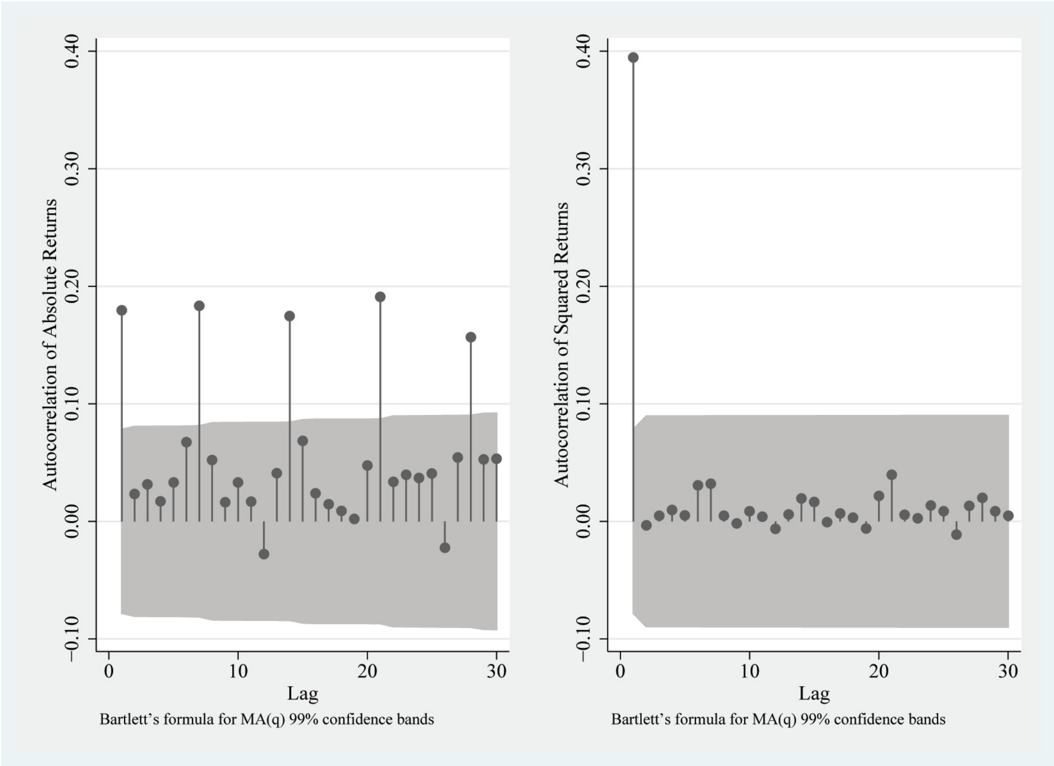

Figure 3.10: Autocorrelations of Absolute and Squared Returns for EUR after adjustment

statistically indifferent from zero. Also, in case of LTC, we present the whole results in Table A.16, Table A.17 and Figure B.6.

From the generated correlograms, autocorrelation and partial autocorrelation functions, one can see that in the same fashion as usual financial time series also returns on crypto-currencies tend to have quite distinctive clusters of high volatility. Nevertheless, volatility clustering of BTC and other crypto-currencies seems to be slightly different.

Initially, the clustering might seem stronger, as the coefficient estimates of correlations are much higher for first lags, but then the autocorrelation functions tend to decay a lot faster in case of XRP and LTC than they do in case of EUR or S&P500. BTC has overall high autocorrelation with high first lag autocorrelation coefficient and slower rate of decay than S&P500. Also, we do not see any weekly trend in case of crypto-currencies.

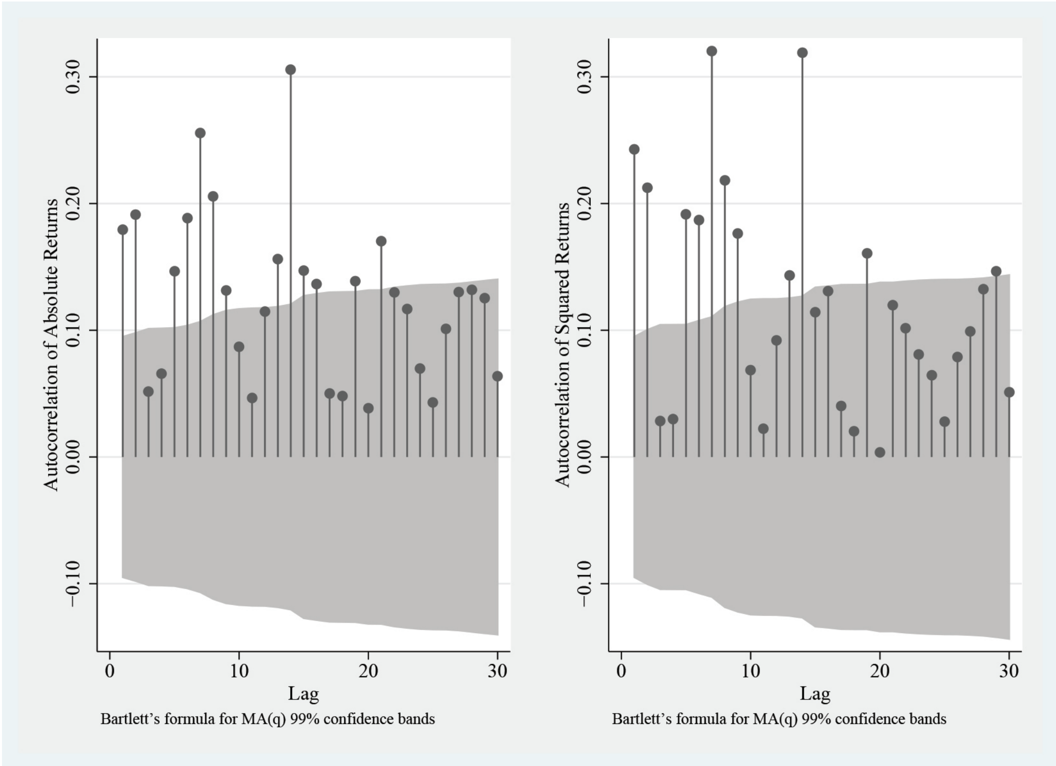

Figure 3.11: Autocorrelations of Absolute and Squared Returns for S&P500

3.7 Slow Decay of Autocorrelation in Absolute Returns

As described in previous section, decay of absolute returns' autocorrelation is slower in case of crypto-currencies than in case of EUR currency. The difference can be seen especially when looked at data of BTC, which absolute values' autocorrelation decays at the lowest rate of the observed crypto-currencies. On the other hand, we were not able to recognize decay for autocorrelation in absolute returns on S&P500 with all seven lags taken into an account. It is most probable that we would find a decay, if we have taken more lags into consideration, but it is definitely slower then for crypto-currencies.

To be more specific and accurate in our description, there is autocorrelation estimate of 0.3607 for XRP's first lag and second highest estimate is 0.1375 (12^{th} lag). Similarly in LTC's case there is first lag autocorrelation estimate of 0.4216, while the second highest correlation estimate is 0.1447 (8^{th} lag). These show quite fast decay in the autocorrelation function, due to large difference between the first and second highest observed values. Besides that, both function are statistically indifferent at other than first lag. All in all, if it is safe, as we assume, to take data from S&P500 as a benchmark for time series of stock indices generally, then XRP and LTC seem to not exhibit slow decay of autocorrelation in their absolute returns. The difference between the correlation of first lag and the rest is quite high for both XRP and LTC and their autocorrelation functions (as well as that of BTC) have only one peak (at first lag). Meanwhile, for EUR the autocorrelation functions (with adjusted data) decays from 0.1795 at first lag to 0.0171 (not significantly different from zero) at second lag and then sharply recovers to 0.1836 at seventh lag. Similarly, S&P500's autocorrelation functions recover and have multiple peaks.

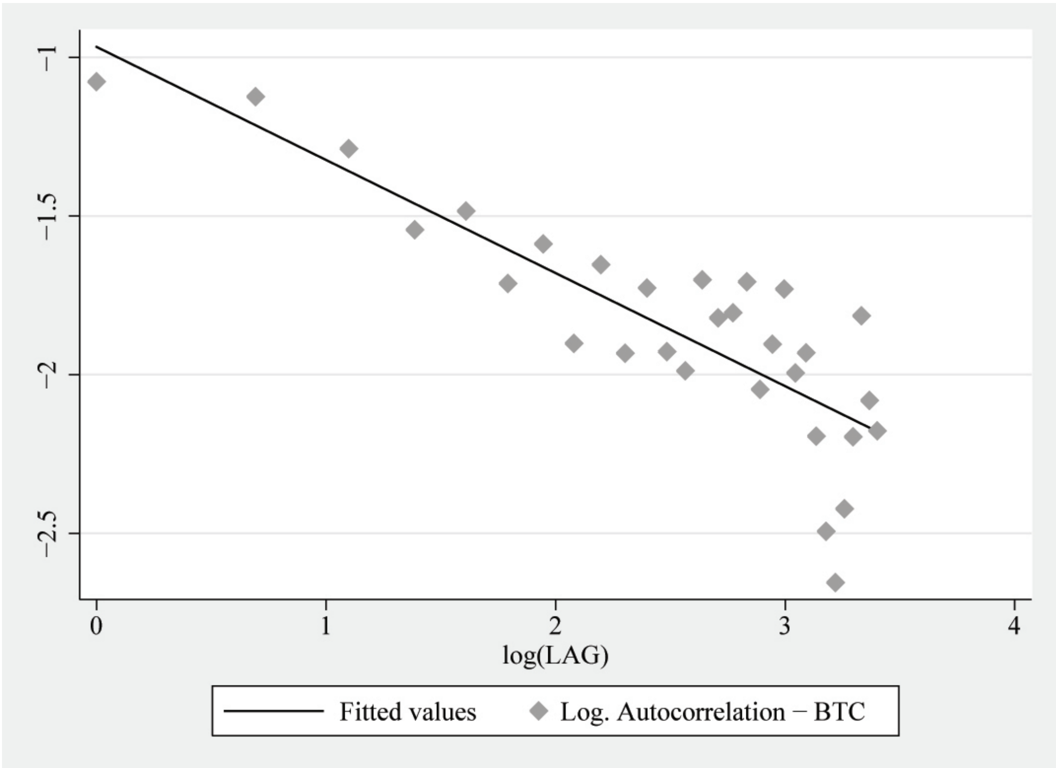

Figure 3.12: Decay of autocorroletaion of day-to-day absolute returns on BTC

Although BTC's autocorrelation in absolute returns decays slower than XRP's or LTC's, it is still quite fast in comparison with data of EUR. It seems that crypto-currencies, other than BTC, have faster decay of autocorrelation than stock indices and common fiat currencies. Assuming that, they do not fulfil the stylized fact.

We wanted to look even deeper into this issue and therefore the next step was to determine approximate pace at which the autocorrelation functions de-

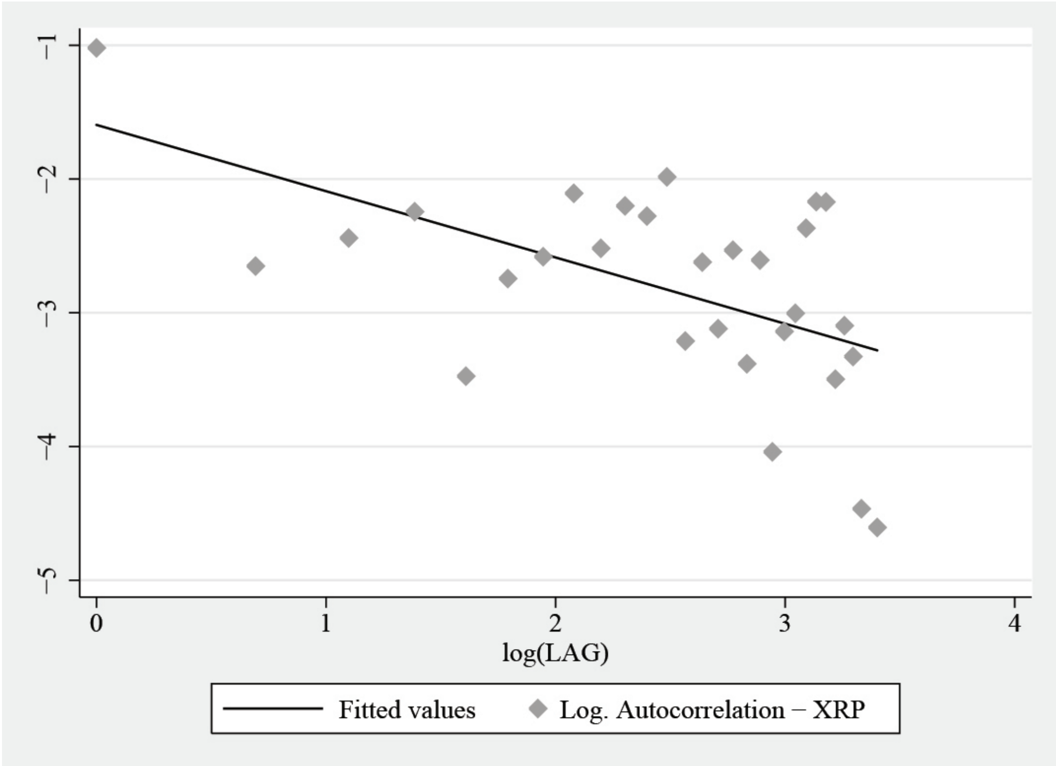

Figure 3.13: Decay of autocorrelation of day-to-day absolute returns on XRP

cay. For that we have used log-log regression of autocorrelation values on their respective lags. This approach has been chosen specifically due to the assumption of the relationship between the given function and respective lags being closer to power-law rather than linear. The assumed model for cryptocurrencies has the following form:

$$\log \mathrm{AC}_{\mathrm{BTC,XRP,LTC}} = \alpha + \beta \log \mathrm{Lag} + \varepsilon$$

, where AC represents the autocorrelations in question, LAG is a time interval between the two points between which we have computed the autocorrelation (in days), α and β are constants, and ε describes the error term of our model. A graphical representation can be seen in Figure 3.12, Figure 3.13 and Figure 3.14.

Coefficients from the regressions can be found in Table 3.11, Table 3.12 and Table 3.13. For BTC the performed regression shows power-law relationship with coefficient of approximately -0.356 between variables AC and Lag, which represents a 35.6% decrease in autocorrelation function with any 100% increase in lag variable (also 3.56% decrease for 10% increase in lag etc.). For XRP the coefficient is higher at -0.496, showing faster decay in case of the XRP currency.

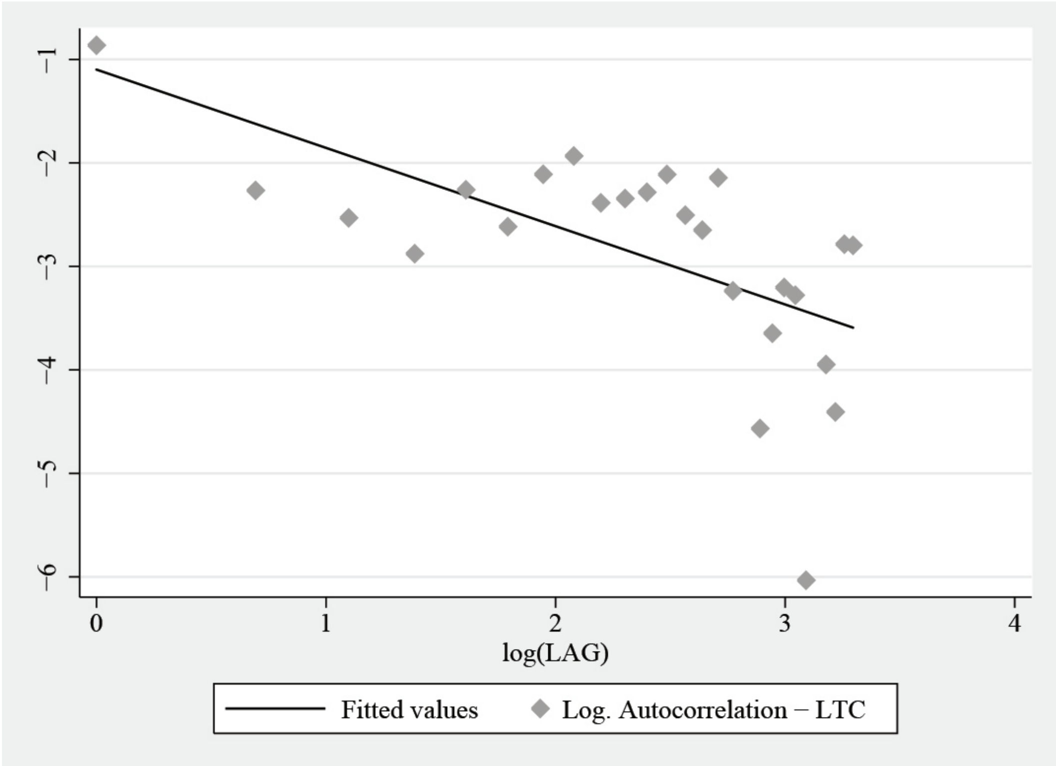

Figure 3.14: Decay of autocorrelation of day-to-day absolute returns on LTC

As expected, results for LTC show the highest coefficient of all three (-0.756). However Std. Err. is biased, we can still take the coefficient estimates as and indicator of the actual shape and especially pace of decay.

Table 3.11: Linear Regression for BTC

Variable	Coefficient	(Std. Err.)
log_LAG	-0.356	(0.043)
Intercept	-0.967	(0.114)
N	30	
R^2	0.708	
$F_{(1,28)}$	67.915	

We can not use the same model for EUR currency nor S&P500 index. Shape of their autocorrelation functions is no power-law (Figure 3.9 and Figure 3.11) and therefore log-log regression does not represent the decay well enough. The actual shape is quite complicated and does not seem to decrease at exponential

Table 3.12: Linear Regression for XRP

Variable	Coefficient	(Std. Err.)
log_LAG	-0.496	(0.144)
Intercept	-1.595	(0.375)
N	29	
R^2	0.304	
$F_{(1,27)}$	11.803	

Table 3.13: Linear Regression for LTC

Variable	Coefficient	(Std. Err.)
log_LAG	-0.756	(0.198)
Intercept	-1.098	(0.493)
N	25	
R^2	0.388	
$F_{(1,23)}$	14.573	

rate at any point (not even for low lags), additionally, the function's tendency to recover makes the modelling even more difficult.

From both approaches, it is obvious that the decay in autocorrelation functions of the three crypto-currencies is hardly as slow as that of EUR, which autocorrelation function does not even seem to decay when we observe first thirty lags. S&P500 currency seems to decay slightly faster but it changes at twelfth lag, where the autocorrelation function starts to rise again and actually surpass the initial first-lag value. This behaviour is due to the weekly trend in S&P500's and EUR's absolute returns' autocorrelation functions (the same trend can be seen even for GBP, which has been tested to check for possibly similar shape of autocorrelation function to EUR's). From all of the above, we can claim that crypto-currencies generally do not fulfil this stylized fact because we have found two crypto-currencies (XRP and LTC) where autocorrelation functions of their absolute return decay at much higher rate than those of regular financial assets. On the other hand, BTC's data are in accordance with the stylized fact. The difference is also in the shape of decay as there are no weekly trends (higher positive correlation of absolute returns which are approximately one week from each other) or generally any upward spikes in autocorrelation of crypto-currencies' absolute returns.

3.8 Leverage Effect

Leverage effect, firstly noted is a negative correlation between a given asset's past returns and squared subsequent returns of the same asset (Black 1976). This effect is also a cause of gain/loss asymmetry, because assets which is under the leverage effect generally have higher returns in times of low volatility and relatively lower returns in wilder periods of high volatility. Bouchaud & Potters (2001) claim that the low and high prices respectively, influence the future volatility and therefore specify the direction in which the influence goes. Based on that, we will take value of day-to-day returns as our independent value and the square values representing volatility as dependent.

In this section, a cross-correlation measure has been used to check whether crypto-currencies really have higher than zero correlation between its volatility and returns and therefore to check for presence of leverage effect. Moreover, computation of the cross-correlation at different lags, gives us more information about the specific correlations and paints a better picture about the actual shape of cross-correlation function. Obtaining p-values is technically more difficult, but software was able to reproduce them using calculation of individual pairwise correlations. Finally, we listed and focused on the positive lags only, whereas it is the past returns which influence current volatility.

For BTC, the correlation that is significantly different from zero is only for second lag, and computed at -0.1242. Correlation at fourth lag of 0.0701 is positive and might show absence of the leverage effect, but the first value is higher in absolute values and p-value of the fourth lag's correlation is, although quite low, above the significance level of 0.01. That is sufficient for us to claim that the data are in an agreement with the aforementioned stylized fact. Complete results can be seen in Table 3.14.

The stylized fact holds true for XRP in a similar fashion (Table 3.15). However, there are, two significant values of correlation, one being positive and one negative, the positive one is as in the previous case larger and has lower p-value. LTC seems to go with the other two crypto-currencies and leverage effect seems to be present too. Negative correlation of -0.2230 between its squared logarithmic returns and logarithmic returns on the day before has been measured using the cross-correlation. Litecoin's cross-correlation results are in Table 3.16.

Table 3.14: Cross-Correlation Between BTC's Lagged Value of Returns and Its Square Returns

Lag	CORR	Sig
1	-0.0305	0.2656
2	-0.1242	0.0000
3	-0.0109	0.6922
4	0.0701	0.0105
5	0.0078	0.7757
6	0.0239	0.3847
7	0.0462	0.0920

Table 3.15: Cross-Correlation Between XRP's Lagged Value of Returns and Its Square Returns

Lag	CORR	Sig
1	-0.2836	0.0000
2	0.0125	0.7961
3	0.1349	0.0053
4	-0.0325	0.5051
5	-0.0416	0.3978
6	0.0447	0.3657
7	0.0568	0.2521

Table 3.16: Cross-Correlation Between LTC's Lagged Value of Returns and Its Square Returns

Lag	CORR	Sig
1	-0.2230	0.0000
2	0.0293	0.5377
3	0.0368	0.4399
4	0.0897	0.0603
5	0.0452	0.3456
6	-0.0225	0.6389
7	0.0660	0.1706

Now, both EUR's and S&P500's cross-correlations — of volatility measure (squared returns) and regular returns — also came up with significant negative values (summed up in Table 3.17 below). Because of that, it is possible to accept the statement that the leverage effect is an usual phenomenon which holds for all financial assets, including crypto-currencies.

Table 3.17: Cross-Correlation Between EUR's and S&P500's Lagged Values of Returns and Their Respective Square Returns

Lag	$\mathbf{CORR}_{\mathrm{EUR}}$	$\mathbf{Sig}_{\mathrm{EUR}}$	$\mathbf{CORR}_{\mathrm{SP500}}$	$\mathbf{Sig}_{\mathrm{SP500}}$
1	-0.1701	0.0000	-0.0470	0.2700
2	-0.0047	0.8688	-0.0055	0.9164
3	0.0115	0.6861	-0.0227	0.7597
4	-0.0197	0.4876	-0.1495	0.0428
5	-0.0212	0.4562	-0.2430	0.0000
6	-0.0640	0.0246	-0.1074	0.0117
7	-0.0378	0.1860	-0.0315	0.3937

3.9 Volume/Volatility Correlation

The last stylized fact, which is in scope of this thesis is generally positive correlation between change in trading volume and volatility of returns. Mostly, financial returns increase in absolute value with increase in volume traded. This fact is supported and nicely described (besides Cont (2001)) by Zolotoy & Melenberk (2009), who finds a relationship between increase in lagged values of volume and current variance in returns of stocks and stock indices. The link between those two is most probably due to appearance of *information flow* (Clark 1973). The relationship and forces behind it in case of stocks is also well described by Darrat *et al.* (2007).

For purposes of this thesis, less sophisticated approaches have been applied to find out how crypto-currencies fare in comparison and in similar fashion to the previous section about Leverage effect, sample cross-correlation function has been used. Independent variable is in this case a change in logarithmic volume ($\log \mathrm{V(t)}$, where $V(t)$ is change in trading volume) and dependent variable is the volatility — represented by absolute values of logarithmic returns ($|\log \mathrm{R(t)}|$. From the previously mentioned studies, it would be most intuitive to expect crypto-currencies to also have positive correlation between volatility and trading volume.

What is important to note, is that due to the fact that trading volume data are not daily, an unit of lag in this subsection is not exactly one day. The lags are not periodical in any way (although, usually two or three days) either. Therefore, 4th lag might not be exactly two times further from zero than 2nd lag etc. Lags play rather indicative role in this section.

Basically, both BTC's and LTC's sample cross-correlation functions are mostly positive and are negative only at lags where the results are at the same time not significantly different from zero (significance level = 0.01). The highest correlation, in case of BTC, can be seen at 2nd and 4th lags — Table 3.18. Also, in case of LTC it is in 2nd and 4th lags where the cross-correlation function is positive and significantly different from zero — Table 3.19.

Table 3.18: Cross-Correlation Between BTC's Lagged Values of Change in Trading Volume and BTC's Absolute Returns

Lag	CORR	Sig
1	-0.0966	0.1971
2	0.2271	0.0022
3	0.1049	0.1636
4	0.2157	0.0039
5	0.1611	0.0327
6	0.1584	0.0362
7	0.1394	0.0665

Table 3.19: Cross-Correlation Between LTC's Lagged Values of Change in Trading Volume and LTC's Absolute Returns

Lag	CORR	Sig
1	0.0595	0.4138
2	0.2078	0.0040
3	-0.0170	0.8167
4	0.2686	0.0002
5	-0.0402	0.5852
6	-0.0819	0.2667
7	0.1664	0.0236

If we take a look at XRP's cross-correlation function between volume and volatility measures (Table 3.20), we see that although it deviates far less from zero and that there are no results significantly different from zero at 0.01 significance level. There are still two lags where correlation is significantly positive using 0.05 significance level. It seems that even in case of XRP its volume does correlate positively with its volatility, but the relationship seems to be much weaker than in case of BTC or LTC.

Table 3.20: Cross-Correlation Between XRP's Lagged Values of Change in Trading Volume and XRP's Absolute Returns

Lag	CORR	Sig
1	0.1893	0.0126
2	0.0550	0.4735
3	0.1533	0.0452
4	-0.0844	0.2739
5	-0.1039	0.1786
6	-0.0212	0.7853
7	0.0576	0.4599

To follow up on the previous finding and to better see how correlation between volume and volatility behaves, Granger causality Wald test has been used on the crypto-currencies. Aim of this test has been to find out how the two variables influence each other and in which direction the influence actually goes. The idea similar to that used by Brooks (1998). To apply the test, Vector Autoregression (VAR) models with 2 lags has been used first to see the interdependence between the two variable for each crypto-currency.

For BTC, the interdependence is seen already from the VAR, which shows multiple significant relationships between the volume and volatility variables. Additionally, Granger causality Wald test, depicted in Table 3.21, does reject the null hypothesis for both of the two variables, meaning that lagged values of one do cause the other and vice versa. The test is significant with p-values of 0.000 for both volume and volatility measures.

LTC data also show some level of interdependence, although not as high and R-squared is quite low (R-squared of 0.0507 for volume equation and 0.0685 for volatility equation; adjusted R-squared for BTC's VAR is 0.1362 and 0.2539 respectively) compared to the BTC's model. There is a significant influence of second lag of volatility changes on volume increases values. There is also visible

Table 3.21: Granger Causality Wald Test for BTC

Equation	**Excluded**	χ^2	**df**	Prob > χ^2
$\log \Delta V(t)$	$\lvert\log R(t)\rvert$	21.354	2	0.000
$\log \Delta V(t)$	ALL	21.354	2	0.000
$\lvert\log R(t)\rvert$	$\log \Delta V(t)$	23.202	2	0.000
$\lvert\log R(t)\rvert$	ALL	23.202	2	0.000

some level of autocorrelation in the return data, which is already covered in the first section of this thesis and quite high, although insignificant, influence of two-lagged change in volume on absolute returns.

The Granger test results, shown in Table 3.22 describe causality in the direction from absolute logarithmic returns towards logarithmic change in volume, with significant p-value of 0.008, therefore significant on 0.01 significance level. Causality in the opposite direction is insignificant and with p-value of exactly 0.050.

Table 3.22: Granger Causality Wald Test for LTC

Equation	**Excluded**	χ^2	**df**	Prob > χ^2
$\log \Delta V(t)$	$\lvert\log R(t)\rvert$	9.7066	2	0.008
$\log \Delta V(t)$	ALL	9.7066	2	0.008
$\lvert\log R(t)\rvert$	$\log \Delta V(t)$	5.9849	2	0.050
$\lvert\log R(t)\rvert$	ALL	5.9849	2	0.050

VAR model in the case of XRP fits even worse and there are, as in case of cross-correlation, no significant relations. As seen in Table 3.23, Granger causality Wald test does not recognize any significant influence either. Only after we loose significance level to 0.05, there are signs of possible influence of change in volume on XRP's absolute returns - its volatility.

Table 3.23: Granger Causality Wald Test for XRP

Equation	**Excluded**	χ^2	**df**	Prob > χ^2
$\log \Delta V(t)$	$\lvert\log R(t)\rvert$	0.68769	2	0.709
$\log \Delta V(t)$	ALL	0.68769	2	0.709
$\lvert\log R(t)\rvert$	$\log \Delta V(t)$	6.4266	2	0.040
$\lvert\log R(t)\rvert$	ALL	6.4266	2	0.040

Basically, the three currencies differ quite a lot in the way VAR model fits their data as well as in the outcomes of Granger test. From the results, it seems that generally, the crypto-currencies' volume and volatility positively correlate and especially in cases of BTC/LTC. The causality goes both ways for BTC but in LTC and XRP, there is not enough evidence for causality to be determined on 0.01 significance level. Relationship going in way of volume influencing the volatility, can be seen if we loose our significance level to 0.05, but we do not consider that sufficient for us. XRP stands out from the others as the one with least correlated volume and volatility and there seems to be none when we accept the 0.05 significance level as insufficient. Therefore, in the case of XRP, it is not safe to assume anything just on an outcome of this exact VAR model. Other than that, for both LTC and BTC, it is possible to see the link between volume and volatility.

Chapter 4

Conclusions

Although, it is possible to see differences between data of day-to-day returns on crypto-currencies in question — BTC, XRP and LTC, and data of day-to-day returns on EUR and S&P500, there is barely any hard hitting difference which would go straight against the stylized facts (Cont 2001). The biggest difference can be seen in the way autocorrelation of return data behaves. Cont (2001) claims that financial time series generally lack autocorrelation, but in the first section of this thesis we have come to a conclusion that crypto-currencies tend to exhibit higher levels of autocorrelation than benchmark assets. This correlation is then mostly in the form of an influence of first-lagged returns on the current values. There is no such a strong influence found in the benchmark values and therefore we find this to be the most distinctive sign of crypto-currencies covered by this thesis. The possible explanation behind this fact might be that people rely on today's change in value of crypto-currencies to predict its future values more than they do so in other cases of financial assets. That could be caused by lack of real life events which directly influence values of crypto-currencies. It is important to note is that both ripple and Litecoin exhibit negative autocorrelation, while Bitcoin showed significant positive autocorrelation.

On the other way, crypto-currencies seem to lack weekly trends in autocorrelation of absolute returns, which we have found in data for both EUR and S&P500 (also in data for GBP to reduce possibility of bad calculations on our side). This fact influences a behavior of volatility clustering. While crypto-currencies have higher first lag autocorrelation in their absolute values, their absolute value autocorrelation does not significantly stretch below the first Although, present in autocorrelation of absolute returns, weekly trends do not seem to be present in autocorrelation of regular returns.

Another differences found during the analysis of crypto-currencies' data are less disrupting for the initial idea of crypto-currencies fulfilling Cont's stylized facts. Crypto-currencies seem to have statistical properties closer to that of stock indices than to properties of fiat currencies? data. That can be seen in distribution tails of logarithmic day-to-day returns, which seem to be heavier and in aggregation gaussianity, which is not as strong as in case of EUR currency and also resembles more the way in which S&P500 returns converge to normal distribution.

For easier overview of the topic we attach a summary in Table 4.1 on next page.

Table 4.1: Summary

- **Absence of autocorrelation** — Crypto-currencies BTC, XRP and LTC display higher levels of autocorrelation in their return data than benchmark assets EUR and S&P500. This is most noticeable for the first lag, where autocorrelation coefficient is higher than 0.1 and significantly different from zero for all of the currencies. BTC seems to be the only one of the three crypto-currencies to have positive autocorrelation. *Not* in accordance with stylized fact.

- **Heavy tails** — BTC, XRP and LTC all have heavier tails than normal distribution. That said, tails of their return distribution functions are approximately as heavy as tails of S&P500 stock index' return distribution. In accordance with stylized fact.

- **Gain/loss asymmetry** — Gain/loss asymmetry holds in case of crypto-currencies too. Only difference has been found in data of LTC, but after sample from a more recent period had been taken the difference in gain/loss asymmetry was no more present. In accordance with stylized fact.

- **Aggregational Gaussianity** — Although at slower rate than benchmark financial assets, crypto-currencies' return distribution function tends to converge to normal distribution with increase in time span over which returns are computed. The pace at which they do so is closer to that of S&P500 than EUR, as EUR converges a lot faster. In accordance with stylized fact.

- **Volatility Clustering** — Volatility clustering has been measured by autocorrelation in absolute and squared returns. This autocorrelation is higher at first lag, but lower as the lags increase. Especially in cases of XRP and LTC, where autocorrelation is not significantly different from zero in all lags but the first. Big difference is absence of a periodicity which can be seen in data for both EUR and S&P500. In accordance with stylized fact.

- **Slow Decay of Autocorrelation in Absolute Returns** — As mentioned in previous point, crypto-currencies' autocorrelation in absolute returns decays really fast for XRP and LTC. Decay in case of BTC currency is slower and comparable to that of S&P500, but thanks to the other crypto-currencies two we can reject the null hypothesis of slow decay. *Not* in accordance with stylized fact (exceptions).

- **Leverage Effect** — Leverage effect is clearly present in crypto-currencies' data as all three of them display significant negative correlation between their returns and absolute returns (volume). Analysis of benchmark data confirms the hypothesis. In accordance with stylized fact.

- **Volume/Volatility Correlation** — On one hand, BTC shows strong significant correlation between its volume and volatility, and LTC shows weaker, but still significant correlation. On the other one, XRP's data do not display such characteristic while using 0.01 significance level. While the XRP's volume/volatility correlation is significant using significance level of 0.05, we do believe that in context of this thesis it is appropriate to use 0.01 significance level. *Not* in accordance with stylized fact (exceptions).

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Appendix A

Tables

Table A.1: Correlogram for BTC

Lag	**AC**	**PAC**	Q	Prob $> Q$
1	0.1198	0.1204	19.206	0.0000
2	0.0137	0.0002	19.458	0.0001
3	-0.0625	-0.0679	24.691	0.0000
4	0.0696	0.0875	31.184	0.0000
5	0.0632	0.0532	36.554	0.0000
6	0.0538	0.0341	40.44	0.0000
7	-0.0594	-0.0641	45.182	0.0000

Table A.2: Correlogram for XRP

Lag	**AC**	**PAC**	Q	Prob $> Q$
1	-0.2950	-0.3003	39.6	0.0000
2	0.0101	-0.0683	39.646	0.0000
3	-0.0649	-0.0910	41.573	0.0000
4	0.0056	-0.0525	41.587	0.0000
5	0.0575	0.0412	43.104	0.0000
6	-0.0401	-0.0084	43.845	0.0000
7	0.0284	0.0419	44.216	0.0000

Table A.3: Correlogram for EUR

Lag	**AC**	**PAC**	Q	Prob > Q
1	-0.0760	-0.0762	7.3229	0.0068
2	-0.0001	-0.0058	7.3229	0.0257
3	0.0180	0.0176	7.7318	0.0519
4	-0.0134	-0.0104	7.9592	0.0931
5	-0.0338	-0.0372	9.4101	0.0938
6	0.0363	0.0338	11.082	0.0859
7	-0.0062	-0.0032	11.132	0.1330

Table A.4: Correlogram for S&P500

Lag	**AC**	**PAC**	Q	Prob > Q
1	-0.0385	-0.0502	1.098	0.2947
2	0.0146	0.0228	1.2568	0.5334
3	0.0346	0.1189	2.1437	0.5431
4	-0.0005	.	2.1438	0.7093
5	-0.0962	.	9.0343	0.1077
6	0.0155	.	9.2131	0.1619
7	-0.1510	.	26.217	0.0005

Table A.5: Skewness-Kurtosis Test for S&P500

Variable	**Obs**	**Pr(Skewness)**	**Pr(Kurtosis)**	**adj** χ^2 **(2)**	Prob > χ^2
two-day log. return	276	0.0000	0.0000	35.80	0.0000
four-day log. return	139	0.0001	0.0000	28.61	0.0000
eight-day log. return	92	0.0006	0.0056	15.55	0.0004
sixteen-day log. return	35	0.0917	0.2592	4.26	0.1187
thirty-two-day log. return	17	0.0197	0.0931	7.20	0.0273

Table A.6: Correlogram for EUR's Absolute Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.2334	0.2340	68.992	0.0000
2	-0.0075	-0.0658	69.064	0.0000
3	-0.0479	-0.0330	71.979	0.0000
4	-0.0491	-0.0323	75.037	0.0000
5	-0.0037	0.0144	75.055	0.0000
6	0.1258	0.1270	95.195	0.0000
7	0.2378	0.2163	167.18	0.0000
8	0.1102	0.0609	182.64	0.0000
9	-0.0204	-0.0474	183.17	0.0000
10	-0.0342	0.0004	184.66	0.0000
11	-0.0554	-0.0395	188.57	0.0000
12	-0.0629	-0.0655	193.63	0.0000
13	0.0971	0.0972	205.7	0.0000
14	0.2282	0.2057	272.36	0.0000
15	0.1282	0.0487	293.43	0.0000
16	-0.0114	-0.0361	293.6	0.0000
17	-0.0454	-0.0331	296.24	0.0000
18	-0.0586	-0.0191	300.65	0.0000
19	-0.0340	0.0067	302.14	0.0000
20	0.1060	0.0865	316.6	0.0000
21	0.2447	0.1915	393.7	0.0000
22	0.0966	-0.0103	405.71	0.0000
23	0.0000	0.0181	405.71	0.0000
24	-0.0258	0.0059	406.57	0.0000
25	-0.0282	0.0389	407.59	0.0000
26	-0.0581	-0.0658	411.96	0.0000
27	0.1108	0.1029	427.84	0.0000
28	0.2115	0.1029	485.77	0.0000
29	0.1103	0.0197	501.52	0.0000
30	0.0218	0.0370	502.14	0.0000

Table A.7: Correlogram for EUR's Squared Values

Lag	**AC**	**PAC**	Q	Prob $> Q$
1	0.4004	0.4006	203.17	0.0000
2	-0.0032	-0.1949	203.19	0.0000
3	-0.0092	0.0872	203.29	0.0000
4	-0.0025	-0.0422	203.3	0.0000
5	0.0049	0.0264	203.33	0.0000
6	0.0391	0.0327	205.28	0.0000
7	0.0386	0.0106	207.17	0.0000
8	0.0126	0.0088	207.38	0.0000
9	-0.0031	-0.0219	207.39	0.0000
10	-0.0014	0.0095	207.39	0.0000
11	-0.0057	-0.0306	207.43	0.0000
12	-0.0081	-0.0265	207.52	0.0000
13	0.0131	0.0789	207.74	0.0000
14	0.0255	0.0838	208.57	0.0000
15	0.0246	0.0726	209.35	0.0000
16	-0.0016	-0.0574	209.35	0.0000
17	-0.0021	0.0208	209.36	0.0000
18	-0.0060	-0.0367	209.4	0.0000
19	-0.0074	-0.0147	209.47	0.0000
20	0.0291	0.1585	210.57	0.0000
21	0.0456	0.1344	213.24	0.0000
22	0.0135	-0.0352	213.47	0.0000
23	0.0008	0.0300	213.47	0.0000
24	0.0044	0.0225	213.5	0.0000
25	-0.0010	-0.0101	213.5	0.0000
26	-0.0123	-0.0694	213.7	0.0000
27	0.0205	0.1353	214.24	0.0000
28	0.0259	0.0031	215.1	0.0000
29	0.0162	0.0520	215.45	0.0000
30	0.0042	-0.0084	215.47	0.0000

Table A.8: Correlogram for EUR's Absolute Values (adjusted)

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.1795	0.1995	35.037	0.0000
2	0.0233	-0.0300	35.629	0.0000
3	0.0314	0.0334	36.702	0.0000
4	0.0171	0.0309	37.021	0.0000
5	0.0331	0.0153	38.215	0.0000
6	0.0676	.	43.207	0.0000
7	0.1836	.	80.08	0.0000
8	0.0520	.	83.039	0.0000
9	0.0162	.	83.329	0.0000
10	0.0332	.	84.54	0.0000
11	0.0169	.	84.853	0.0000
12	-0.0279	.	85.709	0.0000
13	0.0409	.	87.553	0.0000
14	0.1748	.	121.21	0.0000
15	0.0684	.	126.37	0.0000
16	0.0238	.	126.99	0.0000
17	0.0144	.	127.22	0.0000
18	0.0088	.	127.31	0.0000
19	0.0021	.	127.31	0.0000
20	0.0476	.	129.82	0.0000
21	0.1912	.	170.34	0.0000
22	0.0336	.	171.59	0.0000
23	0.0395	.	173.33	0.0000
24	0.0367	.	174.83	0.0000
25	0.0408	.	176.68	0.0000
26	-0.0223	.	177.23	0.0000
27	0.0545	.	180.55	0.0000
28	0.1568	.	207.98	0.0000
29	0.0529	.	211.1	0.0000
30	0.0531	.	214.25	0.0000

Table A.9: Correlogram for EUR's Squared Values (adjusted)

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.3945	0.3986	169.36	0.0000
2	-0.0033	-0.2090	169.38	0.0000
3	0.0047	0.2074	169.4	0.0000
4	0.0097	0.0115	169.5	0.0000
5	0.0051	0.0042	169.53	0.0000
6	0.0307	.	170.56	0.0000
7	0.0321	.	171.69	0.0000
8	0.0048	.	171.71	0.0000
9	-0.0018	.	171.72	0.0000
10	0.0087	.	171.8	0.0000
11	0.0040	.	171.82	0.0000
12	-0.0064	.	171.86	0.0000
13	0.0058	.	171.9	0.0000
14	0.0196	.	172.32	0.0000
15	0.0165	.	172.62	0.0000
16	-0.0007	.	172.62	0.0000
17	0.0067	.	172.67	0.0000
18	0.0033	.	172.68	0.0000
19	-0.0062	.	172.72	0.0000
20	0.0218	.	173.25	0.0000
21	0.0398	.	175.01	0.0000
22	0.0056	.	175.04	0.0000
23	0.0028	.	175.05	0.0000
24	0.0137	.	175.26	0.0000
25	0.0086	.	175.34	0.0000
26	-0.0112	.	175.48	0.0000
27	0.0133	.	175.68	0.0000
28	0.0201	.	176.13	0.0000
29	0.0087	.	176.21	0.0000
30	0.0047	.	176.24	0.0000

Table A.10: Correlogram for S&P500's Absolute Values

Lag	AC	PAC	Q	Prob > Q
1	0.1792	0.2234	23.773	0.0000
2	0.1912	0.3682	50.873	0.0000
3	0.0518	0.1078	52.865	0.0000
4	0.0659	.	56.092	0.0000
5	0.1465	.	72.053	0.0000
6	0.1885	.	98.522	0.0000
7	0.2558	.	147.33	0.0000
8	0.2056	.	178.9	0.0000
9	0.1315	.	191.84	0.0000
10	0.0869	.	197.5	0.0000
11	0.0467	.	199.13	0.0000
12	0.1147	.	209.02	0.0000
13	0.1562	.	227.39	0.0000
14	0.3056	.	297.72	0.0000
15	0.1470	.	314.02	0.0000
16	0.1366	.	328.12	0.0000
17	0.0500	.	330.01	0.0000
18	0.0481	.	331.76	0.0000
19	0.1387	.	346.35	0.0000
20	0.0384	.	347.47	0.0000
21	0.1702	.	369.5	0.0000
22	0.1298	.	382.34	0.0000
23	0.1168	.	392.75	0.0000
24	0.0697	.	396.46	0.0000
25	0.0429	.	397.87	0.0000
26	0.1010	.	405.68	0.0000
27	0.1301	.	418.67	0.0000
28	0.1321	.	432.07	0.0000
29	0.1254	.	444.16	0.0000
30	0.0638	.	447.29	0.0000

Table A.11: Correlogram for S&P500's Squared Values

Lag	**AC**	**PAC**	Q	Prob $> Q$
1	0.2428	0.2769	43.63	0.0000
2	0.2125	0.4435	77.078	0.0000
3	0.0284	0.0474	77.675	0.0000
4	0.0301	.	78.348	0.0000
5	0.1916	.	105.68	0.0000
6	0.1869	.	131.69	0.0000
7	0.3202	.	208.18	0.0000
8	0.2184	.	243.82	0.0000
9	0.1765	.	267.12	0.0000
10	0.0685	.	270.63	0.0000
11	0.0224	.	271.01	0.0000
12	0.0919	.	277.36	0.0000
13	0.1432	.	292.79	0.0000
14	0.3191	.	369.48	0.0000
15	0.1144	.	379.35	0.0000
16	0.1308	.	392.28	0.0000
17	0.0403	.	393.51	0.0000
18	0.0203	.	393.83	0.0000
19	0.1607	.	413.42	0.0000
20	0.0035	.	413.43	0.0000
21	0.1199	.	424.36	0.0000
22	0.1015	.	432.2	0.0000
23	0.0808	.	437.18	0.0000
24	0.0645	.	440.36	0.0000
25	0.0278	.	440.95	0.0000
26	0.0789	.	445.73	0.0000
27	0.0991	.	453.26	0.0000
28	0.1324	.	466.72	0.0000
29	0.1467	.	483.28	0.0000
30	0.0511	.	485.29	0.0000

Table A.12: Correlogram for BTC's Absolute Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.3408	0.3422	155.47	0.0000
2	0.3251	0.2417	297.06	0.0000
3	0.2759	0.1340	399.14	0.0000
4	0.2137	0.0461	460.44	0.0000
5	0.2267	0.0880	529.48	0.0000
6	0.1804	0.0347	573.25	0.0000
7	0.2044	0.0749	629.45	0.0000
8	0.1494	-0.0038	659.48	0.0000
9	0.1914	0.0799	708.85	0.0000
10	0.1448	0.0037	737.13	0.0000
11	0.1779	0.0663	779.84	0.0000
12	0.1455	0.0152	808.42	0.0000
13	0.1370	0.0041	833.77	0.0000
14	0.1825	0.0635	878.83	0.0000
15	0.1619	0.0290	914.29	0.0000
16	0.1645	0.0212	950.93	0.0000
17	0.1814	0.0514	995.55	0.0000
18	0.1292	-0.0521	1018.2	0.0000
19	0.1490	0.0142	1048.3	0.0000
20	0.1773	0.0699	1091	0.0000
21	0.1361	0.0041	1116.2	0.0000
22	0.1450	0.0095	1144.8	0.0000
23	0.1115	-0.0259	1161.7	0.0000
24	0.0826	-0.0271	1171	0.0000
25	0.0703	-0.0301	1177.8	0.0000
26	0.0887	0.0118	1188.5	0.0000
27	0.1113	0.0184	1205.4	0.0000
28	0.1630	0.0619	1241.7	0.0000
29	0.1248	0.0095	1263	0.0000
30	0.1134	-0.0143	1280.6	0.0000

Table A.13: Correlogram for BTC's Squared Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.1465	0.1467	28.727	0.0000
2	0.2936	0.2813	144.24	0.0000
3	0.2070	0.1519	201.71	0.0000
4	0.1188	0.0071	220.66	0.0000
5	0.1578	0.0585	254.08	0.0000
6	0.0601	-0.0190	258.95	0.0000
7	0.1325	0.0642	282.56	0.0000
8	0.0440	-0.0150	285.17	0.0000
9	0.1222	0.0701	305.27	0.0000
10	0.0425	-0.0150	307.71	0.0000
11	0.0704	0.0154	314.39	0.0000
12	0.0541	0.0058	318.34	0.0000
13	0.0445	0.0116	321.02	0.0000
14	0.0597	0.0095	325.85	0.0000
15	0.0460	0.0115	328.71	0.0000
16	0.0637	0.0091	334.2	0.0000
17	0.0583	0.0260	338.82	0.0000
18	0.0429	-0.0241	341.31	0.0000
19	0.1123	0.0642	358.43	0.0000
20	0.0931	0.0683	370.19	0.0000
21	0.0707	0.0149	376.98	0.0000
22	0.1183	0.0582	396.02	0.0000
23	0.0465	-0.0219	398.96	0.0000
24	0.0469	-0.0356	401.95	0.0000
25	0.0204	-0.0415	402.52	0.0000
26	0.0123	-0.0227	402.73	0.0000
27	0.0305	-0.0049	404	0.0000
28	0.0797	0.0419	412.68	0.0000
29	0.0346	0.0024	414.32	0.0000
30	0.0481	-0.0052	417.48	0.0000

Table A.14: Correlogram for XRP's Absolute Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.3607	0.3669	59.194	0.0000
2	0.0706	-0.0460	61.466	0.0000
3	0.0871	0.0867	64.93	0.0000
4	0.1060	0.0575	70.081	0.0000
5	0.0310	-0.0115	70.521	0.0000
6	0.0643	0.0560	72.422	0.0000
7	0.0757	0.0169	75.065	0.0000
8	0.1216	0.0258	81.903	0.0000
9	0.0806	-0.0292	84.913	0.0000
10	0.1107	0.0823	90.604	0.0000
11	0.1026	0.0348	95.501	0.0000
12	0.1375	0.1075	104.32	0.0000
13	0.0403	-0.0529	105.08	0.0000
14	0.0727	0.0307	107.55	0.0000
15	0.0442	-0.0252	108.47	0.0000
16	0.0795	0.0837	111.45	0.0000
17	0.0340	-0.0033	111.99	0.0000
18	0.0738	0.0723	114.57	0.0000
19	0.0176	-0.0521	114.72	0.0000
20	0.0433	0.0162	115.61	0.0000
21	0.0496	0.0297	116.78	0.0000
22	0.0937	0.0047	120.97	0.0000
23	0.1143	0.0448	127.22	0.0000
24	0.1140	0.0243	133.45	0.0000
25	0.0303	0.0121	133.89	0.0000
26	0.0452	0.0356	134.87	0.0000
27	0.0359	0.0398	135.49	0.0000
28	0.0115	-0.0297	135.56	0.0000
29	-0.0121	-0.0288	135.63	0.0000
30	0.0100	-0.0211	135.68	0.0000

Table A.15: Correlogram for XRP's Squared Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.4221	0.4234	81.077	0.0000
2	-0.0025	-0.1753	81.08	0.0000
3	0.0257	0.1179	81.383	0.0000
4	0.0501	-0.0142	82.532	0.0000
5	0.0182	0.0162	82.684	0.0000
6	0.0161	0.0057	82.803	0.0000
7	0.0142	-0.0005	82.896	0.0000
8	0.0841	-0.0018	86.166	0.0000
9	0.0630	-0.0314	88.006	0.0000
10	0.0244	0.0251	88.284	0.0000
11	0.0568	0.0607	89.783	0.0000
12	0.0549	0.0267	91.191	0.0000
13	-0.0078	-0.0246	91.219	0.0000
14	0.0038	0.0093	91.226	0.0000
15	0.0217	0.0180	91.447	0.0000
16	0.0328	0.0324	91.953	0.0000
17	-0.0054	-0.0221	91.967	0.0000
18	0.0251	0.0394	92.266	0.0000
19	0.0033	-0.0325	92.271	0.0000
20	0.0015	0.0096	92.272	0.0000
21	0.0140	0.0153	92.365	0.0000
22	0.0429	0.0009	93.245	0.0000
23	0.0629	0.0242	95.138	0.0000
24	0.0700	0.0445	97.489	0.0000
25	0.0137	0.0020	97.579	0.0000
26	0.0088	0.0202	97.617	0.0000
27	0.0028	0.0036	97.62	0.0000
28	-0.0173	-0.0232	97.765	0.0000
29	-0.0233	-0.0199	98.027	0.0000
30	-0.0073	0.0042	98.053	0.0000

Table A.16: Correlogram for LTC's Absolute Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.4216	0.4302	81.953	0.0000
2	0.1037	-0.0879	86.919	0.0000
3	0.0796	0.1029	89.856	0.0000
4	0.0563	-0.0064	91.329	0.0000
5	0.1044	0.1321	96.397	0.0000
6	0.0731	-0.0345	98.89	0.0000
7	0.1212	0.1605	105.75	0.0000
8	0.1447	0.0588	115.55	0.0000
9	0.0920	0.0018	119.52	0.0000
10	0.0959	0.0235	123.84	0.0000
11	0.1019	0.0410	128.74	0.0000
12	0.1210	0.0764	135.65	0.0000
13	0.0817	0.0024	138.81	0.0000
14	0.0707	0.0541	141.18	0.0000
15	0.1172	0.0364	147.71	0.0000
16	0.0393	-0.0842	148.44	0.0000
17	-0.0042	0.0389	148.45	0.0000
18	0.0104	-0.0350	148.5	0.0000
19	0.0261	0.0024	148.83	0.0000
20	0.0406	0.0147	149.62	0.0000
21	0.0377	0.0184	150.31	0.0000
22	0.0024	-0.0368	150.31	0.0000
23	-0.0213	-0.0460	150.53	0.0000
24	0.0193	0.0365	150.71	0.0000
25	0.0122	-0.0460	150.79	0.0000
26	0.0617	0.0717	152.64	0.0000
27	0.0610	-0.0167	154.46	0.0000
28	-0.0032	-0.0099	154.47	0.0000
29	-0.0111	-0.0362	154.53	0.0000
30	-0.0366	-0.0234	155.19	0.0000

Table A.17: Correlogram for LTC's Squared Values

Lag	**AC**	**PAC**	Q	Prob > Q
1	0.4614	0.4644	98.15	0.0000
2	0.0377	-0.2298	98.806	0.0000
3	0.0288	0.1572	99.191	0.0000
4	0.0236	-0.0713	99.448	0.0000
5	0.0751	0.1582	102.07	0.0000
6	0.0367	-0.1083	102.7	0.0000
7	0.0377	0.1304	103.36	0.0000
8	0.0698	-0.0073	105.64	0.0000
9	0.0338	0.0010	106.18	0.0000
10	0.0589	0.0232	107.81	0.0000
11	0.0617	0.0177	109.61	0.0000
12	0.0486	0.0366	110.72	0.0000
13	0.0330	-0.0028	111.24	0.0000
14	0.0201	0.0333	111.43	0.0000
15	0.0812	0.0353	114.56	0.0000
16	0.0340	-0.0498	115.11	0.0000
17	-0.0218	0.0198	115.34	0.0000
18	-0.0087	-0.0405	115.38	0.0000
19	-0.0044	0.0322	115.39	0.0000
20	0.0060	-0.0209	115.4	0.0000
21	-0.0068	-0.0089	115.43	0.0000
22	-0.0252	-0.0135	115.73	0.0000
23	-0.0183	-0.0172	115.9	0.0000
24	-0.0074	0.0008	115.92	0.0000
25	-0.0020	-0.0136	115.92	0.0000
26	0.0200	0.0038	116.12	0.0000
27	0.0044	-0.0150	116.13	0.0000
28	-0.0192	-0.0166	116.31	0.0000
29	-0.0204	-0.0117	116.51	0.0000
30	-0.0205	-0.0141	116.72	0.0000

Table A.18: VAR for BTC

Variable	Coefficient	(Std. Err.)
Equation 1 : $\log \Delta \mathrm{V}(t)$		
1^{st} Lag of $\log \Delta \mathrm{V}(t)$	-0.1473405*	(0.0722836)
2^{nd} Lag of $\log \Delta \mathrm{V}(t)$	0.1877619**	(0.0724446)
1^{st} Lag of $\lvert\log \mathrm{R}(t)\rvert$	0.7164479**	(0.1694261)
2^{nd} Lag of $\lvert\log \mathrm{R}(t)\rvert$	0.0487642	(0.1770715)
Intercept	-0.0256593**	(0.0087954)
Equation 2 : $\lvert\log \mathrm{R}(t)\rvert$		
1^{st} Lag of $\log \Delta \mathrm{V}(t)$	-0.1053222**	(0.0305879)
2^{nd} Lag of $\log \Delta \mathrm{V}(t)$	0.0942769**	(0.030656)
1^{st} Lag of $\lvert\log \mathrm{R}(t)\rvert$	0.3783418**	(0.0716951)
2^{nd} Lag of $\lvert\log \mathrm{R}(t)\rvert$	0.1666404*	(0.0749304)
Intercept	0.0144897**	(0.0037219)
N	179	
Log-likelihood	542.7405	

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

Table A.19: VAR for LTC

Variable	Coefficient	(Std. Err.)
Equation 1 : $\log \Delta$ Volume		
1st Lag of $\log \Delta \mathrm{V(t)}$	0.0520198	(0.0728686)
2nd Lag of $\log \Delta \mathrm{V(t)}$	-0.0664955	(0.0724421)
1st Lag of $\lvert \log \mathrm{R(t)} \rvert$	-0.0260652	(0.0897247)
2nd Lag of $\lvert \log \mathrm{R(t)} \rvert$	0.2788267**	(0.0895332)
Intercept	-0.0180667	(0.0141913)
Equation 2 : $\lvert \log \mathrm{R(t)} \rvert$		
1st Lag of $\log \Delta \mathrm{V(t)}$	0.036742	(0.0587468)
2nd Lag of $\log \Delta \mathrm{V(t)}$	0.1367027*	(0.058403)
1st Lag of $\lvert \log \mathrm{R(t)} \rvert$	0.0413287	(0.0723363)
2nd Lag of $\lvert \log \mathrm{R(t)} \rvert$	0.1458086*	(0.0721818)
Intercept	0.0645154**	(0.0114411)
N	190	
Log-likelihood	247.3436	

Significance levels : † : 10% * : 5% ** : 1%

Table A.20: VAR for XRP

Variable	Coefficient	(Std. Err.)
Equation 1 : $\log \Delta$ Volume		
1st Lag of $\log \Delta$ Volume	0.4600027	(0.6246843)
2nd Lag of $\log \Delta$ Volume	-0.3286515	(0.6358771)
1st Lag of $\|\log \mathrm{R(t)}\|$	0.4982969	(0.8579283)
2nd Lag of $\|\log \mathrm{R(t)}\|$	0.491426	(0.8413331)
Intercept	-0.2217062	(0.1801991)
Equation 2 : $\|\log \mathrm{R(t)}\|$		
1st Lag of $\log \Delta \mathrm{V(t)}$	0.1319151*	(0.0549993)
2nd Lag of $\log \Delta \mathrm{V(t)}$	0.0273143	(0.0559848)
1st Lag of $\|\log \mathrm{R(t)}\|$	0.0092369	(0.0755349)
2nd Lag of $\|\log \mathrm{R(t)}\|$	0.1302045†	(0.0740738)
Intercept	0.0963356**	(0.0158653)
N	172	
Log-likelihood	-214.011	

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

Appendix B

Figures

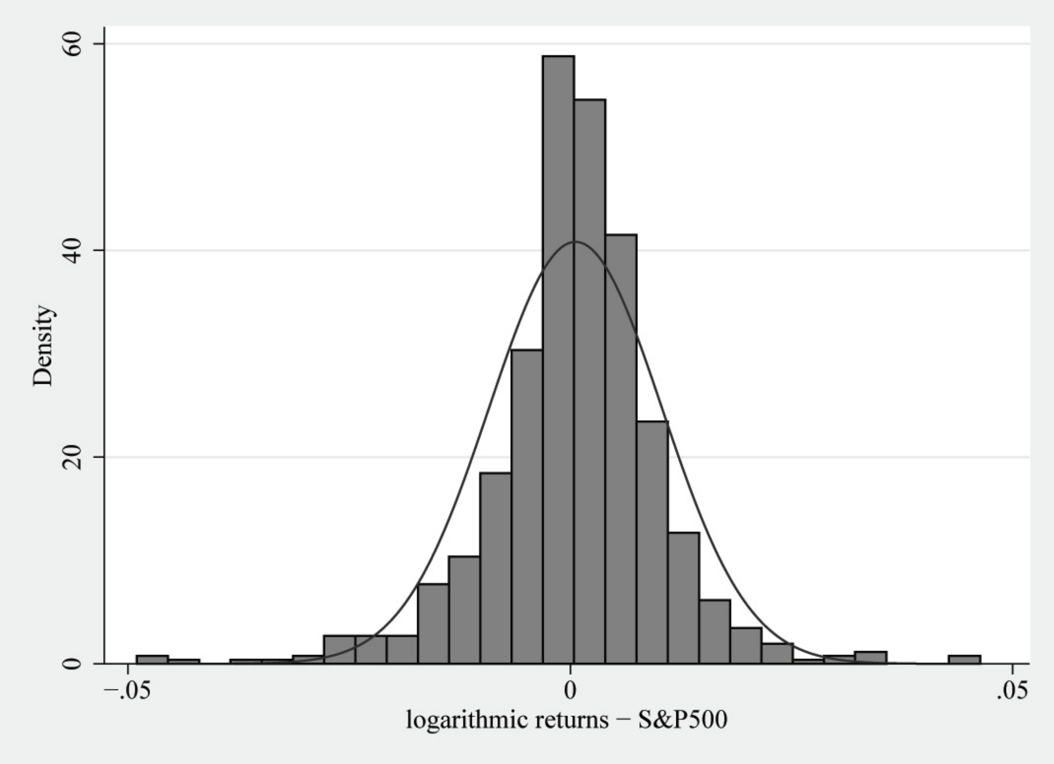

Figure B.1: Histogram for S&P500

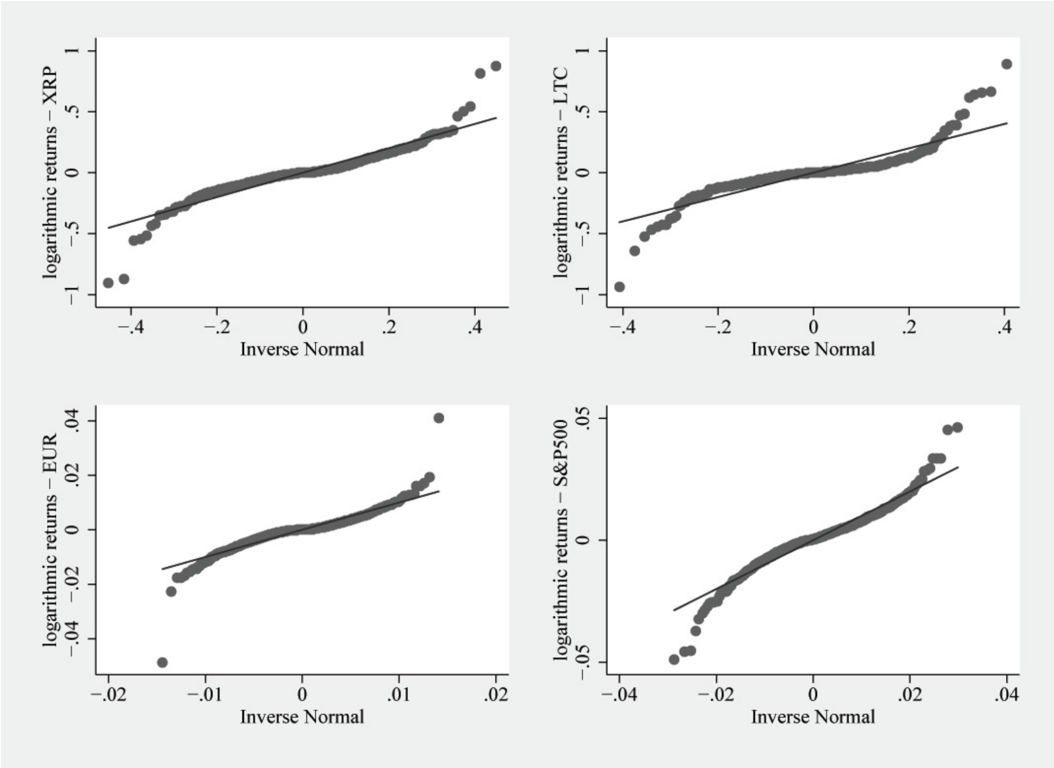

Figure B.2: Quantile-normal plot for XRP, LTC, EUR and S&P500

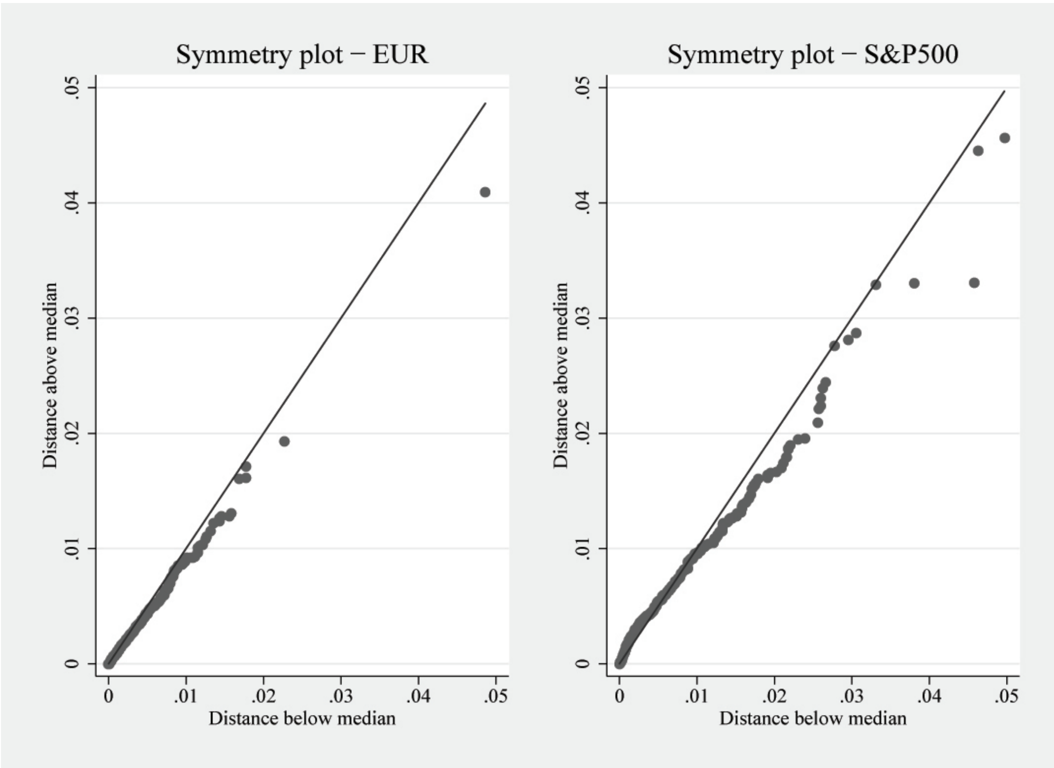

Figure B.3: Symmetry plot for EUR and S&P500

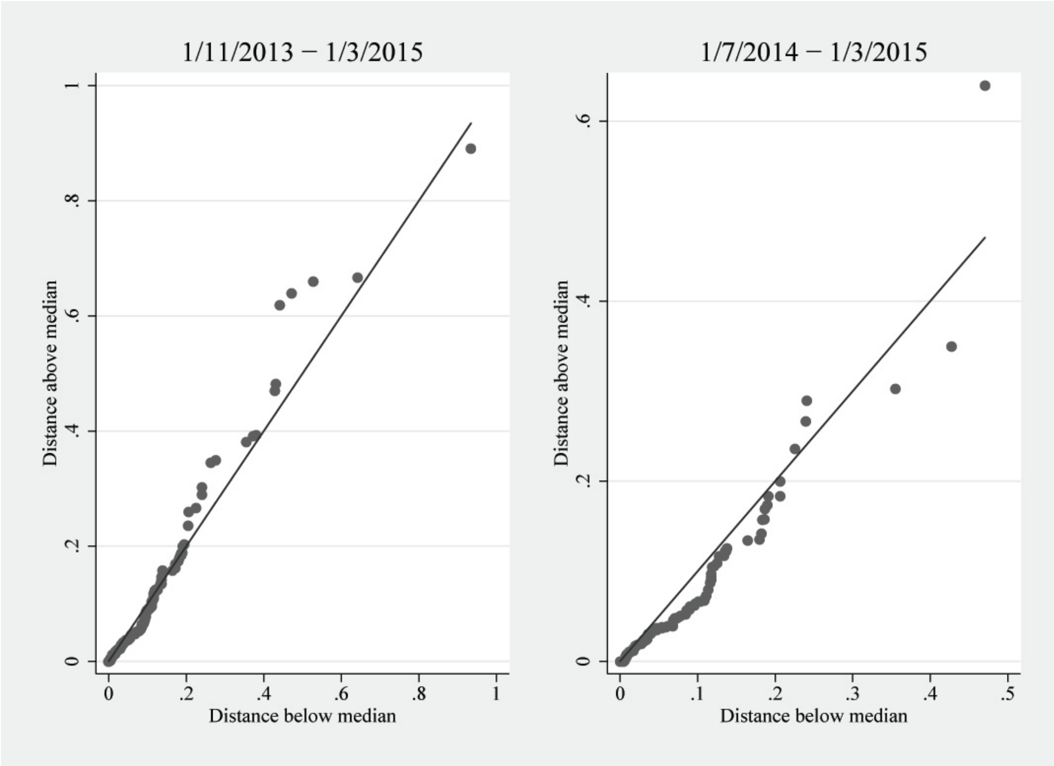

Figure B.4: Symmetry plot for LTC

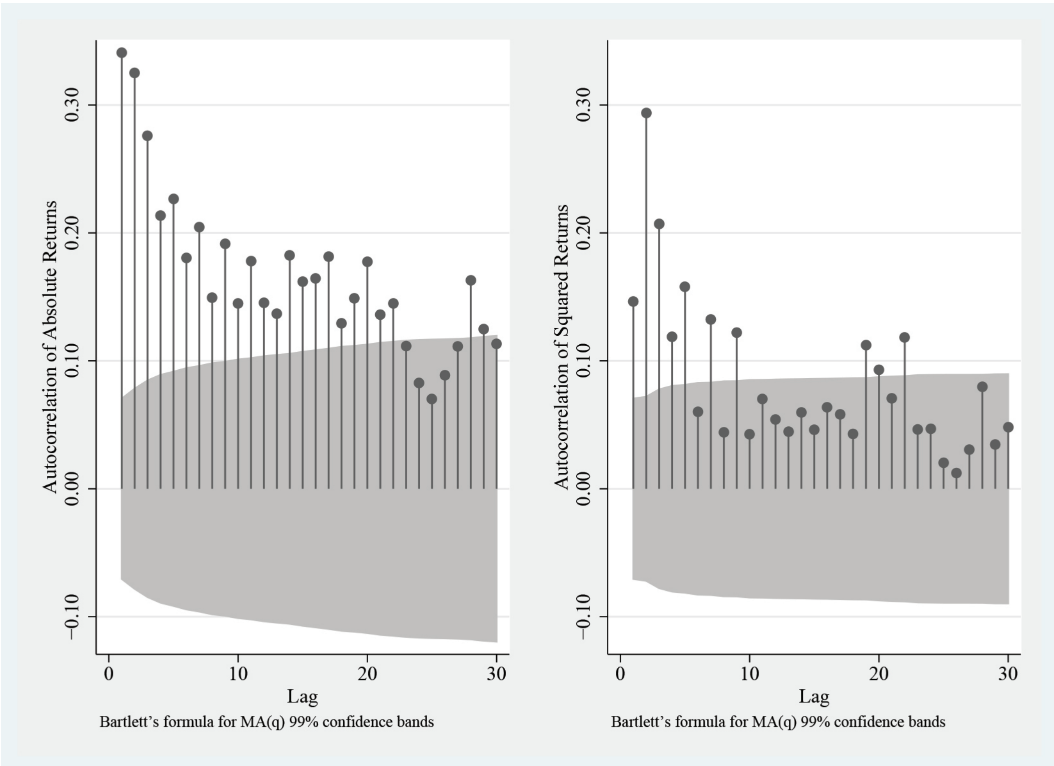

Figure B.5: Autocorrelations of Absolute and Squared Returns for BTC

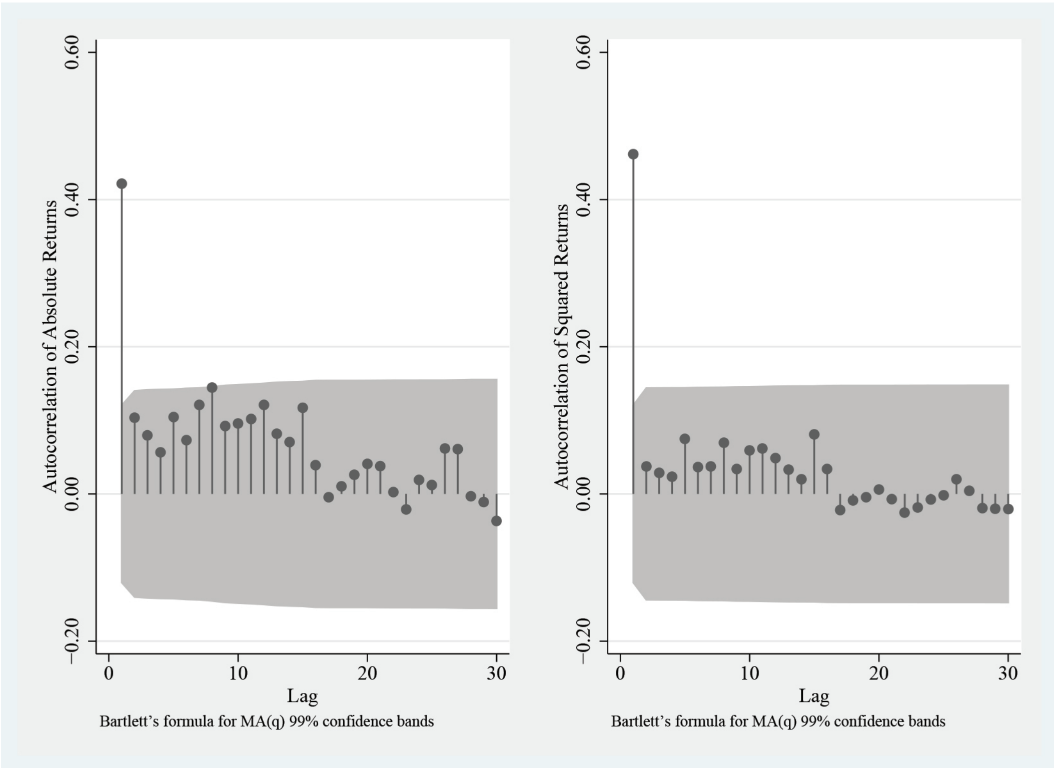

Figure B.6: Autocorrelations of Absolute and Squared Returns for LTC

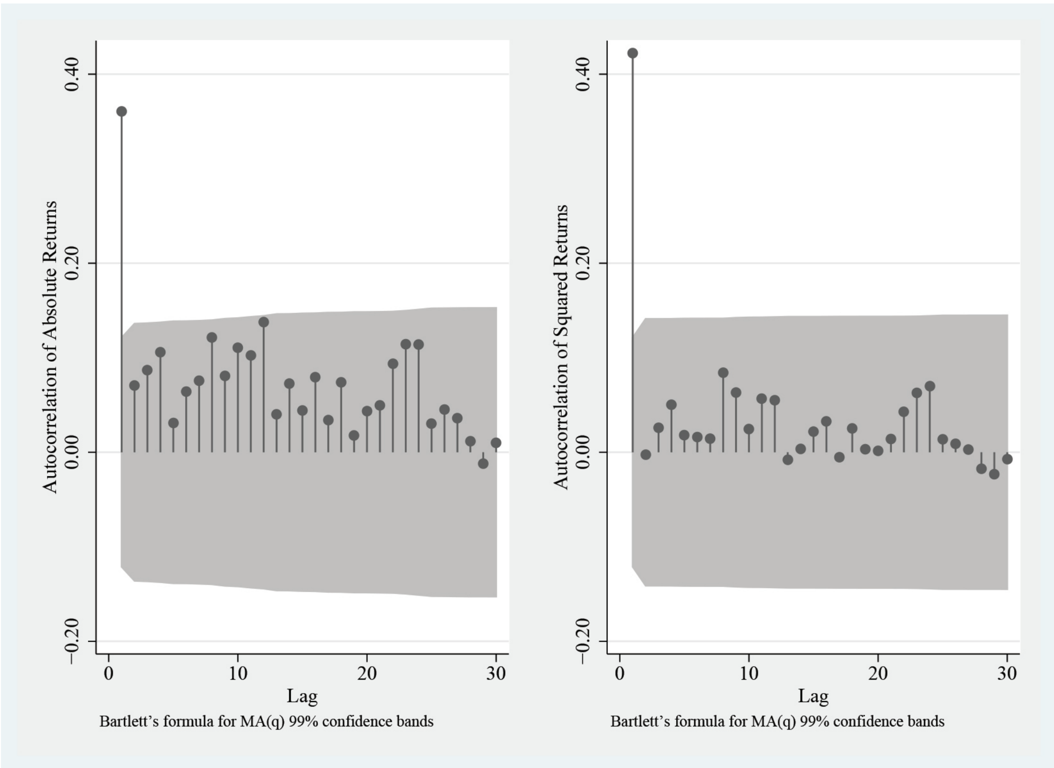

Figure B.7: Autocorrelations of Absolute and Squared Returns for XRP