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The Ph.D. thesis of Gabriel Pathó named *Mathematical modeling of thin films of martensitic materials* consists of five chapters. A thorough introduction describes shape memory alloys (SMAs) and explains mechanisms responsible for their unique behavior. In particular, it addresses various modeling approaches available in the literature. Depending on the scale, a different levels of complexity are considered. This is exploited in Chapter 2 where a mesoscopic thermomechanical model of SMAs is derived and analyzed. The starting point is a microscopic bulk model. Then a dimension-reduction procedure brings us to a microscopic thin film model. This one is then upscaled (for vanishing interfacial energy) to a mesoscopic level which is more suitable for computational experiments. These are then performed in Chapter 3 in isothermal situations. Here, Young measures are used to capture main features as, e.g., conditions on compatibility of austenitic/martensitic variants of the material.

These considerations brought us to the question how to ensure invertibility of elastic deformations in a framework of relaxation. As a starting point he considered a model of a hyperelastic material whose energy density depends, besides the deformation gradient, also on its inverse. First, an explicit characterization of Young measures generated by matrix-valued L^p maps such that inverted matrices are also bounded in the L^p space is given in general dimensions. The gradient case is, inevitably, much more involved. The main problem is that standard available techniques might destroy invertibility of gradients. Instead, theory of partial differential inclusions is used to overcome this difficulty. Altogether, the main result of Chapter 4 characterizes gradient Young measures generated by gradients of Lipschitz maps such that inverted gradients are also bounded in the L^∞ norm. This result has enjoyed international feedback and has called attention of various groups to this problem. For example, Koumatos, Rindler, and Wiedemann then characterized gradient Young measures generated for maps with positive Jacobians in the case p not greater than the dimension. Conti and Dolzmann resolved the relaxation problem for orientation-preserving maps, however, severe constraints on the resulting relaxed energy density must be employed. The results in Chapter 4 are, in my best knowledge, first attempts to characterize Young measures under differential/algebraic constraints.

The involvement of Gabriel in my international project on weak lower semicontinuity of integral functionals under differential constraints resulted in Chapter 5 of the thesis. Conditions are isolated which are necessary and sufficient conditions ensuring sequential weak lower semicontinuity of functionals along sequences satisfying a first-order linear partial differential constraint. The main novelty is that we allow for integrands whose negative and positive part have the same growth. Besides $\mathcal{A}$-quasiconvexity, it involves new conditions on the boundary of the domain. The results strongly depend on existence/nonexistence of suitable extensions and the chapter formulates also a few open questions from the theory of function spaces, for example. In the case of the curl operator and a smooth domain, the new condition coincides with the so-called quasiconvexity at the boundary due to Ball and Marsden.

Altogether, in my opinion, the thesis of Gabriel brings many new and interesting results, most of them are already published in international and impacted journals. During all years of his PhD studies, Gabriel worked hard and came with many ideas which turned out to be fundamental for the reached results. It has always been joy to work with Gabriel and he has proven that he is able of independent scientific work. I strongly recommend to award the Ph.D. degree to him.

Martin Kružík (Advisor)