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Report on the PhD thesis:

Weighted rearrangement-invariant spaces and their basic properties

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The main objective of this PhD dissertation is to study functional properties of weighted rearrangement-invariant spaces and to present some applications.

Some of the techniques of proof used have been developed in recent years, but some new techniques are also introduced. Moreover, previous open questions concerning the dual space of generalized weighted Lorentz spaces have been answered. Old results are generalized and improved. Applications are pointed out.

The thesis is written as a collection of research papers. It consists of an abstract, a self-contained Introduction, a Summary of papers attached to the thesis, five research papers and a Bibliography. One of the papers is accepted for publication in a highly ranked international mathematical journal and the two other are submitted for publication. The PhD candidate is coauthor of three of these papers.

The Introduction contains a summary of the papers, the background and motivation of the studied problems, insight in previous research connected with the problems and many historical remarks about the Hardy inequality, maximal operator, Lorentz and Sobolev spaces.

The first article is written in collaboration with two co-authors and it is published in the prestigious journal Studia Matematica. The main objective of the paper is to characterize the associate space of a very general, two-parameter weighted Lorentz space. This generalized Lorentz space contains, among other, the classical Gamma-spaces and small Lebesgue spaces, which are of big importance in mathematical analysis. As application, there are given necessary and sufficient conditions for an embedding between some general Sobolev spaces (of arbitrary order) into L^∞. It is known that Sobolev spaces are crucial in the theory of differential equations, and hence to study their properties is very important. Some functional properties of the generalized Lorentz spaces or of their associate spaces are also studied.

The second paper is written together with one coauthor. For a space with a general lattice-like structure, the authors give a sufficient condition for its associate space to be a Banach function space. The result is used then to give an alternative proof of Sawyer's celebrated characterization

of normability of a classical Lorentz space of type Lambda. This general method is also used for the characterization of normability in the weak case. Finally, the authors characterize the linearity of the weak-type Lorentz spaces of the type Gamma.

In the **third paper**, necessary and sufficient conditions of linearity and quasi-norm property via dilation operator on the representation space of rearrangement-invariant lattices are given. The results are applied to Orlicz-Lorentz lattices. The candidate is the sole author of this paper.

The **fourth paper** consists of two, different characterizations of the weights for which the classical weighted Lorentz spaces of type Gamma are normable, in the case $0<p<1$. The result is not new but the proof is more transparent than the previous one. The technique is based on discretization arguments. The candidate is the sole author of this paper.

Embeddings between two classical Lorentz spaces of type Gamma defined with respect to two different weighted means are characterized in **the fifth paper**. In particular, the authors give two-sided estimates of the optimal constant in the inequality between two such norms. The new part consists of the fact that different inner weights are allowed. Proofs are based on a combination of duality techniques with various kinds of weighted inequalities for iterated operators. Some of these inequalities were known and others are proved here. All but one possible cases are solved. The results of this article may be regarded as generalizations of older results.

The scientific standard of the thesis, in international perspective, is high. The techniques used to prove the results are deep. The Bibliography is complete. All papers of the thesis are well-written and I could find only few insignificant misprints in the Summary.The candidate has a very good perspective on the research area. He has demonstrated a very good knowledge of previous results both from technical and historical point of view. He has also the ability to view his own research in a greater context by pointing out important open problems and applications. The applications are mainly in theory of function spaces, a branch of functional analysis but also in the theory of differential equations.

My conclusion is that the candidate has brought important contributions to the research area. The articles are well-written and complete. Some results are completely new whereas others are improvements of older results. More transparent alternative proofs of classical results are also given. The author has shown abilities to create original scientific work of good quality. The thesis has a very good international level and I findthat it is well-worth of being defended for the PhD degree.

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