

EXTERNAL REFEREE REPORT

Name of the referee: Javier Soria
Category: Full Professor
Department, University to which s/he belongs: Department of Applied Mathematics and Analysis, University of Barcelona (Spain)

Title of the thesis: Weighted rearrangement-invariant spaces and their basic properties
Name of the student presenting the thesis: Filip Soudský

Specify reasons endorsing the quality of the above-mentioned thesis for its public reading:

This monograph achieves a far-reaching contribution in the study of classical functional properties of well-known rearrangement invariant (r.i.) spaces. These spaces are of great interest in Analysis, in particular in areas like Interpolation Theory, optimal Sobolev embeddings, or the boundedness of Hardy-type operators.

In recent years, starting with the groundbreaking works of Ariño and Muckenhoupt [1] and Sawyer [31] (references correspond to the Bibliography in the monograph), different aspects (for example, weighted norm inequalities, Köthe duality and associate spaces, or normability properties), of the so-called Lorentz spaces (probably the most studied among r.i. spaces, introduced in [23]) are being deeply considered by many authors in high level publications (see, e.g., [3,4,6,7,8,9,10,15,29,32,34]), which is a clear indication of the rich activity and interest of the questions resolved in this thesis.

The results proved by Filip are non-trivial in all of these topics and show that he has really been able to comprehend the main ideas behind this theory. It is also clear that he can masterfully deal with all the Real and Functional Analysis techniques needed to successfully undertake his project.

The thesis is clearly written, with a thorough Introduction and a detailed description of the main results on each of the five papers prepared by the candidate.

What objectives have been achieved with the thesis?

The main achievements of this work can be summarized as follows:

- Paper 1: The authors give a complete characterization of the associate space for the *Generalized Gamma Space* (Theorem 1.1). They also describe when it is a Banach function space, with an absolutely continuous norm, and whether it is reflexive (Theorems 1.3, 1.5, 1.6, and 1.7), thus extending the main results in [12]. Finally, they consider the higher-order Sobolev embeddings into the space of bounded functions (Theorem 1.4).
- Paper 2: In this paper the authors present an alternative approach to the normability problem, using duality methods based on properties of associate spaces on rather general structures (Theorem 3.1). As an application they obtain new results for the

weighted Lorentz spaces (Theorems 4.1 and 5.1), recovering some of the main results in [6,32], as well as the characterization of the linearity property for the end-point case $p = \infty$ (Theorem 5.2).

- Paper 3: For some rearrangement-invariant functional having the lattice property, the author gives a characterization of the linearity of the corresponding set where it is finite (Theorem 2.3). He then applies this argument to the case of Orlicz-Lorentz spaces (Theorem 3.3), thus extending the main result in [4].
- Paper 4: This short paper describes the normability property for the Gamma spaces, in the non-standard case $0 < p < 1$ (Theorem 1). This was already proved in [20], using a different argument.
- Paper 5: The authors characterize, under some restriction on the indices, embeddings between two classical Lorentz spaces of type Gamma defined with respect to two different weighted means (Theorem 1.1). For this purpose they develop an approach consisting of a duality argument combined with estimates of optimal constants in inequalities involving iterated operators of either integral or supremal type.

Originality of the work:

The main results proved in theses manuscripts, and the techniques used for such purposes, enjoy a high degree of originality and deepness.

Absolute and/or relative assessment of the thesis in comparison with other works of research:

In comparison with other theses I have evaluated, this monograph is at the top level.

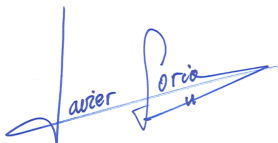
In consideration of all the above, is the thesis judged to be suitable for public reading?

Yes No

Observations:

My only observation is that some of the papers are in collaboration with other authors besides his thesis' advisor (in paper 5 even with another graduate student), which makes things a bit difficult to evaluate his own independent work. However, since Filip did in 2014 a one month research stay at my Department, during this period I had the opportunity to learn first hand his abilities as a well prepared graduate student and a clever mathematician.

Signature and date

A handwritten signature in blue ink that reads "Javier Soria". The signature is stylized with a long horizontal stroke at the end.

Javier Soria

Barcelona, August 5, 2015