Referee's report on the Ph.D. thesis of Tomáš Gavenčiak

Structural and complexity questions of graph theory

(MFF UK Praha)

Referee: **prof. RNDr. Petr Hliněný, Ph.D.** May 16, 2016

The submitted thesis deals with a so called *cops and robber game* search on graphs and hypergraphs. Although this looks like a very uniform topic, actually there exist two deeply different research directions about this game: the (former) one assuming both the robber and the cops move on the graph, and the (newer) one allowing the robber to move infinitely fast and the cops "flying" above the graph. This thesis contains interesting and significant new results in both of these directions.

Thesis content. In the first part the author studies the former game on certain classes of intersection graphs related to string graphs. The new results have the form of proving general upper bounds on the cop number for the considered classes – these are mathematical results. In the second part, while still studying the former game (with a slight generalization to an ∞-fast robber), the results turn algorithmic. The main outcome is a cubic-time algorithm determining the cop number on interval graphs. This is quite nontrivial given that for slight generalizations of interval graphs the same problem is already known to be hard. These two parts I find as the strong points of the thesis work (modulo some minor remarks below).

The third part studies the newer form of the game. It is inspired by the natural game-characterization of *tree-depth* and aims to extend this to hypergraphs. The author gives a basic definition of hypertree-depth which is naturally related to a *marshals and robber game* (where marshals present a natural counterpart of cops in hypergraphs – the difference being that marshals occupy edges instead of vertices). The related definitions are then generalized from hypergraphs to hypergraph pairs, and the main new contributions of this part concern this generalization and mimic the several known characterizations of tree-depth. To my taste, these generalizations are very artificial, and the new contribution in this regard is mainly in solving associated technical problems. Though, the concept of hypergraph minor may, perhaps, find wider applications in the future.

Remarks. While the core of the thesis consists of already refereed papers, and so all the important outcomes are mathematically correct and sound, at some places the text suffers from not-so-good unification of the texts of different papers. A prime example is that in several proofs the words "Proof of. . . " are missing (a TEX-related problem, e.g., page 35,39). At several places (e.g., page 12, 51), the author assumes that the ends of intervals are integers $1, \dots, 2|V|$, but I cannot find any justification for this assumption in your text. Also, the very special notation defined in 1.1.1 and used only much later from page 65 should be recalled since the reader cannot remember it for so long.

There are also some local problems with math arguments, such as in Section 2.7. Do you mean *orientable* surfaces? You should define a non-contractible closed curve. While I would be happy if you just said *non-contractible simple loop*, the subsequent arguments show that you allow self-intersecting curves and even curves which traverse the same point set there and back. With such a general treatment, a proper formal definition is surely needed. The same concerns the concept of *imitating a non-contractible curve.* In the proof of Lemma 34, you do not specify where π_1, π_2 have their ends, which is not trivial. Consequently, it is certainly not trivial to "observe that $\pi_1 - \pi_3$ is imitated by...". In Lemma 35, you should precisely define "shortest W".

Section 3.4.2 is nearly unreadable since it lacks a proper formal definition. The concept seems quite complex, and just a loose description is not enough. I also have some problems following the many technical conditions and concepts of Chapter 4 and, as mentioned above, I find part of them as too artificial and not well motivated in a broader sense.

Some further very minor remarks follow:

- Page 7; I would prefer to see an undirected edge as $\{u, v\}$ instead of (u, v).
- Page 7; I would say that, in CS, trees "grow down", not "up" as in your explanation.
- Page 10; the definition of planar graphs is very imprecise!
- Page 11; "firs".
- Page 16; directed tree-width does not have a game characterization, it is just a loose relation.
- Page 30; you cannot protect the closed neighbourhood in "general graphs", only in string graphs.
- Page 43; graph parameters instead of "game parameters".
- Page 52; "Maneuvers".

Conclusion. The submitted thesis contains results from 5 original research papers, three of which have been published at international refereed conferences (two at an A-level conference ISAAC), one in an international journal (TCS) and another one submitted to a top-level international journal (JGT). The author declares his contribution to these papers as "significant", and the referee agrees with this self-assessment. Furthermore, the student has another three research papers on different topics in the DBLP database.

The text of the submitted thesis is, overall, written in a proper formal mathematical language, and in good English. The author has shown good research skills and proficiency in writing scientific text.

While I have expressed some minor reservations about parts of this submitted thesis, altogether, I can confirm that Tomáš Gavenčiak is capable of an independent research work and fulfills all the standard requirements on doctoral theses in Computer science. I am happy to recommend his work to be **accepted as doctoral thesis** at the Faculty of Mathematics and Physics of Charles University in Prague, and Tomáš Gavenčiak to be **awarded the Ph.D. title**.

In Brno, May 16, 2016,

Petr Hliněný