Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

BACHELOR THESIS

Spatial agent-based models of common pool resources

Author: **Dominik Vach**

Supervisor: **PhDr. Martin Gregor, Ph.D.**

Academic Year: **2015/2016**

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 13, 2016

Signature

Acknowledgments

The author is grateful especially to his supervisor, PhDr. Martin Gregor Ph.D., for his valuable comments, good advice, and insightful ideas which were very helpful in writing the thesis. The author is very thankful also to his family, who supported him throughout his university studies for the past three years.

Abstract

This thesis examines the application of the spatial aspect applied in the competitive models in the context of the natural resource economics. At first, the spatial models are thoroughly derived in one dimension. Then also their general properties such as the choice of the agents' location or their payoff function are examined. These properties are investigated for various distributions of the resource, and therefore they depend also on their parameters. The Nash equilibrium and local stability conditions are derived for the basic setups. In the second part, these competitive models are numerically tested also in a two-dimensional space. One of the results also suggests, that in the setup where the players have perfect information, the beginning player is not necessarily always better off than the second player. Throughout the entire thesis it is also comprehensively examined whether the existence of corners of the strategy space has an impact on the existence of the competition which was successfully demonstrated on several cases.

JEL Classification	Q20, Q22, C62, C63, C68, C72
Keywords	spatial models, natural resource exploitation, Nash equilibrium, fishery, computer simulations

Author's e-mail	`vach.dominik@gmail.com`
Supervisor's e-mail	`martin.gregor@fsv.cuni.cz`

Abstrakt

Tato práce pojednává převážně o aplikování prostorového aspektu v kompetitivních modelech v kontextu ekonomie přírodních zdrojů. Tyto modely jsou nejprve detailně odvozeny a jsou hledány jejich obecné vlastnosti pro různé distribuce jako je volba strategie jednotlivých hráčů, případně jejich výplatní funkce v závislosti na parametrech použitých distribucí. V druhé části práce jsou tyto kompetitivní modely testovány numerickými simulacemi v jednodimenzionálním prostoru, ale také v dvojdimenzionálním prostoru. V těchto simulacích je kromě hledání Nashovy rovnováhy zkoumán také koncept podmínek lokální stability. Jako jeden z výsledků simulací bylo mimo jiné také zjištěno, že ve hře, kde hráči disponují kompletní informací i zdroji, může být za určitých podmínek nevýhodné pro hráče táhnout jako první. V průběhu celé práce je také rozsáhle zkoumán vliv existence okrajů prostoru na rozhodování jednotlivých hráčů, kdy v některých případech tento efekt podnítil vznik soutěže, která by na otevřeném prostoru bez okrajů nevznikla.

Klasifikace JEL	Q20, Q22, C62, C63, C68, C72
Klíčová slova	prostorové modely, využití přírodních zdrojů, Nashova rovnováha, rybolov, počítačové simulace

E-mail autora	vach.dominik@gmail.com
E-mail vedoucího práce	martin.gregor@fsv.cuni.cz

Contents

List of Figures

Chapter 1

Introduction

Spatial distributed common pool resource can be understood as any resource distributed over a space shared by multiple agents who exploit it. Such a common pool resource can face the problems of limited usage in case of over-exploitation. The aim of this thesis is to analyze common pool stocks especially in the analogy to the fisheries economics and try to emphasize the importance of the spatial dimension in the analysis of the competitiveness of the individual agents.

The fisheries analogy was chosen especially due to the recent calls for the spatial extensions which was emphasized besides others for example by Behringer & Upmann (2014). The most intuitive application could be represented by the case where two agents (e.g. fishers) compete along the same space (e.g. lake) over the resource stock (e.g. fish) which is distributed spatially around the space.

In the first part of the thesis, the goal is to propose an innovative general description of a competition in the spatial models when the resource is exploited by two agents. This analysis should provide a simple overview of how the basic resource distributions differ and what effects can be observable for what types of distributions. The usage of the common pool stock is limited to its size and therefore the fundamental question which should be examined by this thesis is what the factors determining the competitiveness of the agents are in the environment, where the spatial dimension is regarded.

The emphasis is, therefore, put on the analysis of the resource distributed in one dimension along a line segment or a circle. These two shapes of space can help to analyze two different effects regarding the competition. In the first one the distance from some central point (e.g. harbour) is important whereas for the second space the intuition is more about competing between the players who have similar rather than different characteristics in terms of the distance from the central point.

After the derivation of the general equilibrium and stability conditions and an application of them on the selected resource stock distributions, the aim is to simulate some more realistic scenarios also numerically, which could be hard to examine analytically. Such scenarios may demand the numerical analysis for example due to involvement not only of one dimension but two dimensions. In such a two-dimensional spatial model the competition over a resource stock can be simulated numerically far more easily than analyzing it strictly mathematically.

The numerical simulations are aimed to be utilized also in case of simulating the role of the information in the scenarios where there is not perfect information, which is the second most important goal of this thesis. The imperfect information is hard to examine analytically with the usage of only mathematical tools as it usually requires a lot of iterations, therefore in this part the thesis would ideally use the numerical methods for an analysis of such scenarios with imperfect information. The fundamental question for the numerical simulations is to find what the role of the information about the distribution of the resource is and how it affects agents' payoffs in case of various distributions. The aim is also to compare what amount of information is how efficient for attaining of the specific amount of the payoff.

One of the aims of the numerical simulations is also the scenario where the players alternate under perfect information until they find the Nash equilibrium and analyze thus, what conditions must be met for both players in order to optimally maximize their payoff.

The thesis structure consists of the two main chapters called Theoretical part and Numerical simulations, in which the main results are contained, and three minor chapters which should help the reader to orientate, namely, Introduction, Literature review, and Summary and conclusion.

Chapter 2

Literature review

In this chapter, the aim is to explain what the reasons were for investigating the topic chosen in this thesis and how it is related to the literature that has been written on this matter. This chapter also should help the reader to orientate in the field of natural resource economics and especially in economics of fisheries with emphasis on the spatial aspect. This literature review should help as a guide in case of more interest in the areas covered by this work.

2.1 The historical roots of fisheries economics

Natural resource economics is a relatively well established interdisciplinary field of study which goal is to find the linkages between human economies and natural ecosystems. This thesis should provide a theoretical framework which finds its application especially in the case of fisheries, however, with possible extension also towards exploitation of other natural resources.

The most significant papers for the case regarding economics of fisheries are represented by the papers of Gordon (1954) and Scott (1955) which serve as base articles for the current analysis of the exploitation of the fishery stocks. Unlike the former author who focuses his work mainly on over-exploitation of the common pool fishery stocks, the latter uses more mathematical approach when applied it on modelling of the optimal resource management and thus paved way for many further research efforts. Basically, these two papers formed the background for a wide-ranging literature of natural resource economics that flourished especially in the 1970s and 1980s.

In contrast to the two papers mentioned which are predominantly theoretical, Deacon *et al.* (1998) are advocates of extensions of these models to more realistic usage. The problems which they deal with are, therefore, more based on what the fisheries managers or biologists are concerned with rather than to make the analysis very technical as did Scott (1955) and many other economists who chose to emphasize the optimization problem.

2.2 Integration of the spatial dimension concept

The most important and urgent extension Deacon *et al.* (1998) points on is the involvement of the spatial dimension in the fishery models. Nevertheless, not many papers are devoted to this kind of extension even though it is obviously extremely relevant in case of fisheries. Practically no evidence of this topic is being examined even in the recent textbooks related to the topic (e.g. Conrad (2010), R. Perman & McGilvary (2011)).

The vast majority of authors included the dynamical aspect in their works which was relatively exhaustively studied, however, they neglected also the importance of the spatial dimension. The dynamical aspect adds quite a labourious solution of differential equations to the problems solved whereas the spatial dimension added does not necessarily complicate the equations much. Even authors M. D. Smith & Wilen (2009) note that there is a long tradition of both aspects, however, they are too separated from each other and no authors combine them in the single model.

Historically, the spatial dimension has been examined already since the 19th century. For the first time it was in the model created by the famous farmer and amateur economist von Thünen in his work The Isolated State (1826) (Hall 1966) which is often referred to in land economics and economic geography. The more comprehensively studied spatial economics was also by Hotelling where he separated the dynamical and spatial part in his two seminal papers (Hotelling 1929) and (Hotelling 1931) .

The idea of Hotelling to solve classical monopolistic competition model with relevance to the location of the consumers by putting them on a line of fixed length was crucial for further development of this concept. The spatial competition introduced by Hotelling was extended by classic circle model by Salop (1979) who solved the problem of product location similarly as Hotelling, however, with emphasis on the neighbourhood of the particular consumers by distributing them on the perimeter of a circle. This idea was innovative in the sense of description of some situations which could not be described by the Hotelling model. Although both models have their applications usually in industrial organization where it may describe product differentiation, the fundamental ideas of these models to examine both a line segment and a circle space are handled in this thesis as a very important spatial issue.

2.3 Game theoretical approach

Along with the natural resource economics the fisheries were comprehensively examined also by game theory tools. This approach uses mathematical formalism to describe player strategies when there are any conflicts or common interests. Modern approach is usually attributed to Neumann & Morgenstern (1947) who laid its foundations and were followed by John Nash who worked on the non-cooperative (Nash 1951) and cooperative (Nash 1953) solutions.

In the fisheries, game theory has found its place in the two-player game analysis of sharing the fishery resource between two different coastal countries (Munro 1979). Munro's conclusion was that in the cooperative games of this kind players usually do not have the same preferences and therefore simplifies the joint management of a resource exploitation by introducing a concept of transferable utility between the cooperative players using so-called side payments.

Following the approach that was presented by Munro, a lot of further works related to fishery management occurred. These works usually utilized cooperative or competitive game theoretical framework where the positive effect of the cooperation of the players was consequently shown.

As the two player setups were examined, the condition for stability in such scenarios is given first by the concept of Pareto Optimality, which says that no player can achieve better payoff without decreasing the other cooperative player's payoff, and second by the concept of Individual Rationality Constraint, which says the players who are cooperating must have the same or higher payoff than in the case of no cooperation (Sumaila *et al.* 2010).

A very important conclusion for this thesis is made by Trisak (2005) who showed that the size of the common pool fish stock influences the decisions of the particular players whether to cooperate or not which will be analyzed later, however, in a spatial setting. Almost all early contributions were assuming two player game whereas in reality there can occur a more complex scenario. This problem was usually solved by aggregating the players into two groups or introducing the concept of the fishing coalitions (Kronbak & Lindroos 2007). A coalition framework helps to handle large number of cooperating players, however, in this thesis we will use rather two-player analysis and simulations as a trade-off in order to make it simpler both in analytical description and also in numerical simulations. This approach is chosen even though there are some notes that there might be limitations in the description of the real world scenarios when using only two-player game analysis. (Hannesson 1995)

The literature on fisheries regarding two player games utilize a single stage structure the most (i.e. both players make decisions at the beginning of the game, where they are aware of what the future stock will look like). But there are also papers where two stage or multiple stages are regarded (Hannesson 1995). In this thesis, more of these variants are simulated in the numerical part and it is applying them predominantly in competition cases without perfect information about the future stages where the players play sequentially. According to Sumaila *et al.* (2010) possible development in this field could be in use of game theory in a broader ecosystem-based context. Whilst the majority of works is relating to single fish stocks, this thesis aims to describe the fish stocks continuously and spatially in one-dimensional cases and also some simulations in two-dimensional context which could help to link the importance of spatial analysis with the utilization of the game theory framework in natural resource economics.

2.4 Recent literature

The lack of investigation of spatial aspect in the natural resource economics is according to Wilen (2007) contrasting with the very comprehensively exploited spatial dynamic systems in the hard sciences such as mathematics or physics. Recently there have been some efforts e.g.(Neubert & Herrera 2008) to employ the spatial factor by implementing the diffusion coefficient which enables fish to move from one place to another. Some other authors on the other hand link the agent's harvesting speed with the amount of resource that can be extracted which reduces the complexity of the analysis. This approach was chosen by Robinson et al. (2002) in a resource extraction (timber gathering) model. Last but not least, the spatial dimension was examined in this spirit also by A. O. Belyakov & Veliov (2013) and very similarly to them also by (Behringer & Upmann 2014) where both groups of authors used a model assuming that the agent (fisher) is moving along a circle where he is harvesting the resource (fish stocks).

The purpose of thesis is to continue in investigation of the spatial dimension from the very basic perspective of a two-player game where the dynamic aspect of the resource stock is not regarded. More emphasis is given on the analysis of the agents' behaviour if their information about the resource is not perfect and how this factor contributes to the optimal locations of agents. The spatial dimension is thoroughly examined in the theoretical chapter where one-dimensional spatial distribution of the resource was assumed, which could have intuitively a good real application to either a lake or a river fishing and also to other natural resource extraction. The role of information is due to its analytical difficulty simulated in the second part of the thesis. That part continues also in studying the problem of agents' choice of location from the theoretical part by using numerical simulations and also extends the spatial dimension to a two-dimensional space in order to introduce a broader utilization of the analysis to the real world cases.

Chapter 3

Theoretical part

This chapter analyzes a spatial competition game where two agents (e.g. fishing nations, fleets or vessels) are competing along a one-dimensional space. The space can be distinguished according to an existence of corners to either a line segment space or a circle space. The aim of this chapter is not only to analyze Nash equilibrium of the arising competition but also local stability conditions, thus finding necessary and if possible also sufficient equilibrium conditions. The agents' payoff is dependent on the distribution of the natural resource the agents are exploiting, therefore, the analysis proceeds from the most simple uniform distribution to more complicated distributions.

In the following model each agent (denoted by subscript $i \in \{1; 2\}$) can choose a strategy variable l_i from the set of strategies $L = [0, 1]$ which intuitively describes agent's location. The goal of both agents is to maximize their individual payoff h_i which can depend on the choice of their strategies, on their range described by an exogenous parameter e, and thus also on the area in which the players' range overlaps. Last but not least, it depends also on the shape of the space (existence of corners). In the cases where agents' range overlaps it is assumed that the payoff is split between the players equally.

3.1 Uniform distribution

The most simple distribution that can be mentioned is the uniform distribution. In this chapter, a line segment space is assumed (with an exception of the section 3.6) and hence also an existence of corners. As it is very intuitive and literature related, from this point on will the agents be sometimes denoted as "vessels" but the model could be applied in many diverse fields regarding exploitation of some resource.

In equilibrium, both players will try to maximize the payoff they are able to acquire. First, the model assumes no costs of transport. Second, the distribution of the resource stock is a uniform distribution on a line segment. Therefore, there is no location which should naturally attract more activity. The length of the line segment is normalized to 1. Without loss of generality we denote the vessel 1 the vessel which is closer to the origin (left corner of the line segment). The other vessel is the vessel 2.

Both players can choose their strategies l_i independently, however, they will tend to respect two observations. First, they will try to avoid the space corners because if they moved marginally beyond the corner, they would not increase their payoff. Second, their willingness to enter a competition is low as they can obtain only a half the payoff they could get alternatively, thus they will avoid the competition too. In case of $e_1 + e_2 \leq 0.5$ both observations can be fulfilled. Hence, in these cases the players will choose such locations they will not compete with each other. In other cases ($e_1 + e_2 > 0.5$), the marginal gain from entering the competition equals $\frac{1}{2}$ which is higher in comparison to avoiding the competition and entering corner location with marginal gain zero. In these cases the players maximizing their payoff will choose to enter a competition rather than to avoid it by escaping towards the corner.

There are many different possibilities how the agents can choose their strategies. As was written in the previous paragraph, the players will enter a competition if $e_1 + e_2$ exceeds the bound 0.5. Until that moment, if they are rational they can choose any strategies that respect the two observations stated above. Formally that means:

$$(l_1, l_2) \in [e_1, l_2 - e_2 - e_1] \times [l_1 + e_1 + e_2, 1 - e_2] \tag{3.1}$$

In this analysis we will focus especially on the case where the convergence is maximal. All other choices of (l_1, l_2) are either irrational (those not matching the relation (3.1)) or with the same outcome as in the case where the players converge to each other (and thus share one border). The reason why this is the most interesting case is that in this case the players' location can be understood as a cutoff location where both players are maximizing their payoff and still respecting the corner and competition evasion. As the $e_1 + e_2 = 0.5$ bound is exceeded, both players' strategy alternates to avoid the corners but also avoid the area shared with the competitor as much as possible (divergence).

With the focus on the cutoff case, if both players' initial point is in the middle of the line segment, the harvest of each player rises proportionally with the increase of e_i (see Fig. 3.1).

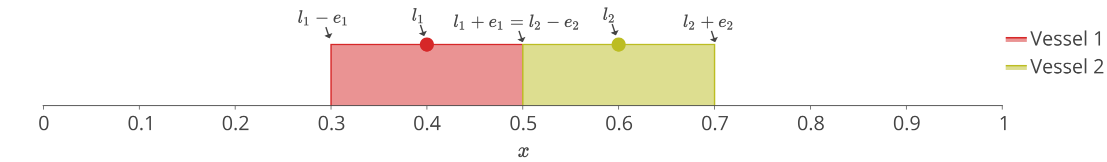

Figure 3.1: Vessels are not competing
Source: Author's computation

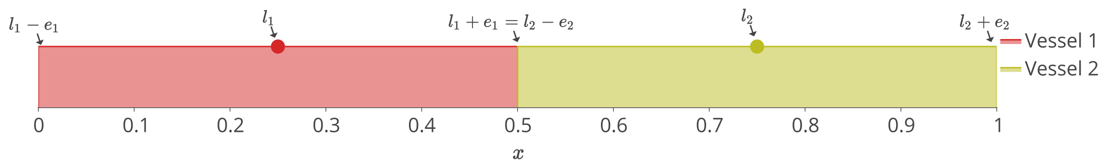

Figure 3.2: Vessels start to compete
Source: Author's computation

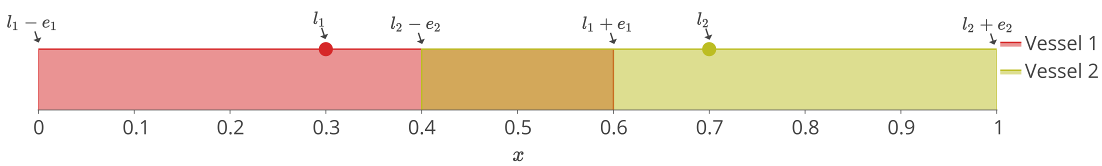

Figure 3.3: Vessels are competing
Source: Author's computation

As the e_i variables are gradually rising (in figures also symmetrically), the whole model gets into the point where the vessels have to start to compete for the shared area of the resource stock (see Fig. 3.2 and Fig. 3.3). The equilibrium location of the individual vessels l_i will be a function of both e_1, e_2. The cutoff location of the vessels diverges from the middle of the line segment towards the point where neither of vessels can increase its resource area without disrupting the opponent's harvesting area.

When this point is reached, the equilibrium locations will start to gravitate back towards the middle of the segment as the e_i increases. The equilibrium locations of the vessels can be described by the following formulas

$$l_1(e_1, e_2) = \begin{cases} \frac{1}{2} - e_2 & \text{if } e_1 + e_2 \leq 0.5 \\ e_1 & \text{if } e_1 + e_2 > 0.5 \end{cases}, \tag{3.2}$$

$$l_2(e_1, e_2) = \begin{cases} \frac{1}{2} + e_1 & \text{if } e_1 + e_2 \leq 0.5 \\ 1 - e_2 & \text{if } e_1 + e_2 > 0.5 \end{cases}. \tag{3.3}$$

This can be also rewritten in the form with absolute values

$$l_1(e_1, e_2) = \frac{1}{2}\left(\frac{1}{2} + e_1 - e_2 + \left|e_1 + e_2 - \frac{1}{2}\right|\right), \tag{3.4}$$

$$l_2(e_1, e_2) = \frac{1}{2}\left(\frac{3}{2} + e_1 - e_2 - \left|e_1 + e_2 - \frac{1}{2}\right|\right). \tag{3.5}$$

The next step is to characterize the harvest of each vessel. Let us denote r_1, r_{12}, r_2 the proportions of resource harvested by the vessel 1 only, both vessels simultaneously, and the vessel 2 only, respectively. The amount of resource r_1 is harvested solely by the vessel 1, and therefore it does not belong to the shared area unlike the amount r_{12} which is shared equally by both vessels. The amount r_2 is harvested only by the vessel 2. The payoffs of both vessels can be described in the following way

$$h_1(r_1, r_{12}) = r_1 + \frac{r_{12}}{2}, \tag{3.6}$$

$$h_2(r_2, r_{12}) = r_2 + \frac{r_{12}}{2}, \tag{3.7}$$

where the amounts of resource r_1, r_{12}, r_2 can be derived geometrically from the strategic variable l_i and the exogenous parameter e_i of the vessels. If the players are competing in the shared area, the equilibrium harvests r_1, r_{12}, r_2 will be

$$r_1(e_1, e_2, l_1, l_2) = (l_2 - e_2) - (l_1 - e_1), \tag{3.8}$$

$$r_{12}(e_1, e_2, l_1, l_2) = (l_1 + e_1) - (l_2 - e_2), \tag{3.9}$$

$$r_2(e_1, e_2, l_1, l_2) = (l_2 + e_2) - (l_1 + e_1), \tag{3.10}$$

whereas if the players are not competing (the condition $l_1 + e_1 \leq l_2 - e_2$ holds), the formulas will be slightly different as there will be no shared area.

$$r_1(e_1, e_2, l_1, l_2) = (l_1 + e_1) - (l_1 - e_1) = 2e_1, \tag{3.11}$$

$$r_{12}(e_1, e_2, l_1, l_2) = 0, \tag{3.12}$$

$$r_2(e_1, e_2, l_1, l_2) = (l_2 + e_2) - (l_2 - e_2) = 2e_2, \tag{3.13}$$

If the equations (3.8),(3.9),(3.10) are substituted into the equations (3.6), (3.7), the equations holding for the cases when the shared area exists are obtained

$$\begin{aligned} h_1(e_1, e_2, l_1, l_2) &= (l_2 - e_2) - (l_1 - e_1) + \frac{(l_1 + e_1) - (l_2 - e_2)}{2} = \\ &= \frac{3e_1 - e_2 - l_1 + l_2}{2}, \end{aligned} \tag{3.14}$$

$$\begin{aligned} h_2(e_1, e_2, l_1, l_2) &= (l_2 + e_2) - (l_1 + e_1) + \frac{(l_1 + e_1) - (l_2 - e_2)}{2} = \\ &= \frac{-e_1 + 3e_2 - l_1 + l_2}{2}. \end{aligned} \tag{3.15}$$

If the equations (3.4),(3.5) are substituted into the equations (3.14),(3.15) then it is obtained the formula for payoff depending only on the specific values of the exogenous parameter e_i for both vessels

$$h_1(e_1, e_2) = \frac{3e_1 - e_2 + \frac{1}{2} - \left|e_1 + e_2 - \frac{1}{2}\right|}{2}, \tag{3.16}$$

$$h_2(e_1, e_2) = \frac{-e_1 + 3e_2 + \frac{1}{2} - \left|e_1 + e_2 - \frac{1}{2}\right|}{2}. \tag{3.17}$$

Similarly, in the cases without shared area we will obtain the particular payoffs by substituting the equations (3.11),(3.12),(3.13) into the equations (3.6),(3.7)

$$h_1(e_1, e_2, l_1, l_2) = r_1(e_1, e_2, l_1, l_2) = 2e_1, \tag{3.18}$$

$$h_2(e_1, e_2, l_1, l_2) = r_2(e_1, e_2, l_1, l_2) = 2e_2. \tag{3.19}$$

The total harvest H can be computed as the sum of h_1 and h_2

$$H(e_1, e_2) = e_1 + e_2 + \frac{1}{2} - \left|e_1 + e_2 - \frac{1}{2}\right|. \tag{3.20}$$

If we suppose that $e_1 = e_2 = e$ the equations will simplify to

$$h_1(e) = h_2(e) = e + \frac{1}{4} - \left|e - \frac{1}{4}\right| = \begin{cases} 2e & \text{if } e \leq 0.25 \\ \frac{1}{2} & \text{if } e > 0.25 \end{cases}, \tag{3.21}$$

$$H(e) = 2e + \frac{1}{2} - \left|2e - \frac{1}{2}\right| = \begin{cases} 4e & \text{if } e \leq 0.25 \\ 1 & \text{if } e > 0.25 \end{cases}. \quad (3.22)$$

These results imply that from the point when $e \geq 0.25$ the total harvest is not increasing (see Fig. 3.4). The only variables which are changing are the variables denoting locations of vessels l_1 and l_2 which is also depicted in the Fig. 3.4. Also the interval of competitive harvests changed according to the following equation

$$r_{12}(l_1, l_2, e) = 2e - (l_2 - l_1) = 2e - \frac{1}{2} + \left|2e - \frac{1}{2}\right|. \quad (3.23)$$

An interesting implication of adding an equality $e_1 = e_2 = e$ is that the sum of strategy variables gives us always $l_1 + l_2 = 1$ in the cutoff case, which can be easily seen from the Fig. 3.4 or derived from the equations (3.4),(3.5). The location of both players in the cutoff case (the equilibrium with maximal convergence of players) is at first non-competitively diverging as the players want to avoid the competition, however, as the exogenous parameter e exceeds the value 0.25, both players will rather choose corner avoidance than no competition and thus will converge towards the middle of the line segment space entering the competition while minimizing the shared area as much as possible.

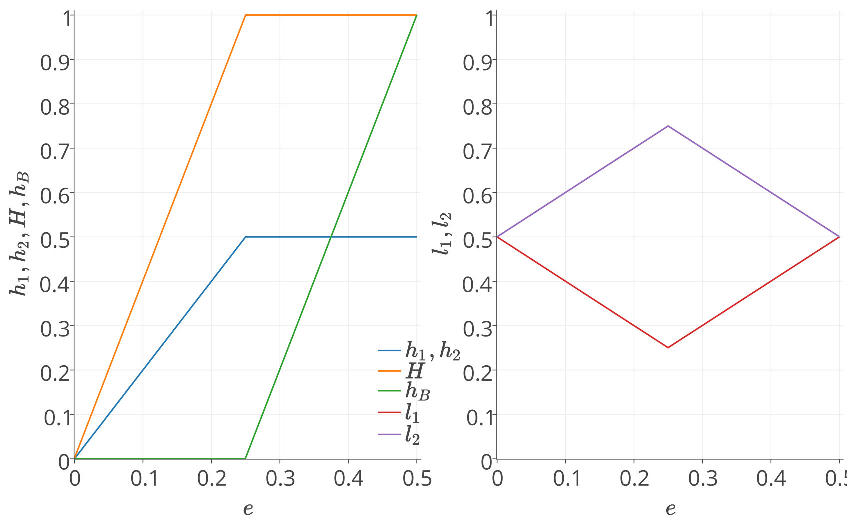

Figure 3.4: Uniform distribution payoff and location summary
Source: Author's computation

3.2 Local stability conditions under the assumption of agents' symmetric range

In the model above it is assumed that the distribution of the resource stock is uniform. Let us relax this assumption and suppose arbitrary distribution with a density function $f(x)$ and a cumulative distribution function given by its definition $F(x) = \int_0^x f(t)\mathrm{d}t$. In the uniform distribution setup above, it held that

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases} . \tag{3.24}$$

$$F(x) = \begin{cases} 1 & \text{if } x > 1 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x < 0 \end{cases} . \tag{3.25}$$

With the arbitrary distribution given by density $f(x)$, the general conditions of equilibrium local stability can be examined. These conditions hold if there does not exist any one-side marginal deviation of player strategy l_i which leads to better payoff than the previous strategy. In other words it means that if a player makes a marginal shift of strategy and it does not change the payoff, then the local stability conditions hold.

Let us assume again that $e_1 = e_2 = e$. Now three different scenarios can occur.

In the first scenario, the players do not share any area of the resource stock (there is no competition). The local interior stability conditions for both vessels are then

$$\lim_{x \to l_i -} f(x - e) \leq \lim_{x \to l_i -} f(x + e), \tag{3.26}$$

$$\lim_{x \to l_i +} f(x - e) \geq \lim_{x \to l_i +} f(x + e). \tag{3.27}$$

In the case of the continuous density function $f(x)$ these conditions can be simply rewritten as

$$f(l_i - e) = f(l_i + e). \tag{3.28}$$

These stability conditions are not valid in the cases where other constraints occur. This can happen for example in the cases of corners of the line segment. If the player's range is outside the line segment space ($l_i - e_i < 0$ or $l_i + e_i > 1$) then it is locally stable equilibrium only if the marginal gain is zero at both boundaries of player's range. Therefore, the rational players will avoid the locations ranging beyond the corners as there is no payoff incentive.

The left corner ($l_i - e = 0$) equilibrium is locally stable if

$$\lim_{x \to l_i+} f(x+e) \leq f(0). \tag{3.29}$$

The right corner ($l_i + e = 1$) equilibrium is locally stable if

$$\lim_{x \to l_i-} f(x-e) \leq f(1). \tag{3.30}$$

In the second scenario, there exists an area where both players are harvesting ($l_2 - e < l_1 + e$). The payoff in the shared area is split equally between both players. Therefore, in order to achieve locally stable equilibrium, the marginal payoff of the divergence option must be equal to the marginal payoff of the convergence option (which is, however, divided between both players). As it is assumed that $0 \leq l_1 < l_2 \leq 1$ the local interior stability conditions can be written formally

$$\lim_{x \to l_1-} f(x-e) \leq \frac{1}{2} \lim_{x \to l_1-} f(x+e), \tag{3.31}$$

$$\lim_{x \to l_1+} f(x-e) \geq \frac{1}{2} \lim_{x \to l_1+} f(x+e), \tag{3.32}$$

$$\frac{1}{2} \lim_{x \to l_2-} f(x-e) \leq \lim_{x \to l_2-} f(x+e), \tag{3.33}$$

$$\frac{1}{2} \lim_{x \to l_2+} f(x-e) \geq \lim_{x \to l_2+} f(x+e). \tag{3.34}$$

Again, if the density function $f(x)$ is continuous, the previous conditions simplify to

$$f(l_1 - e) = \frac{1}{2} f(l_1 + e), \tag{3.35}$$

$$\frac{1}{2} f(l_2 - e) = f(l_2 + e). \tag{3.36}$$

If the vessel 1 is in the left corner ($l_1 - e = 0$), the local constrained stability condition is

$$\frac{1}{2} \lim_{x \to l_1+} f(x+e) \leq f(0). \tag{3.37}$$

If the vessel 2 is in the right corner ($l_2 + e = 1$), the local constrained stability condition is

$$\frac{1}{2} \lim_{x \to l_2-} f(x-e) \leq f(1). \tag{3.38}$$

Finally, in the third scenario, we assume full competition ($l_1 = l_2$). Under the assumption of $e_1 = e_2 = e$ it holds that all the payoff is split between both players equally ($r_1 = r_2 = 0, h_1 = h_2 = \frac{r_{12}}{2}$). This scenario is very similar to the previous one, therefore the formulas used in the previous scenario will be used in this scenario using the substitution $l_1 = l_2 = l$. The local interior stability conditions are thus formally

$$\lim_{x \to l-} f(x-e) \leq \frac{1}{2} \lim_{x \to l-} f(x+e), \tag{3.39}$$

$$\frac{1}{2} \lim_{x \to l+} f(x-e) \geq \lim_{x \to l+} f(x+e). \tag{3.40}$$

If the density function $f(x)$ is continuous, the previous conditions simplify to

$$f(l-e) \leq \frac{1}{2} f(l+e), \tag{3.41}$$

$$\frac{1}{2} f(l-e) \geq f(l+e), \tag{3.42}$$

but this is the system of two linear inequalities and can be solved. After substituting first inequality into the second the following result can be obtained

$$\frac{1}{4} f(l+e) \geq f(l+e), \tag{3.43}$$

which holds if and only if $f(l+e) = 0$. This implies also $f(l-e) = 0$. Therefore, the full competition in the interior can occur if and only if the density function $f(x)$ is discrete and matching the conditions (3.39),(3.40) or if the density function $f(x)$ is continuous but its functional value $f(l-e) = f(l+e)$ is equal to zero.

If the players are both in the left corner ($l - e = 0$), the local constrained stability condition can be written as

$$\lim_{x \to l+} f(x+e) \leq \frac{1}{2} f(0). \tag{3.44}$$

If they are in the right corner ($l + e = 1$), the local constrained stability condition is

$$\lim_{x \to l-} f(x-e) \leq \frac{1}{2} f(1). \tag{3.45}$$

Unlike the interior case, these stability conditions can be matched even if the density function $f(x)$ is continuous. It is sufficient to have enough fast declining $f(x)$ near the edges for the conditions to hold.

The conclusion is that it is impossible to have a full competition in the case of continuous density function unless 1) both players are near one of the edges, 2) the players are harvesting along one peak in the interior which is circumscribed by points with zero density. On the other hand, in the case of discrete density function, the conditions are generally not as much restricted as in the continuous case.

3.3 Symmetric strictly quasi-concave distribution

Let us now suppose a single-peaked symmetric distribution (i.e. density function is strictly quasi-concave and meets condition $f(x) = f(1-x)$). In this case the results will be very similar to the uniform case which was examined before if the peak density is not very high in comparison to corners density. The reason for this is that the location which naturally attracts more attention is located in the middle of the line segment. Hence, if we compare this scenario to uniform distribution scenario, the player have to be at least as much converging to each other as in the uniform distribution scenario. The intuition behind is that there is increased attractiveness only in the middle of the line segment space, and therefore it simply cannot increase divergence of the agents.

The interesting fact is that players do not have incentives to compete if the peak density is below twice as high as the density at the corner. Formally this means that the analysis is exactly the same as in the uniform distribution if the following condition is met:

$$f(0) = f(1) \geq \frac{f(\frac{1}{2})}{2}. \tag{3.46}$$

The local stability conditions are sufficient to sustain no competition in such a case.

On the other hand, a different scenario can happen if the distribution does not meet the condition (3.46). If we suppose that $e_1 = e_2 = e$, both vessels will start their harvest in the middle (as it is rational to harvest the highest density) and they will diverge from each other while the local stability conditions are met. However, as we assume strictly quasi-concave function with density at the corners less than the half of the peak (i.e. negation of the condition 3.46), the players have incentives to intrude into the competitor's area as long as it is more profitable than divergence. These incentives are driven by the local stability conditions described in the previous subsection.

Note, that if the exogenous parameter e is small, the local stability conditions do not have to imply the competition occurrence. This holds because if they operated nearby the peak and their range did not reach the points where the divergence is not worthy, they would still rather diverge than converge into a competition.

However, as we suppose symmetric distribution, both players will probably mirror their strategies and if one vessel defected and intruded into the area of influence of the other vessel the response would be the same and neither of vessels would be better off as their harvests would cancel out. Furthermore, they would be worse off as they would not use all the resource they would be otherwise able to use if they did not compete. Therefore, the socially best optimum would be if they cooperated and did not intrude into each other area of influence.

The optimal scenario would therefore be to diverge until the entire area is saturated and only after that the players should start to compete. In that case, the equilibrium values of the strategic variables l_i are the same as in the uniform distribution case.

The only difference in comparison to the uniform distribution analysis is that in the strictly quasi-concave case there would not exist more optimal strategies for low e as the player receives the highest payoff if he chooses to converge as much as possible (but still not to compete!). Hence, the players will always choose the equilibrium with the highest convergence, which was in the uniform distribution analysis considered only as a cutoff case and there were also other strategy choices which were admissible, too.

To sum it up, if the condition 3.46 holds or the players start to cooperate, the equilibrium values of the examined variables would be very similar to the uniform case only the equations describing it are generalized. For the detailed analysis, see Appendix, where the equations are derived.

3.4 Symmetric quasi-convex distributions

In the previous section, the situation of two competing players in a symmetric quasi-concave distribution setting was examined. Now, let us take a look at cases where there is a symmetric ($f(x) = f(1-x)$) quasi-convex distribution. Intuitively, these distributions tend to increase divergence between the locations of the two players, therefore the detailed description of players' strategies is quite straightforward.

Unlike the previous case, the players now do not have any incentives to compete as they will be better off at each one's corners of the distribution. Therefore, they will choose the location where they harvest the most, which is

$$l_1 = e \tag{3.47}$$

$$l_2 = 1 - e \tag{3.48}$$

Their payoff can be easily derived too

$$h_1 = F(2e) - F(0), \tag{3.49}$$

$$h_2 = F(1) - F(1 - 2e), \tag{3.50}$$

in the case of $e \leq 0.25$. On the other hand, when there is a partial competition caused by corner restriction ($e > 0.25$) the harvest of the players can be described as

$$h_1 = \frac{F(2e) - F(1-2e)}{2} - (F(1-2e) - F(0)), \tag{3.51}$$

$$h_2 = F(1) - F(2e) + \frac{F(2e) - F(1-2e)}{2}. \tag{3.52}$$

3.5 Linear distribution

Let us now suppose distribution given by formula: $f(x) = ax+b$ on the interval $[0,1]$ and $f(x) = 0$ else. The variables a and b are exogenous parameters. In order to decrease the number of free parameters (degrees of freedom), we can norm the distribution on the line segment so the total quantity of natural resource is 1. Formally this means

$$\int_0^1 f(x)dx = 1. \tag{3.53}$$

If we apply this norm on the distribution given by linear density function the following condition is obtained

$$\int_0^1 (ax+b)dx = \left[\frac{ax^2}{2} + bx\right]_0^1 = \frac{a}{2} + b = 1, \tag{3.54}$$

$$a = 2(1-b). \tag{3.55}$$

Therefore the natural resource distribution is in this case a function not only of a location denoted by x but also of one exogenous parameter b, which is responsible for the slope and for the vertical shift of the density function of the distribution. That means that b denotes the intercept of a vertical axis ($f(x)$) but simultaneously also a parameter negatively related to the slope of the density function.

As we do not suppose cases where the distribution could be negative, we can consider that $b \geq 0$ condition must hold. Similarly, if the parameter b was greater than 2, there would emerge a region where the distribution would be negative. If we think about cases where $1 < b \leq 2$, we can infer that they are very similar to the cases where $0 \leq b < 1$. The only difference lies in the fact that there is an axial symmetry and thus the former case is the mirror image of the latter one (See Fig. 3.5). Without loss of generality we can, therefore, assume $b \in [0; 1]$. The density and cumulative distribution functions are given by

$$f(x) = 2(1-b)x + b, \tag{3.56}$$

$$F(x) = \int_0^x [2(1-b)t + b]dt = (1-b)x^2 + bx. \tag{3.57}$$

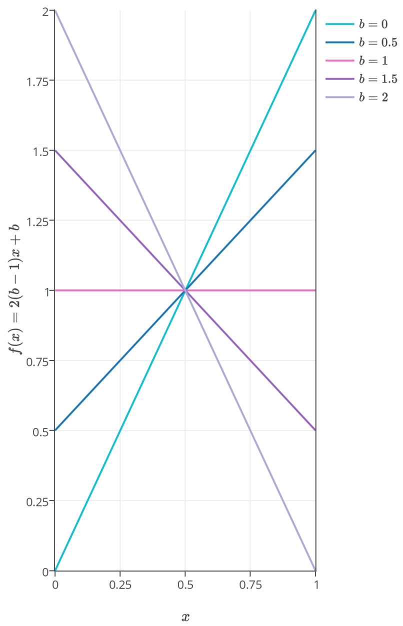

Figure 3.5: Density functions depending on the parameter b
Source: Author's computation

The aim of this subsection is to find Nash equilibrium in pure strategies of two players l_1, l_2 (if it exists). Under the assumption $b \in [0; 1)$, the distribution is upward sloping with a constant slope, therefore the location near the right edge of the line segment naturally attracts attention of both players. In the case of $b = 1$, the uniform distribution is obtained, which was discussed earlier. If we supposed first only one player (i.e. player 2, which operates in the right), he will naturally choose the location where he maximizes his payoff, which is as close to the right corner as possible

$$\begin{aligned} h_2(l_2) &= F(l_2 + e) - F(l_2 - e) = \\ &= (1-b)(l_2+e)^2 + b(l_2+e) - (1-b)(l_2-e)^2 - b(l_2-e) = \\ &= 4l_2e(1-b) + 2be, \end{aligned} \tag{3.58}$$

$$\frac{dh_2}{dl_2} = 4e(1-b) > 0, \tag{3.59}$$

$$\frac{d^2h_2}{dl_2^2} = 0. \tag{3.60}$$

As the slope of the payoff function of the player 2 is positive for all $e > 0$ and $b \in [0, 1)$. The closer the player is to the right edge of the line segment space, the higher the payoff is (except the $b = 1$ case). This fact implies that the player will maximize his harvest when he is in the right corner. It would be nonsense to harvest zero natural resource (beyond the right point of the line segment i.e. $x = 1$), hence he will choose the location where he is 1) in the right as much as possible, 2) where there is a non-zero harvest at the maximum point x which is, however, still in player's range. This means he will choose the location

$$l_2(b, e) = l_2(e) = 1 - e, \tag{3.61}$$

which is interestingly but not surprisingly not dependent on the distribution parameter b. As will be shown later, the location of the player 1 will depend on the parameter b. In contrast to these conditions, if the parameter b was equal to 1, the distribution would be uniform with a density function as stated in the subsection regarding uniform distribution. The Nash equilibrium in a case of two players harvesting the resource with uniform distribution has been already discussed, therefore it will be left here and aimed in this subsection to clearly explain what happens in case of $b \in [0, 1)$.

The position of the player 2 is already known. Now, let us take a look at what happens with the player's 1 decision. If the areas of harvest of individual agents overlapped, the payoff for each of them would reduce to half. Therefore, the problem of how to set the location of the player 1 reduces to the problem without competitor but with only half the resource available in the overlapped area (See Fig.3.6).

The problem of finding the equilibrium strategy of the player 1 can lead to situations where there exists an interior equilibrium (e.g. in Fig. 3.6). The reason for this is that the second player has to balance marginal gains from the non-competitive area with those in the competitive area, simply due to observation of local stability conditions.

The existence of an interior equilibrium depends mainly on the value of parameter b and partially also value of e. If the intercept b would be close to zero (and thus also a slope would be high), the full competition could emerge. In contrast, if the parameter b was close to 1, it would be very similar to the uniform distribution scenario, and therefore no competition would be entered. If the b is somewhere between those two values (depending also on the e value), the case of interior equilibrium can emerge. The more specific conditions for the nature of the equilibrium will be examined later.

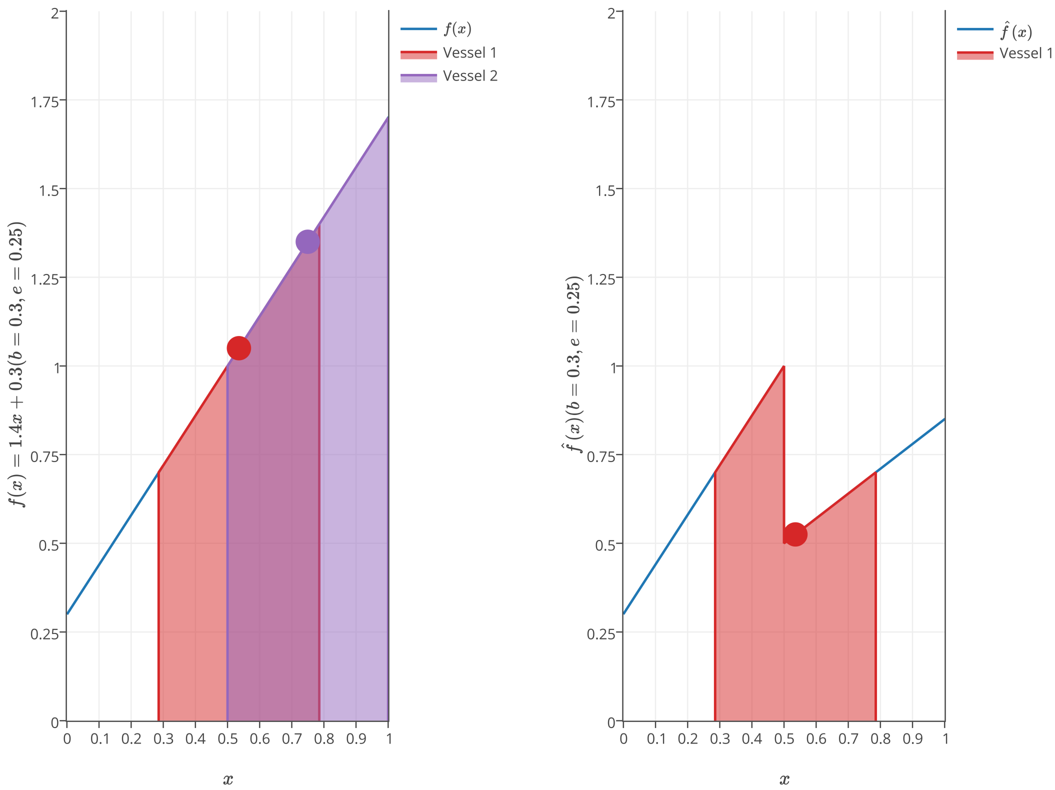

Figure 3.6: Determination of players' location in setting $b = 0.3, e = 0.25$

Source: Author's computation

This attitude towards setting the location of the player 1 in response of the location of the player 2 can be utilized not only in this case with linear density function but generally, which will be discussed later and is the way how the numerical simulations are done. Now, let the rest of density available to the player 1 denote as $\hat{f}(x)$ along with the definition

$$\hat{f}(x) \equiv \begin{cases} f(x)/2 & \text{if } l_2 - e \leq x \leq l_2 + e \\ f(x) & \text{else} \end{cases}, \tag{3.62}$$

where under $f(x)$ it is supposed the linear density function stated formerly $f(x) = 2(1-b)x + b$. The player 1 has to make the same decision about choice of strategy as the player 2 did previously but applies it on the altered distribution instead:

$$\hat{f}(x) = \begin{cases} (1-b)x + \frac{b}{2} & \text{if } 1-2e \leq x \leq 1 \\ 2(1-b)x + b & \text{if } 0 \leq x < 1-2e \\ 0 & \text{else} \end{cases}. \tag{3.63}$$

If we define $\hat{F}$ as the cumulative distribution function of the density function

$$\hat{F}(x) = \int_0^x \hat{f}(t)dt, \tag{3.64}$$

the harvest of the player 1 will be then

$$\begin{aligned} h_1 = r_1 + \frac{r_{12}}{2} &= \hat{F}(l_1+e) - \hat{F}(l_1-e) = \\ &= \frac{F(l_1+e) - F(1-2e)}{2} + F(1-2e) - F(l_1-e) = \\ &= (1-b)\frac{1-4e+3e^2-l_1^2+6l_1e}{2} + b\frac{1+e-l_1}{2}. \end{aligned} \tag{3.65}$$

The objective function we want to maximize is the player 1's harvest h_1. Therefore we solve the maximization problem

$$\max_{l_1 \in [0,1]} h_1(l_1). \tag{3.66}$$

If we assume $b \in [0,1)$, the first and second order conditions are

$$\frac{dh_1}{dl_1} = (1-b)(-l_1+3e) - \frac{b}{2} = 0, \tag{3.67}$$

$$\frac{d^2h_1}{dl_1^2} = b - 1 < 0. \tag{3.68}$$

This means, there exists a maximum as the player 1's harvest function is concave at each point of the line segment. From the first order condition it follows that the player 1 maximizes his payoff (harvest) at the location

$$l_1 = \frac{b}{2b-2} + 3e. \tag{3.69}$$

This is, however, valid only if there exists a competitive area, because the player 1's payoff function h_1 would be defined in non-competitive scenario otherwise than in the competitive one (which was done above). However, this problem can be simplified even without solving the case, where there is no competition. The key lies in the fact that the density function $\hat{f}(x)$ has a positive slope along all the noncompetitive values x. This means, rational player 1 would choose the solution which is not competitive but as near as possible to the player 2, i.e. solution where the player 1 is operating at the point $l_1 = 1-3e$, which is the first point where the shared payoff r_{12} is equal to zero (measured from the location of the player 2). Furthermore, this solution is also consistent with the equations regarding the non-negative r_{12} which were examined above. Therefore, it is not necessary to examine also the cases where $l_1 < 1 - 3e$, because rational player 1 would not choose such positions and would rather choose the most convergent no competition strategy possible. The choice of the strategy l_1 can be rewritten more precisely as

$$l_1 = \max\left(\frac{b}{2b-2} + 3e, 1 - 3e\right), \tag{3.70}$$

and in its core it means that the player 1 is trying to compete if possible, otherwise he will choose the most convergent non-competitive equilibrium strategy located at $1 - 3e$. It is interesting to determine exactly what conditions must be met in order to have no competition. As was stated before this happens when player 1 decides to harvest exactly at point $l_1 = 1 - 3e$. This means the condition for sustaining no competition is

$$1 - 3e \geq \frac{b}{2b-2} + 3e. \tag{3.71}$$

In order to find e such that there will not be competition for given exogenous parameter b, it is necessary to think about the "marginal player 1" (i.e. the player 1 who is not competing but can enter competition if he moved marginally along the line segment space). Let denote e_m such cutoff parameter e which is owned by the "marginal player 1", then the following equation holds

$$1 - 3e_m = \frac{b}{2b-2} + 3e_m. \tag{3.72}$$

This can be rewritten as

$$e_m(b) = \frac{b-2}{12(b-1)}. \tag{3.73}$$

From the definition of e_m it is only meaningful to find it if there can be no competition. It is trivial that if the condition $e > \frac{1}{4}$ is met, then it is certain there will occur a competition (similarly as in the uniform distribution case). Furthermore, by substituting eq.(3.73) into this condition the following relation is obtained

$$b > \frac{1}{2}. \tag{3.74}$$

This means that if there is $b > \frac{1}{2}$, then there will always exist a competition between the player 1 and the player 2 if the e exceeds $\frac{1}{4}$. In other cases there can exist both competition or no competition depending on the value of b and e.

Similarly, we can discuss also the cases where there will be a full competition. This happens when $l_1 = l_2$. This condition is met when

$$1 - e_{fc} = \frac{b}{2b-2} + 3e_{fc}, \tag{3.75}$$

where e_{fc} denotes the minimum exogenous variable e which is necessary to sustain full competition given the parameter b. The last equation can be also rewritten as

$$e_{fc}(b) = \frac{2-b}{8(1-b)}. \tag{3.76}$$

This is meaningful only in cases where there is not full competition naturally (i.e. $e < 0.5$). In the Fig. 3.7 the discussion about competitive cases is clearly summarized. It can be seen which selections of parameters b and e lead to either no competition, partial competition or full competition. Note that this is only valid if both players are rational (maximize their payoff), otherwise they could choose other locations than the equilibrium ones, which could cause a competition in other cases than are discussed.

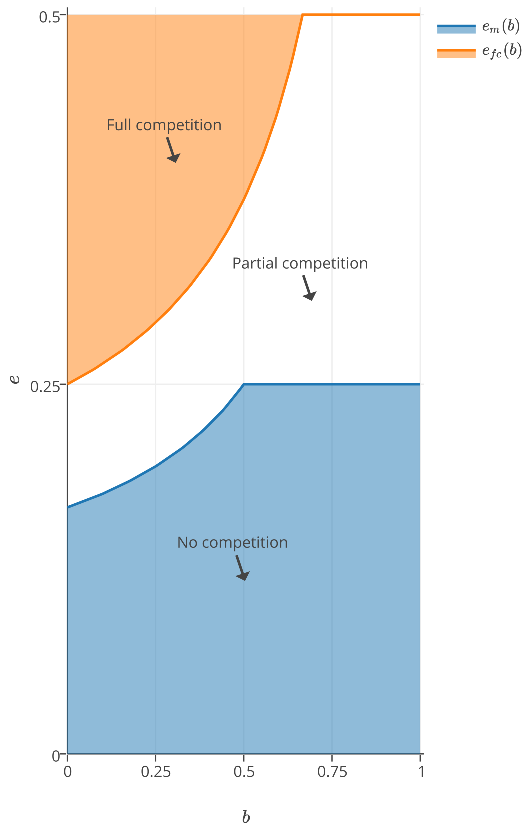

Figure 3.7: Discussion of competitive behaviour depending on parameters b, e

Source: Author's computation

It is interesting that the set of partial competition and also the set of full competition is non-convex. It is driven mainly by the following two observations.

The first one is that two players in equilibrium simply cannot sustain no competition if the $e > 0.25$ as the space they are competing in is not vast enough and very similarly they cannot sustain partial competition if they can harvest along the entire space ($e \geq 0.5$).

The second observation is that as the parameter b drops to zero, there is steeper density function, which forces players to compete more actively for the location, where the resource is concentrated. Therefore, it is not surprising that for lower values of the parameter b the lower e is sufficient to enter a competition (or full competition). The closer the value of b is to 1, the more similar is the case to the uniform distribution case, where the first observation dominates because agents do not have so strong incentives to change their location. Thus, as the parameter e grows marginally with parameter b held constant, it increases the competitiveness of both players.

Similarly, if the parameter b decreases marginally with parameter e held constant, the players may switch from no competition to partial competition, or from partial competition to full competition, again increasing the competitiveness of both players.

The inverse to the parameter b (i.e. $\frac{1}{b}$) can thus be understood as a heterogeneity parameter. Generally, it holds that the higher the heterogeneity parameter, the higher the competitiveness. It is caused by the simultaneous effect of parameter b on an intercept but also a slope. The higher slope in this case causes that the half marginal gain from the competitive area is more attractive than in the cases where the slope is lower. Moreover, the lower the intercept is, the higher willingness to compete occurs. Therefore, in this setting both effects of the parameter b are cumulative and therefore the analysis of its role is clear.

One could deduce from this that it means that raise in e and increase in b at the same time will result in a specific, always the same change of competitiveness. Unfortunately, that is not a correct deduction. The same increase in the linear combination (b, e) can result in two completely different effects (see Fig. 3.7).

The first effect regards the change from partial competition to no competition due to not sufficient slope of the density function (the second observation). The second effect is the change from no competition again to partial competition as there is no more space for both players (the first observation).

Very similar situation happens in case of transition from the full competition to partial competition as the (b, e) grows marginally. It suggests that the slope of the density function is not enough steep to keep players competing in the full competition. However, if the parameters (b, e) are keeping to grow up to the point where there is not enough space for both players, the partial competition will again change into a full competition. The different nature of the exactly same marginal changes in the linear combination (b, e) can be seen as the consequence of the non-convexity of the partial and full competition sets.

In other words, it can be observed that higher b and lower e are both leading to lower competitiveness. These two parameters are not pure substitutes (they are complementary) and therefore the linear combination of their effects can be non-monotonic.

To sum this subsection up, it may be argued that not all cases with linear density function were examined, however, that is not true. Any density function which respects proportionality ($f(x) = ax$) can be normed to the studied case $f(x) = 2x$. Note that we do not suppose $a \leq 0$ as the distribution would be negative at the interval $(0, 1]$ which would not make much intuition. In addition, all the cases where there is a positive shift of density function ($f(x) = ax + b$ where $b > 0$) can be summarized in normed cases $f(x) = 2(1 - b)x + b$. If b exceeded 1, we would get the same solutions, only inverted along the vertical axis in $x = 0.5$.

The last case which we did not discuss is the case where there is zero density in the left corner of the line segment and it continues being zero (there is zero density in the left side of the interval $[0, 1]$) until there is a kink point where the density starts to have positive values and respects the formula for linear function. As we suppose zero density everywhere outside the $[0, 1]$ interval, all these cases can be handled if the origin is shifted towards the kink point and the values at the horizontal axis are rescaled to the interval $[0, 1]$ and afterwards the distribution is normed again.

3.6 Non-existence of corners

Unlike the previous subsections, in this one we change the fundamental premise we have faced until now, namely the space which is regarded can be now seen as a circle. Thus, there are not any corners in which the players could operate. This setting has its substantiation as it can simulate cases where there is not important to distinguish the central part of the area from the periphery. Rather it is useful for a description of the competition in neighbourhood with the same local characteristics (not dependent on the distance from the centre).

The easiest distribution of the natural resource that can be supposed without corners is the resource uniformly distributed along a circle. In order to keep very similar notation as until this moment, let us assume the circle to be normed to the circumference equal to 1. This assumption is again similarly as in the line segment situation completely without loss of generality, because any circle can be normed to a circle with such a circumference.

The reason why it is supposed normed to unit circumference and not to unit radius is that it can be easily mapped on a unit line segment of which some of the scenarios were examined previously. The positions on the circle are therefore parametrized in the original interval $[0,1]$ unlike the intuition which could tempt us to parametrize it by interval $[0,2\pi]$.

In such a setting where there is a uniform distribution without corners, the results do not differ much from the ones which were obtained in version with corners. Players have no incentives to compete until the whole space is filled. Therefore, the competition will start from the moment when the exogenous parameter e describing their range is higher than $e = 0.25$.

In contrast to a line segment space scenario, there is a wider range of possibilities where the players can operate before they start to compete because if the player who is situated more in the right than the other one was for example nearby the origin, the left player has a possibility to escape from the corner because he will "jump" immediately to the other side of the line segment (which is actually a circle mapped to the line segment).

Therefore we may consider the exogenous parameter e as a parameter which forbids the player to get closer to the other player than $2e$ (in absolute value).

The consequences of higher freedom in terms of location choice are, however, not noticeable in terms of harvest which is dependent on the parameter e exactly the same as in the scenario where the line segment space was reflected.

As it is not important what the position of the first player in this scenario is, it is good enough to consider only the difference between the two locations. Without loss of generality we can suppose the location of one player fixed at location $l_1 = 0$. This ensures that the other player will choose his location included in the interval $[2e, 1-2e]$. It is obvious that it holds only if there is no competition ($e \leq 0.25$). When there is a partial or full competition, the player 2 will naturally choose the location $l_2 = \frac{1}{2}$ as it minimizes the area which is shared with the player 1. The equations referring to players harvests in this setting are the same as in the section regarding the line segment space, thus we will skip it and focus on more interesting cases.

There are two naturally interesting cases of distributions along the circle. The first one's density function is a continuously differentiable function and can be described by a function of sine. The second one could be summarized by a symmetric linear density function with kink in the middle ($x = \frac{1}{2}$).

In the first scenario, the players are operating on a circle space, thus it is not very important to distinguish, whether the distribution is sine or cosine, because both of these functions can be mapped onto a line segment identically. Furthermore, any horizontally shifted sine (cosine) function can be mapped onto the same line segment too. Therefore, the sine function is chosen without loss of generality. The density function could be chosen in the following form

$$f(x) = \begin{cases} a\sin(2\pi kx) + 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}, \tag{3.77}$$

where a is an exogenous parameter and k is positive integer regarding number of peaks.

The reason for this form is that it is normed again to the area of 1 which is ensured by a vertical shift of the sine to 1. This is also done in order to avoid cases where the distribution could be negative. However, this forces us also to do a restriction on the distribution parameter a, which has to be less than or equal to 1. There is also not much sense in supposing the parameter a negative, as the identity

$$-\sin(2\pi kx) + 1 = \sin(2\pi kx + \pi) + 1 \tag{3.78}$$

holds and since it was written previously that the horizontal shift of the sine function does not change the distribution at the circle space.

Let us suppose $k = 1$. The analysis of the players location choices is simple as long as the sine amplitude is not very high. The distribution can be shifted horizontally, if we shifted it so that the peak was in the middle of the density function,

$$f(x) = a\sin\left[2\pi\left(x - \frac{1}{4}\right)\right] + 1, \tag{3.79}$$

we would get a quasi-concave function at interval $[0, 1]$. There is no reason why the players should operate in the locations $x = 0$ or $x = 1$ since there is a minimum of the natural resource. Therefore the players will make decisions exactly the same as in the case of quasi-concave density function on the line segment which was already discussed.

First, for simplicity let us assume the cases where the parameter $a \in (0, \frac{1}{3}]$ as in that case the maximal value of the density function is $\frac{4}{3}$ and the minimal value $\frac{2}{3}$. This choice of parameter a ensures that according to quasi-concave distribution analysis the players will behave exactly the same as in the uniform case along the line segment (in the most converging equilibrium possible but not competing).

When the players will start to compete ($e > 0.25$), they would intrude into the competitors area not at the minimum value of the density function but at its peak ($x = 0.5$), as it seems more beneficial for both players. The part of natural resource which is not shared until the full competition is the part close to the minimum of the density function ($x = 0$ or $x = 1$). The players' harvests can be described with the same equations as the equations from the section Quasi-concave distribution available in the Appendix.

The problem emerges if the $a \in (\frac{1}{3}, 1]$ as the players will start to compete with each other earlier than the entire space is covered. In such cases, the equilibrium strategies can be found by balancing both players marginal gains according to local stability conditions. This analysis would be relatively cumbersome and very technical, therefore we will not go into that much detail analytically in this work, however, in the numerical simulations chapter, the effects in that cases are investigated, simulated and discussed.

In contrast, if we still suppose $a \in (0, \frac{1}{3}]$ but higher k than 1, the analysis could be interesting. For example for $k = 2$, the results will be slightly different. There will exist two peaks, where in each one one player will operate. As these peaks are exactly the same distance no matter which direction the player moves, the players will keep operating in the peaks even if a competition exists there ($e > 0.25$). In case of competition, at one of the two minimums a fixed boundary will be established whereas the players will intrude into each other peaks via the second minimum.

The similar scenario will occur in case of even k, the players will continue to operate so they are able to cover the whole peak (circumscribed by the minimum density) and if their range is higher than the range covering exactly one peak, they will start to cover the second. As we assume the players are rational and are fully aware of how the other player behaves, both players will choose the peak where the other player does not operate as their second peak. This process continues (as e grows) until all the peaks are covered by the two players. After the whole area is covered ($e > 0.25$) the players will compete similarly as described in case of $k = 1$.

For odd k the discussion is similar as for even k, the only difference is that the peaks are covered one by one until the last one rests as e grows. Then the situation is exactly the same as in case of $k = 1$ and two players are sharing the last peak where they also enter the competition in case of $e > 0.25$. The case, where $k = 2$ and $a = 1$ is managed and discussed in the numerical simulations chapter. The detailed analytical analysis for these distributions would be very extensive and would not bring no new concept as the analysis would be driven by the local stability conditions.

The second scenario which could be quite realistic distribution along the circle is, as was mentioned above, linear distribution with peak (kink-point) in the middle ($x = 0.5$) described by formula for density function

$$f(x) = \begin{cases} a|x - \frac{1}{2}| + b & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}, \tag{3.80}$$

where a and b are exogenous parameters of the distribution. It can be normed to area of 1 as in the previous case. The following characteristics also apply for all distributions that are equivalent to a horizontal shift of this distribution in the interval $[0, 1]$.

Again, the players do not have any incentives to operate in the locations where the $f(x)$ is minimal, therefore they will behave exactly the same as in the sine case.

The same also applies for all distributions which are monotonic in the first half of the interval $[0, 1]$ and axially symmetric around the axis $x = 0.5$. The harvests of the individual players will be similarly described by the formulas used in Appendix describing the quasi-concave case. Note, that if the condition (3.46) from the chapter 3.3 did not hold, the analysis would be more complicated by following only the local stability conditions. This distribution is not simulated in the numerical simulations chapter as it has similar characteristics to the sine density function and therefore we only suggest it as a natural second choice after the sine function.

Chapter 4

Numerical simulations

4.1 Methodology

In the previous chapter, the general equilibrium and stability conditions were examined in a two player game for various distributions of the resource and two different shapes of the strategy space (a line segment or a circle). In this chapter, the aim is to make numerical simulations which will support the theoretical framework from the previous chapter but also to simulate some observations more generally, in two dimensions. The further goal of this chapter is to create a pattern which will be applied by individual agents to predict the information about the density of the natural resource. This pattern (or agent heuristics) can be understood as an algorithm which agents use to predict the information in a setup with imperfect information about the density function.

Consequently, the players' payoffs in optimal scenarios with perfect information will be compared to the players' payoffs in scenarios where a restriction on information exists. The difference between the optimal payoff and the payoff under imperfect information will be evaluated as an incentive to buy such information. The goal is to evaluate whether the information is worth of purchase and how gainful it is for various scenarios given different parameters of players' range and amount of information.

The discretization of the simulations is chosen dependent on its numerical difficulty. According to results in figures, if there is a step in parameters e and s 0.01, then the discretization chosen was 101×101 tiles. If the step is 0.02, the simulation was made on 51×51 tiles. The last one possibility used is the step 0.05 in case of the most difficult calculations used on the 21×21 tiles, which was used if there would be too many iterations otherwise.

4.1.1 The choice of a two-dimensional space

There is a lot of different space shapes that could be supposed. To enumerate some of the most useful we can consider for example an equilateral triangle, a square, a hexagon or a circle. There is no specific reason why one of these shapes (or a different one) should be preferred. However, in order to match the two-dimensional simulations with the theoretical background, which we investigated in one dimension formerly, we stick to a square shaped two-dimensional space. The reason to choose a square arises from the existence of both analogies to one dimensional simulations (a line segment as well as a circle mapped onto a line segment). If we supposed the two opposite edges passable (i.e. if player moves through one edge, he will instantaneously "jump" onto the opposite edge), it would be similar to the one dimensional case of a circle space.

It arises a question whether to suppose both pairs of edges passable or leave one of them impassable. However, the space with both all passable edges is relatively unrealistic. If one imagined such a space with connected edges in three dimensions, it can be geometrically understood as a torus which is at least in natural resource economics probably unusable.

The more realistic space is the space with connected only one pair of edges, which can be understood as a space where some of the distribution characteristics depend only on the radial direction from some central point (e.g. harbour) but some other characteristics are not and alternate as the distance from the point changes.

The last choice is probably the most realistic space as it can be simply used for any natural resource which has a spatial distribution and can be depicted on a two-dimensional map. Therefore, this is the space which will be assumed from this point on in the simulations.

4.1.2 Agents' range

For all simulations the assumption $e = e_1 = e_2$ which held throughout almost the whole previous chapter (except the uniform distribution) will be assumed. However, it is ambiguous what shape should this range parameter cover in a two-dimensional space. The two most logical shapes are a square and a circle, that means the parameter e could specify player's range either radially (i.e. the greater the parameter is, the greater the player's the radial range is) or squarely (i.e. player's coordinates x and y are ranging from $x - e$ to $x + e$ or $y - e$ to $y + e$, respectively).

As the two-dimensional space that is chosen is square, it would make sense to suppose also square range, however, the radial range makes sense too as it can describe symmetrical characteristics of players' harvest and seems more realistically. Therefore, the parameter e is an exogenous parameter describing player's radial range.

4.1.3 Agent heuristics

A very important part of the simulations refers to the players' choice of strategies. In the scenario, where we assume imperfect information about the density of the natural resource, it is crucial to determine what the players' information is. Similarly as with the range issue, also here can be a variety of choices what is agents' information. We chose as an imperfect information scheme the scheme where players have a restricted visibility over the natural resource. However, their visibility can vary and is described by parameter s. This parameter is supposed radial very similarly as the parameter e.

Basically that means the players see the same amount of information about the density in any direction. Therefore, both exploitation and observation depend on Euclidean distance, where the observation range naturally exceeds exploitation range (i.e. $s \geq e$). As an intuition, the parameter s could represent for example an effective range of sonar (which could be used for finding large fish stocks). Another analogy could be for example an existence of a fog which exists in areas which are too far and therefore players do not know what the density is out of their visibility range and therefore they must predict it.

The prediction of the density in areas where the players do not have any information could be chosen in various ways, for example by extrapolating the unknown values from the information the agents have. Other heuristics can be that the density is predicted by extending the density at the boundary between known and unknown density. In this work we stick to this kind of prediction and therefore the agents extends the most distant value, which is still observable for them, in all direction in all remaining points with unknown density.

If it is supposed the predicted point has coordinates $[x, y]$ and the distance d from the point is the Euclidean distance defined as $d = \sqrt{(x_i - x)^2 + (y_i - y)^2}$ where x_i and y_i are the components of vector $l_i = (x_i, y_i)$ and describe i-th player's strategy (location) in both coordinates. The density function is then predicted outside the player's observation range by extending the density function that is known for him. Formally it can be derived for predicted density $g(x, y)$ as follows

$$g(x,y) = \begin{cases} f(x,y) & \text{if } d \leq s \\ f(x_i + r(x - x_i), y_1 + r(y - y_i)) & \text{else} \end{cases}, \tag{4.1}$$

where r is the ratio of parameter s to the distance d ($r = \frac{s}{d}$). The predicted part consists basically of the player's location point $[x_i, y_i]$ and the distance which must be added in horizontal and vertical direction to get to the point which is on the borderline of the circle which is the furthest but still observable for the i-th player.

In one-dimensional case it reduces to extending the last point visible to player at both sides to all unknown points. Formally it reduces to:

$$g(x) = \begin{cases} f(x_i) & \text{if } |x - x_i| \leq s \\ f(x_i + r(x - x_i) & \text{else} \end{cases}, \tag{4.2}$$

where r is the ratio of parameter s to the distance d which is this time defined as $d = |x_i - x|$.

The heuristics is slightly different for a circle space scenario, where we suppose the player connects the boundary values of the observable density function in the unobserved points linearly.

4.2 One player simulations in one dimension

First, we start with the most simple simulations in one dimension. These simulations are aimed to support the theoretical results from the previous chapter and also to find out what the incentives of the player are to buy additional information about the density function.

The most simple distribution that can be investigated is the uniform distribution (as was done in the theoretical chapter above). However, the results are not very interesting for the examination of the agent's heuristics because no matter how much information the player buys, the prediction will be under our algorithm of prediction always correct and will not result in any change of behaviour in comparison to the case where no information is bought.

Nevertheless, the distribution with the linear density function was examined in the theoretical chapter, too. This distribution can be more interesting in predicting the density function as it is not constant.

Let us denote x^* the player's location after he adjusts to the predicted most efficient location. Similarly as in the theoretical part, we suppose increasing linear density function $f(x) = 2(1-b)x + b$, where $b \in [0, 1)$.

Due to the fact that the observation range is s, the player will always predict the distribution correctly in the interval $[x_0 - s, x_0 + s]$, where x_0 is player's initial strategy variable (location). In the remaining intervals he will predict the density by extending the density in the boundary point to the rest of the intervals. If we denote $g(x)$ the predicted density, it follows that

$$g(x) = \begin{cases} f(x_0 - s) = 2(1-b)(x_0 - s) + b & \text{if } 0 \le x < x_0 - s \\ f(x) = 2(1-b)x + b & \text{if } x_0 - s \le x \le x_0 + s \\ f(x_0 + s) = 2(1-b)(x_0 + s) + b & \text{if } x_0 + s < x \le 1 \\ 0 & \text{else} \end{cases} . \qquad (4.3)$$

As the function $g(x)$ is maximal and constant in the interval $x \in [x_0 + s, 1]$, the player is indifferent between the choice of points in interval $[x_0 + s + e, 1 - e]$ where he maximizes his payoff h_1. There are two possibilities in change of player's strategy after the prediction.

First, let assume that player's initial strategy x_0 is far enough from the point $x = 1$, that means there is a higher predicted payoff for the player if he shifts towards the point $x = 1$. Second, the player is already close enough to the point $x = 1$ and hence his prediction cannot help him to make any better decision than to be as close to the corner as possible.

The player in this scenario does not have any incentive to converge towards the point $x = 0$ as the predicted density function $g(x)$ is upward sloping, and thus non-increasing in that direction. If we set, as was already said earlier, x^* the agent's strategy (location) with the highest possible predicted payoff (and in case of multiple maxima we pick up the nearest one), then the agent's optimal location x^* can be rewritten as a function of player's initial position x_0

$$x^*(x_0) = \begin{cases} x_0 + s + e & \text{if } 0 \le x_0 < 1 - 2e - s \\ 1 - e & \text{if } 1 - 2e - s \le x_0 \le 1 \end{cases} . \qquad (4.4)$$

The agent's location adjustment for the initial point $x_0 = 0.5$ is depicted in the figure 4.1 for various parameters of e and s.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.54																								
0.04	0.56	0.58																							
0.06	0.58	0.60	0.62																						
0.08	0.60	0.62	0.64	0.66																					
0.10	0.62	0.64	0.66	0.68	0.70																				
0.12	0.64	0.66	0.68	0.70	0.72	0.74																			
0.14	0.66	0.68	0.70	0.72	0.74	0.76	0.78																		
0.16	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82																	
0.18	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.82																
0.20	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.84	0.82	0.80															
0.22	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.84	0.82	0.80	0.78														
0.24	0.76	0.78	0.80	0.82	0.84	0.86	0.86	0.84	0.82	0.80	0.78	0.76													
0.26	0.78	0.80	0.82	0.84	0.86	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74												
0.28	0.80	0.82	0.84	0.86	0.88	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72											
0.30	0.82	0.84	0.86	0.88	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70										
0.32	0.84	0.86	0.88	0.90	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68									
0.34	0.86	0.88	0.90	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66								
0.36	0.88	0.90	0.92	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64							
0.38	0.90	0.92	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62						
0.40	0.92	0.94	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60					
0.42	0.94	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60	0.58				
0.44	0.96	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60	0.58	0.56			
0.46	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60	0.58	0.56	0.54		
0.48	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60	0.58	0.56	0.54	0.52	
0.50	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60	0.58	0.56	0.54	0.52	0.50

Figure 4.1: Player's location adjustments x^* if the initial point $x_0 = 0.5$

Source: Author's computation

The importance of the additional information is measured by players' incentives to buy it. The incentives $I(x_0)$ to buy some information given the initial location x_0 are defined as:

$$I(x_0) = (F(x^*(x_0) + e) - F(x^*(x_0) - e)) - (F(x_0 + e) - F(x_0 - e)), \quad (4.5)$$

where $x^*(x_0)$ is the agent's location after he adjusts from his initial point x_0 to predicted the most efficient location, and $F(x) = (1-b)x^2 + bx$ is player's cumulative distribution function as follows from the section in theoretical chapter referring to the case with linear density function. Therefore the equation (4.5) can be rearranged

$$\begin{aligned} I(x_0) &= (1-b)((x^*(x_0)+e)^2 - (x^*(x_0)-e)^2) + b((x^*(x_0)+e) - (x^*(x_0)-e)) - \\ &- ((1-b)((x_0+e)^2 - (x_0-e)^2) + b((x_0+e) - (x_0-e))) = \\ &= 4e(1-b)(x^*(x_0) - x_0). \end{aligned} \quad (4.6)$$

Let us denote TI the total incentives player has to buy additional information given specific parameter e. Formally they can be defined as

$$TI = \int_{e}^{1-e} I(x_0)dx_0 = \int_{e}^{1-e} 4e(1-b)(x^*(x_0) - x_0)dx_0. \qquad (4.7)$$

These incentives can be understood as a difference in payoff between the case where the player is predicting his optimal location and the case where he is not and leaves his location unchanged. The initial location x_0 can be chosen purely random from the interval $[e, 1-e]$ (the corner cases were not included because we suppose the player would naturally not choose such strategies). The incentives $I(x_0)$ are summed by the integral over an interval of possible initial positions in order to count for all the random cases that can happen for specific parameters e and s (in simulations, the integral is discretized into a sum).

If we substitute eq.(4.4) into eq.(4.7) we can obtain

$$\begin{aligned} TI &= \int_{e}^{1-2e-s} 4e(1-b)(x^*(x_0) - x_0)dx + \int_{1-2e-s}^{1-e} 4e(1-b)(x^*(x_0) - x_0)dx_0 = \\ &= \int_{e}^{1-2e-s} 4e(1-b)(s+e)dx_0 + \int_{1-2e-s}^{1-e} 4e(1-b)(1-e-x_0)dx_0 = \\ &= 2e(s+e)(2-5e-s)(1-b). \end{aligned} \qquad (4.8)$$

However, this equation holds only in case of $e \leq 1 - 2s - e$. Otherwise, the resulting incentives are defined only as the second integral in eq.(4.8) from e to $1-e$ and the result is no longer dependent on the parameter s.

$$\begin{aligned} TI &= \int_{e}^{1-e} 4e(1-b)(x^*(x_0) - x_0)dx_0 = \int_{e}^{1-e} 4e(1-b)(1-e-x_0)dx_0 = \\ &= 4e(\frac{1}{2} - 2e + 2e^2)(1-b) \end{aligned} \qquad (4.9)$$

The resulting total player's incentives for various parameters e and s are depicted in the figure 4.2.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.00																								
0.04	0.01	0.01																							
0.06	0.01	0.02	0.03																						
0.08	0.01	0.02	0.03	0.04																					
0.10	0.01	0.02	0.03	0.05	0.06																				
0.12	0.01	0.03	0.04	0.05	0.07	0.08																			
0.14	0.02	0.03	0.04	0.06	0.07	0.08	0.10																		
0.16	0.02	0.03	0.05	0.06	0.07	0.09	0.10	0.11																	
0.18	0.02	0.03	0.05	0.06	0.08	0.09	0.11	0.12	0.12																
0.20	0.02	0.04	0.05	0.07	0.08	0.10	0.11	0.12	0.13	0.13															
0.22	0.02	0.04	0.06	0.07	0.09	0.10	0.11	0.12	0.13	0.14	0.14														
0.24	0.03	0.04	0.06	0.08	0.09	0.11	0.12	0.13	0.13	0.14	0.14	0.13													
0.26	0.03	0.04	0.06	0.08	0.10	0.11	0.12	0.13	0.14	0.14	0.14	0.13	0.12												
0.28	0.03	0.05	0.07	0.08	0.10	0.11	0.13	0.14	0.14	0.14	0.14	0.13	0.12	0.11											
0.30	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.14	0.15	0.14	0.13	0.12	0.11	0.10										
0.32	0.03	0.05	0.07	0.09	0.11	0.12	0.13	0.14	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09									
0.34	0.03	0.05	0.07	0.09	0.11	0.12	0.14	0.14	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07								
0.36	0.03	0.06	0.08	0.10	0.11	0.13	0.14	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06							
0.38	0.04	0.06	0.08	0.10	0.12	0.13	0.14	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05						
0.40	0.04	0.06	0.08	0.10	0.12	0.13	0.14	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03					
0.42	0.04	0.06	0.08	0.10	0.12	0.13	0.15	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03	0.02				
0.44	0.04	0.06	0.09	0.11	0.12	0.14	0.15	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03	0.02	0.01			
0.46	0.04	0.07	0.09	0.11	0.13	0.14	0.15	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03	0.02	0.01	0.01		
0.48	0.04	0.07	0.09	0.11	0.13	0.14	0.15	0.15	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03	0.02	0.01	0.01	0.00	
0.50	0.04	0.07	0.09	0.11	0.13	0.14	0.15	0.16	0.15	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.07	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00

Figure 4.2: Player's incentives to adjust to the prediction instead of remaining on the initial random point for various parameters e and s

Source: Author's computation

To find for what parameters e,s the player has maximal incentives to buy information the first order conditions must hold. Given fixed parameter b, the highest total incentives TI the player has if both following terms are equal to zero

$$\frac{\partial TI(e,s)}{\partial e} = 2(1-b)(2s + 4e - 12es - 15e^2 - s^2), \tag{4.10}$$

$$\frac{\partial TI(e,s)}{\partial s} = 4e(1-b)(1 - 3e - s). \tag{4.11}$$

If both derivatives are equal to zero, by solving the system of two equations we can obtain the following results (Assuming $e > 0$,$b \in [0, 1)$). There are two possible extremes, the first one is $[e, s] = [\frac{1}{6}, \frac{1}{2}]$, the second extreme is $[e, s] = [\frac{1}{2}, -\frac{1}{2}]$ which obviously does not have any tangible interpretation as it has been assumed that the parameter s is at least as large as parameter e.

The numerical simulations were run for the setup, where the density function was specially chosen $f(x) = 2x$ (parameter b was set 0). This choice was done because the values of the function $TI(e, s)$ describing incentives is only amplified proportionally by changing the parameter b (it can be seen from the functional form of eq.(4.8) and (4.9)). The line segment space was discretized into 51 tiles, where each tile represented a point with its specific density function. This was simulated for different parameters e and s ranging from 0.02 to

0.50, with step 0.02 matching the condition $s \geq e$. The results of the simulations are depicted in the figure 4.2. It is noticeable that the simulations are matching the theory outlined above perfectly. The highest incentives namely occur approximately for the parameter $e \approx 0.16$ which is the nearest discretized value to the theoretical maximal value of e which is as was stated earlier $e = \frac{1}{6}$.

Similar approach is done in case of density function

$$f(x) = a \sin(2\pi kx) + 1, \tag{4.12}$$

where $a \in (0, 1]$ and $k \in \mathbb{N}$. This density was also examined in the theoretical chapter. This distribution is examined on a circle space where we specify the agents' heuristics in a way that the density predicted in an unknown point is given by weighted average of the two furthest boundary points which the agent can still observe. The weights are given proportionally according to distance to the predicted point from the boundary points.

As a first scenario we suppose one-player in one dimensional circle space whose location is given in interval $[0, 1)$. Based on value of the parameter s, agent then makes prediction about the density function. After this is done, the strategy (location) of the agent is changed immediately to the point where the payoff is the highest. This procedure is very similar to what has been already examined in this section in case of distribution with linear density function. At first we assume the density function parameters to be $a = 1, k = 1$. These parameters are later altered to show also their effect.

The total incentives to buy information given by parameter s are, however, in this case given by summing over incentives at all possible initial locations as there are no corners (not only over $[e, 1 - e]$)

$$TI = \int_0^1 I(x_0) dx_0, \tag{4.13}$$

where $I(x_0)$ are the incentives to buy additional information defined according to equation (4.5) with cumulative distribution function of this "sine" density and x_0 are possible initial locations. The highest total incentives were found by numerical simulations in case of $e \approx 0.24$ and $s \approx 0.50$. For various parameters e and s the results are depicted in figure 4.3.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.01																								
0.04	0.01	0.03																							
0.06	0.02	0.04	0.06																						
0.08	0.02	0.04	0.07	0.09																					
0.10	0.03	0.05	0.07	0.10	0.13																				
0.12	0.03	0.05	0.08	0.11	0.14	0.16																			
0.14	0.03	0.06	0.09	0.12	0.14	0.17	0.20																		
0.16	0.04	0.06	0.09	0.12	0.15	0.18	0.21	0.23																	
0.18	0.04	0.07	0.1	0.13	0.16	0.19	0.22	0.24	0.26																
0.20	0.04	0.07	0.11	0.14	0.17	0.20	0.22	0.25	0.27	0.28															
0.22	0.05	0.08	0.11	0.14	0.17	0.21	0.23	0.26	0.28	0.29	0.30														
0.24	0.05	0.08	0.12	0.15	0.18	0.21	0.24	0.26	0.28	0.30	0.31	0.31													
0.26	0.05	0.09	0.12	0.15	0.19	0.22	0.24	0.27	0.29	0.30	0.31	0.31	0.31												
0.28	0.05	0.09	0.12	0.16	0.19	0.22	0.25	0.27	0.29	0.30	0.31	0.31	0.31	0.30											
0.30	0.05	0.09	0.13	0.16	0.19	0.22	0.25	0.27	0.29	0.31	0.31	0.32	0.31	0.31	0.29										
0.32	0.06	0.09	0.13	0.16	0.20	0.23	0.25	0.28	0.29	0.31	0.31	0.32	0.31	0.31	0.29	0.28									
0.34	0.06	0.09	0.13	0.17	0.20	0.23	0.26	0.28	0.29	0.31	0.31	0.32	0.31	0.31	0.29	0.28	0.26								
0.36	0.06	0.10	0.13	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.31	0.32	0.31	0.31	0.30	0.28	0.26	0.23							
0.38	0.06	0.10	0.13	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.31	0.32	0.31	0.31	0.30	0.28	0.26	0.23	0.20						
0.40	0.06	0.10	0.13	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.31	0.31	0.30	0.28	0.26	0.23	0.20	0.17					
0.42	0.06	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.31	0.31	0.30	0.28	0.26	0.23	0.20	0.17	0.14				
0.44	0.06	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.32	0.31	0.30	0.28	0.26	0.23	0.20	0.17	0.14	0.10			
0.46	0.06	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.32	0.31	0.30	0.28	0.26	0.23	0.20	0.17	0.14	0.10	0.06		
0.48	0.06	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.32	0.31	0.30	0.28	0.26	0.23	0.20	0.17	0.14	0.10	0.06	0.02	
0.50	0.06	0.10	0.14	0.17	0.20	0.23	0.26	0.28	0.30	0.31	0.32	0.32	0.32	0.31	0.30	0.28	0.26	0.23	0.20	0.17	0.14	0.10	0.06	0.02	0.00

Figure 4.3: Sine density: Agent's total incentives TI given fixed density parameters $a = 1, k = 1$ and various parameters e and s

Source: Author's computation

In order to show how well the player predicts the location x^*, the figure 4.4 is depicted. As according to the functional form of the density function the maximal density is in point $x = 0.25$, the player will in ideal case tend to move towards this point. In the mentioned figure, it is apparent that only for low e and s the ideal location x^* is not predicted well. Note, however, that the figure 4.4 shows location adjustments only for initial position $x_0 = 0.5$, therefore there might exist also initial positions (e.g. minimum $x = 0.75$) where the prediction adjustment of agent is significant for more combinations of parameters e and s.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.46																								
0.04	0.44	0.42																							
0.06	0.42	0.40	0.38																						
0.08	0.40	0.38	0.38	0.36																					
0.10	0.38	0.38	0.36	0.34	0.34																				
0.12	0.36	0.36	0.34	0.34	0.32	0.32																			
0.14	0.34	0.34	0.34	0.32	0.32	0.30	0.30																		
0.16	0.34	0.32	0.32	0.32	0.30	0.30	0.30	0.28																	
0.18	0.32	0.32	0.32	0.30	0.30	0.30	0.28	0.28	0.28																
0.20	0.30	0.30	0.30	0.30	0.30	0.28	0.28	0.28	0.26	0.26															
0.22	0.28	0.30	0.28	0.28	0.28	0.28	0.28	0.28	0.26	0.26	0.26														
0.24	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.26	0.26	0.26	0.26	0.24													
0.26	0.26	0.26	0.28	0.28	0.28	0.28	0.26	0.26	0.26	0.26	0.26	0.26	0.24												
0.28	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24											
0.30	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24										
0.32	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24									
0.34	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24								
0.36	0.24	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24							
0.38	0.24	0.24	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24						
0.40	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24					
0.42	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24				
0.44	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.26	0.26	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24			
0.46	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.26	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24		
0.48	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	
0.50	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

Figure 4.4: Sine density: Agent's location adjustments x^* if the initial point $x_0 = 0.5$

Source: Author's computation

The parameter a from the theoretical chapter lies in the interval (0,1] where if we admitted to be equal to zero, the distribution would change to the uniform distribution, which we already discussed earlier. Let us now suppose the parameter is not $a = 1$ but $a = 0.5$. A numerical computation is made to find out, how the incentives of the agent change in comparison to the case where $a = 1$. The results of the simulations are depicted in the figure 4.5.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.00																								
0.04	0.01	0.01																							
0.06	0.01	0.02	0.03																						
0.08	0.01	0.02	0.03	0.05																					
0.10	0.01	0.02	0.04	0.05	0.06																				
0.12	0.02	0.03	0.04	0.05	0.07	0.08																			
0.14	0.02	0.03	0.04	0.06	0.07	0.09	0.10																		
0.16	0.02	0.03	0.05	0.06	0.08	0.09	0.10	0.11																	
0.18	0.02	0.03	0.05	0.06	0.08	0.09	0.11	0.12	0.13																
0.20	0.02	0.04	0.05	0.07	0.08	0.10	0.11	0.12	0.13	0.14															
0.22	0.02	0.04	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.14	0.15														
0.24	0.02	0.04	0.06	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.15													
0.26	0.03	0.04	0.06	0.08	0.09	0.11	0.12	0.13	0.14	0.15	0.15	0.15	0.15												
0.28	0.03	0.04	0.06	0.08	0.09	0.11	0.12	0.13	0.14	0.15	0.15	0.16	0.15	0.15											
0.30	0.03	0.05	0.06	0.08	0.10	0.11	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.14										
0.32	0.03	0.05	0.06	0.08	0.10	0.11	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14									
0.34	0.03	0.05	0.07	0.08	0.10	0.11	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13								
0.36	0.03	0.05	0.07	0.08	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.11							
0.38	0.03	0.05	0.07	0.08	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10						
0.40	0.03	0.05	0.07	0.08	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.08					
0.42	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.08	0.07				
0.44	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.08	0.07	0.05			
0.46	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.09	0.07	0.05	0.03		
0.48	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.09	0.07	0.05	0.03	0.01	
0.50	0.03	0.05	0.07	0.09	0.10	0.12	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.15	0.15	0.14	0.13	0.12	0.10	0.09	0.07	0.05	0.03	0.01	0.00

Figure 4.5: Sine density: Agent's total incentives TI given fixed density parameter $a = 0.5, k = 1$ and various parameters e and s

Source: Author's computation

It is apparent that the incentives for the agent decreased. Moreover, they decreased by half in comparison to the scenario where $a = 1$. This might be interesting but not surprising because if the parameter a decreased down to the zero, the distribution would be uniform, and there would be no incentives to buy any parameter s. Hence, the parameter a has a proportional function i.e. the higher the parameter a is, the higher are the incentives to buy information about the distribution. This is very intuitive as the parameter a essentially describes the concentration of the resource at one place ($x = 0.25$). As was explained formerly in the theoretical chapter, due to non-existence of corners, the distribution could be shifted to move the maximum, where the resource concentrates, also to different point than $x = 0.25$.

In the next figure (Fig. 4.6) it is simulated what the agent's incentives to buy information s are if there is no single peak but there are two of them ($k = 2$).

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.02																								
0.04	0.03	0.05																							
0.06	0.03	0.06	0.09																						
0.08	0.04	0.07	0.1	0.12																					
0.10	0.05	0.08	0.11	0.13	0.14																				
0.12	0.05	0.09	0.12	0.14	0.15	0.14																			
0.14	0.05	0.09	0.12	0.14	0.15	0.14	0.12																		
0.16	0.06	0.09	0.12	0.14	0.15	0.14	0.12	0.09																	
0.18	0.06	0.09	0.12	0.14	0.14	0.14	0.12	0.09	0.06																
0.20	0.06	0.09	0.12	0.14	0.14	0.13	0.11	0.08	0.05	0.03															
0.22	0.06	0.09	0.12	0.13	0.14	0.13	0.11	0.08	0.05	0.03	0.01														
0.24	0.06	0.09	0.12	0.13	0.13	0.12	0.1	0.08	0.05	0.02	0.01	0.00													
0.26	0.06	0.09	0.12	0.13	0.14	0.13	0.11	0.08	0.05	0.03	0.01	0.00	0.01												
0.28	0.06	0.09	0.12	0.14	0.14	0.13	0.12	0.09	0.06	0.03	0.01	0.01	0.01	0.03											
0.30	0.06	0.09	0.12	0.14	0.15	0.14	0.13	0.1	0.07	0.04	0.02	0.01	0.02	0.04	0.07										
0.32	0.06	0.09	0.12	0.14	0.15	0.15	0.13	0.11	0.08	0.05	0.02	0.01	0.02	0.05	0.08	0.11									
0.34	0.06	0.09	0.12	0.14	0.15	0.15	0.14	0.12	0.09	0.06	0.03	0.01	0.03	0.06	0.09	0.11	0.13								
0.36	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.12	0.1	0.07	0.03	0.02	0.03	0.06	0.10	0.12	0.14	0.15							
0.38	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.1	0.07	0.03	0.02	0.03	0.07	0.10	0.13	0.15	0.16	0.16						
0.40	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.07	0.04	0.02	0.04	0.07	0.10	0.13	0.15	0.16	0.16	0.14					
0.42	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.07	0.04	0.02	0.04	0.07	0.11	0.13	0.15	0.16	0.16	0.14	0.12				
0.44	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.08	0.04	0.02	0.04	0.08	0.11	0.13	0.15	0.16	0.16	0.14	0.12	0.09			
0.46	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.08	0.04	0.02	0.04	0.08	0.11	0.13	0.15	0.16	0.16	0.14	0.12	0.09	0.06		
0.48	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.08	0.04	0.02	0.04	0.08	0.11	0.13	0.15	0.16	0.16	0.14	0.12	0.09	0.06	0.02	
0.50	0.06	0.09	0.12	0.14	0.16	0.16	0.15	0.13	0.11	0.08	0.04	0.02	0.04	0.08	0.11	0.13	0.15	0.16	0.16	0.14	0.12	0.09	0.06	0.02	0.00

Figure 4.6: Sine density: Agent's total incentives given fixed density parameter $a = 1, k = 2$ and various parameters e and s

Source: Author's computation

In this setup, the agent has the highest incentives at two different points in parametric space (e, s). The first one has the coordinates $[e, s] \approx [0.12, 0.5]$. The second one has them $[e, s] \approx [0.36, 0.5]$. These are probably not the peaks themselves precisely because of the numerical discretization. If there was a higher number of tiles in discretization, the resulting highest incentives would be probably described by the parameters e which are exactly as high to cover the highest part of either one peak or both two peaks of the density function $(e = 0.125, e = 0.375)$. Furthermore, the minimal incentives are in case of $[e, s] \approx [0.24, 0.24]$. This is again intuitive because if $e = 0.25$, then the player should be indifferent between his strategies, as his harvest will cancel out and will harvest always a half of the resource $h_1 = 0.5$.

Very interesting point in (e, s) parametric space is also point with coordinates $[e, s] \approx [0.10, 0.14]$ which is local maximum (not global). The reason this maximum exists is that there are lower incentives generally for $s \approx 0.24$, which is probably caused by the fact that if $s = 0.25$ the agent would predict all remaining points with the correct average expected harvest. However, if the agent predicted all points correctly, his incentives to buy information represented by parameter s are lower and therefore this local maximum is separated from the global maximum by a saddle point of lower incentives located approximately at $[e, s] \approx [0.10, 0.24]$. The parametric spaces (e, s) for density functions with higher parameter k would probably look very similarly as this one, however, the higher the parameter k is, the better discretization is needed to make the differences between the points with low and high incentives still observable.

4.3 Two player simulations in one dimension

In the theoretical chapter, we analyzed predominantly the situation where there were two agents. Let us now suppose again the situation with the linear density function: $f(x) = 2(1-b)x + b$ (in simulations assuming $b = 0$). The numerical simulations for the setup with two player are made with the following characteristics.

In the beginning, the first agent chooses location randomly and then the location is chosen also for the second agent. After they set their initial location, the second player will predict in one stage the best optimal response on the location of the first agent under limited information given the parameter s. The payoff is then compared between the player who adjusted the strategy and the player who remained inactive. The difference of their payoffs is depicted in the Fig. 4.7.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.00																								
0.04	0.01	0.01																							
0.06	0.01	0.02	0.03																						
0.08	0.01	0.02	0.03	0.04																					
0.10	0.01	0.02	0.03	0.04	0.05																				
0.12	0.01	0.03	0.04	0.05	0.06	0.06																			
0.14	0.02	0.03	0.04	0.05	0.06	0.07	0.07																		
0.16	0.02	0.03	0.04	0.06	0.06	0.07	0.07	0.07																	
0.18	0.02	0.03	0.05	0.06	0.07	0.07	0.07	0.07	0.07																
0.20	0.02	0.04	0.05	0.06	0.07	0.08	0.08	0.08	0.07	0.07															
0.22	0.02	0.04	0.05	0.06	0.07	0.08	0.08	0.08	0.08	0.07	0.07														
0.24	0.02	0.04	0.06	0.07	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.06													
0.26	0.03	0.04	0.06	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.06												
0.28	0.03	0.04	0.06	0.07	0.08	0.08	0.09	0.09	0.08	0.08	0.08	0.07	0.06	0.05											
0.30	0.03	0.05	0.06	0.07	0.08	0.09	0.09	0.09	0.09	0.08	0.08	0.07	0.06	0.05	0.04										
0.32	0.03	0.05	0.06	0.08	0.08	0.09	0.09	0.09	0.09	0.08	0.08	0.07	0.06	0.05	0.04	0.03									
0.34	0.03	0.05	0.07	0.08	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.07	0.06	0.05	0.04	0.03	0.02								
0.36	0.03	0.05	0.07	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02							
0.38	0.03	0.05	0.07	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01						
0.40	0.04	0.06	0.07	0.08	0.09	0.09	0.10	0.10	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01					
0.42	0.04	0.06	0.07	0.09	0.09	0.10	0.10	0.10	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.00				
0.44	0.04	0.06	0.08	0.09	0.09	0.10	0.10	0.10	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.00	0.00			
0.46	0.04	0.06	0.08	0.09	0.10	0.10	0.10	0.10	0.09	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.00	0.00	0.00		
0.48	0.04	0.06	0.08	0.09	0.10	0.10	0.10	0.10	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	
0.50	0.04	0.06	0.08	0.09	0.10	0.10	0.10	0.10	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00

Figure 4.7: Linear density: Agent's total incentives given fixed density parameter $b = 1$ and various parameters e and s
Source: Author's computation

It might seem that nothing has changed in the parametric space (e, s) in comparison to the case with one player (see Fig. 4.2), nevertheless, there is a slight difference. The difference in the payoff incentives of these two cases (one player v. two players) is depicted in the Fig. 4.8.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.00																								
0.04	0.00	0.00																							
0.06	0.00	0.00	0.00																						
0.08	0.00	0.00	0.00	0.00																					
0.10	0.00	0.00	0.00	0.00	0.01																				
0.12	0.00	0.00	0.00	0.00	0.01	0.02																			
0.14	0.00	0.00	0.00	0.00	0.01	0.02	0.03																		
0.16	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.04																	
0.18	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05																
0.20	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06															
0.22	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.06	0.07														
0.24	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.07													
0.26	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.07	0.07												
0.28	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.07	0.06	0.06											
0.30	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.07	0.06	0.06	0.06										
0.32	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.07	0.06	0.06	0.06	0.05									
0.34	0.00	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05								
0.36	0.00	0.00	0.01	0.01	0.02	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04							
0.38	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03						
0.40	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03					
0.42	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03	0.02				
0.44	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.01			
0.46	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.01	0.01		
0.48	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.01	0.01	0.00	
0.50	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.04	0.03	0.03	0.02	0.01	0.01	0.00	0.00

Figure 4.8: Linear density: Difference between the agents' total incentives in two players case and in one player case given fixed density parameter $b = 1$ and various parameters e and s

Source: Author's computation

This figure can be interpreted as the summary of where the difference between the one player scenario and the two players scenario is significant and where it is not. The highest difference is concentrated around the point $[e, s] \approx [0.24, 0.24]$. It can be understood as in case of both relatively low e and s the adjustment of the second player is not very dependent on the location of the first player. Similarly, in case of high e and s both players are covering almost the entire space and therefore, there is not much added value in high parameter s. Obviously, the most significant difference is in case of relatively middle-sized parameters e and s. This is due to the players do not yet necessarily know the optimal location without prediction but already harvest relatively large amount of the resource stock with likely intrusion in the competitor's area. It is also very important to note, that the difference in incentives to buy the information given by s may be also driven by impossibility of relocation of the second player, the results would be lower if he relocated if he had payoff below the average.

Exactly the same analysis is done in case of "sine" density function to compare the effect of the second player also in case of distributions, where there are no corners. In the figure 4.9, the incentives are depicted for the scenario, where the first player is inactive and the second player adjusts his position to maximize his payoff. The parameters a and k are now chosen equal to 1 for simplicity. To compare what the difference between two stage procedure with two players and one stage procedure with one player is, the differences in the total incentives take place in the figure 4.10.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.01																								
0.04	0.01	0.03																							
0.06	0.02	0.04	0.06																						
0.08	0.02	0.05	0.07	0.09																					
0.10	0.03	0.05	0.08	0.1	0.12																				
0.12	0.03	0.06	0.08	0.11	0.13	0.14																			
0.14	0.03	0.06	0.09	0.11	0.13	0.15	0.16																		
0.16	0.04	0.07	0.09	0.12	0.14	0.16	0.17	0.17																	
0.18	0.04	0.07	0.1	0.13	0.15	0.16	0.18	0.19	0.19																
0.20	0.04	0.07	0.1	0.13	0.15	0.17	0.19	0.2	0.21	0.21															
0.22	0.05	0.08	0.11	0.14	0.16	0.18	0.2	0.21	0.22	0.23	0.23														
0.24	0.05	0.08	0.11	0.14	0.17	0.19	0.2	0.22	0.23	0.24	0.25	0.25													
0.26	0.05	0.09	0.12	0.15	0.17	0.19	0.21	0.23	0.25	0.26	0.27	0.27	0.27												
0.28	0.05	0.09	0.12	0.15	0.17	0.2	0.22	0.24	0.26	0.27	0.28	0.29	0.29	0.29											
0.30	0.05	0.09	0.12	0.15	0.18	0.2	0.22	0.24	0.26	0.27	0.29	0.29	0.29	0.29	0.28										
0.32	0.06	0.09	0.13	0.15	0.18	0.2	0.23	0.25	0.27	0.28	0.29	0.30	0.3	0.3	0.28	0.27									
0.34	0.06	0.09	0.13	0.16	0.18	0.2	0.23	0.25	0.27	0.28	0.29	0.30	0.3	0.3	0.29	0.27	0.25								
0.36	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.27	0.28	0.29	0.30	0.3	0.3	0.29	0.27	0.25	0.23							
0.38	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.27	0.29	0.29	0.30	0.3	0.3	0.29	0.27	0.25	0.23	0.20						
0.40	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.27	0.29	0.29	0.31	0.31	0.3	0.29	0.27	0.25	0.23	0.20	0.17					
0.42	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.28	0.29	0.29	0.31	0.31	0.3	0.29	0.27	0.25	0.23	0.20	0.17	0.14				
0.44	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.28	0.29	0.29	0.31	0.31	0.3	0.29	0.28	0.25	0.23	0.20	0.17	0.14	0.1			
0.46	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.28	0.29	0.29	0.31	0.31	0.3	0.29	0.28	0.25	0.23	0.20	0.17	0.14	0.1	0.06		
0.48	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.28	0.29	0.29	0.31	0.31	0.3	0.29	0.28	0.25	0.23	0.20	0.17	0.14	0.1	0.06	0.02	
0.50	0.06	0.1	0.13	0.16	0.18	0.21	0.23	0.25	0.28	0.29	0.29	0.31	0.31	0.3	0.29	0.28	0.25	0.23	0.20	0.17	0.14	0.1	0.06	0.02	0.01

Figure 4.9: Sine density: Agent 2's total incentives given fixed density parameter $a = 1, k = 1$ and various parameters e and s

Source: Author's computation

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50
0.02	0.00																								
0.04	0.00	0.00																							
0.06	0.00	0.00	0.00																						
0.08	0.00	0.00	0.00	0.00																					
0.10	0.00	0.00	0.00	0.00	0.01																				
0.12	0.00	0.00	0.00	0.00	0.01	0.02																			
0.14	0.00	0.00	0.00	0.00	0.01	0.02	0.04																		
0.16	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.05																	
0.18	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.05	0.06																
0.20	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.05	0.06	0.07															
0.22	0.00	0.00	0.00	0.01	0.01	0.03	0.04	0.05	0.06	0.07	0.07														
0.24	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.04	0.05	0.06	0.06	0.06													
0.26	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.03												
0.28	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02											
0.30	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.01	0.01										
0.32	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.01	0.01	0.01									
0.34	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.02	0.03	0.02	0.02	0.01	0.01	0.01	0.01	0.00								
0.36	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00							
0.38	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00						
0.40	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00					
0.42	0.00	0.00	0.00	0.01	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00				
0.44	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			
0.46	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
0.48	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
0.50	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 4.10: Sine density: Difference between the agents' total incentives in two players case and in one player case given fixed density parameter $a = 1$ and various parameters e and s

Source: Author's computation

The difference is now the greatest in the proximity of point $[e, s] \approx [0.22, 0.22]$. In comparison to the case where the distribution was linear the difference between the one-player scenario and two-player scenario is now more concentrated in cases where $s \approx e$ (in Fig. 4.10 near the diagonal). This is given by the fact that if the agents are operating on the "sine" density, it is easier for them to locate the point where the density is the highest, as they can move both directions towards the peak. The highest difference can be seen in case where the second player can be most better off in comparison to the first player who did not adjust his position, i.e. in case of lower e and s than in case of linear distribution.

Further analysis of the 2 player scenarios involve also the analysis of the agents' optimal payoff if there is complete information about the density. The uniform and linear distribution have been already examined in the theoretical chapter. The most difficult one to analyze analytically is the last one broadly examined, namely "sine" distribution. The players are competing along the peak and do not necessarily have incentives to diverge when there is extra space (but with lower density).

In the theoretical chapter we examined that the competition arises when

$a > \frac{1}{3}$. In these cases the optimal strategy would be if both players cooperated. The competition is depicted in the Fig. 4.11, where the optimal locations of agents are denoted x_1 and x_2. The player one chooses his optimal location first, therefore chooses the location $x_1 = 0.25$ (in simulation 0.24, because of not sufficient discretization). The second player then makes his best response on player 1's strategy. The two players keep to alternate until the Nash equilibrium is found and they change their strategies no longer. Theoretically, there could be the case where there is no Nash equilibrium but this distribution is not the case. After this finite number of stages, the optimal location of both players is found.

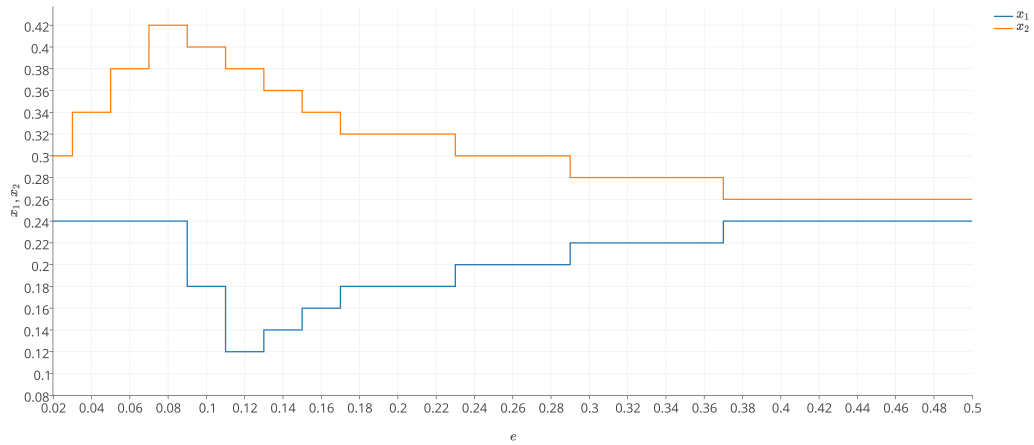

Figure 4.11: Sine density: Optimal locations of agents in Nash equilibrium (discretized)

Source: Author's computation

Note, that in an ideal case the locations in Fig. 4.11 are smooth and all imperfections are caused by the discretization into 50 tiles. It is interesting that for the small parameter e, the player 1's position remains unchanged and the player 2 accepts inferior strategy (in terms of payoff) and diverge from the player 1, however, as the e rises, the player 2 stops to accept the inferior location and the local stability conditions force him to enter a competition with player 1. If the parameter e is high enough, both players are harvesting at the peak ($x = 0.25$). The payoffs of the individual players and their difference are depicted in the Fig. 4.12 (The figure is discretized as well, however, smoothed additionally in order to get closer to theoretical outcomes). The greatest difference in players' payoffs is if the parameter $e \approx 0.12$ and then, as e rises, the both payoffs converge to each other.

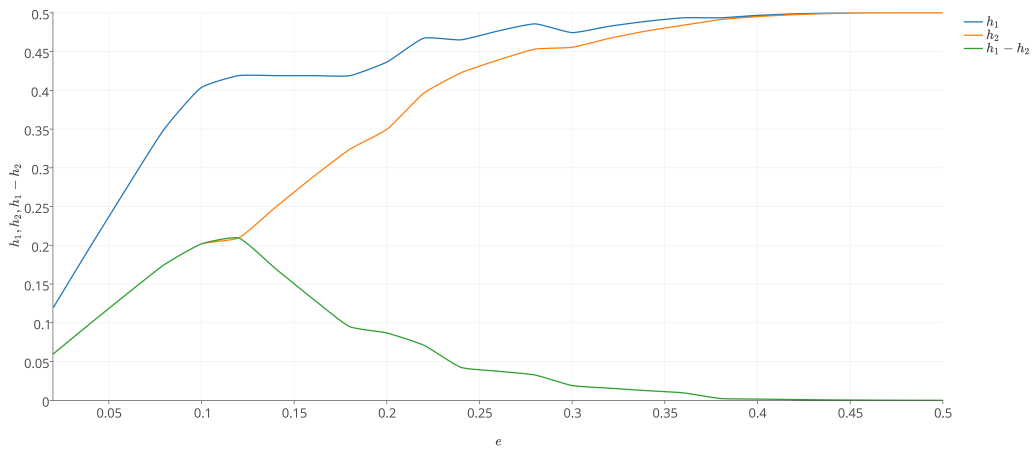

Figure 4.12: Sine density: Optimal payoffs of agents in Nash equilibrium (discretized and smoothed)
Source: Author's computation

4.4 One player simulations in two dimensions

Let us simulate the setup where one player chooses his location in a two-dimensional square space with impassable edges. First, the emphasis is put on the optimal case under perfect information. Second, the parameter s is introduced in the scenario where the player at first appears randomly in any point and then relocates to the point with predicted highest payoff.

In this section in two-dimensional simulations we suppose the density function of the resource the easiest possible for testing the prediction characteristics. From the theoretical part it is obvious that the most simple density function is linear. Such a distribution can be helpful as the agent with the agent heuristics which was introduced in the beginning of this chapter cannot predict the full information about the resource unless he has large parameter s and therefore the effect of s is noticeable.

In two dimensions the question what slope and what direction should the linear density function have. Although there is a huge number of possibilities and it is also possible e.g. to have a density function linear in one direction linear and in other different, which could also generate interesting results. We leave, however, this topic for possible extension of this work and focus more rigorously on the distribution growing in both directions, horizontally and vertically. The possible density function can be written as

$$f(x, y) = ax + by + c. \tag{4.14}$$

The overall density can normed again to area of 1

$$\int_0^1 \int_0^1 f(x)dx = 1, \tag{4.15}$$

which reduces number of free parameters to 2. In the numerical simulations covered by this thesis the only one case is examined, namely $f(x) = x + y$. The reason why to choose this one originates from the idea that in one-dimensional simulations it has been examined that the higher the slope, the higher the effects of the potential information asymmetry (or imperfection) which is amongst the objects of this study. One could argue that the distribution could increase only in one direction and in the second one could be uniform, however, that would be very similar to already analyzed scenario in one dimension and therefore not examined.

It is relatively clear that the player maximizes his payoff by choosing the location as much close as possible to the corner with maximal density. However, as the edges are impassable, the player would not get any reward if stepped away from the square space. On the other hand, the concept of circle exploitation range can be slightly tricky and give incentives to the player to step away in order to get closer to the highest density points in the corner. Note that if the player's exploitation parameter e affected the agents range "squarely" and not radially, the rational agent would not leave the square of coordinates ranging in $[e, 1 - e] \times [e, 1 - e]$.

However, even with the radial parameter e there is a rational bound above which the player will not go. If the player is as close to the corner as possible, the Euclidean distance from the corner would still be e which corresponds to the point $[1 - \frac{\sqrt{2}}{2}e, 1 - \frac{\sqrt{2}}{2}e]$.

The optimal location can be found analytically, however, it is quite laborious and not very important, therefore we skip it and the ideal location is found numerically. The ideal location for the player given the parameter $e = 0.2$ is depicted in the Fig. 4.13, where the density function is depicted in a grid for the points distanced 0.05 far from each other. It is considerable that the player chooses rather to cut part of his payoff in order to get more closer to the corner where the density is the highest. The exact payoff and agent's coordinates are depicted in Fig. 4.13.

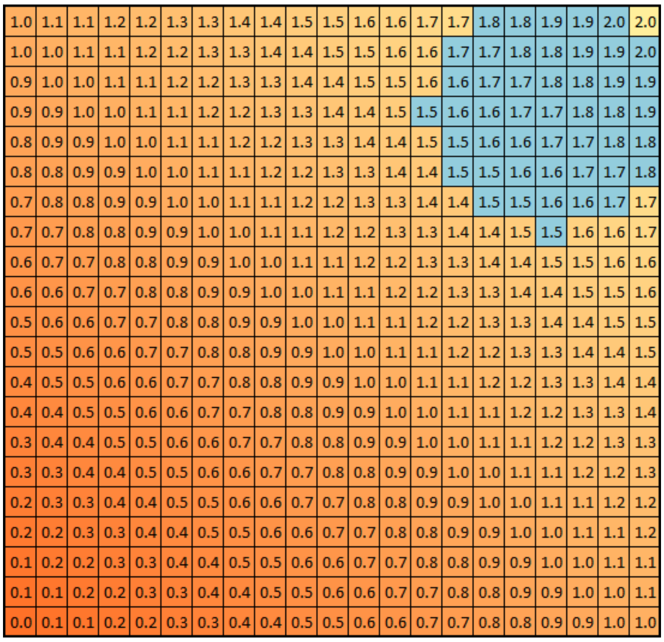

Figure 4.13: Two-dimensional density function with optimal agent's position at $[x, y] = [0.85, 0.85]$ for parameter $e = 0.2$

Source: Author's computation

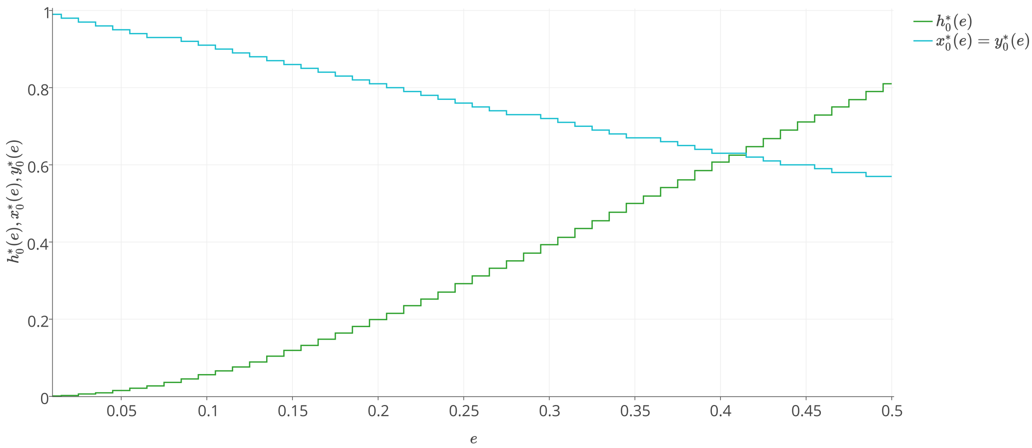

Figure 4.14: Agent's optimal position and payoff in a one player scenario depending on parameter e

Source: Author's computation

It is noticeable that the player gives up some tiles in favour of boosting the payoff by exploiting the tiles closer to the corner with the highest density. The frequency of this trade-off (exchanging tiles for higher payoff by shifting beyond the edges of the square space) rises as the parameter e grows. In the Fig. 4.14 there are points where the strategy variable x_0^* describing agent's optimal location does not change when the parameter e rises which is an indication of such a trade-off happening.

The highest payoff for the agent, who is not restricted by any information inefficiency, should be theoretically growing quadratically with increasing parameter e (as the area harvested depends on πe^2), however, due to the effect which was described in the previous paragraph, the agent will not operate in the interior of the space and will maximize the payoff by moving towards the corner as far as it is gainful. This effect slows the quadratic growth of the payoff and for large e it is more similar to linear dependency which is shown in Fig. 4.14.

The next scenario which is analyzed is similar to what has been simulated previously in the one-dimensional setting. The procedure consists of at first randomly assigned location for the agent, then this agent adjusts his position according to his information, which is in this new setting assumed imperfect, and is measured by the exogenous parameter s describing radial observation range which was introduced earlier in this chapter. The agent makes his best response and chooses location which maximizes the predicted payoff. Note, that this predicted payoff usually differs from the optimal payoff. The efficiency of his adjustment can be measured by the difference between payoffs before and after the relocation. This concept has been already stated in one-dimensional analysis and is denoted as agent's incentives to buy some information described by s. In the Fig. 4.15 these incentives are summed for all agent's initial locations for given parameters e and s.

s\e	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.05	0.02									
0.10	0.05	0.15								
0.15	0.09	0.21	0.35							
0.20	0.09	0.22	0.35	0.44						
0.25	0.07	0.16	0.33	0.44	0.51					
0.30	0.10	0.23	0.40	0.47	0.53	0.44				
0.35	0.11	0.25	0.41	0.48	0.55	0.45	0.30			
0.40	0.10	0.24	0.43	0.51	0.56	0.45	0.30	0.16		
0.45	0.12	0.26	0.44	0.52	0.57	0.46	0.30	0.16	0.05	
0.50	0.12	0.26	0.44	0.53	0.57	0.46	0.30	0.16	0.05	0.00

Figure 4.15: Agent's total incentives given various parameters e and s in a two dimensions

Source: Author's computation

There are evidently incentives to increase the parameter s in situations of low e. This can be seen in the Fig. 4.15 where the payoff incentives are generally non-decreasing with the exception of results matching $s = 0.25$ which are caused in this case by numerical inaccuracy. The highest increase in payoff player has if $[e, s] = [0.5, 0.25]$ which is relatively high in comparison with one-dimensional case. It could be explained by the fact, that in the two-dimensional space it is harder to find the optimal location and it barely happens randomly whereas in one dimension the player usually cannot improve his position much in case of higher e.

4.5 Two player simulations in two dimensions

The goal of this subsection is to simulate the setup which tries to approach a more realistic space to some extent but is too complex to analyze it analytically (surfaces of lakes, rivers, etc. are two-dimensional). We stick in this setup to a two-dimensional square space with impassable edges. There is a huge number of variants how to simulate the two player behaviour. However, to make this work more consistent with the previous simulations the agent's heuristics remain exactly the same as was said in the beginning of this chapter. There emerges also a choice whether to make players move and make decisions sequentially or simultaneously. In this case the sequential movement was chosen in order to better understand the movements as reactions on the previous actions of the opponent.

The resolution of the discretization was chosen depending on numerical difficulty. The optimal scenario where only changes in e are regarded (the information is perfect) is discretized on 101×101 tiles. The scenario where the player makes best response on the location of the other player and this time with imperfect information was discretized to 21×21 tiles. In the last part of this section, also one special scenario has been examined where the discretization is 51×51.

The density function of the distribution on which the agents are operating is as in the previous section $f(x, y) = x + y$ which gives us, similarly as in the case with one-dimensional linear function, the highest density in the corner (i.e. $[x, y] = [1, 1]$).

The first scenario examined in this section has two stages. In the first stage, both agents are set their initial locations randomly. In the second stage, one of the players changes his position according to his information in order to maximize his payoff. The overall difference in payoff between these two players is measured as incentives to buy information and relocate. These incentives are in terms of payoff depicted in the Fig. 4.16.

s\e	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.05	0.02									
0.10	0.05	0.12								
0.15	0.09	0.17	0.25							
0.20	0.09	0.17	0.25	0.24						
0.25	0.07	0.14	0.25	0.24	0.23					
0.30	0.10	0.19	0.28	0.25	0.24	0.13				
0.35	0.11	0.20	0.29	0.26	0.24	0.13	0.08			
0.40	0.10	0.19	0.30	0.27	0.25	0.14	0.08	0.03		
0.45	0.12	0.21	0.31	0.28	0.25	0.14	0.08	0.03	0.03	
0.50	0.12	0.21	0.32	0.28	0.26	0.14	0.08	0.03	0.03	0.00

Figure 4.16: Agent's total incentives given various parameters e and s in a two player game in two dimensions
Source: Author's computation

The incentives have according to the Fig. 4.16 very similar shape to the incentives measured for an analysis with only one player. The difference lies in the point where are these incentives maximal. This point is located in parametric space (e, s) at coordinates $[e, s] \approx [0.15, 0.50]$. This result shows there is a stronger effect of information in a situation where the exploitation range of both players is low. It can be seen that there are similar numerical defects as in the previous analysis of only one player prediction. These imperfections are again probably caused by low numerical resolution (which was chosen due to its high numerical difficulty) and they affect the parametric space (e, s) so that the total incentives are not monotonic in neither of parameters, however, they would probably be in parameter s given fixed parameter e if there was better resolution of tiles.

The incentives are also influenced by another effect, namely the incentives are regarded only at tiles which are not in corner because we assume, that even the "naive" player, who does not change his location, would not choose his position near the edges or corners. This is the reason why for the high parameter e the incentives diminish as there is higher probability the players will end up at the same point.

This model can be also compared to the scenario with only one player, examined previously, and by making differences in the incentives in these two scenarios, the effect of evading the location of second player and the incentives for doing so can be captured. It is, however, again very important to note, that the difference in incentives to buy the information given by s may be also driven by the non-relocating second player. All of this analysis has been already done in one-dimensional scenario, where this effect was observable. For two dimensions, the effect is depicted in the Fig. 4.17. The most significant difference between the scenario with two players and with only one player is for a relatively high parameter $e \approx 0.30$. This could be interpreted as the existence of much higher incentives to accommodate agent's position in a single player scenario because there would not be any overlapped zones or a neighbouring area where the potential payoff would be harvested by another player. The player who adjusts the position in a two player scenario cannot be much better off if the parameter e is too high because the same exploitation range is covered also by the second player.

s\e	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.05	0.00									
0.10	0.00	0.03								
0.15	0.00	0.04	0.10							
0.20	0.00	0.05	0.10	0.20						
0.25	0.00	0.02	0.08	0.20	0.28					
0.30	0.00	0.04	0.12	0.22	0.29	0.31				
0.35	0.00	0.05	0.12	0.22	0.31	0.32	0.22			
0.40	0.00	0.05	0.13	0.24	0.31	0.31	0.22	0.13		
0.45	0.00	0.05	0.13	0.24	0.32	0.32	0.22	0.13	0.02	
0.50	0.00	0.05	0.12	0.25	0.31	0.32	0.22	0.13	0.02	0.00

Figure 4.17: Difference between the agents' total incentives in two players case and in one player case in two dimensions given various parameters e and s

Source: Author's computation

The next scenario examined is the situation where the agents try to maximize their payoff and sequentially alternate until the optimal choice of location (Nash equilibrium) is found. The first agent starts with choice of the location exactly according to the analysis done in one player analysis section, then the second player makes his best response. The series of best responses continues until no player can improve his payoff by changing his strategy. The equilibrium locations for both players (denoted $x_1^*, y_1^*, x_2^*, y_2^*$ for the first and second agent, respectively) are depicted in the Fig. 4.18.

A very interesting question arises here, namely, under what conditions the players are competing and who is better off. In the mentioned figure, the player 1 is the player whose turn was first and therefore the player 1 captured the most profitable area. If the parameter e was high enough, the payoff of the second player would be without intruding the opponent's territory very low. This can be seen as an incentive to compete for the corner with highest density. As the parameter e increases, the zero payoff beyond the boundaries will force the player 1 to choose the location closer to the middle of the space which enables the player 2 to oust the player 1 out of his position and partially improves the payoff of player 2. In an extreme case of $e \approx 0.43$ or higher, the simulations suggest that the player 2 would oust the player 1 as an reaction on player 1's optimal location choice by entering the competition and a moderate cut of player 1's payoff. The lost payoff pushes the player 1 away from the diagonal, where would be (from the one player analysis) the location optimal if there was not a competition. The interesting thing about these extreme cases is that as the player 1 leaves his optimal position, it enables player 2 to capture more advantageous location than the player 1 after relocation has. This causes switch in players' payoffs and actually says that being an incumbent player (player 1) is worse than being an entrant.

If the player 1 could detect that there is a danger in terms of other player playing after him, the best strategy would be to not make the optimal choice of location and satisfy with lower payoff than is optimal in a one-player case.

The competition between the two players, however, occurs already around $e \approx 0.30$ as there can be noticed that the location parameter $x_2^*(e)$ stops to diverge from the other coordinates in the Fig. 4.18. From the Fig. 4.19 one can also deduct that around $e \approx 0.30$ the optimal payoff of the first player h_1^* starts to be concave and the difference between the payoffs of both players starts to diminish as the e increases. The interesting case happens also in scenarios of $e \approx 0.46$ or higher, where both players have approximately the same payoff. In that situation their distance from the diagonal would be practically symmetric and their exploitation range will cover almost the entire area which is why they try to harvest the density at the corners further from the other player.

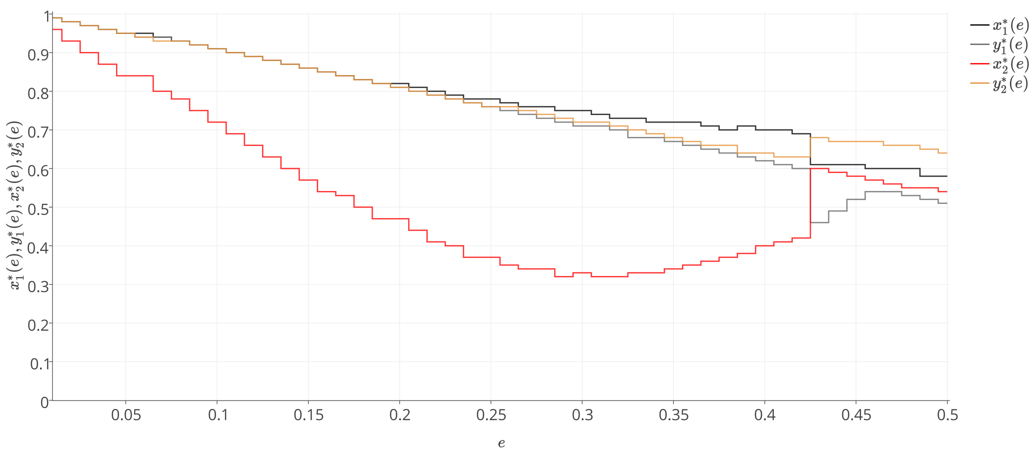

Figure 4.18: The optimal locations of the players in two dimensions under perfect information

Source: Author's computation

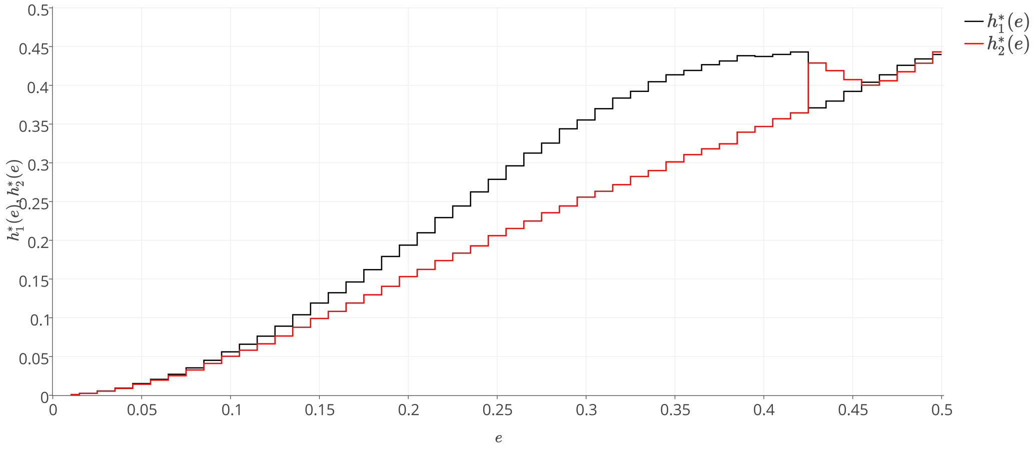

Figure 4.19: The optimal payoffs of the players in two dimensions under perfect information

Source: Author's computation

In the last part of this chapter the special scenario is analyzed which has not been investigated in the one-dimensional case. The aim of that scenario is to examine, how efficiently the player can find out what parameter s is an optimal for him in order to maximize the payoff or the difference between the payoff of both players if there is only limited amount of observations available. The setup tries to compare the situation from the reality where one player has a better technology (e.g. sonar) and can more efficiently observe the information about the density function. It is assumed that in this scenario both players can use their predictive skills, however, the second player has only limited information i.e. the player 2 does not radially observe any additional information above the distance e whereas the player 1 does. The player 1 is also favourized as he chooses his location first.

The game starts with the random setting of x and y coordinate of both players ($[x_1, y_1], [x_2, y_2]$). Thereafter it is followed by the prediction of the density function in case of player 1. His predictive skills depend on the parameter s which can be interpreted as the radial observation range. In order to control for the effect of this special predictive factor, we suppose the player 2 has these predictive skills as was mentioned in the previous paragraph very limited, namely by the parameter e. Therefore, the player 2 does not have as precise information as the player 1 and bases his intuition about the density function purely from the harvest that he can gather.

In this setup, we consider both players are aware of the opponent's strategy (location) and it is assumed that the payoff (and their predicted payoff as well in their decisions) is split between the players equally at the places where the harvesting of two players overlaps.

First turn is the player's 1 marginal move directly towards the point with the highest predicted payoff. The real highest predicted payoff can differ from the player's prediction and the accuracy of the prediction depends on the value of parameter s. In the discretized grid, the marginality of his movement is exhibited by either a horizontal, vertical or diagonal shift of one tile. The distance of diagonal movement is slightly higher than horizontal or vertical movements, however, as the number of steps is finite, and both players moves the same speed, it does not influence the results much (which may be seen mainly in terms of differences between the players' outcomes).

The player 2 responds immediately after player's 1 first movement is done. The procedure is exactly the same as in case of player 1 except for one thing, the predictive parameter is not s but e. The player 2 has significant disadvantage, because he is one turn delayed in movement and therefore is on average more distant from the optimal point, this effect will be shown later in figures. The player 2 moves again marginally to the point where predicts the highest payoff, which could be somewhere else than was in the player 1's prediction due to the fact that the prediction was done in different location and under different prediction parameter.

The players alternate until the equilibrium is found and the players no longer change their positions. In some cases there can emerge a situation, where neither player can find his equilibrium location and are changing their position in a loop. These loops are only on a marginal level as the players are competing for the tiles with imperfect information, therefore the simulation is stopped after the players get into a loop and the payoff in a random of these points is calculated.

This game is repeated 10 times with the same parameters only different initial locations, for every choice of parameter s given parameter e. After this procedure the player 1 counts what choice s is the most gainful for him as an average payoff. It is important to note, that the player is not calculating his payoff during the travel before he gets into the final point (and it thus does not depend what path he chooses towards his final point).

The players' payoffs per one tile are depicted in the figures 4.20 and 4.21. The difference between these payoffs is depicted in the figure 4.22.

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48
0.02	1.90																							
0.04	1.96	1.92																						
0.06	1.96	1.92	1.85																					
0.08	1.90	1.83	1.74	1.84																				
0.10	1.96	1.92	1.76	1.66	1.77																			
0.12	1.96	1.91	1.75	1.80	1.62	1.75																		
0.14	1.96	1.92	1.75	1.68	1.64	1.55	1.72																	
0.16	1.90	1.91	1.88	1.66	1.62	1.75	1.72	1.69																
0.18	1.96	1.92	1.88	1.66	1.62	1.55	1.48	1.40	1.66															
0.20	1.96	1.91	1.88	1.82	1.62	1.75	1.72	1.43	1.36	1.62														
0.22	1.95	1.91	1.74	1.68	1.79	1.73	1.72	1.40	1.33	1.25	1.57													
0.24	1.96	1.91	1.87	1.77	1.77	1.73	1.72	1.69	1.63	1.25	1.19	1.53												
0.26	1.96	1.82	1.87	1.76	1.79	1.73	1.72	1.69	1.66	1.29	1.19	1.11	1.45											
0.28	1.96	1.92	1.74	1.78	1.79	1.75	1.72	1.43	1.66	1.62	1.49	1.53	1.41	1.38										
0.30	1.96	1.90	1.75	1.77	1.79	1.55	1.72	1.66	1.66	1.62	1.57	1.53	1.45	0.99	0.94									
0.32	1.95	1.92	1.87	1.68	1.77	1.73	1.70	1.69	1.66	1.62	1.57	1.53	1.41	1.38	1.29	1.24								
0.34	1.96	1.92	1.88	1.78	1.79	1.73	1.50	1.69	1.66	1.62	1.57	1.53	1.45	1.38	1.26	1.24	1.15							
0.36	1.96	1.92	1.74	1.78	1.77	1.55	1.72	1.69	1.66	1.59	1.57	1.53	1.45	1.38	1.29	1.24	1.15	1.06						
0.38	1.95	1.91	1.88	1.78	1.79	1.57	1.50	1.69	1.63	1.62	1.57	1.53	1.45	1.38	1.29	1.24	1.12	1.06	0.98					
0.40	1.96	1.92	1.87	1.80	1.79	1.75	1.48	1.69	1.63	1.59	1.57	1.53	1.41	1.34	1.29	1.20	1.15	1.04	0.98	0.86				
0.42	1.96	1.90	1.74	1.79	1.62	1.73	1.70	1.69	1.66	1.62	1.53	1.53	1.45	1.34	1.22	1.20	1.15	1.06	0.98	0.90	0.83			
0.44	1.96	1.91	1.88	1.79	1.79	1.75	1.70	1.49	1.33	1.62	1.57	1.53	1.45	1.38	1.29	1.24	1.15	1.06	0.98	0.88	0.82	0.66		
0.46	1.96	1.92	1.88	1.79	1.79	1.73	1.48	1.64	1.66	1.62	1.57	1.49	1.41	1.38	1.29	1.24	1.15	1.06	0.98	0.86	0.82	0.72	0.62	
0.48	1.95	1.92	1.87	1.79	1.77	1.75	1.48	1.69	1.66	1.62	1.53	1.53	1.45	1.34	1.29	1.24	1.15	0.79	0.96	0.86	0.83	0.73	0.62	0.59

Figure 4.20: Average payoff per tile for player 1 in two dimensions

Source: Author's computation

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48
0.02	1.96																							
0.04	1.90	1.82																						
0.06	1.90	1.82	1.77																					
0.08	1.96	1.91	1.88	1.66																				
0.10	1.90	1.82	1.86	1.80	1.64																			
0.12	1.90	1.83	1.87	1.66	1.79	1.55																		
0.14	1.90	1.82	1.87	1.79	1.77	1.75	1.48																	
0.16	1.96	1.83	1.74	1.80	1.79	1.55	1.48	1.40																
0.18	1.90	1.82	1.74	1.80	1.79	1.75	1.72	1.69	1.33															
0.20	1.90	1.83	1.74	1.66	1.79	1.55	1.48	1.66	1.63	1.25														
0.22	1.91	1.83	1.88	1.79	1.62	1.57	1.48	1.69	1.66	1.62	1.19													
0.24	1.90	1.83	1.75	1.70	1.64	1.57	1.48	1.40	1.36	1.62	1.57	1.11												
0.26	1.90	1.92	1.75	1.71	1.62	1.57	1.48	1.40	1.33	1.59	1.57	1.53	1.06											
0.28	1.90	1.82	1.88	1.68	1.62	1.55	1.48	1.66	1.33	1.25	1.26	1.11	1.10	0.99										
0.30	1.90	1.84	1.87	1.69	1.62	1.75	1.48	1.43	1.33	1.25	1.19	1.11	1.06	1.38	1.31									
0.32	1.91	1.82	1.75	1.78	1.64	1.57	1.50	1.40	1.33	1.25	1.19	1.11	1.10	0.97	0.94	0.88								
0.34	1.90	1.82	1.74	1.68	1.62	1.57	1.70	1.40	1.33	1.25	1.19	1.11	1.06	0.97	0.97	0.88	0.84							
0.36	1.90	1.82	1.88	1.68	1.64	1.75	1.48	1.40	1.33	1.29	1.19	1.11	1.06	0.97	0.94	0.88	0.84	0.79						
0.38	1.91	1.83	1.74	1.68	1.62	1.73	1.70	1.40	1.36	1.25	1.19	1.11	1.06	0.97	0.94	0.88	0.87	0.79	0.75					
0.40	1.90	1.82	1.75	1.66	1.62	1.55	1.72	1.40	1.36	1.29	1.19	1.11	1.10	1.01	0.94	0.92	0.84	0.82	0.75	0.75				
0.42	1.90	1.84	1.88	1.68	1.79	1.57	1.50	1.40	1.33	1.25	1.22	1.11	1.06	1.01	1.01	0.92	0.84	0.79	0.75	0.71	0.69			
0.44	1.90	1.83	1.74	1.68	1.62	1.55	1.50	1.61	1.66	1.25	1.19	1.11	1.06	0.99	0.94	0.88	0.84	0.79	0.75	0.73	0.70	0.72		
0.46	1.90	1.82	1.74	1.68	1.62	1.57	1.72	1.46	1.33	1.25	1.19	1.15	1.10	0.97	0.94	0.88	0.84	0.79	0.75	0.75	0.70	0.65	0.62	
0.48	1.91	1.82	1.75	1.68	1.64	1.55	1.72	1.40	1.33	1.25	1.22	1.11	1.06	1.01	0.94	0.88	0.84	1.06	0.78	0.75	0.69	0.65	0.62	0.61

Figure 4.21: Average payoff per tile for player 2 in two dimensions

Source: Author's computation

s\e	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48
0.02	-0.1																							
0.04	0.06	0.10																						
0.06	0.06	0.10	0.08																					
0.08	-0.1	-0.1	-0.1	0.18																				
0.10	0.06	0.10	-0.1	-0.1	0.14																			
0.12	0.06	0.08	-0.1	0.14	-0.2	0.20																		
0.14	0.06	0.10	-0.1	-0.1	-0.1	-0.2	0.25																	
0.16	-0.1	0.08	0.14	-0.1	-0.2	0.20	0.25	0.29																
0.18	0.06	0.10	0.14	-0.1	-0.2	-0.2	-0.2	-0.3	0.33															
0.20	0.06	0.08	0.14	0.16	-0.2	0.20	0.25	-0.2	-0.3	0.37														
0.22	0.04	0.08	-0.1	-0.1	0.17	0.16	0.25	-0.3	-0.3	-0.4	0.38													
0.24	0.06	0.08	0.11	0.08	0.14	0.16	0.25	0.29	0.27	-0.4	-0.4	0.42												
0.26	0.06	-0.1	0.11	0.05	0.17	0.16	0.25	0.29	0.33	-0.3	-0.4	-0.4	0.39											
0.28	0.06	0.10	-0.1	0.10	0.17	0.20	0.25	-0.2	0.33	0.37	0.23	0.42	0.31	0.39										
0.30	0.05	0.06	-0.1	0.08	0.17	-0.2	0.25	0.23	0.33	0.37	0.38	0.42	0.39	-0.4	-0.4									
0.32	0.05	0.10	0.11	-0.1	0.14	0.16	0.20	0.29	0.33	0.37	0.38	0.42	0.31	0.41	0.35	0.36								
0.34	0.06	0.10	0.14	0.10	0.17	0.16	-0.2	0.29	0.33	0.37	0.38	0.42	0.39	0.40	0.28	0.36	0.30							
0.36	0.06	0.10	-0.1	0.10	0.14	-0.2	0.25	0.29	0.33	0.30	0.38	0.42	0.39	0.41	0.35	0.36	0.30	0.27						
0.38	0.05	0.08	0.14	0.10	0.17	-0.2	-0.2	0.29	0.27	0.37	0.38	0.42	0.39	0.41	0.35	0.36	0.24	0.27	0.22					
0.40	0.06	0.10	0.11	0.14	0.17	0.20	-0.2	0.29	0.27	0.30	0.38	0.42	0.31	0.33	0.35	0.29	0.30	0.22	0.22	0.12				
0.42	0.06	0.06	-0.1	0.11	-0.2	0.16	0.20	0.29	0.33	0.37	0.30	0.42	0.39	0.33	0.21	0.29	0.30	0.27	0.22	0.19	0.15			
0.44	0.06	0.08	0.14	0.10	0.17	0.20	0.20	-0.1	-0.3	0.37	0.38	0.42	0.39	0.39	0.35	0.36	0.30	0.27	0.22	0.15	0.12	-0.1		
0.46	0.06	0.10	0.14	0.11	0.17	0.16	-0.2	0.17	0.33	0.37	0.38	0.34	0.31	0.41	0.35	0.36	0.30	0.27	0.22	0.12	0.12	0.06	0.00	
0.48	0.05	0.10	0.11	0.11	0.14	0.20	-0.2	0.29	0.33	0.37	0.30	0.42	0.39	0.33	0.35	0.36	0.30	-0.3	0.18	0.12	0.15	0.08	0.00	0.0

Figure 4.22: Difference between the average payoff per tile of player 1 and player 2 in two dimensions

Source: Author's computation

It can be noticed that the most significant is the knowledge of the information for low difference between values of s and e (in the figures near the diagonal). The number of positive differences between the payoff of player 1 and player 2 increases for high s which can be seen if the Fig. 4.22 is examined to the detail. It is, therefore, higher probability that the player 1 will have higher payoff than player 2 if he buys more s.

However, the effect of the difference between the players' information is not very high. From the figure 4.22 it follows that the highest difference between the payoffs of the players is concentrating around points where $e \approx 0.24$ no matter what the parameter s is. The results suggest that far more important (in the composition of the payoffs) is the size of the parameter e which shows that in this scenario, the advantage of having more knowledge about the density is totally unimportant as the player 1 would have higher payoff almost by the same level even if his information would be the same as the player 2's ($s = e$). It means that it is very important to be the beginning player if the steps can be done only marginally, especially when $e \approx 0.24$.

Other situation could be, if the moves were the best responses instead and the steps would be not restricted by the marginality of movement. The analysis would be then more similar to the analysis in an optimal scenario, where the second player was enable to have higher payoff than the first player.

Chapter 5

Summary and conclusion

This thesis has analyzed the spatial dimension added to the problem of finding the optimal strategies of natural resource exploitation. There is only a few works emphasizing also the importance of the spatial aspect in a process of making the decision how to harvest the resource. The most of literature related to this topic considered rather dynamic aspect of the harvest instead of making a spatial analysis which is strongly neglected. The special attention has been paid to the field of economics of fisheries in this thesis, however, the analysis is very abstract and applicable also to many other common pool resources.

In this work, the theoretical framework for the analysis of a spatial competition game has been rigorously analyzed and developed. In the game, where two agents (e.g. fishing nations, fleets or vessels) are competing along a one-dimensional strategy space, there have been assumed various distributions of the common pool resource. These distributions of the natural resource (e.g. fish stocks) were aimed to cover the most basic possibilities of this game and thus the resources allocated along the one-dimensional space were studied the most comprehensively.

The space where the agents compete was assumed to be a line segment of a unit length or a circle of a unit circumference. Both shapes of space were focusing on a different application. The resource distributed spatially along the line segment can be characterized by the distance from its corners and thus emphasize the effects of the difference between the center and the periphery (e.g. distance from the harbour), whereas the circle space does not have any corners and hence may control for the effects linked to the competition in areas with the same characteristics (e.g. between the fishers at the same distance from the harbour).

A few special distributions of the natural resource were analyzed to the detail (uniform, linearly increasing, sinusoidal, etc.). However, more importantly also some general observations have been made and the effects which determine the agents' competitiveness according to their strategies were studied. Probably the most interesting result from the theoretical analysis is that the set of strategies where the players do not compete is non-convex in the space of the player's range parameter e and the parameter b describing the inverse concentration of the resource.

This in other words means that the same increase in the amount of resource that is in player's range together with the decrease of resource concentration can result in two different effects even though the marginal change in the parameters is the same, there can emerge a partial competition from no competition as well as the partial competition can disappear and no competition will emerge. The same results are also obtained for the transition from the partial competition to the full competition. The different effects are caused by two observations, the first one is the existence of corners which the players want to naturally evade, the second one says that when the concentration of the resource diminish, it can lead to lost incentives to compete.

Other interesting results of this work are found in the studied two-player game where there was examined apart from the general equilibrium analysis also the concept of local stability conditions of players' strategies and was found out that the full competition of both players cannot be sustained in interior of the space if the resource is continuous unless the resource is bounded by locations with no resource.

The second part of this thesis focused on the numerical simulations, which were done to understand better the effects of competitive behaviour of the agents in more realistic spatial cases in two-dimensions (e.g. a lake). The simulations also broadly investigate what the role of player's information about the resource is and how it determines the payoff if the information is different for each player.

Basically, two basic scenarios have been simulated for one-dimensional case with corners, without corners and for two-dimensional case in a square space. The first one is the scenario where all (one or two) players have perfect information and their aim is to find Nash equilibrium in a game where the agents adjust their locations sequentially. In this scenario the most interesting result is that if the exploitation range is covering large amount of the resource in the two-dimensional space, the beginning player may be in the end worse off than the second player even though he can choose better initial position.

The second scenario examined was the optimization of the player's location under imperfect information about the resource stock. Generally, there are two basic contradictory effects which both lowers the player's willingness to pay for the additional information (e.g. purchase of better sonar). The first one is that the observation range (the area which is visible for the player) is too low and therefore the player cannot correctly predict the optimal location, the second one is when the exploitation range (the area which is harvestable by the player) is too high for the player and the role of information in such a system inflates as the payoff would be similar even without the information.

There was also shown the effect how the existence of a competing player diminishes the player's payoff from the gained information. In the thesis the most efficient amount of information and exploitation was found for specific setups when compared to the "naive" strategy where the player did not adjust according to his information.

The further research on this topic could involve more advanced distributions of the resource. There could be also added the dynamic aspect which was in this thesis neglected due to the focus on the spatial dimension. The most interesting would be further extensions in the two-dimensions and finding general conclusions resulting from such an analysis. That is however very difficult and often almost impossible to analyze without numerical methods. The suggestion for the works related to natural resource exploitation and economics of fisheries is to involve more of the spatial aspect as this thesis shows it can explain the reasons for competitive behaviour significantly.

Bibliography

A. O. Belyakov, A. D. & V. M. Veliov (2013): "Optimal cyclical exploitation of renewable resources." *ORCOS Research Report* .

Behringer, S. & T. Upmann (2014): "Optimal harvesting of a spatial renewable resource." *Journal of Economic Dynamics and Control* **42**: pp. 105–120.

Conrad, J. M. (2010): *Resource Economics.* Cambridge University Press, second edition.

Deacon, R. T., D. S. Brookshire, A. C. Fisher, A. V. Kneese, C. D. Kolstad, D. Scrogin, V. K. Smith, M. Ward, & J. Wilen (1998): "Research trends and opportunities in environmental and natural resource economics." *Environmental and Resource Economics* **11**: p. 383–397.

Gordon, H. S. (1954): "Economic theory of a common property resource: The fishery." *Journal of Political Economy* **75**: pp. 124–142.

Hall, P. (1966): *Von Thunen's Isolated State.* Oxford: Pergamon.

Hannesson, R. (1995): "Sequential fishing: cooperative and non-cooperative equilibria." *Nat. Res. Model* **9**: pp. 51–59.

Hotelling, H. (1929): "Stability in competition." *The Economic Journal* **39**: pp. 41–57.

Hotelling, H. (1931): "The economics of exhaustible resources." *Journal of Political Economy* **39**: p. 137–175.

Kronbak, L. G. & M. Lindroos (2007): "Sharing rules and stability in coalition games with externalities." *Mar. Resour. Econ.* **22**: pp. 137–154.

M. D. Smith, J. N. S. & J. E. Wilen (2009): "The economics o f spatial-dynamic processes: Applications to renewable resources." *Journal of Environmental Economics and Management* **57**: pp. 104–121.

Munro, G. R. (1979): "The optimal management of transboundary renewable resources." *Can. J. Econ.* **12(3)**: pp. 355–376.

Nash, J. (1951): "The non-cooperative games." *Ann. Math.* **54(2)**: pp. 286–295.

Nash, J. (1953): "Two-person cooperative games." *Econometrica* **21(1)**: pp. 128–140.

Neubert, M. & G. E. Herrera (2008): "Triple benefits from spatial resource management." *Theoretical Ecology* **1**: pp. 5–12.

Neumann, J. V. & O. Morgenstern (1947): *Theory of Games and Economic Behavior.*

R. Perman, Y. Ma, M. C. D. M. & J. McGilvary (2011): *Natural Resource and Environmental Economics.* Pearson, fourth edition.

Salop, S. C. (1979): "Monopolistic competition with outside goods." *The Bell Journal of Economics* **10(1)**: pp. 141–156.

Scott, A. D. (1955): "The fishery: The objectives of sole ownership." *Journal of Political Economy* **63**: pp. 116–124.

Sumaila, U. R., M. Bailey, & M. Lindroos (2010): "Application of game theory to fisheries over three decades." *Fisheries Research* **102(1-2)**: pp. 2–8.

Trisak, J. (2005): "Applying game theory to analyze the influence of biological characteristics on fishers' cooperation in fisheries co-management." *Fish. Res.* **75**: pp. 164–174.

Wilen, J. E. (2007): "Economics of spatial-dynamic processes." *American Jour- nal of Agricultural Economics* **89**: pp. 1134–1144.

Appendix A

The equations describing players' behaviour in case of a symmetric strictly quasi-concave distribution

If the condition (3.46), which holds if the peak density is at most twice as high as the boundary density, matches or the players start to cooperate, the equilibrium values of the examined variables would be very similar to the uniform case, only the equations describing it are generalized.

$$l_1(e) = \frac{1}{4} + \left| e - \frac{1}{4} \right|, \tag{A.1}$$

$$l_2(e) = \frac{3}{4} - \left| e - \frac{1}{4} \right|. \tag{A.2}$$

The payoffs of the vessels still remain the combination of the payoff of the sole vessel and the payoff from the shared area

$$h_1(r_1, r_{12}) = r_1 + \frac{r_{12}}{2}, \tag{A.3}$$

$$h_2(r_{12}, r_2) = r_2 + \frac{r_{12}}{2}, \tag{A.4}$$

The particular payoffs r_1, r_{12}, r_2 can be easily described by the cumulative distribution function

$$r_1(e, F(e)) = F(1 - 2e) - F(0), \tag{A.5}$$

$$r_{12}(e, F(e)) = F(2e) - F(1 - 2e), \tag{A.6}$$

$$r_2(e, F(e)) = F(1) - F(2e). \tag{A.7}$$

Therefore

$$h_1(r_1, r_{12}) = -F(0) + \frac{F(2e) + F(1-2e)}{2}, \tag{A.8}$$

$$h_2(r_{12}, r_2) = F(1) - \frac{F(2e) + F(1-2e)}{2}, \tag{A.9}$$

however, if the players are not competing, the payoffs will be slightly different as there will be no shared area.

$$r_1(e, F(e)) = F\left(\frac{1}{2}\right) - F\left(\frac{1}{2} - 2e\right) = h_1(e, F(e)), \tag{A.10}$$

$$r_{12}(e, F(e)) = 0, \tag{A.11}$$

$$r_2(e, F(e)) = F\left(\frac{1}{2} + 2e\right) - F\left(\frac{1}{2}\right) = h_2(e, F(e)). \tag{A.12}$$