Title: Geometric Problems Solvable without Calculations

Abstract: The thesis focuses on geometric problems which are usually set with a picture and ask for some numerical value. They are solvable by a geometric strategy, i.e., strategy with a small involvement of calculations. Their difficulty lies in the necessity to distinguish partial configurations in the picture. The goal of the research was to describe pupils' solving strategies and find out if they are age dependent. The theoretical background of the thesis consists of the concepts of visualisation, roles of pictures in geometry and their understanding by pupils and the ability to see in geometry.

In the pilot study, tasks of the given type were selected for the main study. In the first phase of the main study, three problems were used with 9 pupils of different ages in semi-structured interviews with the goal to get an insight into the solving strategies and their characteristics and look into the influence of prepared hints on the pupils' solutions. In the second phase, 125 pupils aged 12 till 19 solved independently one problem while in the third one (which was a methodological compromise between the previous two), 130 pupils aged 12 till 19 solved two problems independently but had an opportunity to get hints and could explain their solutions verbally. The pupils' written solutions and transcripts of the interviews were analysed in a qualitative way using the techniques based on the grounded theory and elaborated in a quantitative way, too.

The results of the analysis consist of solving strategies for the three tasks and a list of other phenomena which appeared in the pupils' works. Their frequency was investigated for pupils of the same age and groups of younger and older pupils were compared. It was found out, e.g., that mathematically incorrect solutions based on measuring, manipulation, redrawing of the figure, etc. appeared for all age groups, even the oldest pupils. The hints let to geometric strategies only partially and mainly for older pupils. Substantial number of pupils tended to start the solution automatically by calculations without any prior analysis of the picture. The thesis also describes the limitations of the research, its possible continuation and practical applications.

Keywords: geometry, visualisation, ability to see, solving strategies, representation