Title: Exceptional Sets in Mathematical Analysis

Author: Martin Rmoutil

Department: Department of Mathematical Analysis

Supervisor:
Doc. RNDr. Ondřej Kalenda, Ph.D., DSc.,
Department of Mathematical Analysis

Abstract: The present thesis consists of four research articles. In the first paper we study the notion of σ-lower porous set; our main result is the existence of two closed sets $A, B \subset \mathbb{R}$ which are not σ-lower porous, but their product in $\mathbb{R}^2$ is lower porous. In the second and third article we use a set-theoretical method of elementary submodels involving the Lwenheim-Skolem theorem to prove that certain σ-ideals of sets in Banach spaces are separably determined. In the second article we do so for σ-porous sets and σ-lower porous sets. In the next article we refine these methods obtaining separable determination of a wide class of σ-ideals. In both cases we derive interesting corollaries which extend known theorems in separable spaces to the nonseparable setting; for example, we obtain the following theorem. Any continuous convex function on an Asplund space is Frchet differentiable outside a cone small set. In the fourth article we introduce the following notion. A closed set $A \subset \mathbb{R}^d$ is said to be c-removable if the following is true: Every real function on $\mathbb{R}^d$ is convex whenever it is continuous on $\mathbb{R}^d$ and locally convex on $\mathbb{R}^d \setminus A$. We then give new sufficient conditions for a set to be c-removable and we construct an example proving that these conditions are more general than those previously known.

Keywords: σ-porous set, elementary submodel, Banach space, c-removable set, convex function.