Referee report diploma thesis on the diploma thesis

Name of the candidate: Lukáš Holka

Title of the thesis: Spinorial techniques for constructing quasi-local quantities in general relativity

The diploma thesis by Lukáš Holka is presented as a detailed consistent introduction to the publication written by the candidate with his diploma thesis supervisor and represents the last part of the thesis:
Martin Scholtz, Lukáš Holka, *On the Bondi mass of Maxwell-Klein-Gordon spacetimes,* General Relativity and Gravitation, (2014) 46: 1665.

The subject of the thesis, the calculation of the Bondi mass for coupled system of gravitational, electromagnetic and charged scalar field in asymptotically simple spaces is a topical problem of the theory of gravitation. The thesis consists of two parts:

(I) Theoretical Introduction

(II) Calculations and Results

The bibliography contains 21 references to the most important contributions to various aspects of the investigated problems.

The part (I), approx on 90 pages, can be divided to two sub-parts. In the first one the main tools of the spinorial formalism are introduced (spinors and operation on spinors, Newman-Penrose formalism, partially covariant Geroch-Held-Penrose formalism, twistors and twistor equation, geometry of two-surfaces and Sen curvature). The second sub-part deals with impor-

tant problem of the existence of quasi-local quantities in General Relativity (Penrose construction in twistor formalism) and with the existence of global and energy and momentum in asymptotically simple spaces (Arnowitt-Deser-Misner energy-momentum and Bondi energy-momentum) and finally with the definition of quasi-local energy and momentum (Ludvigsen-Vickers and Dougan-Mason constructions).

This part of the thesis is written very carefully, transparently and consistently. It collects all tolls needen in the second part. This clearly indicates that the candidate understands the subject well.

The part (II), on 50 pages, contains detailed calculations of Bondi energy and momenta for the dynamical system coupled of scalar and electromagnetic fields in asymptotically simple spacetime:

(a) First, the Einstein-electro-scalar equations are rewritten in spinor formalism, and then these equations are projected onto the Newman-Penrose tetrad.

(b) The solutions of Einstein-electro-scalar equations are found near null infinity under assumption that the spacetime with electro-scalar fields is asymptotically flat. They are given as power series in conformal factor Ω. The expansions for all relevant objects (spin coefficients, Ricci and Weyl scalars) are presented. They serve for the calculations of Bondi energy-momentum for the model in question. The details of of construction are given in the mentioned paper by M. Scholtz and L. Holka attached to the diploma thesis.

This part of the thesis is written again very transparently and consistently, the calculations are explained and commented. This clearly indicates that the candidate understands the subject well and his contribution to the research has been adequate.

The main results can be briefly summarized as follows: The spinor equations for the coupled gravitational, electromagnetic and scalar have been derived, their asymptotic solution have been found near future null infinity under assumption that the spacetime is asymptotically flat. This solution

reduces to known special cases found earlier. The Bondi mass-loss formula was given in gauge invariant form which is explicitly semi-definite, and consequently, the Bondi mass is a non-increasing function in time.

The dissertation is written in a nice English, explanations are fluent and clear, and statements consistent. I red it with pleasure. I have just two general questions - comments:

(1) The twistor formalism is closely related to the group $G = SU(2,2)$ which is the smallest simple group that contains the (parabolic) subgroup $H =$ *Poincaré group extended by dilatations*. Implicitly, group H plays an important rôle in the Newman-Penrose formalism or Geroch-Held-Penrose formalism. Perhaps, some elements of the $SU(2,2)$ representations theory could elucidate some symmetry structures behind the found (asymptotic) solutions.

(2) Physically more interesting system as the Einstein-electro-scalar system could be the Einstein-Maxwell-Dirac system that contains the electron-positron field instead of complex scalar field. The inclusion of Dirac field requires tetrad formalism but that is just the main tool used above. Are there any serious obstacles to extend the described approach in that direction?

Summary: The diploma thesis of L. Holka deals with an interesting topical problem of coupled gravitational-electromagnetic-scalar field systems. The found solutions extends known more special results and has been published in a renowned scientific journal. It is an excellent diploma thesis and I evaluate it with the grade **A**.

Bratislava, August 27, 2014

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