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Wavelet analysis of business cycles in the Visegrad Four

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I hereby declare that I have written this thesis using only literature and other sources listed in bibliography. Furthermore, I declare that I have not used this thesis to acquire another academic degree. I acknowledge and agree with lending and publishing of the thesis.

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Abstract

This thesis presents an analysis of business cycles in the Visegrad Four on monthly data during 1991-2013. Using the wavelet analysis techniques, we find that the relationship of output and key macroeconomic indicators is dynamic and varies over time and across all frequencies. Furthermore, we study the output synchronization within the Visegrad Four countries and Visegrad Four with Germany. In the Visegrad region, all countries have highly coherent output during the first years of transition. After 1995, their positive co-movement suffers from policy divergence. The synchronization across the Visegrad group increased again, as the countries prepared for their accession to the EU. Among the Visegrad countries, The Czech Republic and Hungary have highly coherent business cycles with Germany during 2000-2013, while Poland's business cycle is the least synchronized with Germany.

JEL Classification E32, C22, F02, F41
Keywords business cycles, wavelet analysis, synchronization, Visegrad Four

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Abstrakt

Tato práce analyzuje hospodářské cykly v zemích Visegrádské čtyřky. Pomocí nástrojů vlnkové analýzy na měsíčních datech v letech 1991-2013 bylo zjištěno, že produkce a hlavní makroekonomické veličiny mají dynamický vztah vyvíjející se v čase a frekvenci. Dále práce studuje synchronizaci hospodářských cyklů zemí Visegrádské čtyřky a zároveň těchto zemí s Německem. V prvních letech ekonomické transformace měly všechny země velmi podobné hospodářské cykly. Jejich synchronizace po roce 1995 začala oslabovat v důsledku rozdílnosti hospodářských politik. Opětovný nárůst jich sladěnosti nastal s přípravou těchto zemí na vstup do Evropské unie. Česká republika a Maďarsko mají vysoce sladěné hospodářské cykly s Německem počínaje rokem 2000, zatímco polský hospodářský cyklus je sladěný s tím německým nejméně.

Klasifikace JEL	E32, C22, F02, F41
Klíčová slova	hospodářské cykly, vlnková analýza, synchronizace, Visegrádská čtyřka

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Chapter 1

Introduction

One of the most challenging phenomena in economics is the identification, proper understanding, and disentangling of factors and mechanisms influencing dynamics of macroeconomic variables. Number of quantitative econometric techniques was developed to study regularities in fluctuations of macroeconomic indicators and business cycles, which are often summarized as stylized facts. In this thesis, we study the stylized facts of the Visegrad Four using the state-of-art wavelet filtering technique.

Stylized facts of business cycles is a description of the statistical properties characterizing co-movements of deviations from long-term trend of particular macroeconomic variables and output of an economy. Analysis of stylized facts plays a substantial role in explaining magnitudes and lengths of cyclical variations of economies. These measures are then usually taken into account when constructing theoretical models of the business cycles or comparing those models with each other or to their benchmarks. Furthermore, an analysis of the cyclical properties of business cycles behaviour and other macroeconomic indicators may serve as a suitable preliminary tool to assign correct theoretical model to a given economy or to design and identify measures used to construct adequate macroeconomic policies such as inflation targeting.

Moreover, the market-based economy is a dynamic system producing time series structured at different frequencies. Therefore, in any field of economics, while surveying a co-movement among economic variables via the traditional filtering methods, we obtain the information only for time domain, represented by a single correlation coefficient. Acknowledging that the correlation between time series may vary in time and at different time-horizons as well, we decided to employ the wavelet-based instruments that are capable of providing a complex picture of time series interdependence in time-frequency space. This approach yields information on local coherency and cross-correlation, which in turn tell us a lot about time and frequency dynamics of surveyed economy.

It has been more than two decades since the beginning of the transition process of the Central and Eastern European countries, a period of turbulent changes that can justifiably be labelled as a dawn of new economic era for the economies of these countries. Despite the vast amount of related literature, there still remain gaps in knowledge that are to be filled by the means of examination of the stylized facts of the Central and Eastern European countries. In order to contribute to this literature, the first aim of this text is to provide a complex picture of properties of business cycles in the Visegrad Four from the beginning of their cooperation. In terms of international framework, we make this description of cyclical characteristics more complex by additional analysis of business cycles synchronization, evaluating co-movements of economic cycles within the Visegrad region.

The second aim of this thesis is to find out whether cooperation among the four countries brought about better business cycle synchronization within the Visegrad group, or if this settlement was rather of political nature. Furthermore, an important issue the policy-makers in the four countries either faced or will face in the close future is the adoption of the Euro. Third aim is to shed some light on benefits of inclusion of surveyed states into the monetary union by looking at the synchronization level of each country with Germany, as a proxy to the European Union.

The contribution of this thesis to the existing literature is threefold. Firstly, we discuss the properties of business cycles in the Visegrad Four, which are not extensively discussed in other studies. Secondly, the estimation of co-movements within the Visegrad Four countries and their synchronization with Germany is put forward because of the availability of longer time series observed at monthly basis. The third contribution is the analysis itself, stemming from the use of the state-of-art wavelet-based time-frequency analysis in macroeconomics employing non-adjusted monthly often-sampled time series data.

The rest of the text is organized as follows. The second chapter reviews standard techniques used in business cycles analyses and discusses the relevant literature of business cycles properties and synchronization. The third chapter describes the methodology of wavelet analysis, the cornerstone of our analysis. In the fourth chapter, we provide brief data description and first preliminary analysis of the variables. In the fifth chapter, we interpret the results concerning business cycles stylized facts and synchronization analysis. The last chapter concludes with the findings.

Chapter 2

Literature

The purpose of this chapter is to provide a review of relevant literature to business cycles analysis corresponding to both stylized facts and synchronization in the Visegrad Four (V4) and the EU.

The literature studying the business cycles is vast and consists from largely diversified approaches to business cycle examinations. In this review, we start with traditional methods originating in desire to study business cycles as sequences of expansions in many economic activities, which afterwards turn into recessions that, in some time, are followed by a rebound phase leading into an expansion again. This expression is a paraphrase of Burns and Mitchell (1946).

There has been an extensive literature using different instruments to decompose cyclical component in time or frequency, to mention few Benczúr and Rátfai (2010), Lamo, Pérez, and Schuknecht (2013), or Harvey (1985). However, this thesis aims to study stylized facts by a novel approach of wavelet analysis that combines time and frequency space simultaneously, thus, it provides a complex picture of the time series behaviour or about dynamics of more variables.

2.1 Stylized facts of business cycles

The descriptions of statistical properties of co-movements of deviations from long-term trend of particular macroeconomic variables and output of an economy are know as stylized facts of business cycles; this definition of stylized facts comes from Lucas Jr. (1977). To study business cycles the crucial part plays the choice of suitable filtering method. Filtering here means the isolation of cyclical component of macroeconomic variables. In following, the model of real business cycles is shortly introduced. When assembling this type of models, it is always worth to know which stylized facts hold for the type of economy one plans to study.

The text of Kydland and Prescott (1982) is the milestone of the Real business cycles theory (RBC). The real business cycles model, further enriched,[1] postulates the conventional real business cycles model responding to technological changes of different origin. These models show the transmission of shocks to the economy. Furthermore, Long Jr and Plosser (1983) show the predictions cyclical behaviour of some variables affected by output fluctuations originating in change of input, e.g., the labour. They found pro-cyclical behaviour of some output components, i.e., consumption, investment, and other variables such as employment, real wages, and real interest rate. King and Plosser (1984) studied the behaviour of money and prices in RBC models, they show that technology shock to the output projects itself to stylized facts such that prices are counter-cyclical, and money in form of bank deposits pro-cyclically lead output.[2]

Apart from the fundamental modelling issues, the literature also (Baxter, 1991) emphasizes the importance of good chosen filtering method. The most prolific business cycles filter is a univariate filter developed by Hodrick and Prescott (1981). Later studies of Harvey and Jaeger (1993) and Cogley and Nason (1995) show that Hodrick-

[1]There exist a vast of modifications; we name early followers of Kydland and Prescott (1982), e.g. King and Plosser (1984), Hansen (1985), Backus, Kehoe, and Kydland (1992).

[2]Rebelo (2005) elaborates in his work on evolution of RBC models and a broad range of their possible applications.

Prescott (HP) filter may itself create artificial cycles, which do not exist in real time series. Moreover, an ad hoc decision on smoothing parameter lambda has been subject to several studies.[3] Despite the controversies the HP-filter is still widely used probably primarily due to its simplicity and the fact that it became an etalon in RBC practice.

Other authors have also introduced methods extracting the unobserved cyclical and trend components (Harvey, 1985; Harvey and Jaeger, 1993). These models shed light on seasonal and irregular patterns that were hard to find before. Nevertheless, the choice of the filtering methods has to be carefully considered and if possible, filters should be combined.[4]

In the general time series literature, many frequency filters have been developed to deal with similar issues as the previous methods. To name one on behalf all, the model developed by Christiano and Fitzgerald (1999) known as the Band-pass filter allows researchers to filter a desired scale, thus, the application to the low frequencies, often called low pass filter, of the economic data would create suitably filtered time series.[5,6]

First seminal work on stylized facts (Kydland and Prescott, 1990) uses HP-filter to decompose cyclical properties of US economic variables. The findings are very robust because they used many different methods to arrive to them. The authors show that macroeconomic variables appear mostly persistent. They find the pro-cyclical behaviour of investment, consumption, imports, exports, and labour productivity, as well as for money, employment, and real wages. Govern-

[3] Ravn and Uhlig (2002), for instance.

[4] For readers interested in this type of de-trending methods we recommend the work of Canova (1998) to study.

[5] Lamo et al. (2013) serve a short presentation filtering methodology applied to the business cycles in the Euro area.

[6] Other filtering methods such as those of ARIMA family, stochastic switching models, or bivariate filters, are beyond the scope of this thesis.

ment consumption and capital stock are shown to be counter-cyclical. The prices in this study are also counter-cyclical. Further studies confirm their results when evaluating business cycles of developed countries, Backus et al. (1992) for instance. These stylized facts according to the theoretical real business cycles models can also be found in many standard macroeconomic textbooks, e.g., Barro (2007) and Romer (2006). The second business cycles model worth noting is the Keynes sticky wage model developed by Hicks (1937). Briefly, this model, contrary to the classic RBC model, assumes neutral money and rigidity of nominal wages. This implies that labour market does not clear in the short-run. This model differs from the classic RBC model in stylised facts of prices, they are pro-cyclical, real wages and labour productivity, they are theoretically counter-cyclical.

In the literature on empirical evaluation of stylized facts, there are several papers worth noting due to their unorthodox results. The work of Agénor, McDermott, and Prasad (2000) take an outstanding position studying developing countries and their stylized facts.[7] In their findings, they present pro-cyclical behaviour with money aggregates and industrial production. For other macroeconomic variables they do not derive clear conclusion about the cyclical relations. Another example studying economic fluctuations is the work of Tawadros (2011), who shows on the quarterly data that Australia's labour productivity behaves pro-cyclically as well as the real wages. The behaviour of interest rates is counter-cyclical and they appear in demand-leading position to the output.

Ghate, Pandey, and Patnaik (2013), study the structural change of business cycles properties for the case of India. The authors show that business cycles in India resemble more the cycles of developed countries after the liberalization of the economy. The volatility of

[7] The study includes 12 countries as Chile, Columbia, or Turkey, for example.

main macroeconomic indicators decreases the monetary policy appears pro-cyclical. The prices have been found counter-cyclical as in the study of Kydland and Prescott (1990).

The literature focusing on business cycles properties provides only few works studying the former transition countries of Central and Eastern Europe. The work of Benczúr and Rátfai (2010) provides a comprehensive analysis of the CEE stylized facts. They show procyclical behaviour of employment, wages, labour productivity, and money deposits. In the study the nominal exchange rates and nominal interest rates have counter-cyclical behaviour. The consumer prices are persistent and in most of the countries with pro-cyclical pattern, which appears to be common also for G7 countries. The Czech Republic CPI has higher volatility than in other countries because of hyperinflation between 1997-1998. Overall the inflation does not show the stable behaviour in this case. Caraiani (2012) in his work applied wavelet methods to derive business cycles stylized facts of Romania. This work finds similar results as the study of Benczúr and Rátfai (2010).

2.2 Business cycles synchronization

This section gives a brief overview of studies on the business cycles synchronization. It discusses the regional synchronization of transition countries among themselves and analyses of the synchronization of this region with a distinctly larger union. For our case of the Visegrad Four, the motivation is that the business cycles synchronization is a debated requirement affecting the cost of potential accession to an optimum currency area (OCA), the Euro-zone.

The OCA theory was developed by Mundell (1961). The synchronization criterion has more benefit for countries with high business cycle correlation with the rest of area because of giving up their individual monetary policy will be less costly than for those with low correlation. Albeit, we might see issues at the European level when policies ruled by the European Central Bank are applied in countries with low business cycles synchronization (Kolasa, 2013). Literature focusing on evolution and determinants of business cycles synchronization between the Central and Eastern European countries and the EU is extensive, e.g., Darvas and Szapáry (2008), Artis, Marcellino, and Proietti (2004).

Fidrmuc and Korhonen (2006) provide a meta-analysis of business cycles correlation of CEE countries with the EU.[8] The results of the meta-analysis are that there is a high correlation of the new EU members with the Euro area. However, concerning the Visegrad region they show that Hungary and Poland have reached high synchronization in comparison to the others.

Another study, Kutan and Yigit (2004), shows that transition countries should achieve the convergence of monetary variables as well as the economic convergence. The authors analyse the same indicators as Kočenda (2001) but different period, and they obtained the same results about convergence, the lack of nominal variables convergence may postpone the Euro adoption. Backé, Fidrmuc, Reininger, and Schardax (2003) are consistent with these studies by the fact that inflation dynamics of CEE countries converges to EU-12 countries.

2.3 Wavelet analysis and business cycles

After discussing possible approaches, we decided to use data on monthly basis in order to get the longest possible time series for the wavelet analysis. The studies mentioned above suffer from several precondi-

[8]This study lists dozens of texts that may be consult for further overview.

tions. First, when they compare variables between each other they do it in time or frequency domain. The biggest advantage of studies using wavelet analysis is the fact the comparisons or evaluations are done in both time and frequency domains (Rua, 2010). In jargon of signal-processing, wavelets strike the optimal balance between the time-resolution and frequency-resolution, unlike the Fourier transform. Secondly, because the basis function for wavelet analysis is localised in time and has bounded support, the analysis is free from the assumption of covariance-stationarity that almost all current and earlier mentioned business cycles filtering methods suffer from (Raihan, Wen, and Zeng, 2005). Through the process of transformation of time series into desired form using standard techniques, researchers may loose some information. In this text, we obtain results from time-frequency wavelet analysis; thus, we do not risk any loss of information by possible improper time series transformation or adjustment. Wavelet analysis has been achieving its position in many fields, e.g., Aguiar-Conraria, Azevedo, and Soares (2008) analysing the evolution of monetary policy in the US during past 60 years, Cazelles, Chavez, Berteaux, Ménard, Vik, Jenouvrier, and Stenseth (2008) show the application on dynamics of global epidemics, or in energy markets Vacha and Barunik (2012).

Business cycles literature early mentioned has always had to cope with the problems of macroeconomic variable of non-stationary nature. These methods are not able to provide a comprehensive picture of business cycles when their properties vary over time and scale. The wavelet analysis serves well as a suitable methodology to observe all evolutionary aspects of business cycles in time and frequency.

Starting with the work of Yogo (2008), who studies U.S. business cycles uses wavelet analysis. In general, this analysis is able to determine all peaks and troughs in correspondence to their definition by the National Bureau of Economic Research.

Jagrič (2002) and Aguiar-Conraria and Soares (2011a) use wavelet methods to analyse business cycles synchronization across the Europe. Jagrič (2002) takes seven Central Eastern European countries, among them the Visegrad countries, and finds out that the volatility of business cycles was high at the beginning of the transformation process but decreased over time. In countries such as the Czech Republic, Croatia, Slovakia, and Slovenia the stabilization of business cycles was faster than in economies of Hungary and Poland, where the production was less industrially diversified. In Hungary and Poland, the variance of the cycle increases again in the second half of 1990s. The duration of fluctuation differs across the region as well. The cycle in Croatia showed two patterns, from one to two years, and later from four to five. The Czech and Slovak Republic keep their cyclical fluctuation length about two and five years. The cycles in Hungary were long and became shorter due to slower stabilization of economy. The opposite happened in Poland, the evolution went from strong fast cyclical component to stable and slower one. Jagrič (2002) have also determined how well the business cycle resembles the European pattern, eventually determined the influences. In Jagrič (2002) and Jagrič and Ovin (2004), they compare economic cycles of seven economies with Germany, where Germany plays the role of a proxy for the EU. The Czech Republic and Slovakia lagging the German business cycle but otherwise they are synchronized and become better adjusted. The model of Jagrič (2002) implies that economic movement of Slovenia, Hungary, and Poland got very close to be perfectly synchronized. Analogously, study of Bruzda (2011) shows that Poland economy synchronization with the EU increases as well as the synchronization within the EU is stable. More recent study of Aguiar-Conraria and Soares (2011a) takes the industrial production index of the Euro-12 countries and compares it with other countries. This study further compares business cycle synchronizations and distances of analysed regions. Not surprisingly, closer countries manifest higher levels of synchronization. The most similarities for the transition countries occur after 2005. They find the most interesting results in the case of Slovakia, which is already a country of the Euro zone, does not expose any significant pattern of convergence, surely not in long run, eventually, it is close to be in phase in the short-term periods. Contrary to Slovakia, Aguiar-Conraria and Soares (2011a) find Hungarian and Czech business cycles coherent with the EU-12 within last five years.

2.4 Some characteristics of the Visegrad countries

This section slightly steps out of the literature review, however, it provides the background needed before starting an analysis concerning the Visegrad Four. The aim of this section is to describe changes in the Visegrad economies that could lead to alterations in behaviour of business cycles and other macroeconomic variables. Prior to the analysis itself, the first two decades of independent development of the Visegrad Four countries are described in detail. We start our discussion early after the break-up of the "Eastern bloc," when the countries began their independent economic and political journey. The beginning of economic transition is usually associated with shocks to economy. The whole process of transition from planned to market based economy has been always a substantial challenge for political leaders.

Cooperation: Basically after the break-up, Czechoslovakia,[9] Hungary and Poland began a discussion about mutual cooperation during their economic transformations. Despite their originally different economic maturities and development, their willingness and regional proximity guided them to establish the Visegrad group on February 15, 1991. Subsequently in 1992, the countries of the Visegrad group signed the Central European Free Trade Agreement (CEFTA). The agreement captured the countries' willingness to create a small partnership parallel to the European Community[10] (Lukášek, 2010). One of the aims of the group was to help its member states to organize their institutions for faster convergence and integration with the European Union. Institutionally, the Visegrad group negotiations continued in direction of joining to the North Atlantic Treaty Organization (NATO) as well as the European Union in the near future. In 1995-1996, all Visegrad countries applied for membership in the European Union. After the screening process during the next few years, the countries became members of the EU by its enlargement in 2004 (Kočenda and Valachy, 2006). This connection calls for further integration towards the European Monetary Union, which demands good economic stability and better functioning institutions. Until now, only Slovakia has already adopted the Euro as its national currency.

Macroeconomic situation overview: The process of economic transition mainly consisted of the transformation to the market based economy, support of the international trade, opening the market to the foreign capital, liberalization of prices and the privatization of state owned firms. During the transition of the Visegrad countries the first thing they faced was an external shock caused by opening borders.

[9]January 1, 1993, Czechoslovakia was divided into two separate countries, the Czech Republic and Slovak Republic

[10]From 1993 named the European Union.

This may be one of the reasons of an increase in volatility of the output, which was common for all the transition countries. At the first instance, policy makers in all countries were confronted with very high inflation rates linked to the domestic prices distortions that led to economy deterioration. This initial problems led to real exchange rate appreciation because of low foreign capital inflow and further to decline in the output of economies (Dibooglu and Kutan, 2005). From this point of view, the macroeconomic stabilization played a crucial role at the beginning of the transition, because of need of realignment of fiscal and monetary policies (Kočenda, 2001).

The exchange rate was another source of instability in the eyes of policy makers, who wanted to stabilize their economies and make them credible. Thus, at the very beginning they adopted fixed exchange rate regimes. The fixation of the currencies was based on a different baskets of currencies for each of the countries. Each country currency basket was differently weighted and determined in order to correspond to directions of international trade of that given country. The Czech Republic and Slovakia had slightly different target band, but they were tied to keep the peg to this basket through the currency market. However, Hungary and Poland followed a similar setting which they officially labelled as a crawling peg, i.e. their central parities were not constant (Kutan and Brada, 2000; Kočenda and Valachy, 2006). The countries were in different initial positions, therefore despite the fact that they adopted similar policies, their paths were not similar. Nevertheless, within the period of exchange rate targeting they moderately achieved the flexibility of exchange rates and shifted their main monetary strategy towards the inflation targeting. The regime switching is a particular information for our study of stylized facts of business cycles.

In the table 2.1, we provide exact dates of regime switching. Since

Table 2.1: Regime switching

Czech Republic		Hungary	
1994-1997	Exchange rate and monetary targeting	1994-2002	Exchange rate targeting
1998-	Inflation targeting	2002-	Inflation targeting
Poland		Slovakia	
1994-1998	Exchange rate targeting	1994-1998	Exchange rate targeting
1998-	Inflation targeting	1998-2008	Informal inflation targeting
		11/2005-12/2008	ERM-II
		2009-	Euro system

Source: Frömmel, Garabedian, and Schobert (2011)

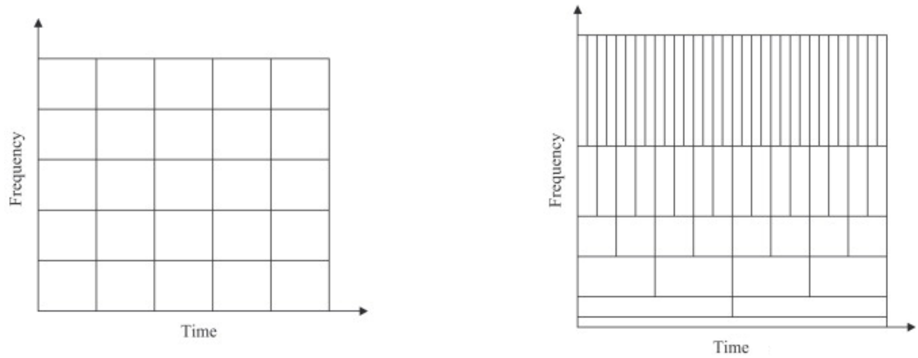
our countries switched to inflation targeting a long time ago, they all now maintain managed floating regimes of exchange rates, except for Poland and Slovakia, whose respective exchange rate floats freely or is the Euro (Frömmel, Garabedian, and Schobert, 2011), for detailed analysis of regime changes in the CEE region one may consult Kočenda and Valachy (2006).

Chapter 3

Theoretical part

This chapter presents the theoretical concepts of time-frequency domain analysis, in this case particularly the wavelet analytical tools. The theory of frequency analysis began in 19^{th} century with Fourier representations. The Fourier transform (FT) is tailored to decompose a signal into a linear combination of sine and cosine functions at different frequencies that can be summed up back to the original function. The Fourier analysis allows observing relations at each of the frequencies, however, this comes at the cost of losing the time information of the signal, as the transform requires the input as a stationary signal. Due to this effect, many time series do not satisfy this constraint, regardless the field they come from. In other words, using the FT makes the analysis time-invariant and hence not suitable to provide any information about dynamics of signal. For this reason, Gabor (1946) developed the short-time Fourier transform (or windowed FT), which is based on applying the Fourier transform on a shorter part of the signal. The width of the window is chosen before the application and its length reflects both required frequency resolution inverse to the window function and window duration, within which the signal most likely fulfils the stationarity assumption. One of the related issues is the inefficiency problem of the short-term Fourier transform, which arises with fixed frequency resolution; it is not possible to change the resolution at different frequencies. However, a series of lower or higher frequencies need lower or higher time resolution, respectively (Gallegati, 2008).

In order to find a better balance between time and frequency resolutions the wavelet transform has been developed. In figure 3.1 it is visible that contrary to the short-time FT (a) with fixed resolution, in case of the wavelet decomposition scheme (b) the window width gets wider at lower frequencies and smaller when the frequency increase.



(a) (b)

Figure 3.1: Time-frequency plane of the short-time Fourier transform (a) and the wavelet transform (b), (Gallegati, 2008).

As shown, the wavelet transform is more flexible in its time and frequency resolutions. The wavelet transform breaks the time series into several functions. These functions belong to the same family, which is based on one particular function known as mother wavelet. The mother wavelet is time-localized in a given time-frequency plane. From that, the wavelet transform using mother wavelet relies on scale (dilation) and time (translation); these are parameters containing the information about the time-scale representation. The dilatation parameter is linked to the frequency, and the translation applies to the location in time. In accordance to each scale, the wavelet transform also produces sets of coefficients where each set relates to a given location.

3.1 Wavelet analysis

The wavelet analysis methodology consists of wide range of instruments designed to break down data on given signals into desired information. This section heavily follows Aguiar-Conraria, Azevedo, et al. (2008) and Grinsted, Moore, and Jevrejeva (2004) because of the depth of discussion of methodology they provide. Reader interested in more details regarding the methodology may consult for example Mallat (1999) and Daubechies (1992). We begin with the wavelet itself, further, the continuous wavelet transform as a former stone of our wavelet methodology will be described, as well as its possible specifications, for more details see Percival and Walden (2006). First of its features is the wavelet power spectrum, which is used for studying the volatility of a time series. The second is the cross-wavelet power that describes a time-varying covariance of a pair of series. Based on the powers, the wavelet coherency estimates local correlations between the series. Lastly, an important interpretation tool of continuous wavelet analysis is the phase difference, which offers information about phase shifts between the variables. Next in the line of wavelet transforms is the discrete wavelet transform; we apply the discrete wavelet transform via its related version called Maximal overlap discrete wavelet transform.

3.1.1 Wavelet

Prior to establishing the wavelet definition, it is convenient to present principal mathematical notation that will be used further on. We employ the symbol ":=" equally to "by definition" and the variables labelled with the asterisk superscript (*) are meant as complex conjugates.

We begin with the set of square integrable functions, $L^2(\mathbf{R})$,

$$\int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t \leq \infty, \tag{3.1}$$

which corresponds to a set of functions defined on the real line. The formulated quantity also represents the energy of the given function, this space therefore symbolises the space of functions with finite energy. In the set of square integrable functions we can define an inner product

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^*(t) \mathrm{d}t \tag{3.2}$$

and related norm $||x|| := \langle x, x \rangle^{\frac{1}{2}}$.

The Fourier transform of $x(t)$, where $x(t) \in L^2(\mathbf{R})$, is given by

$$X(f) := \int_{-\infty}^{\infty} x(t) e^{-i2\pi f t} \mathrm{d}t. \tag{3.3}$$

For all functions of $L^2(\mathbf{R})$, the Perseval relation is valid and stated as

$$\langle x(f), y(f) \rangle = \langle X(f), Y(f) \rangle \tag{3.4}$$

from this relation one can state the Plancherel identity as

$$||x(f)||^2 = ||X(f)||^2, \tag{3.5}$$

claiming that the Fourier transform preserves the energy of a function.

The fundamental function ψ has to fulfil certain requirements before being approved as a mother wavelet (or admissible wavelet) that $\psi \in L^2(\mathbf{R})$.

1. The mother wavelet is habitually normalized to have unit energy,

$$||\psi||^2 = \int_{-\infty}^{\infty} |\psi(t)|^2 \mathrm{d}t = 1. \tag{3.6}$$

The squared integrability to one is a mild condition on the decay of ψ, moreover the usually used wavelet functions have faster decay.

2. The wavelet requires its mean equals to zero.

$$\Psi(0) = \int_{-\infty}^{\infty} \psi(t)\mathrm{d}t = 0 \tag{3.7}$$

3. The admissibility condition is determined such that

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|}\mathrm{d}f < \infty. \tag{3.8}$$

This condition is equivalent to the previous condition for functions with sufficiently fast decay.

The second condition practically says that the wavelet function ψ behaves like a wave, moving up and down around the zero (time axis). To determine the desired mother wavelet, one should keep it in mind along with the decay property.

3.2 Continuous wavelet transform

To get a family of wavelet functions ("wavelet daughters") one needs a mother wavelet ψ is scaled and translated by adequate parameters, s and τ.

$$\psi_{s,\tau}(t) := \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \tag{3.9}$$

where τ is the translation parameter defining time position of ψ and the s is the scale parameter related to the frequency, controlling for the length of the wavelet. The normalization factor $\frac{1}{\sqrt{|s|}}$ guarantees that the wavelet family preserves its unit energy, $||\psi_{s,\tau}|| = 1$, where the parameter s causes dilatation ($|s| > 1$) or compression ($|s| < 1$) of the wavelet (Rua, 2010).

The continuous wavelet transform (CWT) of a signal, $x(t) \in L^2(\mathbf{R})$, with respect to the wavelet ψ, is defined as a convolution of the given signal and the family $\psi_{s,\tau}$,

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{t-\tau}{s} \right) \mathrm{d}t = \int_{-\infty}^{\infty} x(t) \psi^*_{s,\tau} \mathrm{d}t. \tag{3.10}$$

A very important benefit of the wavelet transform is that one can reconstruct the original signal back from obtained wavelet transform. In the reconstruction, the admissibility condition plays a key role allowing the way backward

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \psi_{s,\tau}(t) W_x(s,\tau) \mathrm{d}\tau \right] \frac{\mathrm{d}s}{s^2}. \tag{3.11}$$

Interestingly, these two expressions of the identical signal provide information that we would not be able to observe without such method. The expression

$$||x||^2 = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} |W_x(s,\tau)|^2 \mathrm{d}\tau \right] \frac{\mathrm{d}s}{s^2} \tag{3.12}$$

demonstrates another important property - the energy of a signal x(t) remains the same. Furthermore, for signals of $L^2(\mathbf{R})$ the Parseval type identity is given by

$$\langle x, y \rangle = \frac{1}{C_\psi} \int_{-\infty}^{\infty} [W_x(s,\tau) W_y^*(s,\tau) \mathrm{d}\tau] \frac{\mathrm{d}s}{s^2}. \tag{3.13}$$

There are two ways of approaching the wavelet transform $W_x(s,\tau)$; it can be either complex or real depending on whether the wavelet function ψ is complex or real, respectively. One may divide W_x into its imaginary, $\Im\{W_X\}$, or real, $\Re\{W_X\}$, part. Given these two parts the phase looks like $\phi_x(s,\tau) = \tan^{-1}\left(\frac{\Im\{W_X\}}{\Re\{W_x\}}\right)$, where $\phi \in (-\pi, \pi)$. The position of the phase may look complicated because ϕ_x provides actual position $x(t)$ in its pseudo-cycle. Furthermore, when we have a wavelet only on a real line, its imaginary part equals zero, as does the phase. Another important property of the transform is its amplitude, $|W_x|$. In order to be able to obtain the amplitude and phase of the wavelet transform, one needs to work with complex wavelets. In the case of economics, one mostly deals with real numbers, $x(t) \in \mathbf{R}$, thus, the literature (Daubechies, 1992; Aguiar-Conraria, Azevedo, et al., 2008) recommends to use the wavelet function that for its values $f < 0$ satisfies $\Phi(f) = 0$. Additionally, one may limit the scaling parameter to positive values only, which leads to the formulation of reconstructed signal in following form:

$$x(t) = \frac{2}{C_\psi} \int_0^\infty \left[\int_{-\infty}^\infty \Re(W_x(s,\tau)\psi_{s,\tau}(t))\mathrm{d}\tau \right] \frac{\mathrm{d}s}{s^2}. \tag{3.14}$$

As well, the energy formula limits itself to positive values over the frequency:

$$||x||^2 = \frac{2}{C_\psi} \int_0^\infty \left[\int_{-\infty}^\infty |W_x(s,\tau)|^2 \mathrm{d}\tau \right] \frac{\mathrm{d}s}{s^2}. \tag{3.15}$$

The Parseval type identity completes the list of adjusted properties:

$$\langle x, y \rangle = \frac{2}{C_\psi} \int_0^\infty [W_x(s,\tau)W_y^*(s,\tau)\mathrm{d}\tau] \frac{\mathrm{d}s}{s^2}. \tag{3.16}$$

Choice of the wavelet function

As apparent, the wavelet function is the fundamental object of the whole analysis. The function should be chosen with respect to the characteristics important to the application. There are a number of functions usable in the wavelet analysis with different properties, for example, Daubechies, Haar, Mexican hat, Morlet, etc. In our analysis, we work with the real data, but the most desired feature of the

CWT will be the wavelet coherency and the phase difference analyses, for which we need a complex function. For this reason, we pick the Morlet wavelet, which fits the best with respect to our requirements. Moreover, the Morlet wavelet is a widely supported type of wavelet function, since it offers a good trade-off between time and scale localization (Grinsted et al., 2004).

Right choice of ψ is preceded by correct localization of the wavelet function. Necessarily, the μ_f and σ_f^2, which stand for the centre and variance of the Fourier transform, Ψ, of the wavelet ψ, respectively, can be analogously defined from the centre and variance of the wavelet itself:

$$\mu_t = \int_{-\infty}^{\infty} t|\psi(t)|^2 \mathrm{d}t. \tag{3.17}$$

and

$$\sigma_t^2 = \left\{ \int_{-\infty}^{\infty} (t-\mu_t)^2 |\psi(t)|^2 \mathrm{d}t \right\}. \tag{3.18}$$

It is known that ψ and Ψ reach their "most significant" values in following intervals $[\mu_t - \sigma_t, \mu_t + \sigma_t]$ and $[\mu_f - \sigma_f, \mu_f + \sigma_f]$, respectively (Aguiar-Conraria and Soares, 2011b). Then in the time-frequency plane, one constitutes a rectangle $[\mu_t - \sigma_t, \mu_t + \sigma_t] \times [\mu_f - \sigma_f, \mu_f + \sigma_f]$ that corresponds to the Heisenberg box. Given this, the uncertainty of whether the wavelet ψ is localized around the two centres, (μ_t, μ_f), is equal to $\sigma_t \sigma_f$. The Heisenberg uncertainty principle also says that the lower bound of uncertainty is equal to $\frac{1}{4\pi}$, so that $\sigma_t \sigma_f \geq \frac{1}{4\pi}$.

The Morlet wavelet has a simplified[1] form:

$$\psi_\eta(t) = \pi^{\frac{1}{4}} e^{i\eta t} e^{-\frac{t^2}{2}}, \tag{3.19}$$

where the parameter η is preferably chosen so that $\eta = 6$, in order for the wavelet frequency centre to be approximately equal to one,

[1]The admissibility condition holds for $\eta > 5$.

$\mu_f = \frac{6}{2\pi} \approx 1$. From that, the relationship between the scale and frequency looks like

$$f = \frac{\mu_f}{S} \approx \frac{1}{s}. \tag{3.20}$$

3.2.1 Wavelet power spectrum

In economics, we deal with discrete time series, therefore we need to make the integral from the equation 3.10 discrete as well; this results for a time series $\{x_t, n = 0, 1, \ldots, N-1\}$ of N observations and constant time steps δt in the continuous wavelet transform of x_t as

$$W_m^x(s) = \frac{\delta t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \psi^* \left[(n-m)\frac{\delta t}{s}\right], m = 0, 1, \ldots, N-1. \tag{3.21}$$

For each value of s and m one can compute the wavelet transform using this equation (3.21). Moreover, the computation can be done for all m at once, since the convolution theorem allows to do N' convolutions in Fourier space. Using the convolution of two sequences the continuous wavelet transform for each s is obtained for all m simultaneously (Torrence and Compo, 1998).

It is surely worthwhile to observe energy of a series, in other words, its volatility (local variance) at different frequencies. The single wavelet power spectrum, $|W_x|^2$, is what provides us this piece of information.

When analysing finite length time series by the continuous wavelet transform we always need to care about the border errors, which often plague such kind of analyses. This happens because the edge values of CWT are inappropriately computed, as the CWT is not properly localised in time. To overcome this problem the time series are artificially extended on both sides by zeroes, which is usually referred to as zero padding. The number of zeroes increases with the scale s, as the wider the window the larger the effects of these edges. The padding creates discontinuities at the borders of wavelet spectrum; the cone of influence (COI) is the region where the edge effects have to be carefully interpreted because of the lack of accuracy. Following Aguiar-Conraria, Azevedo, et al. (2008) and Torrence and Compo (1998), we define the COI as the e-folding time of the wavelet at the scale of s. The COI is a space in which the wavelet power, caused by discontinuity at the borders, has declined to e^{-2} of the value at the edge.

To test statistical significance of the wavelet power at time-frequency plane, Torrence and Compo (1998) have proposed to test the wavelet power against the null hypothesis saying that the signal is generated by a stationary process with a particular background power spectrum P_f (Grinsted et al., 2004). For testing processes that are more general, it is necessary to apply Monte-Carlo simulations. Under the null-hypothesis, the distribution corresponding to the local power spectrum has been obtained from the wavelet power spectra of red-noise. The probability that the wavelet power is greater than p, at each scale s and at given time n, is:

$$D\left(\frac{|W_n^x(s)|^2}{\sigma_x^2} < p\right) = \frac{1}{2}P_f\chi_v^2(p), \tag{3.22}$$

v is switching between 1 and 2 whether the wavelet is real or complex, respectively.

3.2.2 Cross-wavelet transform and power

The cross-wavelet transform is a product of two wavelet transforms of two given signals; and it provides information about high common power between them,

$$W_n^{xy} = W_n^x W_n^{y*}. \tag{3.23}$$

The cross-wavelet power (XWT) is defined as $|W_n^{xy}|$. Contrary to the wavelet power spectrum showing local variances, the XWT presents local covariances between signals x and y, as quantified at each frequency. Additionally, the local relative phase between two analysed series can be retrieved as the complex argument of $\arg(W^{xy})$. In order to ascertain the statistical significance of a drawn cross-wavelet power, we follow Torrence and Compo (1998), who have shown that the theoretical distribution of cross-wavelet of two time series x_t and y_t with Fourier spectra P_f^x and P_f^y, respectively, is defined as

$$D\left(\frac{|W_x W_y^*|}{\sigma_x \sigma_y} < p\right) = \frac{Z_v(p)}{v} \sqrt{P_f^x P_f^y}. \tag{3.24}$$

In the distribution above, σ_x and σ_y stand for the respective standard deviations, and the confidence level $Z_v(p)$ relates to the probability p for a probability density function given by the square root of the product of two χ^2 distributions (Grinsted et al., 2004).[2]

3.2.3 Wavelet coherency

To continue, the following measure is a pivotal when analysing the business cycles by the means of wavelets. One can define the wavelet coherency (WTC) between two time series as a proportion of their cross-power spectrum to the product their individual power spectra. The wavelet coherency provides local correlations between the two signals, quantified between 0 and 1. Torrence and Webster (1999) defines the wavelet coherence of x_t and y_t as

$$R_n(s) = \frac{|S\left(s^{-1} W_n^{xy}(s)\right)|}{S\left(s^{-1}|W_n^x|\right)^{\frac{1}{2}} S\left(s^{-1}|W_n^y|\right)^{\frac{1}{2}}}. \tag{3.25}$$

[2]Torrence and Compo (1998) shows that $Z_1(95\%) = 2.182$ for real wavelets ($v = 1$) and for the complex wavelets $Z_1(95\%) = 2.182$, ($v = 2$).

The S represents the smoothing operator, which is necessary and is given by $S(W) = S_{scale}(S_{time}(W_n(s)))$, where the first smoothing convolution is defined along the scale axis and denoted by S_{scale} and the other in time by S_{time}.

Testing the statistical significance level of the wavelet coherence is done by Monte-Carlo methods.[3]

3.2.4 Phase difference analysis

The reason why we study the relative position of two variables in our analysis is to locate situations when one leads or lags after the second, or they coincide. In wavelet analysis, this idea is called phase difference. The function of this concept is given by

$$\phi_{x,y} = \tan^{-1}\left(\frac{\Im\{W_n^{xy}\}}{\Re\{W_n^{xy}\}}\right) \tag{3.26}$$

and its values are $\phi_{x,y} \in [-\pi, \pi]$ (Aguiar-Conraria, Azevedo, et al., 2008). We use the following rules to interpret the phase difference of two series. The interval $[-\pi, \pi]$ is divided into four of the same width. We can say that two variables are positively correlated when $\phi_{x,y} \in [-\pi/2, \pi/2]$, if this is fulfilled then they are in phase. On the other hand, two variables are negatively correlated when $\phi_{x,y} \in [-\pi, -\pi/2]$ or $\phi_{x,y} \in [\pi/2, \pi]$; this also means an anti-phase relationship. Further, the phase difference provides information about leading or lagging relationship between two variables. The variable x leads y if the phase difference value is within $[\pi, 0]$, and on contrary, x is behind y when the phase difference is in $[-\pi, 0]$. These rules are based on the following example which intuitively illustrates the position of two signals and their phase difference in time.

[3]In the paper, the black contour indicates the 5 % significance level against the red noise computed through Monte Carlo simulations. The shaded area is the cone of influence. This holds for plots of wavelet power spectrum, cross-wavelet power, and wavelet coherency.

The phase difference of two artificial series

In our analysis, one of the pivotal instruments for correct clarification of two time series behaviour is the phase difference. Here, we emphasize on a compact elaboration of the accurate interpretation of the phase difference. We choose two artificial signals, x_t and y_t, whose time behaviour is known to us, and therefore we may match this type of behaviour with the output of the packages used in this study. Aguiar and Soares (2011) provide equivalent definition of the phase difference such as $\phi_{xy} = \phi_x - \phi_y$, where ϕ_x and ϕ_y are the phase differences of individual signals against a pseudo cycle.

Our selected signals are defined as follows:

$$x_t = \sin(t), \quad t \in [1, 1000] \tag{3.27}$$

$$y_t = \begin{cases} \sin(t), & t \in [1, 250] \\ \sin(t - \frac{\pi}{4}), & t \in [251, 500] \\ \sin(t + \frac{3\pi}{4}), & t \in [501, 750] \\ \sin(t - \frac{3\pi}{4}), & t \in [751, 1000] \end{cases} . \tag{3.28}$$

We define the second signal in four different forms. Within the first interval both signals are identical, in the second interval, $[252, 500]$, x_t is leading y_t because y_t is shifted to the right by $\frac{\pi}{4}$ of phase difference and the signals are positively correlated. The correlation sign changes when y_t reaches the third interval, where the signals become negatively correlated and the leading position changes, y_t leads x_t. For the last interval, the signals remain negatively correlated and x_t returns to the leading position.

In this framework, we use the package of Grinsted et al. (2004) where the arrows direction in plots of cross-wavelet power and wavelet coherence shows phase difference of two signals. The same interpretation holds for our setting. Arrows pointing to the right show

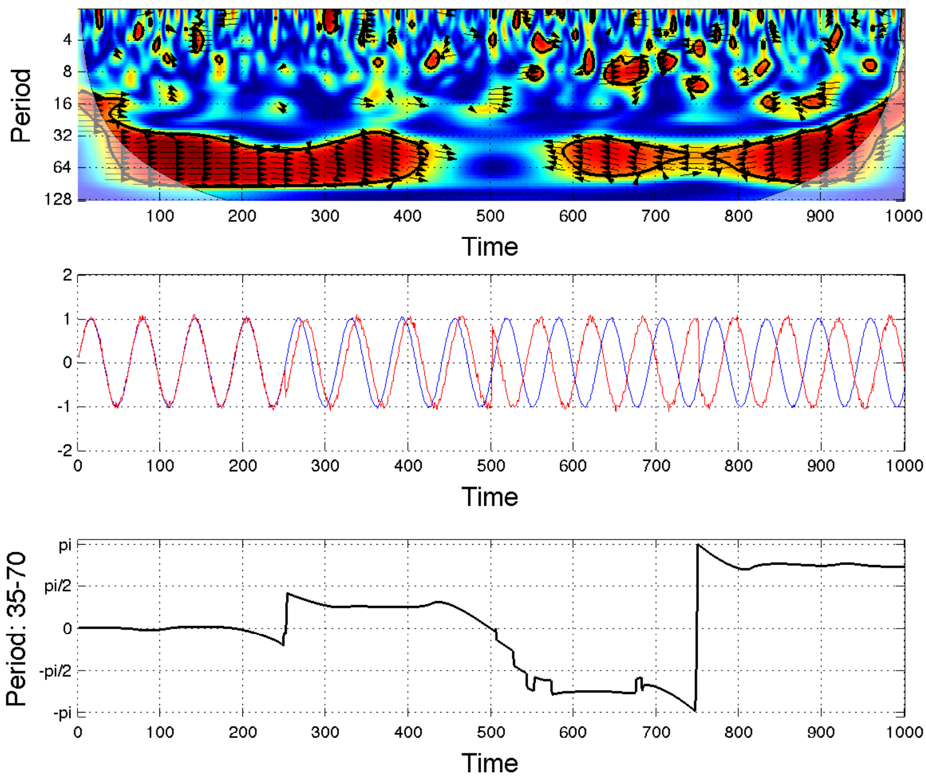


Figure 3.2: Phase difference of two artificial series; At the top: wavelet coherency, In the middle: original artificial series, x_t – blue line, y_t – red line; At the bottom: phase difference plot of ϕ_{xy}; Source: author's computations

that the two series are in phase, whereas arrows pointing to the left denote that they are out-of-phase. When the direction is down, the signal x_t leads y_t by 90°, contrary, when up y_t leads x_t by 90°.

3.3 Discrete wavelet transform

The second type of wavelet transform supplementing the CWT as described in the previous section is the discrete wavelet transform (DWT). Succeeding the CWT, the DWT had been proposed in late 1980s by works of Daubechies (1992) and Mallat (1999), in which they formed several versions of DWT. Our study relies more on the CWT transform, and we only employ the DWT for the purpose of initial analyses of the time series. Particularly, we take advantage of the maximal overlap DWT (MODWT), which has two main advantages over the classical DWT: firstly, the signal does not have to be the length of power of 2, and secondly, the MODWT wavelet and scaling coefficient are time-invariant.

For the purpose of our research we utilize the wavelet filter of the Daubechies 'least asymmetric' family (LA). This type of filters is distinctive due to its very accurate time-localization of wavelet coefficients and the authentic signal.

In what follows we provide brief and necessary components of DWT in order to be able to use the MODWT, especially to draw the energy and variance of variables at each scale. At the beginning, there are two given functions: father wavelet and mother wavelet, whose respective integrals should be one and zero (Crowley, 2007). Daubechies (1992) denotes all filter coefficients such that $\{h_l\}_{l=0}^{L-1}$ and $\{g_l\}_{l=0}^{L-1}$ are the wavelet and scaling filters, respectively, and L is the even number corresponding to the width of the filters. The mother wavelet filters, h_l, coefficients correspond to high-pass filters and the father wavelet filters, g_l, to low-pass filters. Moreover, the relationship of these two filters is a quadrature mirror, that is, $h_l = (-1)^l g_{L-1-l}$ for $l = 0, \ldots, L-1$. The wavelet filter coefficients must fulfil following three properties: zero mean, unite energy and being orthogonal to its even shifts:

$$\sum_{l=0}^{L-1} h_{1,l} = 0, \quad \sum_{l=0}^{L-1} h_{1,l}^2 = 1, \quad \sum_{l=0}^{L-1} h_{1,l}h_{1,l+2n} = 0, \quad \forall n \in \mathrm{Z}, n \neq 0. \tag{3.29}$$

In practice, one uses a pyramid algorithm to get the wavelet $W_{j,t}$ and scaling $V_{j,t}$ coefficients at levels $j = 1, \ldots, J$; this filtering scheme was developed by Mallat (1999). At each level the wavelet and scaling coefficients are obtained from decomposition of the given input. At the first level, we input the signal, x_t, $t \in \{0, \ldots, N-1\}$, to the filters; sequentially, the scaling coefficients are the input for the next level of decomposition up to the last level, where the last scaling coefficient, $V_{J,t}$, remains.

While using the DWT, we should keep in mind that the length of the signal is $N = 2^J$. This implies that J is the maximum level of decompositions and the numbers of coefficients at each scale are $\frac{N}{2}, \frac{N}{4}, \ldots, 1$. Both sets of coefficients obtained from MODWT are defined the same way as from the DWT, only the filters coefficients are rescaled,

$$\tilde{W}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_{j,l} x_{t-l} \tag{3.30}$$

and

$$\tilde{V}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_{j,l} x_{t-l} \tag{3.31}$$

where the filters are $\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$ and $\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}$. In the case of MODWT, one can use the complete non-dyadic length of analysed signal. Furthermore, the MODWT offers more efficiency when estimating the wavelet variance (Gallegati, 2008).

3.3.1 Energy decomposition

From Percival and Walden (2006) we can see that the energy preserving condition in discrete wavelet transform

$$||X||^2 = \sum_{i=1}^{J} ||\tilde{W}_j||^2 + ||\tilde{V}_J||^2, \tag{3.32}$$

where $||X||^2 = \sum_{t=0}^{N-1} x_t^2$ is the energy in the signal x_t, $||\tilde{W}_{j,t}||^2 = \sum_{t=0}^{N-1} W_{j,t}^2$ is the energy contribution by wavelet coefficient at each level, and $||V||^2 = \sum_{t=0}^{N-1} W_{J,t}^2$ provides the information about the energy of the last scaling coefficient.

3.3.2 Variance decomposition

This section is concluded by description of the appropriate formulas used to derive the wavelet variance and the variance of a signal. Percival (2008) furnishes this setting starting with the sample variance of the signal:

$$\hat{\sigma}_x^2 = \frac{1}{N}\sum_{t=0}^{N-1}(x_t - \bar{x})^2, \text{where } \bar{x} = \frac{1}{N}\sum_{t=0}^{N-1} x_t. \tag{3.33}$$

Defining the sample variance based on equation 3.32, one gets

$$\hat{\sigma}_x^2 = \frac{1}{N}\sum_{t=0}^{J} ||\tilde{W}_j||^2 + \frac{1}{N}||\tilde{V}_j||^2 - \bar{x}^2. \tag{3.34}$$

The sample variance regarding to the level j of wavelet coefficients can be written as $\nu_j^2 = ||\tilde{W}_j||^2/N$, this can be viewed as the empirical wavelet variance. Since $\tilde{V}_J$ is a running average of the signal, its sample mean is $\bar{x}$. We can break σ_x^2 into $J+1$ pieces, thus, the sample variance sums up the empirical wavelet variances over the levels as

$$\hat{\sigma}_x^2 = \frac{1}{N}\sum_{j=1}^{J} ||\tilde{W}_j||^2 = \sum_{j=1}^{J} \hat{\nu}_j^2, \tag{3.35}$$

for more details consult Percival (2008).

Chapter 4

Data and preliminary analysis

In our analysis we utilize a single dataset, see table 4.1. We have decided to use publicly accessible time series from the database of the Main macroeconomic indicators (OECD, 2014), originally published by Federal Reserve Bank of St. Louis (FREDII).[1] The period of interest begins by January 1991 and lasts until December 2013. The data we analyse are all aggregated on monthly basis in order to have the maximal possible number of observations for the wavelet analysis. We choose to use the data as they are, without any transformations, because these are not necessary when applying the wavelet methods. All the variables are seasonally non-adjusted, except for the industrial production indices. Most of the series cover the analysed period, but some start later than 1991:M1, which is due to the fact that they were not collected earlier, or in the case of Hungarian and Slovakia's 3-month interest rates, they contain some pitfalls, thus we adjusted the periods.

In whole analysis, the industrial production index represents the proxy to the economic activity, thereby it may not be a perfectly efficient proxy to the output of an economy; the problems may appear when industrial production share in economies becomes lower in the time. That said, the availability of Industrial production index on monthly basis makes it a broadly used substitute for the economic activity, despite its shortfalls (Bruzda, 2011; Gallegati, 2008).

[1]The data are available at https://research.stlouisfed.org and were downloaded on May 3, 2014.

Table 4.1: Data variables

Variable	Definition	Sample Period	Obs.
CZ Production	Production of Total Industry in Czech Republic; Index 2010=100	1991:M1-2013:M12	276
HU Production	Production of Total Industry in Hungary; Index 2010=100	1991:M1-2013:M12	276
PL Production	Production of Total Industry in Poland; Index 2010=100	1991:M1-2013:M12	276
SK Production	Production of Total Industry in Slovak Republic; Index 2010=100	1991:M1-2013:M12	276
Germany Production	Production of Total Industry in Germany; Index 2010=100	1991:M1-2013:M12	276
USD CZK Exchange Rate	US Dollar to National Currency Spot Exchange Rate for the Czech Republic	1991:M1-2013:M12	276
CZ CPI	Consumer Price Index: All Items for Czech Republic; Index 2010=100	1991:M1-2013:M12	276
CZ Inflation	Based on CPI, Percent Change from Year Ago	1992:M1-2013:M12	264
CZ PPI	Domestic Producer Prices Index: Manufacturing for Czech Republic; Index 2010=100	1991:M1-2013:M12	276
CZ Interest Rate	3-Month or 90-day Rates and Yields: Interbank Rates for the Czech Republic	1993:M1-2013:M12	252
CZ 10Y Gov Bond Yields	Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the Czech Republic	2000:M4-2013:M12	165
CZ Unemployment	Registered Unemployment Level for the Czech Republic	1991:M1-2013:M12	276
CZ M1	M1 for the Czech Republic	1993:M1-2013:M12	252
USD HUF Exchange Rate	US Dollar to National Currency Spot Exchange Rate for Hungary	1991:M1-2013:M12	276
HU CPI	Consumer Price Index: All Items for Hungary; Index 2010=100	1991:M1-2013:M12	276
HU Inflation	Based on CPI, Percent Change from Year Ago	1991:M1-2013:M12	276
HU PPI	Domestic Producer Prices Index: Manufacturing for Hungary; Index 2010=100	1998:M1-2013:M12	192
HU Interest Rate	3-Month or 90-day Rates and Yields: Interbank Rates for Hungary	1991:M1-2004:M3	159
HU 10Y Gov Bond Yields	Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Hungary	1999:M2-2013:M12	179
HU Monthly Earnings	Monthly Earnings: Manufacturing for Hungary; National currency	1995:M1-2013:M12	228
HU Unemployment	Registered Unemployment Level for Hungary	1991:M1-2013:M12	276
HU M1	M1 for Hungary	1993:M6-2013:M12	247
USD ZLO Exchange Rate	US Dollar to National Currency Spot Exchange Rate for Poland	1991:M1-2013:M12	276
PL Interest Rate	3-Month or 90-day Rates and Yields: Interbank Rates for Poland	1991:M6-2013:M12	271
PL 10Y Gov Bond Yields	Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Poland	2001:M1-2013:M12	156
PL Monthly Earnings	Monthly Earnings: Manufacturing for Poland; Index 2010=1.00	1995:M1-2013:M12	228
PL Unemployment	Registered Unemployment Level for Poland	1991:M1-2013:M12	276
PL M1	M1 for Poland	1991:M1-2013:M12	276
PL CPI	Consumer Price Index: All Items for Poland; Index 2010=100	1991:M1-2013:M12	276
PL Inflation	Based on CPI, Percent Change from Year Ago	1991:M1-2013:M12	276
PL PPI	Domestic Producer Prices Index: Manufacturing for Poland; Index 2010=100	2000:M1-2013:M12	168
USD SVK Exchange Rate	US Dollar to National Currency Spot Exchange Rate for the Slovak Republic	1993:M1-2013:M12	252
SK 10Y Gov Bond Yields	Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the Slovak Republic	2000:M9-2013:M12	160
SK Monthly Earnings	Monthly Earnings: Manufacturing for the Slovak Republic; Index 2010=1.00	1992:M1-2013:M12	264
SK Unemployment	Registered Unemployment Level for the Slovak Republic	1991:M1-2013:M12	276
SK CPI	Consumer Price Index: All Items for Slovak Republic; Index 2010=100	1991:M1-2013:M12	276
SK Inflation	Based on CPI, Percent Change from Year Ago	1992:M1-2013:M12	264
SK PPI	Domestic Producer Prices Index: Manufacturing for Slovak Republic; Index 2010=100	1994:M1-2013:M12	240
SK Interest Rate	3-Month or 90-day Rates and Yields: Interbank Rates for the Slovak Republic	1999:M6-2013:M12	175

All the time series are seasonally non-adjusted (NSA) except for the five series of Production of Total Industry (SA). Source of the data: OECD (2014), see for details.

Throughout the whole thesis, the economy of Germany is assumed to be a representative of the European Union. This is an obvious choice, given the proximity of Germany to the Visegrad countries and its frequent usage as a reference country (Fidrmuc and Korhonen, 2006). Moreover, Germany is one of the biggest exporters in the world, and therefore also an important partner for export oriented countries of the Visegrad Four; according to Wlazel (2012), more than 25 % of Visegrad export is going to Germany, but 68 percent of this export is not intended for domestic consumption, it gets indirectly re-exported to another country.

All computations running the continuous wavelet tools are done in Matlab software,[2] and results of discrete wavelet analysis are prepared in **R** software.[3]

4.1 Industrial Production Indices

The indices of industrial production (IIP) are fundamental time series of the analysis of business cycles. We thus supply the descriptive statistics of these five series supplemented by the plot of the original indices, all are normalized such that values in 2010 are equal to 100; table 4.2 and figure 4.1 provide first basic statistical facts about the economic evolution in the Visegrad region and about the German economy.

We observe the better economic state of German production with the highest mean and the lowest variance. In comparison with Germany, all Visegrad countries registered more severe downturn at the beginning of the transition. Furthermore, there are periods of strong growth in the cases of Slovakia and Hungary; for Hungary it is most

[2]We use the package developed by Grinsted et al. (2004), which we adjusted to our needs.

[3]In **R** we employ the package *wmtsa* developed by Percival and Walden (2006). All codes are available upon request.

Table 4.2: Data description: Indices of Industrial Production

Variable	min	max	mean	σ^2	skewness	kurtosis
CZ.Production	55.63565	115.7683	81.36170	320.7137	0.26004607	-1.4179334
Germany.Production	73.36172	112.3742	91.29628	126.3242	0.41004868	-1.1213831
HU.Production	32.72147	117.0098	74.18372	706.6913	-0.09839214	-1.4892536
PL.Production	27.18940	113.1906	66.54393	686.4595	0.25086401	-1.2399391
SK.Production	40.51990	132.9204	69.95717	600.6274	0.76043231	-0.6592471

Source: author's computations

evident in the pre-crises period (befor 2008) and for Slovakia after 2009.

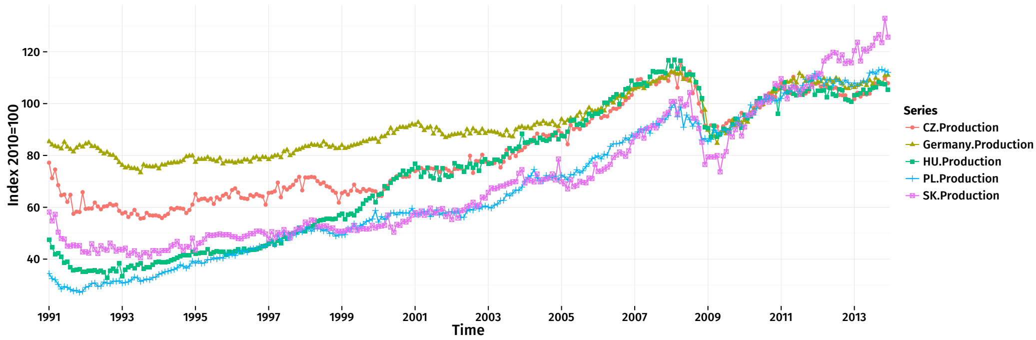


Figure 4.1: Indices of Industrial Production of all five economies; Source: author's computations

4.2 Economic cycles basic filtering

In the upcoming chapter, we will employ the continuous wavelet tools to study local properties and common features of chosen time series. Despite the fact that the wavelet analysis consists of very powerful instruments, we are not able to see the real shape of business cycles. Ergo, we propose to use a classic tool to derive the cyclical components of industrial production indices, which is the Hodrick-Prescott (HP) filter. We follow the basic setting suggested by Hodrick and Prescott (1981), where $\lambda = 14400$ for monthly data.[4]

[4]The smoothing operator λ and its selection have been a subject of study of many authors. An interested reader might consult Ravn and Uhlig (2002), who recommend to use higher λ.

Table 4.3: Variance of HP business cycle components

Variable	CZ.Production	Germany.Production	HU.Production	PL.Production	SK.Production
Variance	0.1547493	0.1118121	0.1830448	0.1052638	0.2417712

Source: author's computations

The table 4.3 provides the pivotal information on how the economies' outputs fluctuate around their long-term trend. The lowest volatility among the five countries was registered in the cases of Poland and Germany, and contrary to that, the Slovakia's economic cycle fluctuates the most.

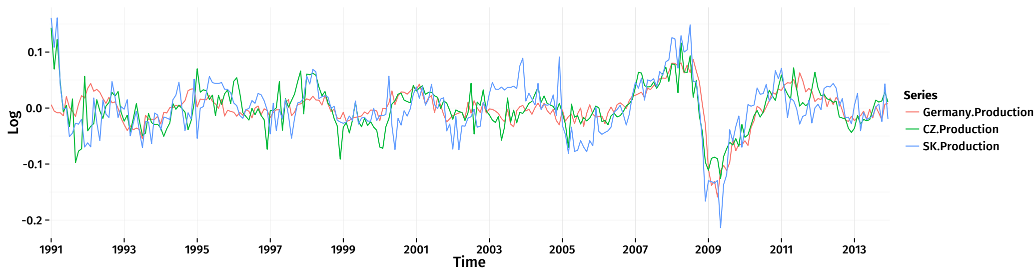


Figure 4.2: Hodrick-Prescott filter of Total Industrial Production of Germany, Czech Republic, and Slovakia; Source: author's computations

In figures 4.2 and 4.3, we compare the cycle of Germany appears to be the most stable one, followed by the Poland business cycle. Furthermore, the comparison reveals that the Visegrad countries began their transition with falling output, which is not a surprising thing; however, a similar pattern is present in the five studied economies in the period after they have been hit by the last crisis, beginning in 2008.

The motive behind the Hodrick-Prescott filter cyclical components plots is that we may be able to reveal the position of an economy within a cycle. Further on, we would like to know whether the production rises or declines; for example, the unemployment and production relationship may vary, they might be in-phase or out-of-phase at different time, therefore, it is desirable to know if an economy grows or falls.

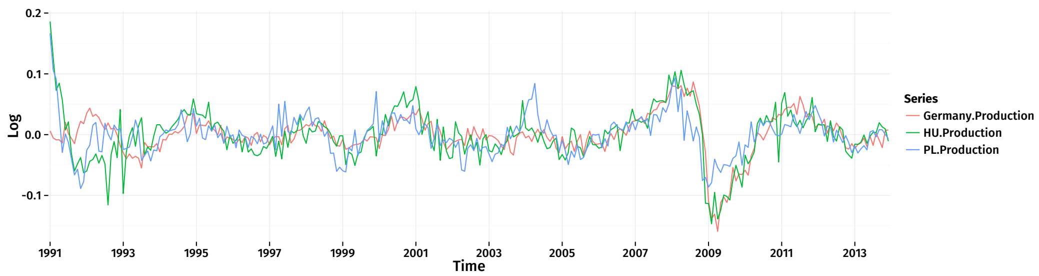


Figure 4.3: Hodrick-Prescott filter of Total Industrial Production of Germany, Hungary, and Poland; Source: author's computations

4.3 Energy and variance decomposition

Our analysis focuses on the business cycles frequencies, it is therefore important to find out which frequency a time series carry out the highest energy. In other words, which frequencies contributes the most to the total variance. Preliminarily, we applied the MODWT described in the section 3.3 using the Daubechies LA(8) wavelet filter to obtain the wavelet coefficients, from which we derived the energy and variance shares over all scales and for all time series. We decomposed time series into seven scales that represent different periods of given signal. Each scale $d1, d2, \ldots, d6$, has its time period specified according to the set of $2-4, 4-8, \ldots, 64-128$ months, and the long-term trend above 128 month for $s6$.

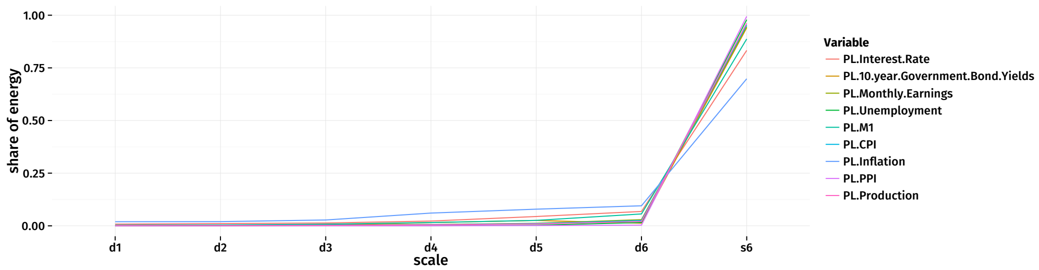


Figure 4.4: Energy decomposition, Poland indicators; Data source: OECD (2014); Source: author's computations

The figure 4.4 presents the energy localization of Poland's economy time series. This pattern of decomposition holds for all the analysed time series of the Visegrad countries.[5] Moreover, this figure supports our assumption that most of the energy, from 60 % to almost 100 %, lies in the long-term trend of the series. Thus, we can expect that if they have a common power it will be localized towards the low frequency of the series.

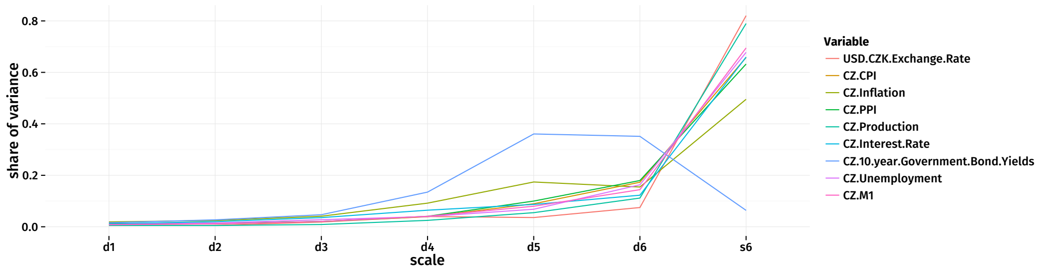


Figure 4.5: Variance decomposition, Czech Republic indicators; Source: author's computations

In the figure 4.5 one can see that most of the macroeconomic indicators contain more volatility at scales devoted to the medium- and long-term. Thus, the volatility of the series is mostly determined in horizons between 32 and 128 months (≈ 3-10 years), and by the long-term trend. As for the case of energy decomposition the variance decomposition pattern remains similar for given countries.[6] One may observe that only the share of variance is higher for 10-year government bond yields at scales about 16 to 64 months. One possible explanation could be these are the variables with higher variation at high frequencies.

[5]See figures in the appendix: A.1, A.2, and A.3
[6]See figures in the appendix: A.4, A.5, and A.6

Chapter 5

Empirical part

This chapter presents results of the continuous wavelet analysis. Our analysis relies on studying relationships of two time series. We begin each analysis by employing the power spectrum that gives us the information about the power of a given time series or between two series. These are useful for the initial information about variances and covariances while decomposing macroeconomic indicators. Another employed instrument is the wavelet coherency, which estimates local correlations between two time series in the time-frequency space. An additional tool of wavelet analysis we find useful for the business cycles analysis is the phase difference of two time series. We provide figures of the phase differences along with all the wavelet coherency outputs in order to support our results.

At the beginning of the analysis we normalized all the time series to have unit variance and zero mean in order to test the phase difference significance (Cazelles et al., 2008). The normalization allows us to exactly quantify the size of noise for Monte Carlo simulations of phase differences. Hence, we added a noise of 5 % to the standard deviation of a time series to this particular time series. The rationale is that the data usually has different variances. Therefore a noise of wrong size would lead to destruction of the original time series, which would result in loss of information. This procedure follows approach of Torrence and Webster (1999); the determination of ”true” phase difference confidence interval is difficult to realise without Monte Carlo simulation. We run 1000 phase difference simulations, reorder, and quantify the 90% confidence interval. To our knowledge, this study is the first to show the confidence intervals of phase differences in wavelet analysis.

The first part of the chapter contains the analysis of stylized facts of business cycles in the Visegrad countries. We begin with a description of price measures, exchange rates and their linkages to the economic cycles. We continue with the relationship of monetary instruments and the economic activity of a given country. The stylized facts analysis is completed by the list of facts concerning unemployment and monthly earnings relation to the business cycles.

In the second part of the chapter, we focus on business cycles synchronization and on co-movements between the countries of the Visegrad Four. Furthermore, we expand the analysis by comparison of business cycles synchronization between the Visegrad Four and the European Union. In this thesis, we use Germany to represent the European Union.

5.1 Stylized facts of the Visegrad countries

Studying the stylized facts relies on well-known key features which are common for both the traditional and the wavelet filtering approaches. Among these features examined in most of the studies there are the volatility of a given time series, the phase relationship of two time series, and the third used stylized fact is the cyclical behaviour, i.e., whether variables are pro-cyclical or counter-cyclical.[1]

As we have already described the properties of the industrial production indices[2] in the preliminary analysis, we start this section by

[1]Whether they are in phase or out-of-phase with the business cycle.

[2]In the text we use ‘Production of Total Industry Index’ as a proxy to the economic activity of a particular country. Further in text, we use shorter label of this index - the production of a given

description of the results concerning the price measures.

5.1.1 Prices

We selected three measures in order to obtain a complex information about price behaviour in the economies. We use consumer price indices (CPI), producer price indices (PPI), and the year-on-year inflation rates based on CPIs. The use of these different measures appears in the literature and it may bring different results (Caraiani, 2012). By the inflation-business cycles relationship we demonstrate changes in monetary policy targeting of economies.

Looking at the dynamics of variances of both CPI and PPI in the figures A.8 and A.9, we see that higher variance is present mostly for all CPIs and PPIs at the beginning of transition in the business cycles period of 2-8 years. Solely, the PPI of Poland does not expose any increased variance during early 1990s but experiences high significant volatility during the last 10 years in at low frequencies. Information

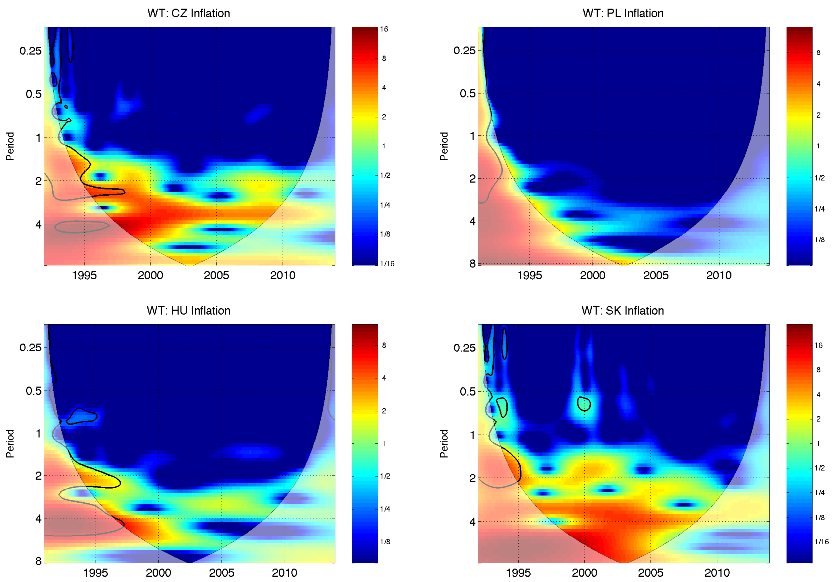


Figure 5.1: Wavelet power spectrum: CPI based inflation; From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia

country.

about inflations volatility is depicted in the figure 5.1, and these results are in line with findings mentioned in the section 2.4. The V4 countries started the transition with very high inflations. All countries encountered at least 4 years of significantly high inflation at 2-8 year period. The Czech Republic experienced high inflation continuously until 2000. Slovakia had come across the most volatile inflation in 4-8 year period, which lasted until its accession to the EU.

Moving further, we start the analysis of relationship of prices and the economic cycles represented by the industrial production of the country. Looking at local covariances in the cross-wavelet power figures, fig. A.17 for CPI and PPI, and fig. A.18 of inflation,[3] we see higher common power at the borders of the time-frequency plane that are not much reliable. However, we need to provide two revealed connections of relatively high common power of the inflation and the production for the Czech Republic and Slovakia during 2000-2010. This variability is linked to the low frequencies and is in accordance with the section 4.3.

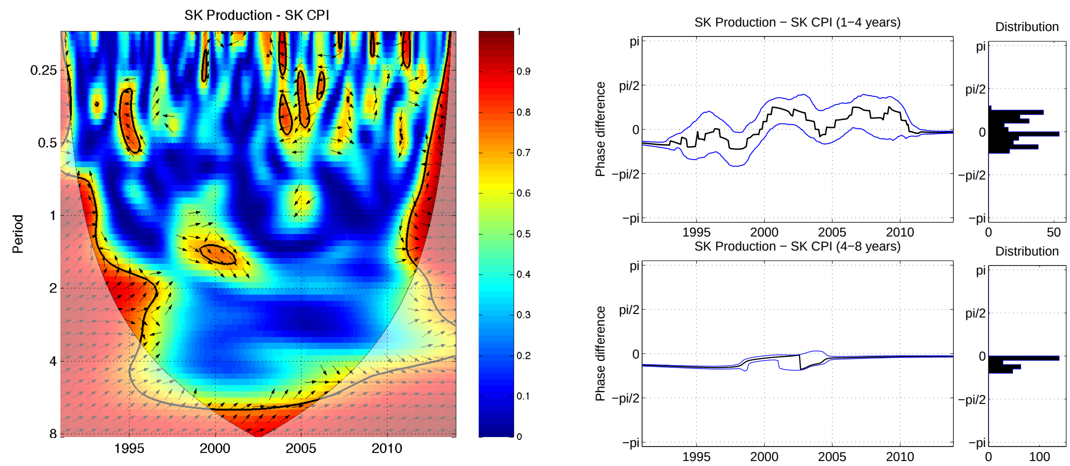


Figure 5.2: Slovakia production and CPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The relationship of CPI and PPI compared to production is shown

[3]These figures do not demonstrate any extraordinary information to be placed in the text.

in the wavelet coherence figures 5.2 and 5.3, these are for countries of Slovakia and the Czech Republic, respectively. They may be seen as representative examples of CPI and PPI coherences with production for all the coutries.[4] All the countries started the transition by common liberalization of prices, hence we observe the high prices-output coherency for all business cycle frequencies. Furthermore, the co-movement of CPI and PPI with the particular productions is clearly pro-cyclical in short- and long-term business cycles.[5] This reveals that the V4 economies are demand driven, which corresponds to Keynesian general theory with liberalized pro-cyclical prices (Caraiani, 2012). The relationship of production and PPI is analogous to the

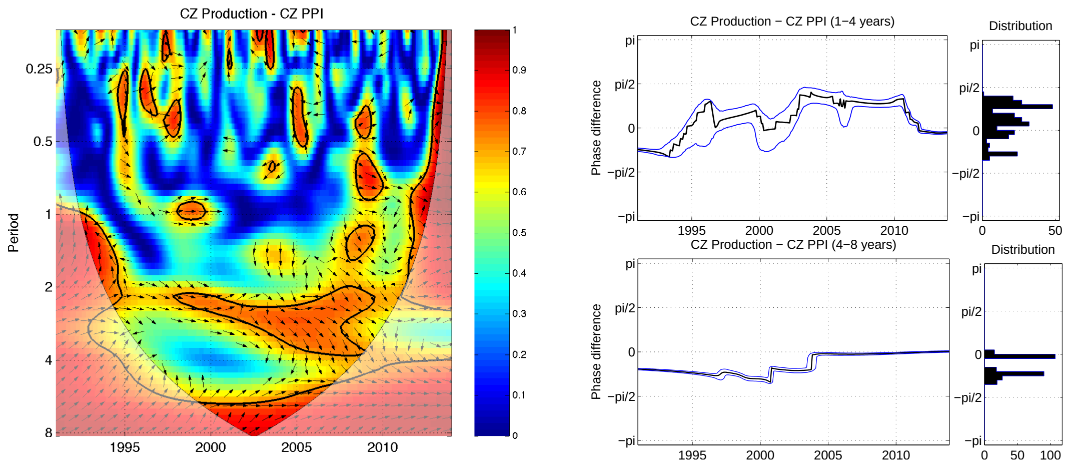


Figure 5.3: The Czech Republic production and PPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

case of CPI. We observe that the relation is strong after the onset of the transformation. In contrast to the CPI, the strong relationship is found only for the case of the Czech Republic and Hungary. The Czech Republic PPI-production interdependence is evident and significant during 1997-2009 in the period of 2-4 years. The PPI of Poland

[4]The rest of figures is placed in the appendix, figures A.21, A.22, A.23, A.24, A.25, and A.26.
[5]During the analysis, we use terms short- and long-term equally to 1-4 year and 4-8 year period band, respectively.

shows high coherence area with the production during 2008-2010 at high frequencies of 4 to 12 months; this happen at the same moment of last crisis. The production and PPI phase difference is dynamic over time but remains consistently pro-cyclical, as for the CPI.

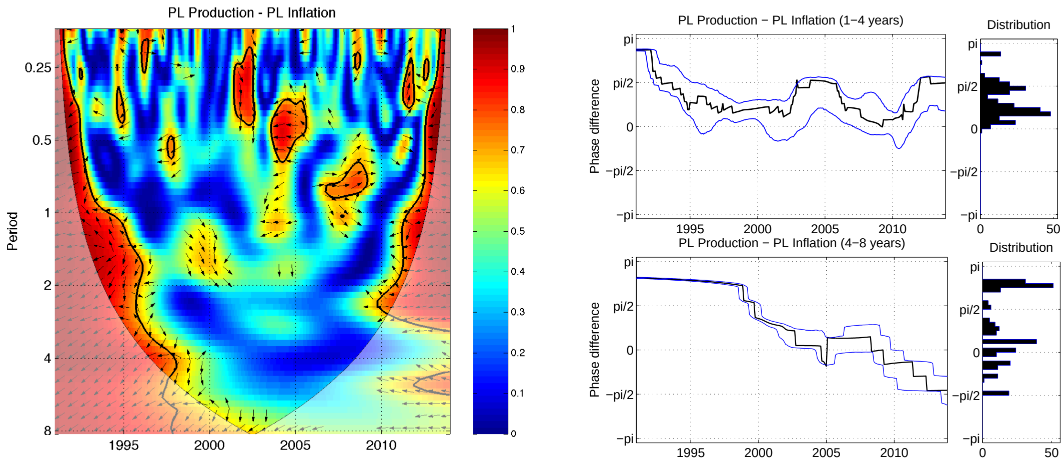


Figure 5.4: Poland production and inflation; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The next measure in the analysis of prices is the inflation. The coherence between production and inflation is surprisingly low most of the time. The exception is Poland, in the figure 5.4 we see the strong counter-cyclical interconnection of production and inflation, when the production is in leading position. That corresponds exactly to what happened in Poland in early 1990s (Bruzda, 2011); high inflation negatively related to the production. The counter-cyclical lasted until Poland changed its monetary policy regime to the inflation targeting in 1998. Analogous dynamics is visible for the Czech Republic inflation, however, it is not supported by any strong coherence, fig. A.27. Hungary experienced, fig. A.28, the same change as the Czech Republic but its long-term phase difference of production and inflation changed later. The Czech Republic started inflation targeting in 1998 and the phase difference change may be observed in 2000. Hungary adopted the same targeting in 2002 but we observe that the phase difference changes a year earlier. In the case of the Slovakia's inflation, the production lags inflation but the level of coherency is fairly low. To be clear, the phase difference based on wavelet coherency is capable to register these regime alterations but when this is not supported with higher coherence we cannot be entirely convinced by the results.

5.1.2 Exchange rate

In this analysis we use the spot exchange rate of the US dollar to the particular national currency as the nominal exchange rate. The exchange rates volatility has different pattern for each of the currencies, fig. A.10. The volatility of the nominal exchange rates of the USD to the Czech and Slovak currency appear high during 2006-2010 in 1-2 year period and during 2003-2006 in 4-5 year period, respectively.

Looking at cross-wavelet power spectra, fig. A.18, the relationship between the production and exchange rates closely reflects the regions of higher volatility in single power of individual exchange rates.

The relationship of exchange rate and the production varies country by country. Slovakia nominal exchange rate shows high coherence with the production during the whole sample in the period of 3-4 years, fig. 5.5. The exchange rate lags the output in this period. In the shorter period of 8-16 months during 2003-2010, we may see an area of high coherence, which may be connected with the time of the Euro expectation. In contrast, the nominal exchange rates of Hungary and Poland, figures A.31 and A.32, show very high coherence with production during 1991-1995 and 1991-1999, respectively, in business cycles period of 1-8 years. This relationship is significant and counter-

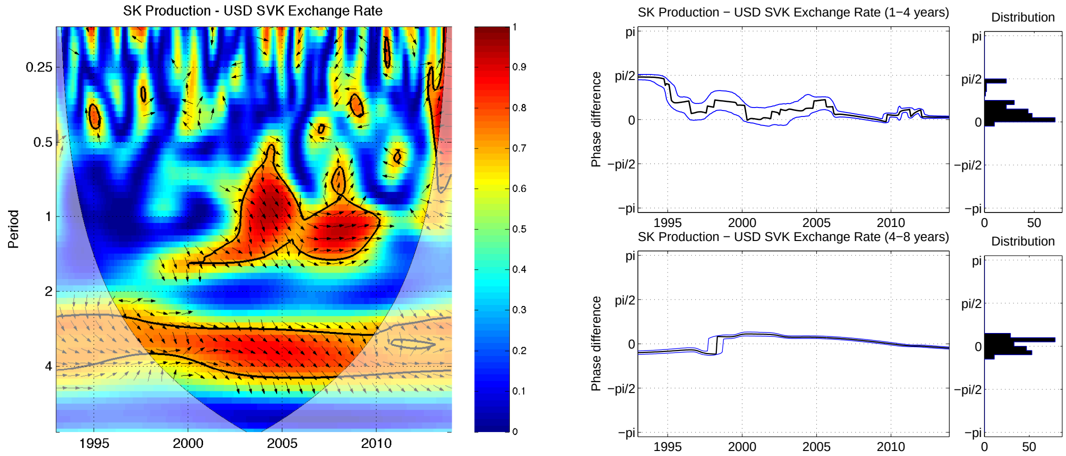


Figure 5.5: Slovakia production and exchange rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

cyclical. These regions of high coherency are almost identical with the coherences of CPI and production of these two countries. The turbulent years of high inflation amplified by counter-cyclical exchange rates that are pegged to the combined currency baskets (Frömmel et al., 2011). The Czech Republic exchange rate dynamics, fig. A.30, is worth studying at 2-4 year period where it has been significant and pro-cyclical since 2005. Furthermore, during the banking crisis which took place within 1995-1999, the exchange rate volatility increases (Kočenda and Valachy, 2006); we might also observe higher coherence between the Czech production and the nominal exchange rate co counter-cyclical behaviour in that time.

5.1.3 Unemployment and earnings

To measure the fluctuations of unemployment we take the time series of registered unemployment level, and according to the theory, as the nominal wages we choose the monthly earnings in manufacturing. It is due to the lack of collection of other data on monthly basis.[6]

[6]The wages collection unfortunately lacks the entry for the Czech Republic.

As in the previous parts, we estimate the power of unemployment and earnings. In figures of A.11 and A.12, we discover two different volatility facts; the monthly earnings do not signalize any higher volatility in the significant part of the power spectra. Opposed to that, the unemployment volatility is higher for all countries in long-term business cycles period. Particularly, the Czech Republic unemployment volatility increases in the 4-6 year period during 2000-2010. The cross-wavelet power indicates areas of common higher variance mirroring the power of the individual time series of unemployment and earnings. For the production and earnings there are not many areas of common power, albeit, for the production and the unemployment we see high common power in the 6-8 year period.

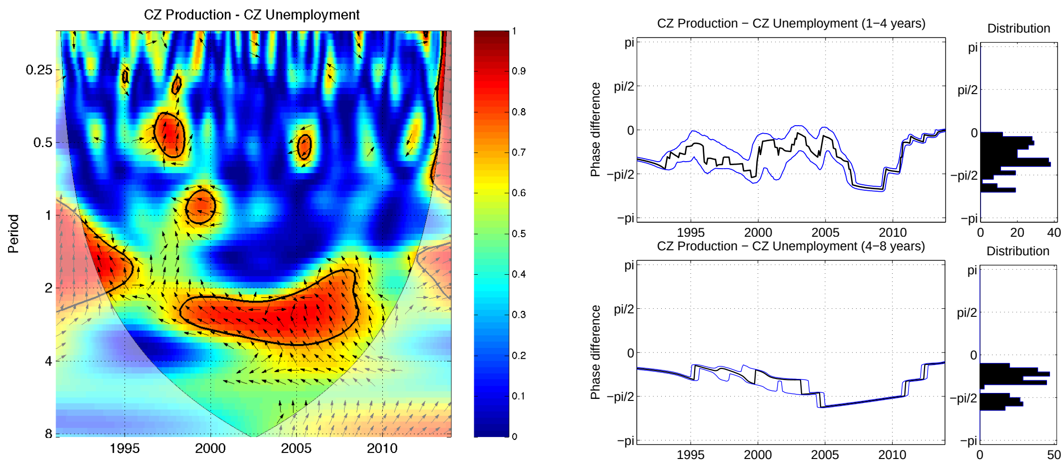


Figure 5.6: The Czech Republic production and unemployment; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The relationship of the Czech Republic production and unemployment is depicted in the figure 5.6. It shows that unemployment is locally correlated with production in period of 2-3 years during 1998-2008 approximately and it exhibits counter-cyclical behaviour. The average phase difference in 1-4 and 4-8 period presents that unemployment is always leading production but the variables may be in phase or out-of-phase. Certainly, they appear being counter-cyclical during the last global crisis. The explanation of the leading position of unemployment seems straightforward; the higher unemployment the lower production with a time delay. In fact, the average phase difference represents also the regions with low coherence, thus, the results illustrated by arrows in the coherency time-frequency plane seem reliable.

The situation is similar for the Hungarian economy, fig. A.33, where the interdependence of production and unemployment is also counter-cyclical in the period of 20-30 months during 2000-2005, and the production lags. The business cycles unemployment relationship in Poland and Slovakia has such low coherency in the complete time-frequency plane that drawing clear results is not possible, fig. A.34 and A.35, respectively. Only during 2 years before 2010, this relationship appears strong and significant in 18-30 month period.

In what follows, we study the production relation to the nominal wages – earnings. In the Keynesian business cycles models the wages have counter-cyclical behaviour contrary to the real business cycles model where they appear pro-cyclical. The production busi-

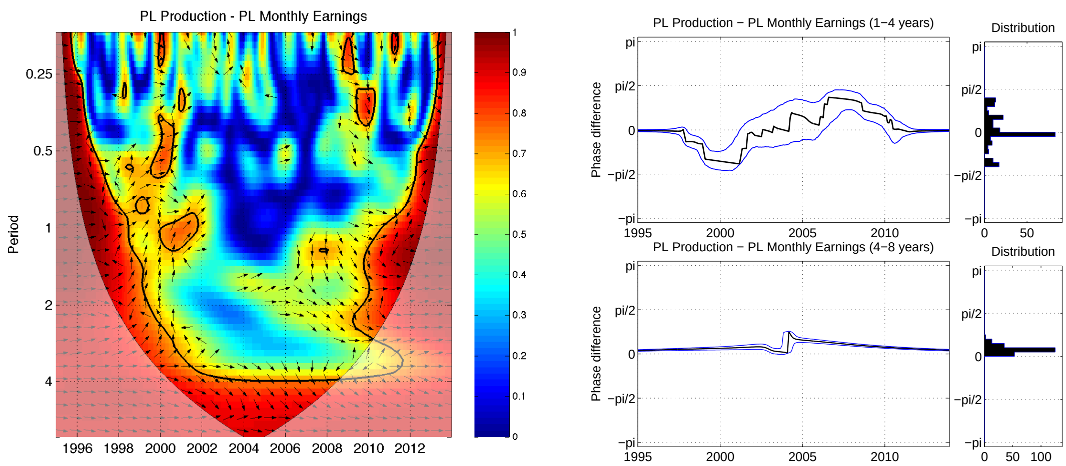


Figure 5.7: Poland production and monthly earnings; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

ness cycles are relatively coherent with the earnings for all the countries in 4-8 year period. In the figure 5.7 we see significant this relationship for the Czech Republic. The wages are all the time in phase with the production. In the short-term period, we distinguish the wide cone of low coherency where it would be advantageous to say anything about the relationship. Figures A.36 and A.37 portray very similar behaviour of nominal wages in Hungary and Slovakia. Both contain the cone of low unsatisfactory coherence and they show pro-cyclical behaviour in the long-term. The pro-cyclical behaviour of nominal wages is more in line with RBC models and it may be connected with labour shifts originating in technological shocks, this can be found in emerging countries as well as in the case of U.S. economy (Agénor et al., 2000; Kydland and Prescott, 1990).

5.1.4 Money and interest rates

The importance of the information about the relationship between money and output in macroeconomics is unquestionable just as the dynamics of interest rates is. In this text we use the narrow money (M1) as the money supply. As a consequence of the Slovakia's Euro adoption we do not analyse M1 of Slovakia because it is not available in the sourced database. Further, we select short- and long-term interest rates, which in our case are 3-month interbank rates and yields and 10-year government bond yields, respectively. The short-term interest rates suffer from pitfalls for countries of Slovakia and Hungary, thus, we shorten the time series. The long-term rates are also shorter, however, the reason is that their monthly collection starts in 1999, approximately.

As we consider two schools of business cycles, the first is New Keynesian stating that money can be pro-cyclical in the short-term, the second is the classical RBC, for which money is theoretically neutral but in some cases it can be pro-cyclical (King and Plosser, 1984). Concerning the volatility, the figure A.13 presents no outstanding higher level of volatility of M1 in the Visegrad countries. The opposite portrait of volatility is shown by figures of interest rates, A.14 and A.15, where regions of high volatility appear. Between 2004 and 2012, there is a significant region of high volatility for long-term interest rates in 2-4 year period for the case of Hungary.

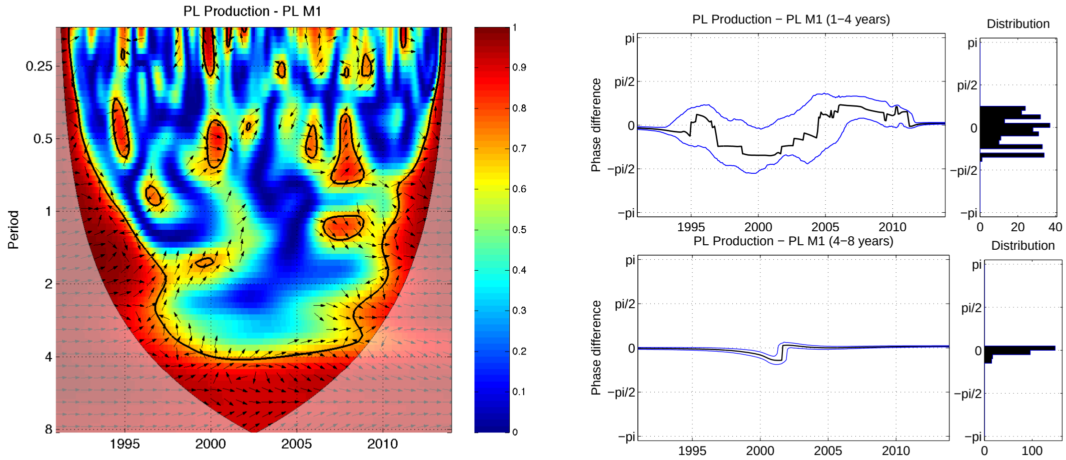


Figure 5.8: Poland production and money supply (M1); On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The coherence of production and money in Poland is provided in the figure 5.8; what we see is 4-8 year business cycle relationship of production and money that is very strong and significant over time. The same relationship is common also for the Czech Republic and Hungary, see figures A.38 and A.39. The relationship of production and M1 in all three countries is strictly pro-cyclical. This is in line with both theoretical models.

The last two macroeconomics indicators that remain to be described are interest rates. Firstly, looking at figure 5.9 of the Czech Republic short-term interbank rates we see almost empty time-frequency

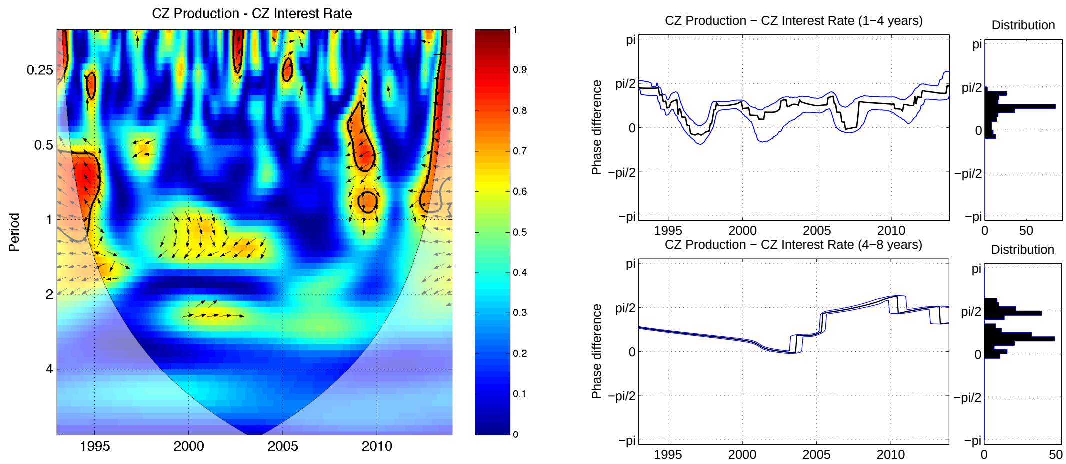


Figure 5.9: The The Czech Republic production and 3-month interbank rates; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

plane of wavelet coherency corresponding to very poor relationship of short-term interest rates and production. The coherences of short-term interest rates of Hungary, Poland, and Slovakia represent similar behaviour, fig. A.40, A.41, and A.42. Analogously to the Czech Republic, Poland has several small coherent areas at very high frequencies that have short persistence. According to phase arrows in the coherence plot, all interest rates coherent regions performs counter-cyclical behaviour but they mostly suffer from the edge effects.

However, the applicability of interest rates in real business cycle models is not certain (Tawadros, 2011), even though, their possible predictability of the future economic activity is mostly considered. The coherency for all the countries productions and 10-year government bond yields shown in the figures 5.10, A.43, A.44, and A.45 further in appendix, is low over time at most of the frequencies.

Nevertheless, all the significant areas of the high coherence of the series show that phase arrows are pointing to the left, corresponding to counter-cyclical relationship of the production and 10-year government bond yields. We find the largest significant region in the time-

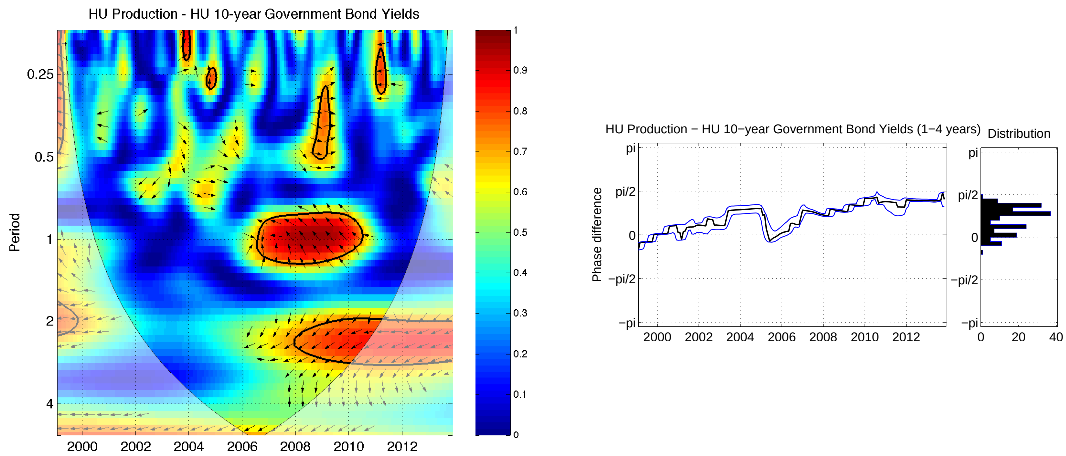


Figure 5.10: Hungary production and 10-year government bond yields; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

frequency plane of Hungary during 2006-2010 at 10-14 month period. This says that when these long-term interest rates exhibit strong relationship with the production the relationship is mostly negative.

5.1.5 Summary

For all variables of the Visegrad Four countries it can be found that high share of the variance is located at the business cycle frequencies, 1-8 year period. The major part of the variance comes from the long-term trends at the beginning and at the end of the sample, hence, many significant parts are influenced by the edge effect.[7] We have to interpret those results carefully, particularly the phase differences corresponding to low coherences. Here, we present an overview of cyclical behaviour observed between the production and selected macroeconomic indicators in the Visegrad region.

Prices: In many industrial economies has been found that prices behave counter-cyclically and thus the economies are supply-driven,

[7]See theoretical part explaining the edge effects, section 3.2.1.

Backus et al. (1992) for instance. The findings of our analysis present the prices in the Visegrad group mostly pro-cyclical implying that economies are demand-driven. Caraiani (2012) obtains the same result for Romanian economy, however, Benczúr and Rátfai (2010), find that prices in CEE countries are counter-cyclical. Further from the inflation behaviour, we are able to identify the structural changes in monetary policies of all the countries, i.e., we observe when they switched from the exchange rate pegs to inflation targeting.

Exchange rate: The nominal exchange rates volatility has different pattern for each country but it is markedly volatile in long-term business cycles. As Agénor et al. (2000) conclude, the nominal and the real exchange rate are highly correlated in the business cycles frequencies, we obtain this result only for Hungary and Poland production and exchange rate relationships for the periods of the exchange rate pegs.

Unemployment and earnings: The volatility of unemployment varies in time and emerges mainly at lower frequencies. In contrast, monthly earning do not show any spike of increased volatility over time. Benczúr and Rátfai (2010) show that employment is highly pro-cyclical and lags the output in CEE countries, this finding is consonant with our results of counter-cyclical unemployment that is leading production, even if the average phase differences show varying, mostly pro-cyclical behaviour, we stay with the significant regions of high coherence in which the unemployment behaviour is counter-cyclical. The nominal wages in our analysis are dominantly pro-cyclical in business cycles period, this corresponds to the classical RBC model.

Money and interest rates: In the case of Visegrad countries, the money supply volatility is at low level in comparison to both short- and long-term interest rates. 3-month interest rates expose higher volatility at the beginning of sample in 2-4 year period. We found the both short- and long-term interest rates experience very weak relationship with the production.Concerning money supply, we obtained the same result as Benczúr and Rátfai (2010), such that M1 is pro-cyclical or coincidental during whole sample in business cycles period, 4-8 years.

5.2 Business cycles synchronization in the Visegrad Four

This section explores the evolution of common dynamics of business cycles of the Visegrad Four (V4) economies. The V4 members began their political and economical cooperation to establish their institutional environment in order to support, among others, the economic growth and synergy.

We use cross-wavelet power spectrum to depict regions of high covariance and the wavelet coherency to inspect local correlations and co-movements between the Visegrad countries. The results of cross-wavelet power are captured in the figure A.46, which explains high common variation captured in business cycles period. We might expect higher coherency in these regions.

Taking alphabetical order, the Czech Republic and Hungary business cycles represented by production demonstrate lower coherency between years 1994 and 2000 in 2-4 year period. The two economies are positively correlated over time, however, a greater coherency of both productions appears when the countries have joined the EU. The Czech Republic leads Hungary in short-term business cycles period during 1995-2000 when the phase difference changes and Hungary leads during 2000-2005. In contrast, we observe opposite phase difference behaviour for the long-term business cycles period. Firstly,

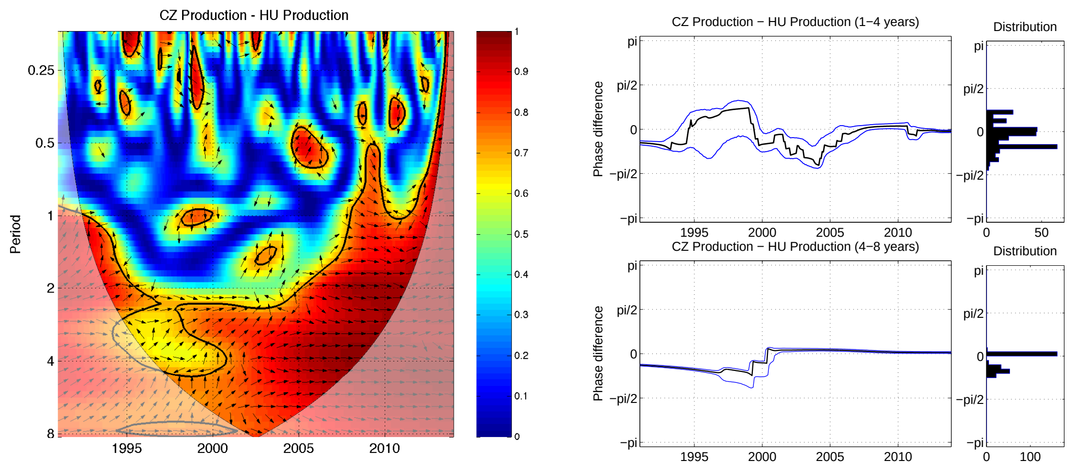


Figure 5.11: The Czech Republic production and Hungary production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

the Czech Republic business cycle leads, and from 2000 onwards their business cycles coincide.

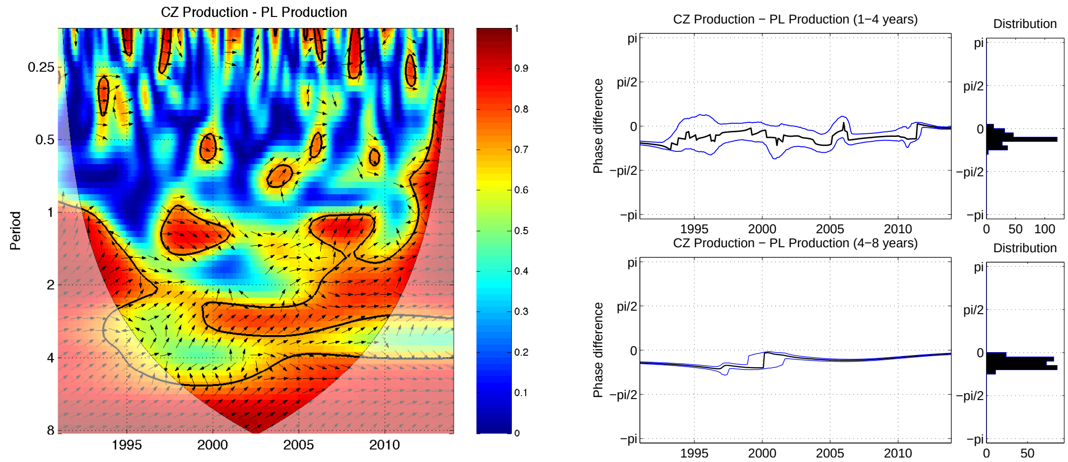


Figure 5.12: The Czech Republic production and Poland production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The coherency of Czech and Polish economies is given by the figure 5.12, it seems that these two countries have the lowest synchronized business cycles in the V4. During 1994-2002, these two countries do not exhibit particularly high and significant coherence in 2-6 year period. In comparison, Poland's long-term business cycle leads the

Czech one at most of the time when the coherency is notably high. This is probably due to the fact that the Polish production grows more in time and converges to the other countries, see 4.1.

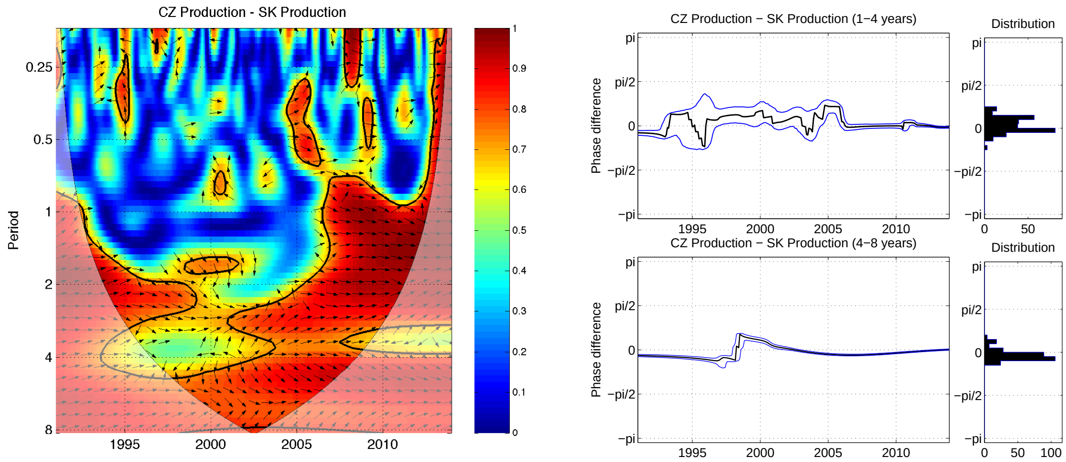


Figure 5.13: The Czech Republic production and Slovakia production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The former Czechoslovakian countries' business cycles coherency is captured in the figure 5.13. The business cycles corresponding to 1-8 year period of both countries coincide all the time while looking at phase difference 90 % confidence intervals.

Inspecting the time-frequency planes of the Czech Republic and other three countries, we may see an area of low coherency during 1994-2002 in 3-4 year period. This means that at the basis of 3-4 year cyclical period, the three countries' business cycles do not have strong relationship with the one of the Czech Republic, at least during the few years after 1995.

The relationship between Hungary and Poland business cycles is ordinary due to the fact that their coherency becomes weaker over time. At the beginning of the transition, when the Visegrad group co-operation started, the figure 5.14 demonstrates very high coherency with clear positive co-movement corresponding to the period from 1

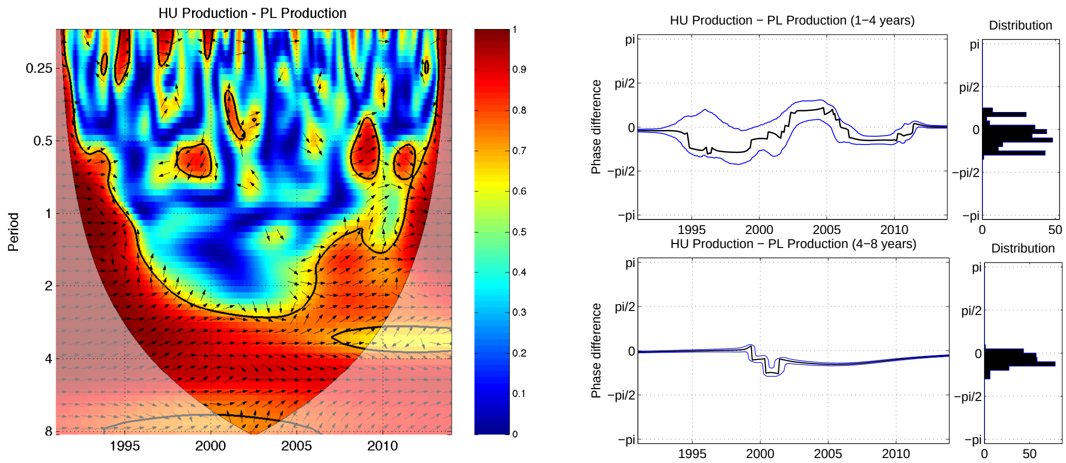


Figure 5.14: Hungary production and Poland production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

to 8 years. The countries' productions remain significantly coherent all the time in 3-7 year period. Poland business cycle is leading during 1995-2000 and 2006-2010 in short-term business cycles period; it also leads from 2000 onwards in the long-term period. This impairing relationship might be caused by the fact that countries do not share borders, or by the different size of economies. Compared to the early 1990s, the coherency is still lower even after 2005, but the period range gets broader. We may therefore link this fact to the effect of EU enlargement on their co-movement.

Following paragraphs are devoted to the study of dynamic relationship of business cycles between Slovakia and its two neighbours: Hungary and Poland.

The business cycles of Slovakia and Hungary are positively correlated over time for the period corresponding to 2-4 years, fig. 5.15. Surprisingly, their coherency in 6-8 year business cycle period is relatively low and mostly insignificant. There are not many relevant regions, however, we may infer that these two business cycles coincide. This relationship can be seen for 2-4 year business cycles period.

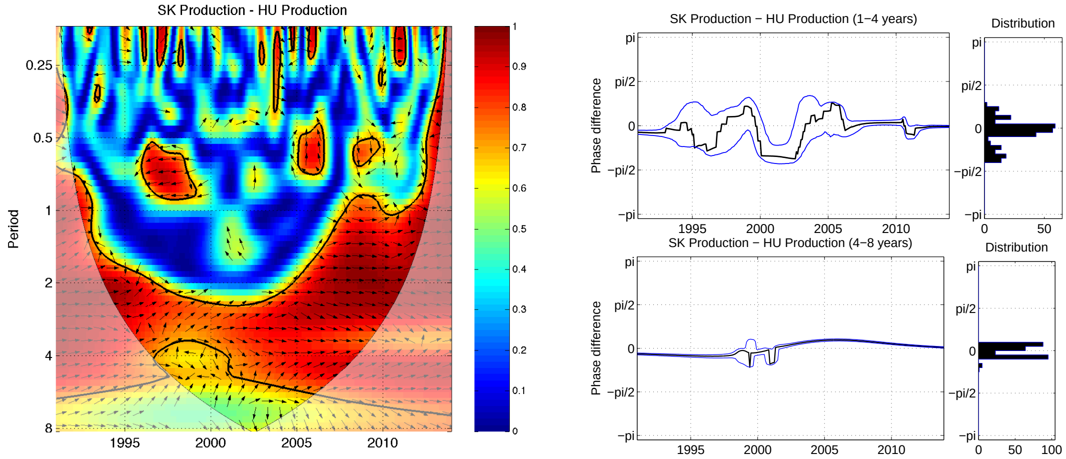


Figure 5.15: Slovakia production and Hungary production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

For the period of 1-4 year, we observe phase difference fluctuations, where Hungary significantly leads within the period of 2000-2003.

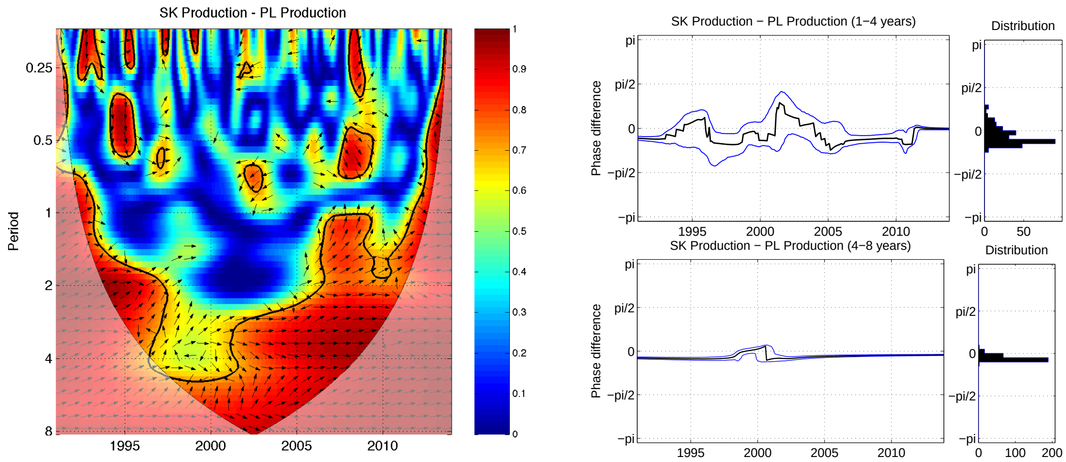


Figure 5.16: Slovakia production and Poland production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The figure 5.16 captures the relationship between Poland and Slovakia productions. Here, the situation of leading positions repeats the relationship between the Czech Republic and Poland business cycles; Poland leads Slovakia over time at all business cycle frequencies, 1-8 year period. Nonetheless, the high significant coherency is not present constantly. These two countries are most coherent between year 1991 and 1995, and they realign again after 2000. Their business cycles coherency has increased since 2002 rapidly.

Summary: This part of the thesis is devoted to business cycles synchronization of the countries in the Visegrad Four. Analysis of business cycles synchronization of the V4 countries uncovered important patterns. At the beginning of transition, we observe very high coherency among all countries, this fact completely supports the result of Jagrič (2002) detecting high common volatility of business cycles across CEE countries. This higher coherency prevail longer then two years of downturn and rebound. Therefore, it may be relevant to started cooperation. In 1995, the coherence between Slovakia and countries of the Czech Republic and Poland declines markedly in 2-4 year period, which may be caused by Slovakia's cold-shoulder participation to the political discussion during 1993-1997 that mirror into the business cycles with a delay Lukášek (2010). Another explanation, accounting for all countries, shows that after few years of formally intensive cooperation the monetary and fiscal policies start diverging, e.g., in the Czech Republic during late 1990s difficult stabilization years take place Antal, Hlaváček, and Holub (2008). This diverging economic situations give rise to asynchrony in business cycles behaviour. Kutan and Yigit (2004) present that convergence of macroeconomic variables matters.We may see an increase of coherency after the EU scan and preparation of the countries for accession to the EU. This happens a short time before 2000.

5.3 Synchronization of the V4 and Germany

This section examines the business cycles synchronization between the countries of the Visegrad Four and Germany. As in the previous section, we start the analysis in 1991, less than two years after the "Eastern bloc " break-up. Since that time, the V4 countries started orienting themselves to have better relationship with the western countries. The knowledge of the level of synchronization yields useful information for the EU as well as the V4. The European Union takes into account how big burden a country will be if it joins the EU; a country may assess its dependence on the EU and possible effect of, e.g., the economic growth or decline, or the same monetary policy on it own. For the Visegrad countries the optimum currency area criteria have to be fulfilled before adopting the Euro; the business synchronization is one of them.

Firstly, we look at the cross-power spectra between each of the V4 countries and Germany, fig. 5.17. From the previous analysis of industrial productions by HP-filter, we know that all V4 countries' business cycles are more volatile than the one of Germany. The volatility of business cycle was very high in 1991; the period of the second highest volatility begins after 2007 and lasts until 2010.

In the figures of cross-wavelet power spectra, fig. 5.17, we see that for all the V4 countries and Germany the local common power is markable and significant for the period of 4-8 years after the 2005. In the case of the Czech Republic and Hungary, the regions of high common power with Germany started in 2004. In the figures, there is apparent that the series have several significant peaks in period of around one quarter, but the power is low. Here, we have observe the high common power of two series of production during the second half of the sample. This information is translated to the coherency

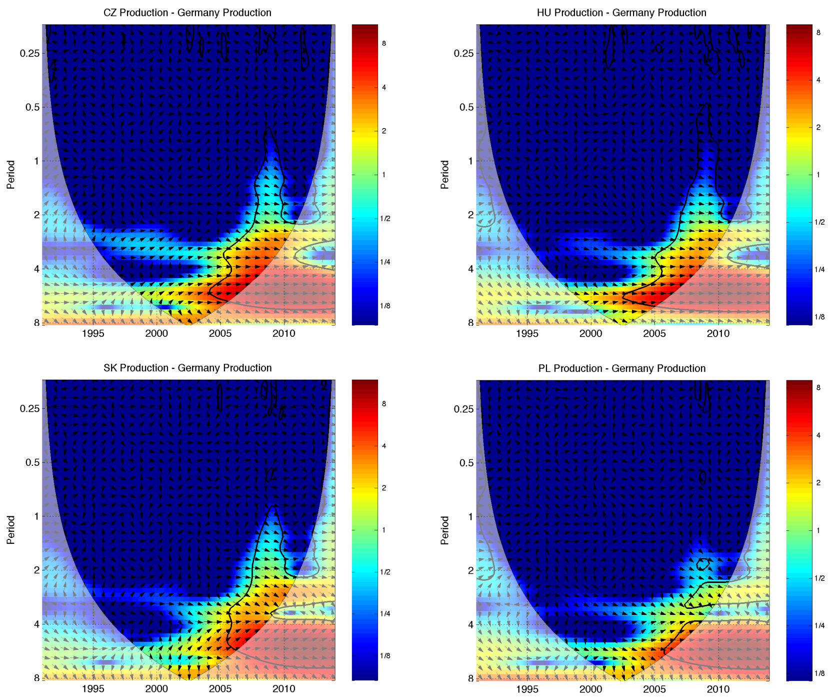


Figure 5.17: Cross-wavelet power: The Visegrad Four countries and Germany. The black contour indicates the 5 % significance level against the red noise computed through Monte Carlo simulations. The shaded area is the cone of influence. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia

complex figures.

We discovered very high co-movement in business cycles period, 2-8 year. For the Czech Republic and Germany productions, figure 5.18. The co-movement between the Czech Republic and Germany is relatively low at high frequencies; from 2 months to 1 year, there is small number of regions with high local correlations. The business cycles period is interesting from the point of view that their coherency increases over time. On top of that, beginning in 2004 the synchronization at 4 year level is almost perfect. In 4-8 year period, the average phase difference shows that the Czech Republic's business cycle leads compared to the Germany's cycle. Although it may seem strange, this information is averaged; when looking at arrows in the coherency plot, the German economy leads the one of the Czech Republic before it joined the EU in 2004. Studying the phase differ-

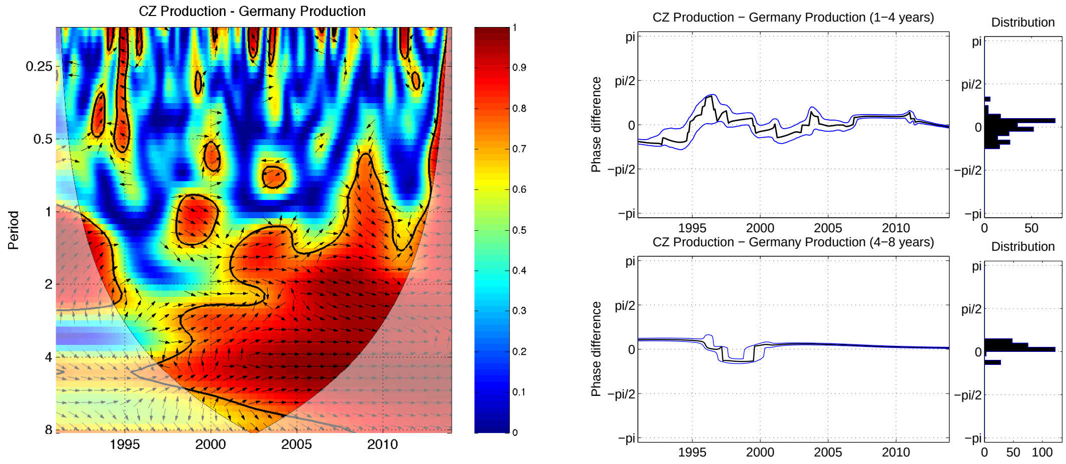


Figure 5.18: The Czech Republic production and Germany production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

ence plot in 1-4 year period, we can see that both countries move together. In fact, in regions of high coherency, Germany is in the leading position, but afterwards the situation changes. Speaking about Germany's production, it leads for the short-term business cycles of 1-4 period during 1991-1995.

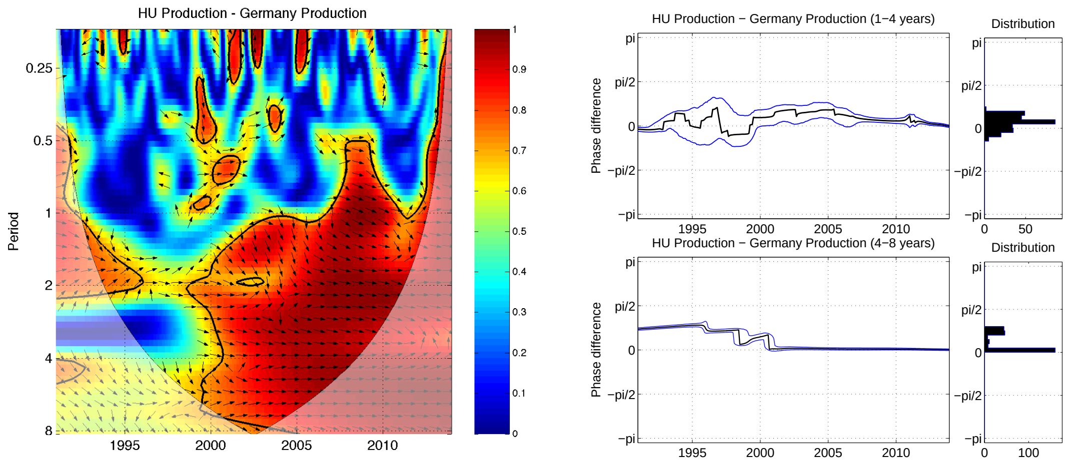


Figure 5.19: Hungary production and Germany production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

The co-movement between Hungary and German productions, fig. 5.19, exhibits similar behaviour as the one of the Czech Republic; the coherency is very high since 2000 and continues to increase until 2013 in 1-8 year period. Both averaged phase differences support that Hungary leads German production for most of the time with an exception of 1996-2000 in 1-4 year period. However, this period is not significant at all. We can find Germany in the leading position between years of 2000 and 2004 in the long-term business cycles period.

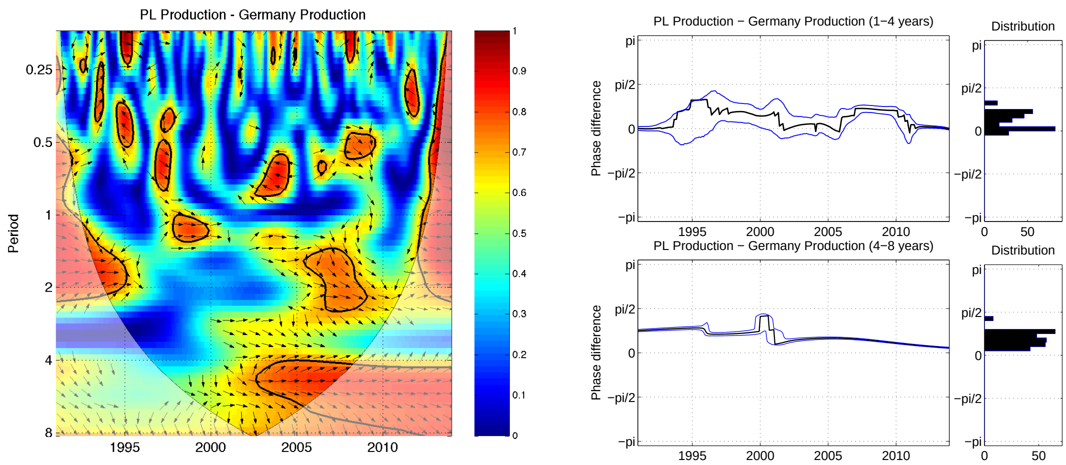


Figure 5.20: Poland production and Germany production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

Among the V4 states, Poland has the lowest industrial production co-movement with Germany, fig. 5.20. Its and Germany's HP-filter cyclical components have the lowest variance, which may indicate their similarity. Nevertheless, the variance of Poland's production series is the highest compared to Germany. The significant regions of higher coherency bring different information. In periods shorter than one year, the economies are out-of-phase in a few local areas, and Germany leads Poland. The phase arrows of the relation between these economies represent both leading and lagging corresponding to 6-8 years period during 2003-2007, but the cone of influence could violate the efficiency of information. The average phase in both 1-4 and 4-8 year periods is not reliable enough to be interpreted because of a few significant regions out of the shaded area – cone of influence.

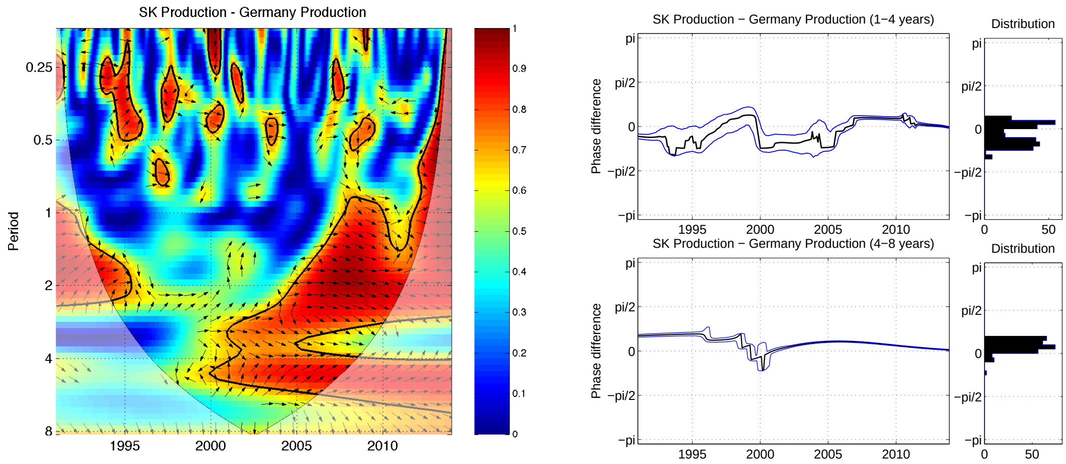


Figure 5.21: Slovakia production and Germany production; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

This paragraph addresses to the coherency of Slovakia and Germany productions, fig. 5.21. As mentioned above, the three other countries does not fully apply for the case of Slovakia. This coherency is not globally as high as those between German and the productions of the Czech Republic and Hungary, fig. 5.18 and 5.19. In both short- and long-term business cycles periods, we see that the economies move in phase, but within the area of short-term, 1-4 year, from 2000 Slovakia lags Germany until 2005, then leads. On the contrary, for long-term period Germany leads Slovakia. It has to be said that this implications are given by significant regions of high coherency from 2000 to 2008 in the long-term period.

To summarize: In the results of synchronization analysis between the Visegrad Four countries and Germany, we observe two coherency regions of the same size for all countries. Firstly, the region of two years at the very beginning of the sample; it partially belongs to the cone of influence, and therefore it is not fully reliable. Nevertheless, it is obvious that the V4 economies were declining during that time, and that during those five years German economy did not fall as much as these economies did. Germany business cycle leads the V4 economies business cycles during their attempts to rebound, see figure 4.1, 4.2, and 4.3. Secondly, all the countries during three years before 2010 appear to have high local correlation with Germany production. And all of them lead Germany's production. Joining this finding with the HP-filter results, we observe that Germany's cycle falls into recession as the last one. Thus, this shows higher vulnerability of V4 economies to this kind of global shocks.

Chapter 6

Conclusion

In this thesis we analyse properties of business cycles in the Visegrad Four since the beginning of economic transformation of this region, from 1991 to 2013. This work takes advantage of the novelty approach of wavelet analysis to study the business cycles. The first part of this analysis provides a comprehensive description of stylized facts of main macroeconomic indicators of each of the four countries. The second part of the analysis is divided into two parts on business cycles synchronization in time and frequency space. The first depicts the pairwise business cycle relationships among the Visegrad countries, while the second is aimed at the same type of interdependence these countries exhibit related to Germany as a proxy for the European Union.

The first goal of the study is to give an overview of stylized facts of Visegrad countries that would serve as a preliminary yet important stepping stone for policy makers assembling their RBC models.

The wavelet analysis allows us to study the relationships that are dynamics in time and frequency. To our best knowledge this study is unique due to its complex description of dynamic behaviour of business cycles properties in the Visegrad Four.

In the analysis of stylized facts, we capture the pro-cyclical manner of prices with the production for all Visegrad Four countries, which implies demand-driven nature of their economies; this is in line with the Keynesian general theory. The phase difference analysis brings striking result concerning the inflation dynamics. We can detect when monetary policies changed to the inflation targeting, for instance. Further, we show that money behaves pro-cyclically with the production in the long-term business cycles. The behaviour of unemployment is counter-cyclical but its coherency with the production is low. The long-term interest rates appear to be more connected to the production than those relevant to the short-term.

We investigate the impact of Visegrad countries cooperation, which began with the aim to help each other to converge faster to the Western Europe countries. We find out that all countries exhibit a transformation shock at the beginning of transition. After the break-up of the "Eastern bloc", they show high co-movement for the first years of their cooperation, up until the economic turbulences during the late 1990s. The period between 1995 and 1999 shows the lowest coherency of their business cycles in the observed time span. After that, the stabilization patterns appear mirrored into the gradual increase of coherency. Only, the relationship of Hungary and Poland productions is declining throughout the surveyed period.

Additionally, we study the business cycles synchronization of the Visegrad Four countries with Germany. The results confirm some already known but interesting patterns. The Slovakia's production synchronization with the EU was poor before its accession to the EU. Further, we reveal that highest coherence between Germany and the Czech Republic and Hungary productions starts in the year 2000. In contrast, the synchronization of business cycles of Poland and Germany is the lowest compared to the remaining countries. For all Visegrad Four countries, we find pro-cyclical behaviour of their productions with that of Germany. Based on our findings, we observe remarkable discovery that all the countries started better synchronization with the Western Europe four years before they joined the

EU.

To conclude, the wavelet-based examination of business cycles stylized facts seems to be equivalent to the traditional filtering methods. Moreover, despite the length of the observed period, we are able to disentangle many key properties of the Visegrad Four business cycles.

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Appendix A

Complementary tables and figures

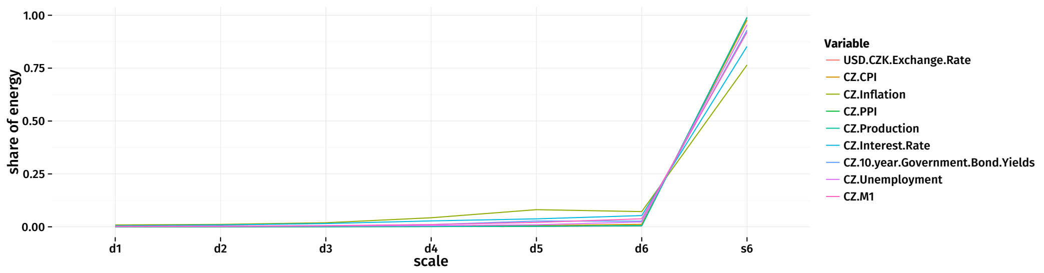


Figure A.1: Energy decomposition, Czech Republic indicators; Data source: OECD (2014); Source: author's computations

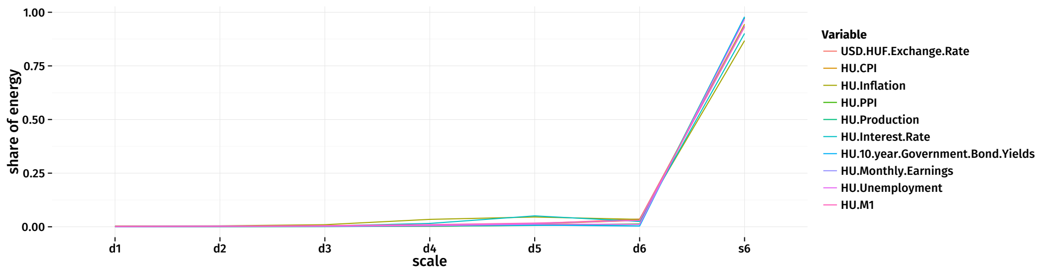


Figure A.2: Energy decomposition, Hungary indicators; Data source: OECD (2014); Source: author's computations

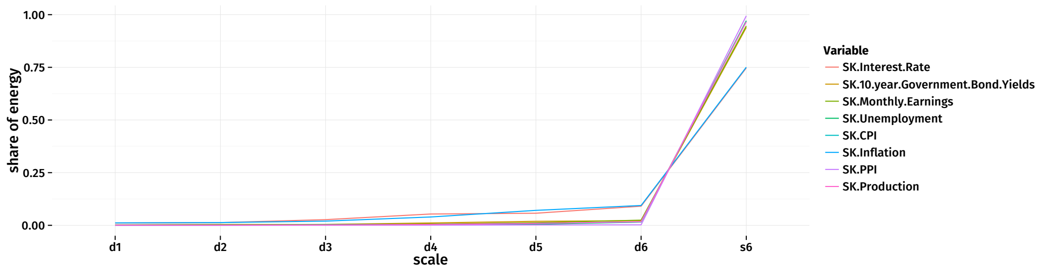


Figure A.3: Energy decomposition, Slovakia indicators; Data source: OECD (2014); Source: author's computations

Table A.1: Energy decomposition at scales

Variable	d1	d2	d3	d4	d5	d6	s6
CZ 10Y Gov Bond Yields	0.0012	0.00204	0.00354	0.01018	0.02727	0.02657	0.9292
CZ CPI	0.00058	0.00079	0.00135	0.00261	0.00599	0.01158	0.9771
CZ Inflation	0.00883	0.01134	0.0191	0.04272	0.081	0.0719	0.76512
CZ Interest Rate	0.00486	0.00858	0.01544	0.02787	0.03757	0.05309	0.85259
CZ M1	0.00216	0.00301	0.00542	0.01071	0.02096	0.03788	0.91986
CZ PPI	4.00E-04	4.00E-04	0.00061	0.00121	0.00297	0.00534	0.98906
CZ Production	0.00024	0.00023	0.00041	0.00114	0.00252	0.00514	0.99032
CZ Unemployment	0.00125	0.00207	0.00389	0.00561	0.00962	0.02302	0.95455
Germany Production	9.00E-05	1.00E-04	0.00021	0.00057	0.00139	0.00223	0.9954
HU 10Y Gov Bond Yields	0.00065	0.00132	0.00178	0.00667	0.00775	0.00334	0.97848
HU CPI	0.00124	0.00175	0.00324	0.00642	0.01351	0.02973	0.94411
HU Inflation	0.00361	0.00444	0.00978	0.03457	0.04551	0.0345	0.8676
HU Interest Rate	0.00157	0.00223	0.00378	0.01568	0.05085	0.02472	0.90117
HU M1	0.00246	0.00307	0.00554	0.01019	0.01722	0.03026	0.93125
HU Monthly Earnings	0.00229	0.00412	0.00419	0.00746	0.01592	0.03017	0.93585
HU PPI	0.00056	0.00076	0.00144	0.00281	0.00759	0.01322	0.97361
HU Production	0.00046	0.00059	0.00112	0.00269	0.00637	0.01056	0.97822
HU Unemployment	0.00038	0.00115	0.00256	0.00479	0.01232	0.01124	0.96757
PL 10Y Gov Bond Yields	0.00117	0.00176	0.00318	0.01529	0.02548	0.01243	0.9407
PL CPI	0.00102	0.00142	0.00262	0.00519	0.01092	0.02502	0.9538
PL Inflation	0.01954	0.01992	0.02766	0.06052	0.07904	0.09512	0.6982
PL Interest Rate	0.00799	0.01074	0.01337	0.02263	0.0442	0.06815	0.83293
PL M1	0.00317	0.00407	0.00783	0.01555	0.02582	0.05637	0.8872
PL Monthly Earnings	0.00145	0.00192	0.00313	0.00609	0.01205	0.02918	0.94618
PL PPI	1.00E-04	0.00014	0.00032	0.00048	0.00122	0.00309	0.99465
PL Production	0.00082	0.00107	0.00214	0.00453	0.00907	0.02048	0.96189
PL Unemployment	0.00014	0.00036	0.00114	0.00107	0.00283	0.01571	0.97876
SK 10Y Gov Bond Yields	0.00155	0.0023	0.00424	0.01127	0.01912	0.02209	0.93942
SK CPI	0.00083	0.00113	0.00203	0.00383	0.00879	0.01602	0.96737
SK Inflation	0.01159	0.01311	0.01995	0.0396	0.07103	0.09413	0.75058
SK Interest Rate	0.01088	0.01295	0.02698	0.05369	0.05741	0.0909	0.74719
SK Monthly Earnings	0.00224	0.00416	0.00382	0.00634	0.01256	0.02458	0.94629
SK PPI	0.00014	0.00018	0.00035	0.00083	0.00173	0.00241	0.99435
SK Production	0.00075	0.00093	0.00175	0.0041	0.0091	0.01742	0.96595
SK Unemployment	0.00057	0.00114	0.00246	0.00408	0.00507	0.01563	0.97105
USD CZK Exchange Rate	3.00E-04	0.00034	0.0011	0.00237	0.00214	0.00445	0.9893
USD HUF Exchange Rate	0.00152	0.00216	0.00341	0.00613	0.01456	0.03609	0.93613
USD SVK Exchange Rate	0.00024	3.00E-04	0.00063	0.00128	0.00217	0.00322	0.99216
USD ZLO Exchange Rate	0.00214	0.00353	0.00658	0.01109	0.0244	0.03569	0.91657

Data source: OECD (2014); Source: author's computations

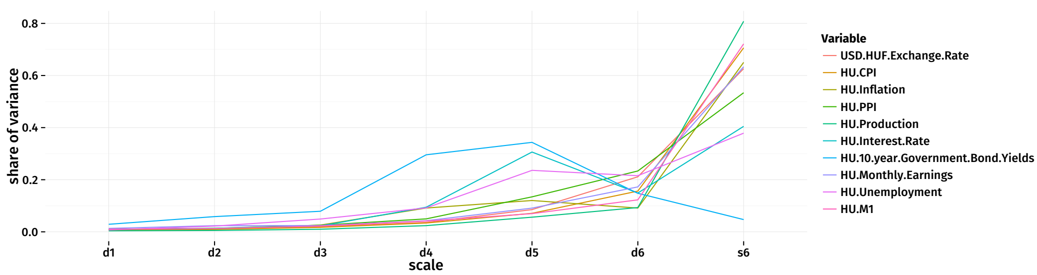


Figure A.4: Variance decomposition, Hungary indicators; Data source: OECD (2014); Source: author's computations

Table A.2: Variance decomposition at scales

Variable	d1	d2	d3	d4	d5	d6	s6
CZ 10Y Gov Bond Yields	0.01585	0.02694	0.04683	0.13469	0.36063	0.35138	0.06368
CZ CPI	0.00864	0.01178	0.02023	0.03904	0.08947	0.17308	0.65777
CZ Inflation	0.01897	0.02436	0.04103	0.09178	0.17402	0.15449	0.49535
CZ Interest Rate	0.01123	0.01983	0.03568	0.06439	0.08679	0.12264	0.65945
CZ M1	0.00823	0.01146	0.02062	0.04077	0.07979	0.14418	0.69496
CZ PPI	0.01348	0.01348	0.02052	0.04064	0.09988	0.17967	0.63233
CZ Production	0.00526	0.00495	0.0088	0.02485	0.05481	0.11156	0.78976
CZ Unemployment	0.00886	0.0147	0.02755	0.03974	0.06813	0.16309	0.67794
Germany Production	0.00623	0.00683	0.01407	0.03842	0.09342	0.1501	0.69094
HU 10Y Gov Bond Yields	0.02891	0.05837	0.07885	0.29562	0.34339	0.14791	0.04695
HU CPI	0.00654	0.0092	0.01702	0.03375	0.07103	0.15628	0.70617
HU Inflation	0.00953	0.01171	0.02583	0.09129	0.12018	0.09111	0.65036
HU Interest Rate	0.00947	0.01342	0.02279	0.09444	0.30624	0.14885	0.40479
HU M1	0.00998	0.01246	0.02246	0.04129	0.0698	0.12268	0.72133
HU Monthly Earnings	0.01308	0.02357	0.02392	0.0426	0.09098	0.17235	0.63351
HU PPI	0.00997	0.0135	0.02542	0.04966	0.13417	0.23349	0.53378
HU Production	0.00403	0.00518	0.00984	0.02373	0.05613	0.09306	0.80803
HU Unemployment	0.00728	0.02199	0.04906	0.09174	0.23595	0.21536	0.37863
PL 10Y Gov Bond Yields	0.01792	0.02695	0.04885	0.2346	0.39104	0.19075	0.08988
PL CPI	0.00752	0.01045	0.01924	0.03808	0.08014	0.18359	0.66097
PL Inflation	0.03017	0.03075	0.04269	0.09343	0.12201	0.14684	0.5341
PL Interest Rate	0.01857	0.02495	0.03105	0.05257	0.10269	0.15835	0.61182
PL M1	0.00779	0.01001	0.01927	0.03825	0.06352	0.13867	0.72248
PL Monthly Earnings	0.0117	0.01542	0.02517	0.04899	0.09683	0.23458	0.56732
PL PPI	0.00936	0.01305	0.02934	0.04446	0.1122	0.28454	0.50706
PL Production	0.0061	0.00796	0.01598	0.03389	0.06777	0.1531	0.7152
PL Unemployment	0.00297	0.00785	0.02461	0.02308	0.0612	0.33994	0.54035
SK 10Y Gov Bond Yields	0.02041	0.03031	0.0558	0.14846	0.2518	0.29094	0.20228
SK CPI	0.00694	0.00946	0.01692	0.03198	0.07339	0.13367	0.72764
SK Inflation	0.03067	0.0347	0.05279	0.10479	0.18797	0.24909	0.33999
SK Interest Rate	0.02759	0.03283	0.0684	0.13612	0.14557	0.23047	0.35903
SK Monthly Earnings	0.01296	0.02409	0.02214	0.03676	0.07281	0.14245	0.68879
SK PPI	0.00883	0.01128	0.02206	0.0514	0.10761	0.15017	0.64864
SK Production	0.00684	0.00858	0.01603	0.03764	0.08352	0.15985	0.68753
SK Unemployment	0.00972	0.01928	0.04172	0.06928	0.08607	0.26521	0.50872
USD CZK Exchange Rate	0.00502	0.00569	0.01849	0.03981	0.03588	0.07475	0.82035
USD HUF Exchange Rate	0.00891	0.01264	0.01994	0.03586	0.0852	0.21117	0.62628
USD SVK Exchange Rate	0.00393	0.005	0.01043	0.02131	0.0361	0.05354	0.8697
USD ZLO Exchange Rate	0.01288	0.02128	0.03967	0.06684	0.14712	0.21518	0.49702

Data source: OECD (2014); Source: author's computations

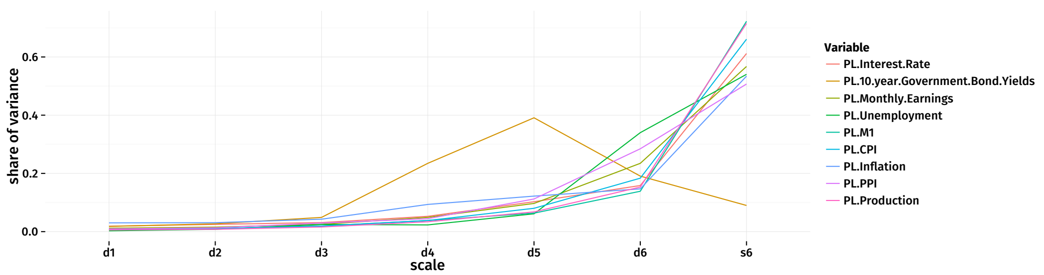


Figure A.5: Variance decomposition, Poland indicators; Data source: OECD (2014); Source: author's computations

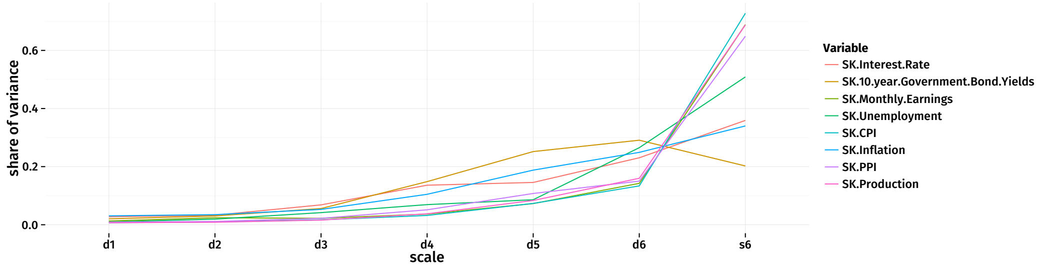


Figure A.6: Variance decomposition, Slovakia indicators; Data source: OECD (2014); Source: author's computations

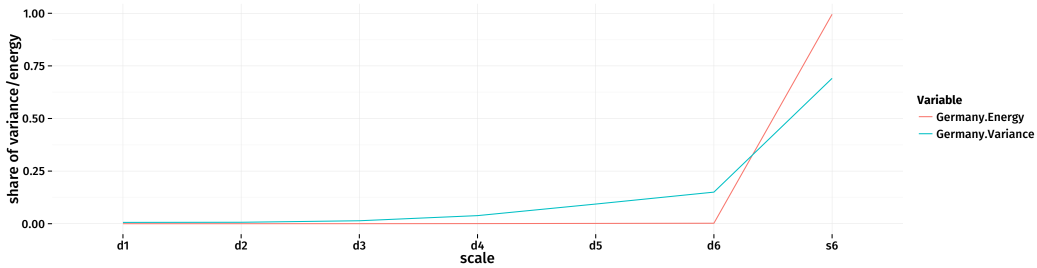


Figure A.7: Variance decomposition, Poland indicators; Data source: OECD (2014); Source: author's computations

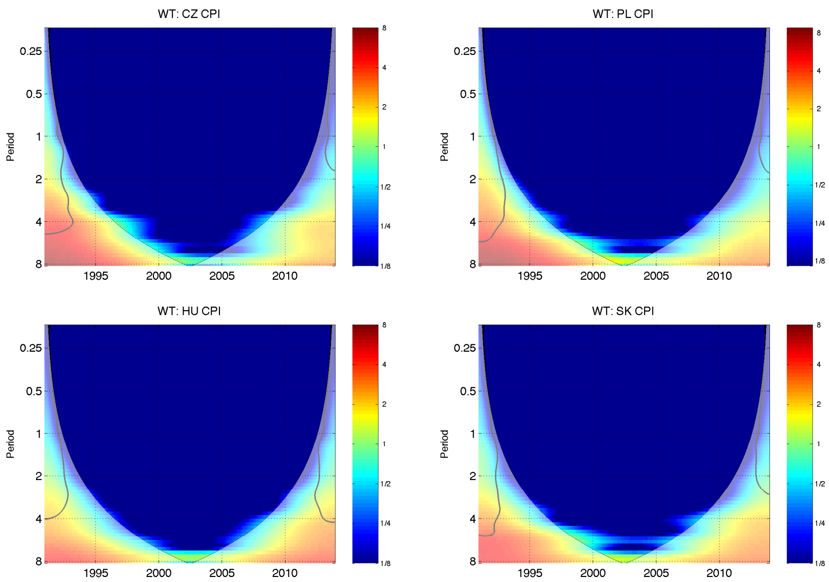


Figure A.8: Wavelet power: V4 - Consumer prices index (CPI). From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.

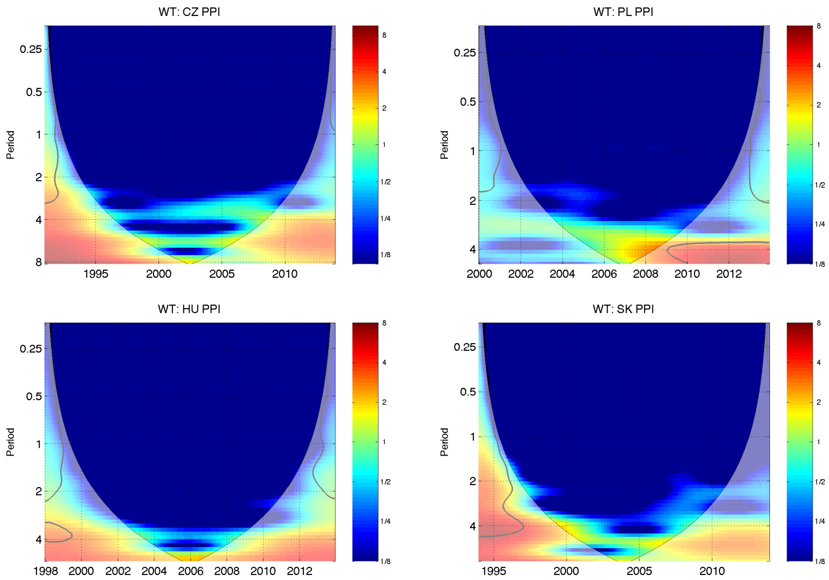


Figure A.9: Wavelet power: V4 - Producer prices index (PPI). From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.

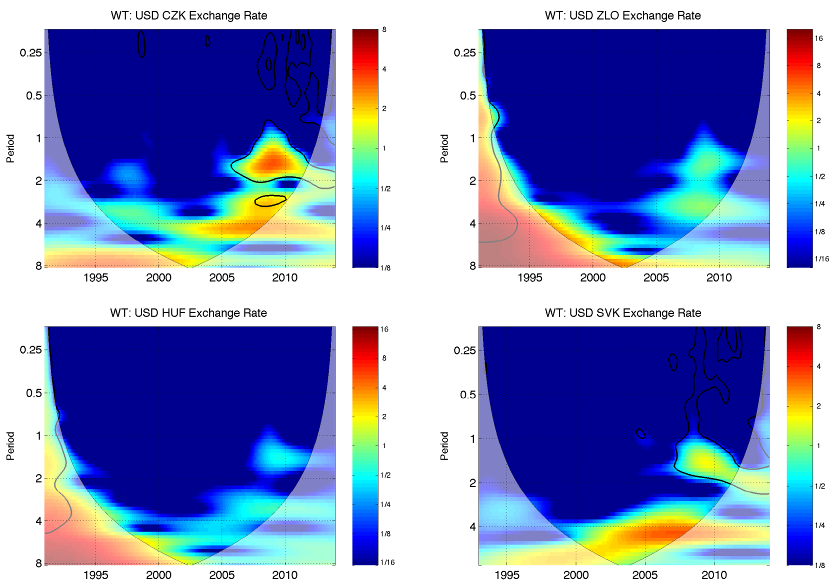
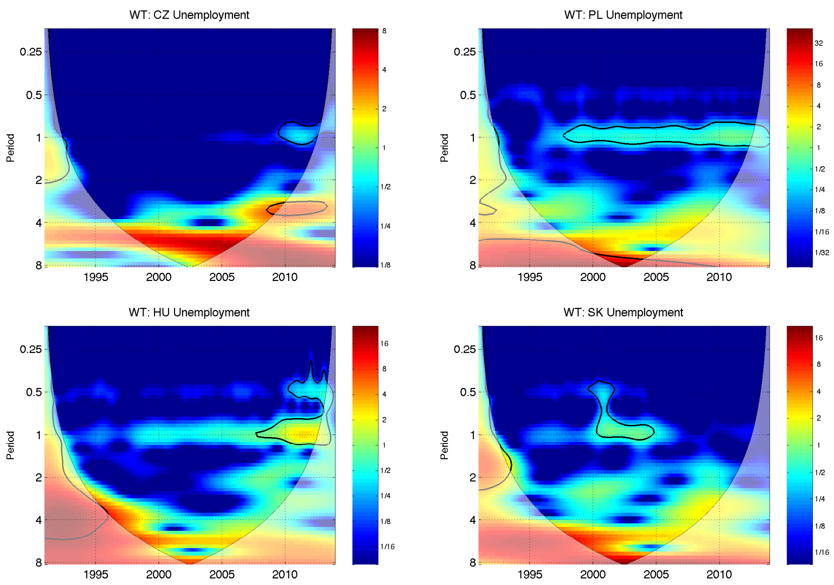


Figure A.10: Wavelet power: V4 - Exchange rates: USD to national currency spot exchange rates. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.



Czech republic and Hungary Poland

Figure A.11: Wavelet power: V4 - Registered unemployment levels. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.

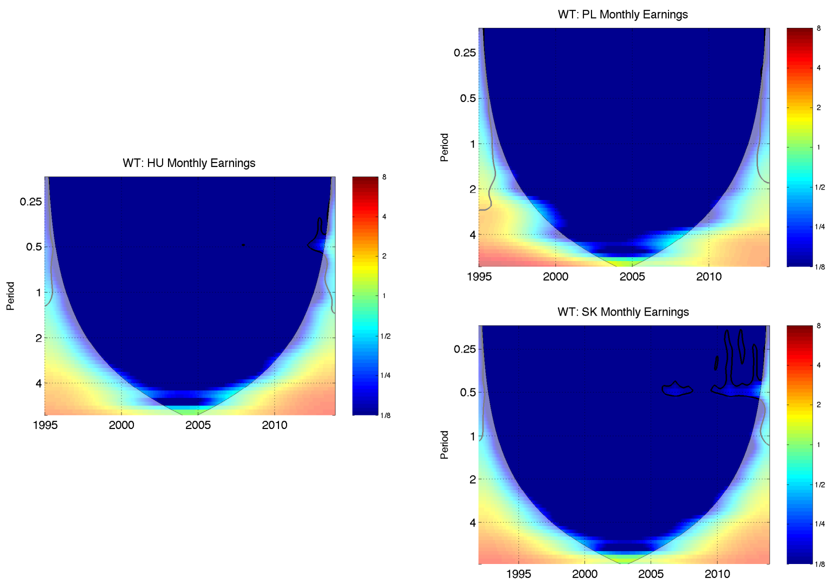


Figure A.12: Wavelet power: V4 - Monthly earnings. From top-left to bottom-right: Hungary, Poland, and Slovakia.

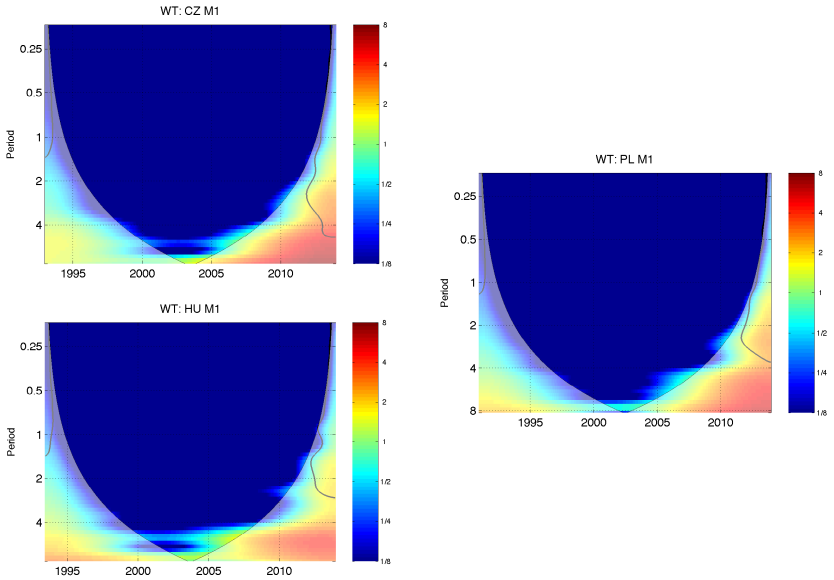


Figure A.13: Wavelet power: V4 - Money supply (M1). From top-left to bottom-right: the Czech Republic, Poland, and Hungary.

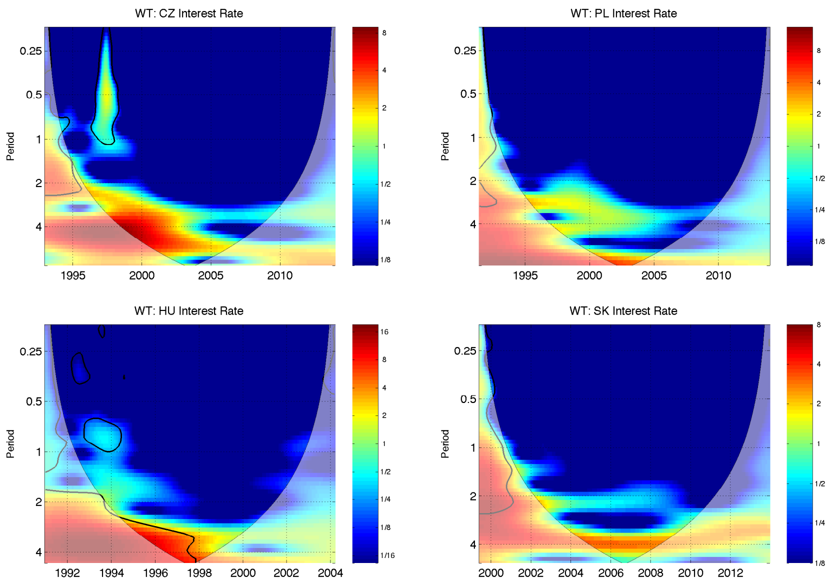
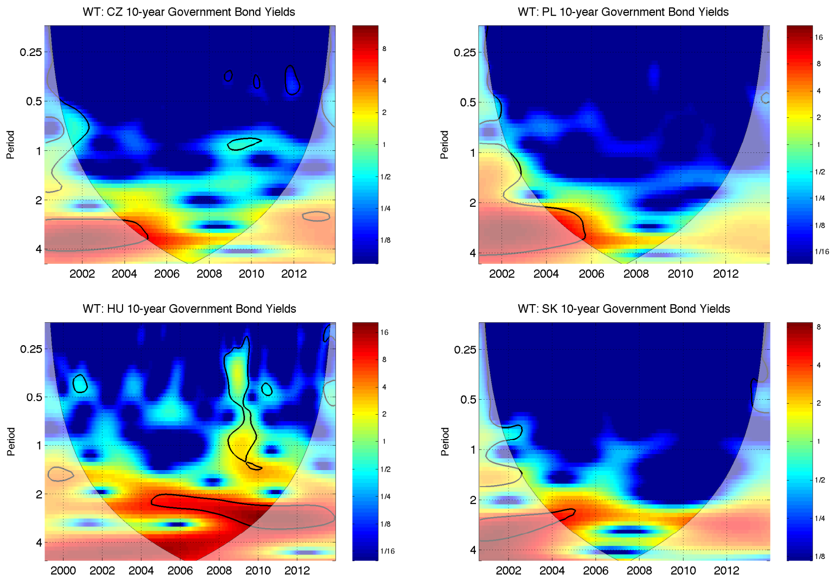


Figure A.14: Wavelet power: V4 - 3-month interbank rates and yields. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.



Czech republic and Hungary   Poland and Slovakia

Figure A.15: Wavelet power: V4 - 10-year Government bond yields. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.

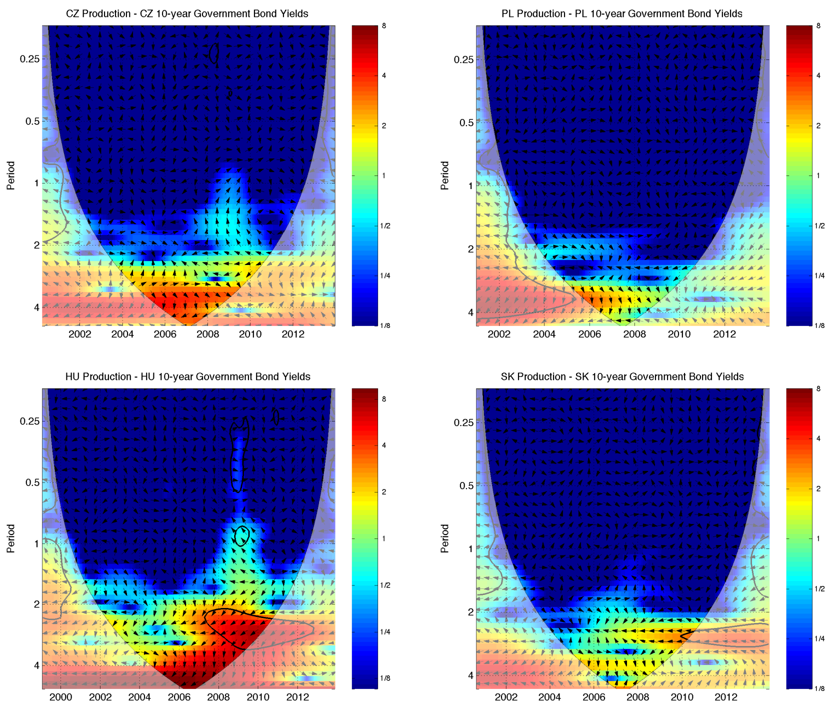
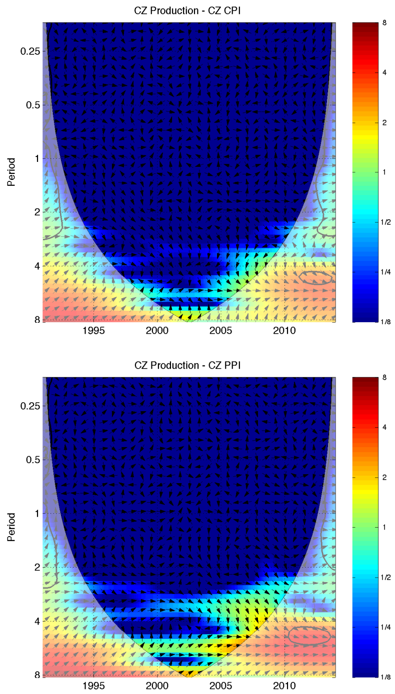
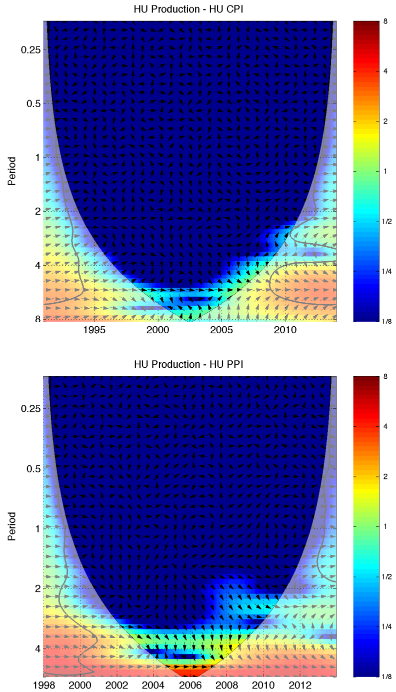


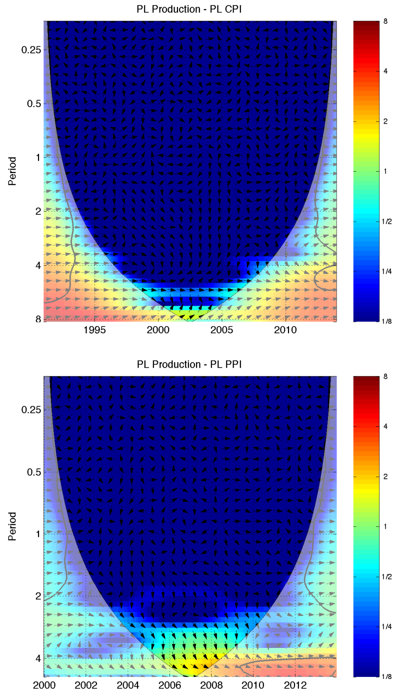
Figure A.16: Cross wavelet power: 10-year government bond yields. From top-left to bottom-right: the Czech Republic, Poland, Hungary, and Slovakia.



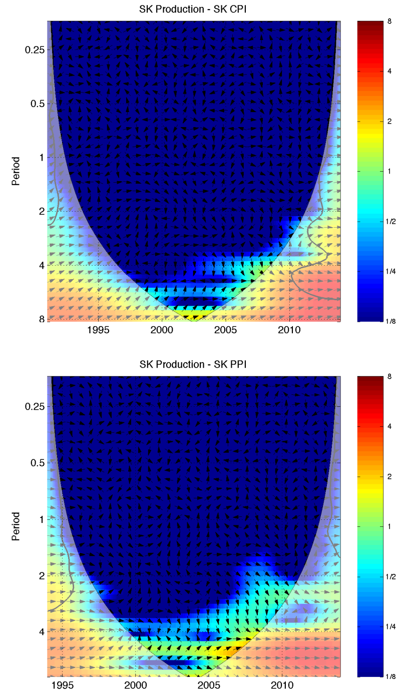
The Czech Republic production and CPI (top), and PPI (bottom)



Hungary production and CPI (top), and PPI (bottom)

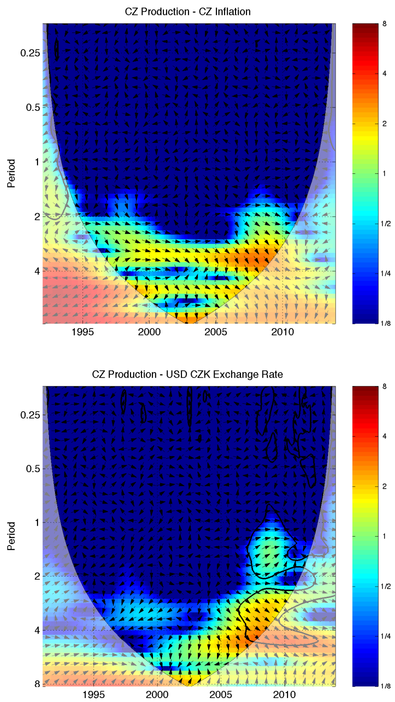


Poland production and CPI (top), and PPI (bottom)

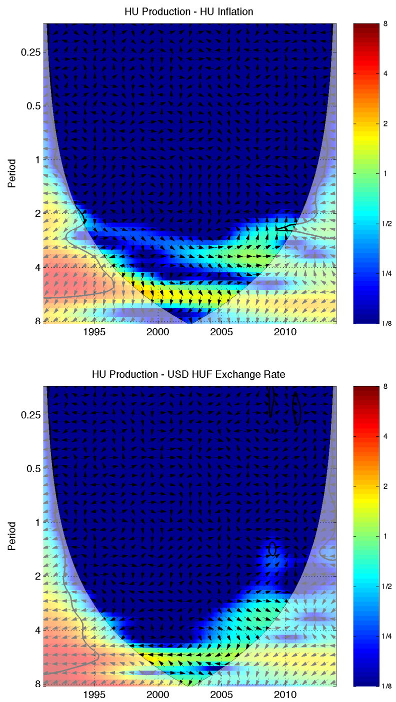


Slovakia production and CPI (top), and PPI (bottom)

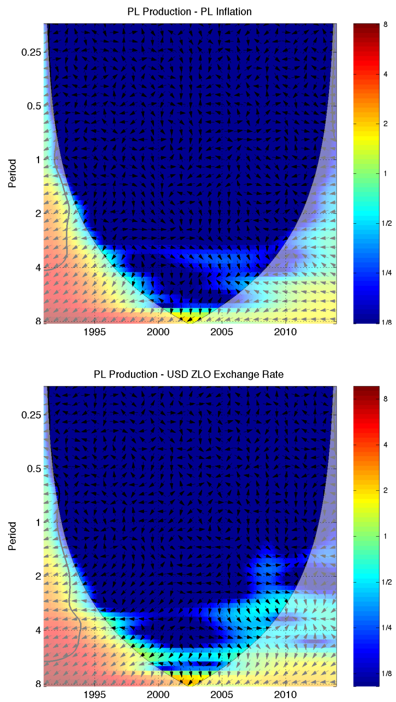
Figure A.17: Cross wavelet power: Top pictures correspond to Consumer Price Indices, bottom to Producer Price Indices.



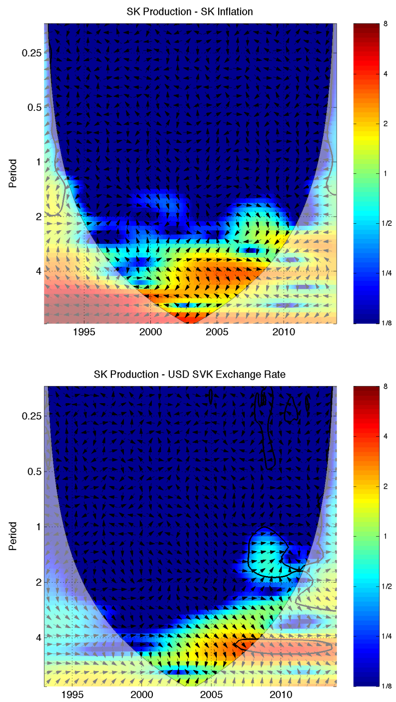
The Czech Republic production and inflation (top), and exchange rate (bottom)



Hungary production and inflation (top), and exchange rate (bottom)

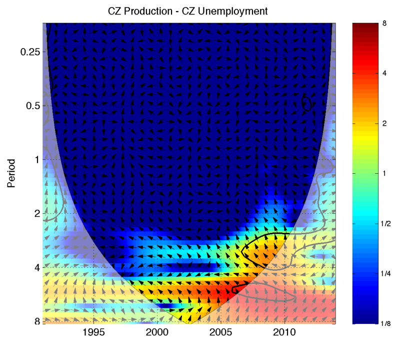


Poland production and inflation (top), and exchange rate (bottom)

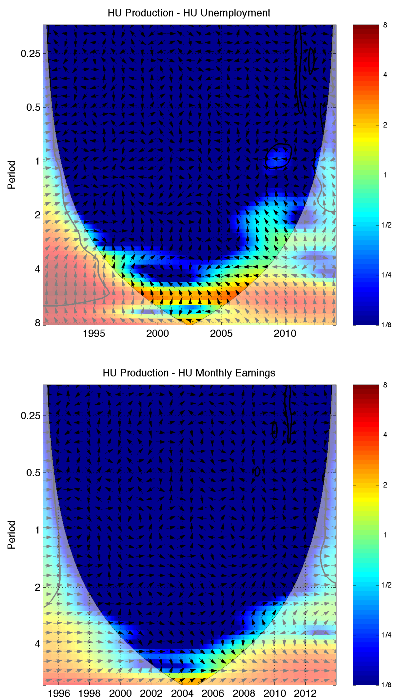


Slovakia production and inflation (top), and exchange rate (bottom)

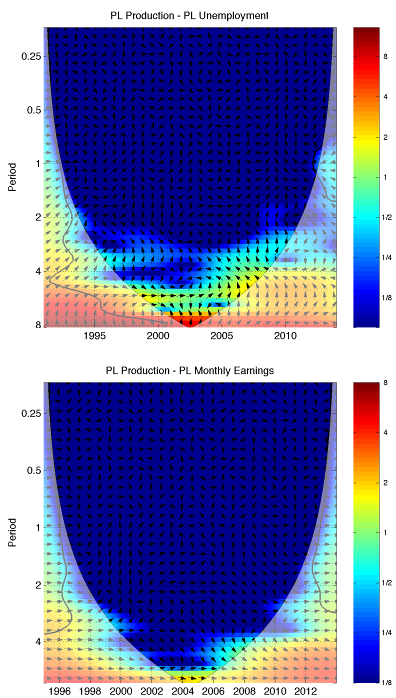
Figure A.18: Cross wavelet power: Top pictures correspond to inflations, bottom to USD to national currency spot exchange rates.



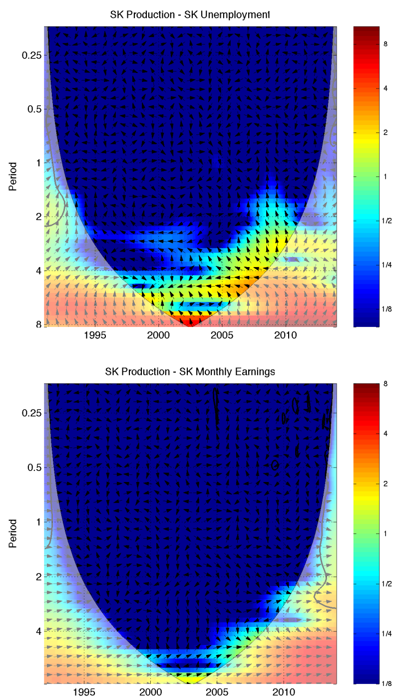
The Czech Republic production and unemployment



Hungary production and unemployment (top), and monthly earnings (bottom)

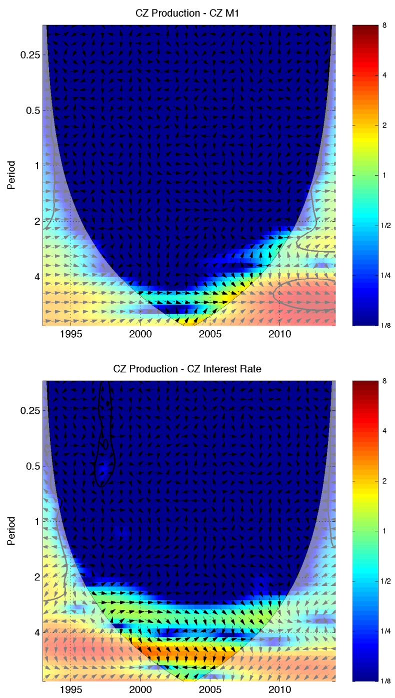


Poland production and unemployment (top), and monthly earnings (bottom)

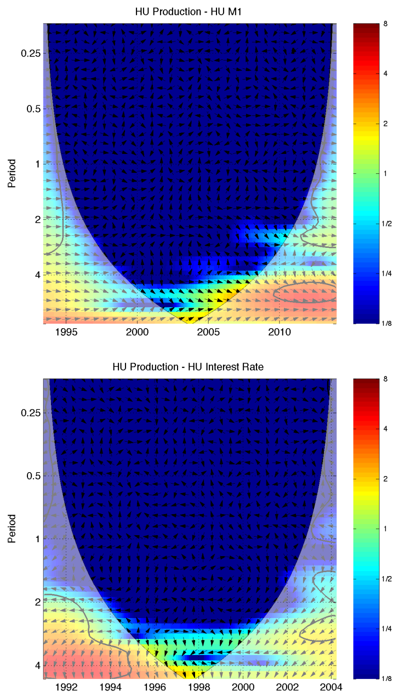


Slovakia production and unemployment (top), and monthly earnings (bottom)

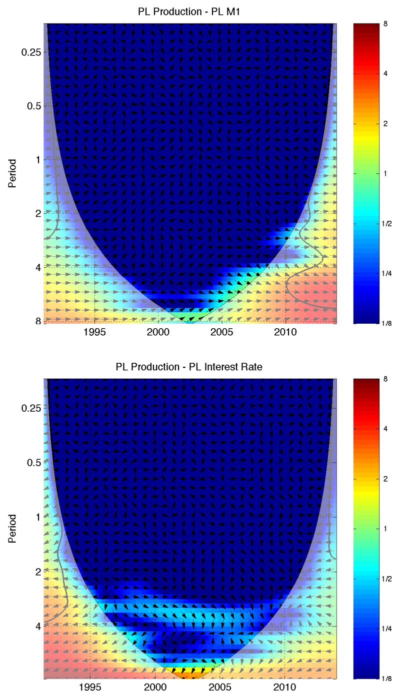
Figure A.19: Cross wavelet power: Top pictures correspond to unemployment level, bottom to monthly earnings, except Czech Republic.



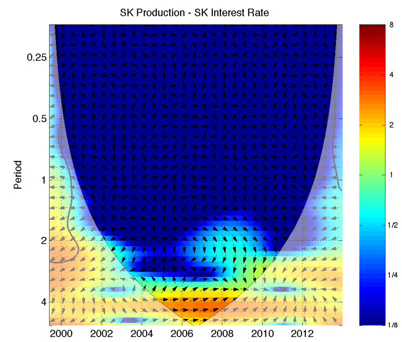
The Czech Republic production and money supply (top), and 3-month interest rate (bottom)



Hungary production and money supply (top), and 3-month interest rate (bottom)



Poland production and money supply (top), and 3-month interest rate (bottom)



Slovakia production and 3-month interest rate

Figure A.20: Cross wavelet power: Top pictures correspond to Money supply (M1), bottom to 3-month interest rates, in to case of Slovakia only the graph of interest rate is present.

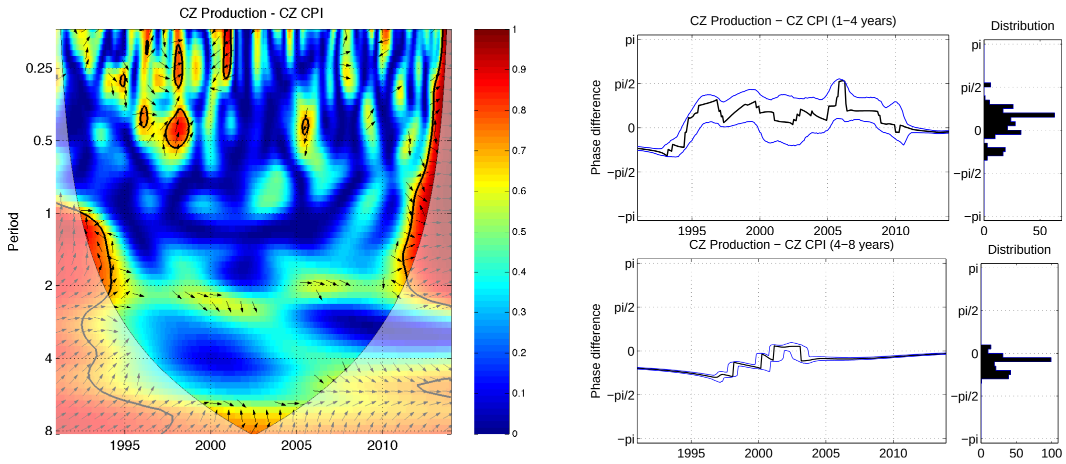


Figure A.21: The Czech Republic production and CPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

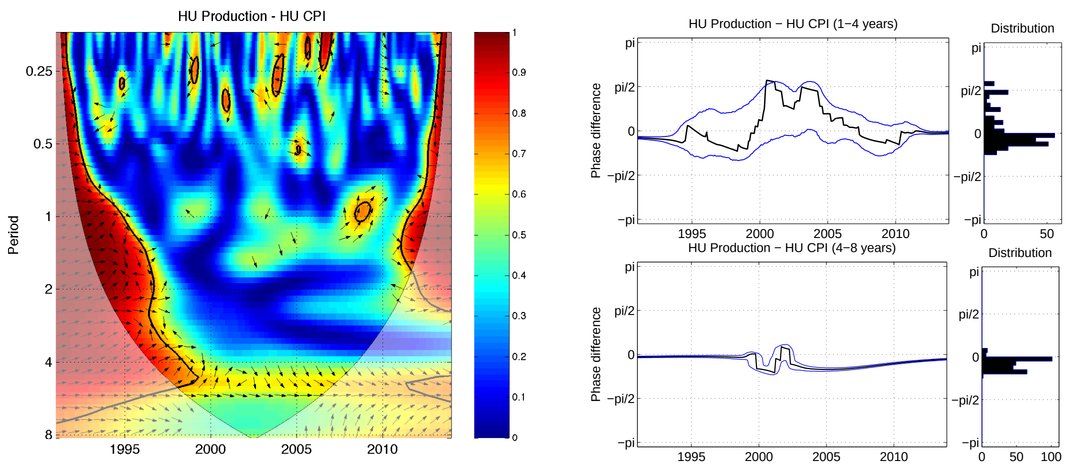


Figure A.22: Hungary production and CPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

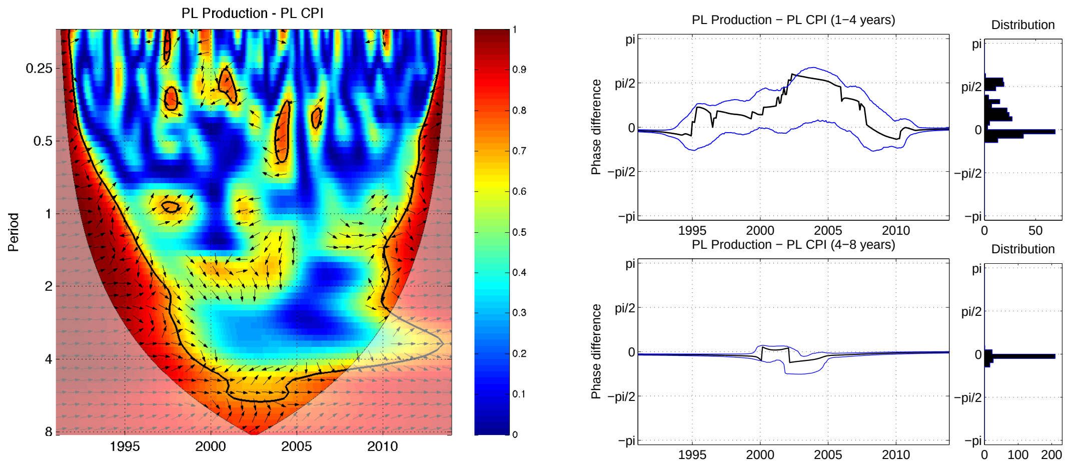


Figure A.23: Poland production and CPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

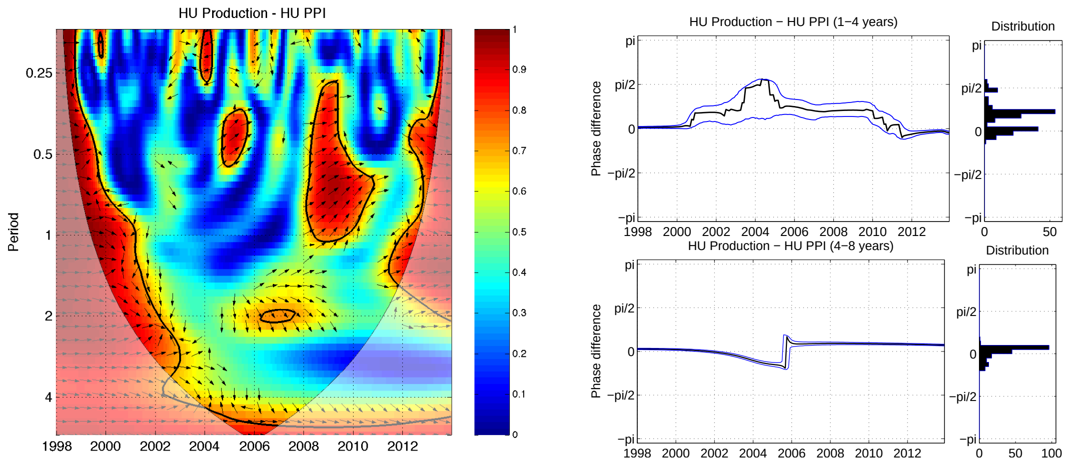


Figure A.24: Hungary production and PPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

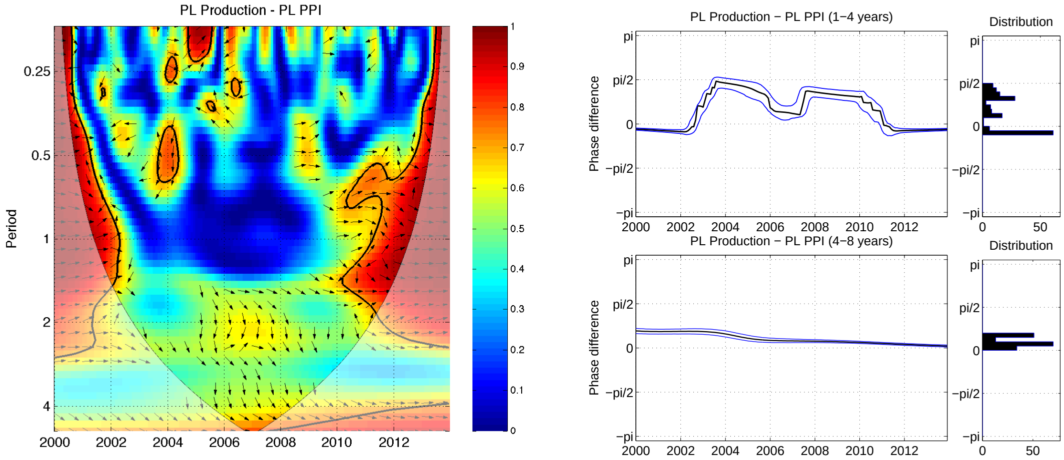


Figure A.25: Poland production and PPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

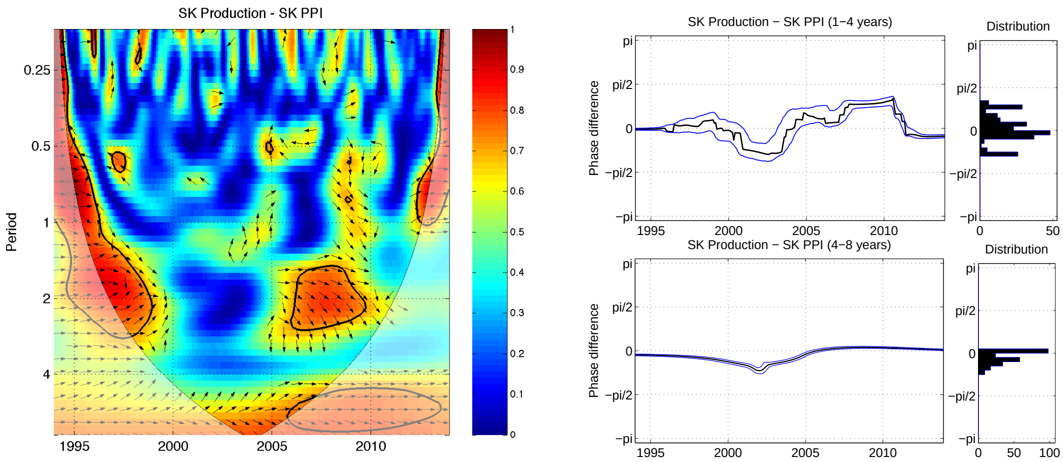


Figure A.26: Slovakia production and PPI; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

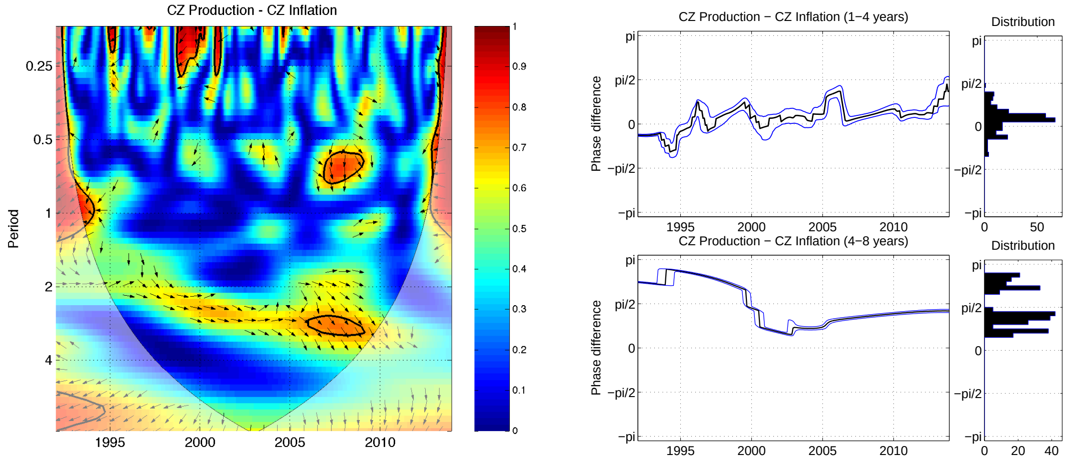


Figure A.27: The Czech Republic production and inflation; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

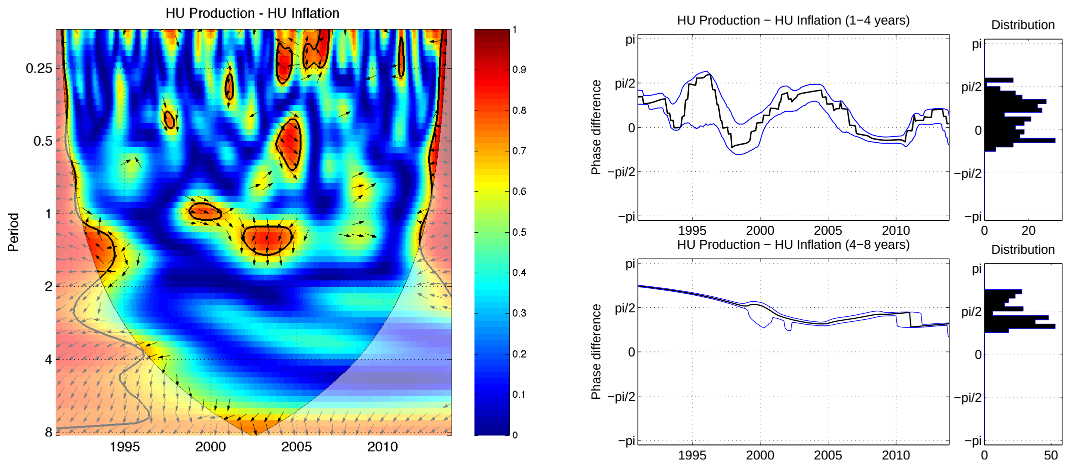


Figure A.28: Hungary production and inflation; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

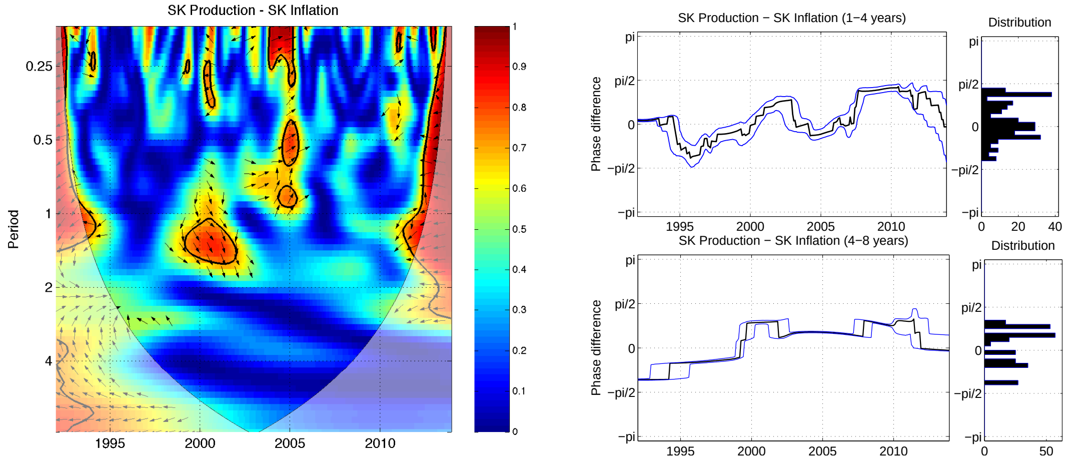


Figure A.29: Slovakia production and inflation; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

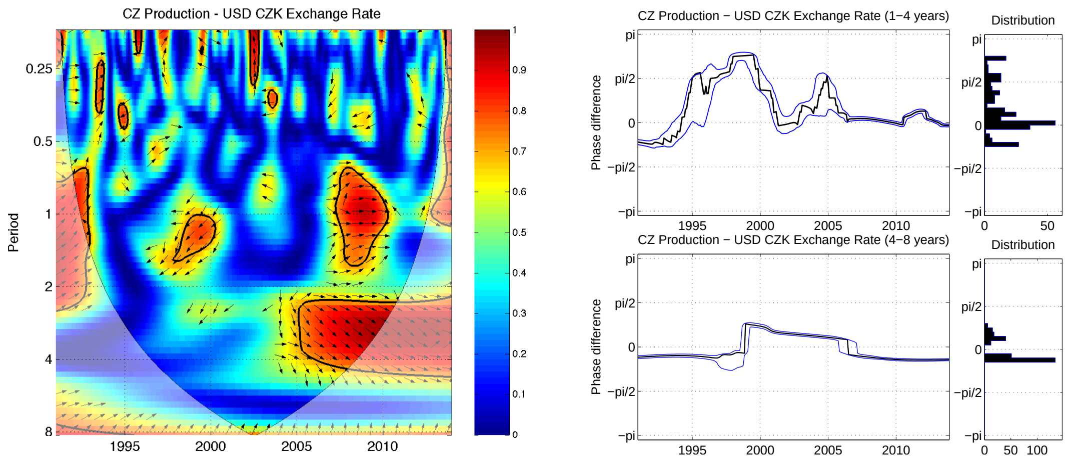


Figure A.30: The Czech Republic and exchange rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

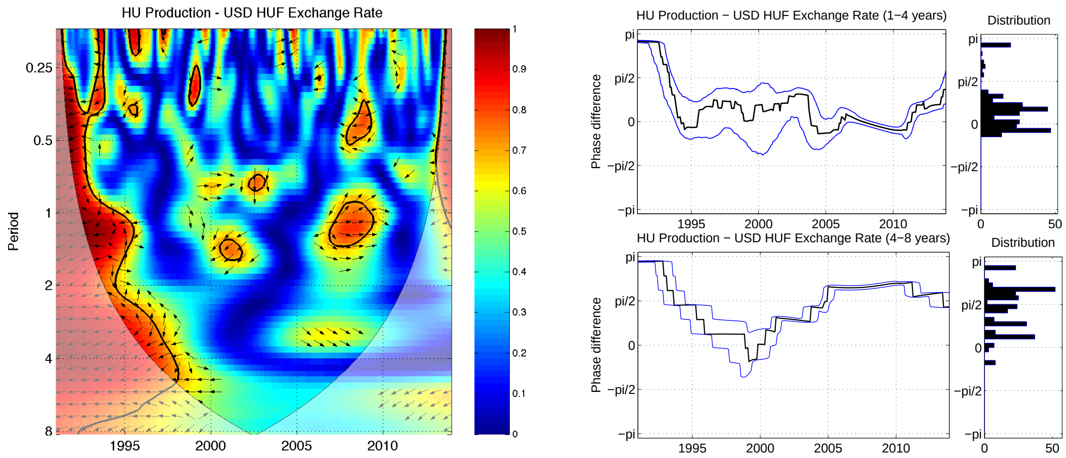


Figure A.31: Hungary production and exchange rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

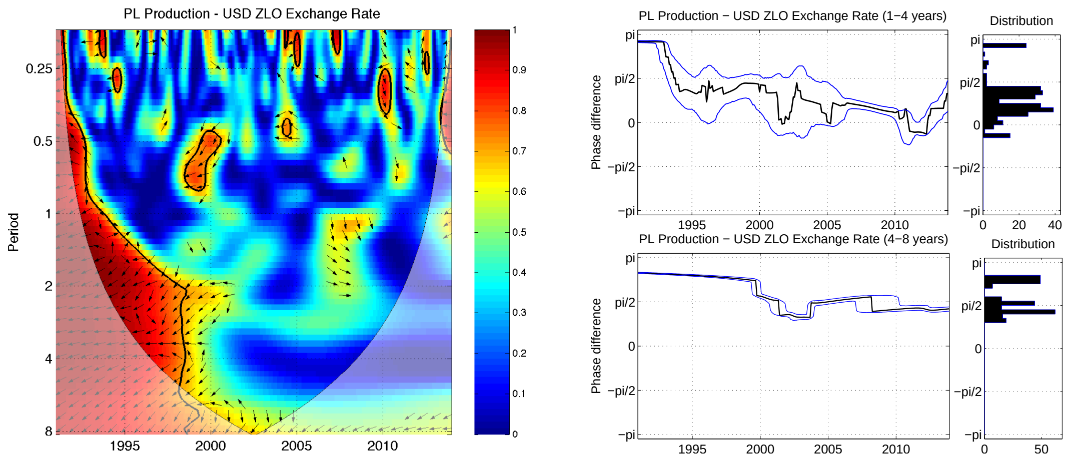


Figure A.32: Poland production and exchange rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

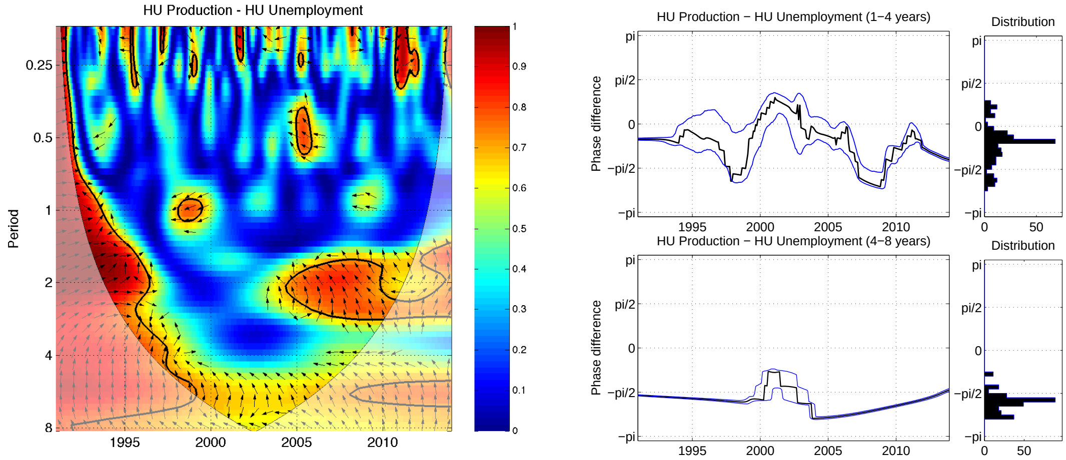


Figure A.33: Hungary production and unemployment; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

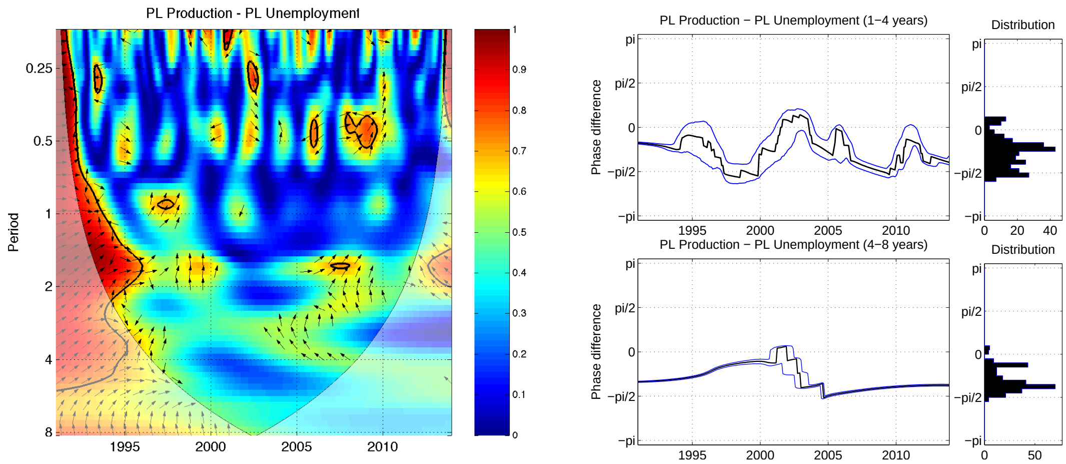


Figure A.34: Poland production and unemployment; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

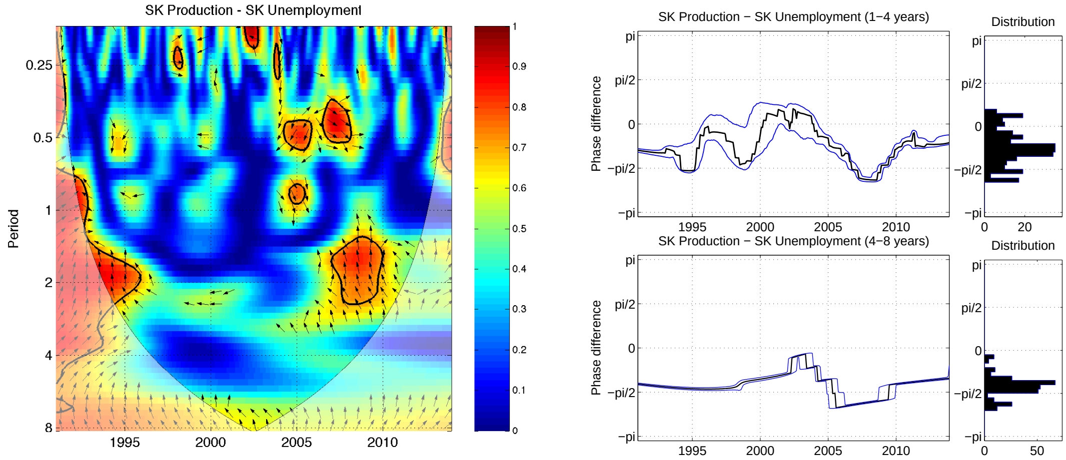


Figure A.35: Slovakia production and unemployment; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

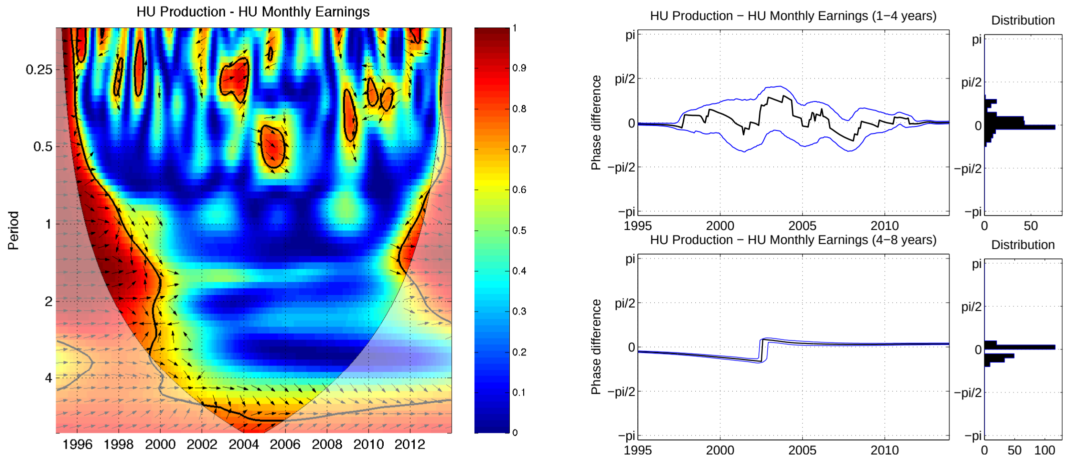


Figure A.36: Hungary production and monthly earnings; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

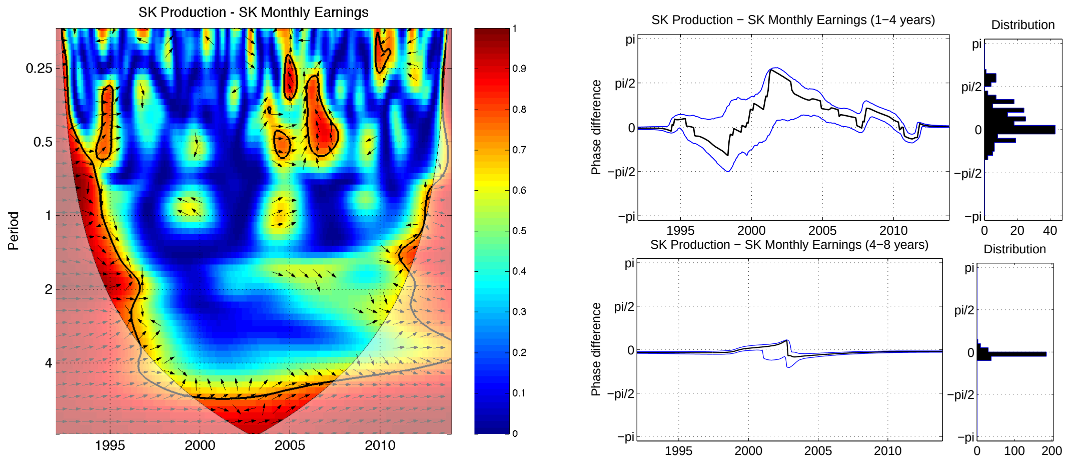


Figure A.37: Slovakia production and monthly earnings; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

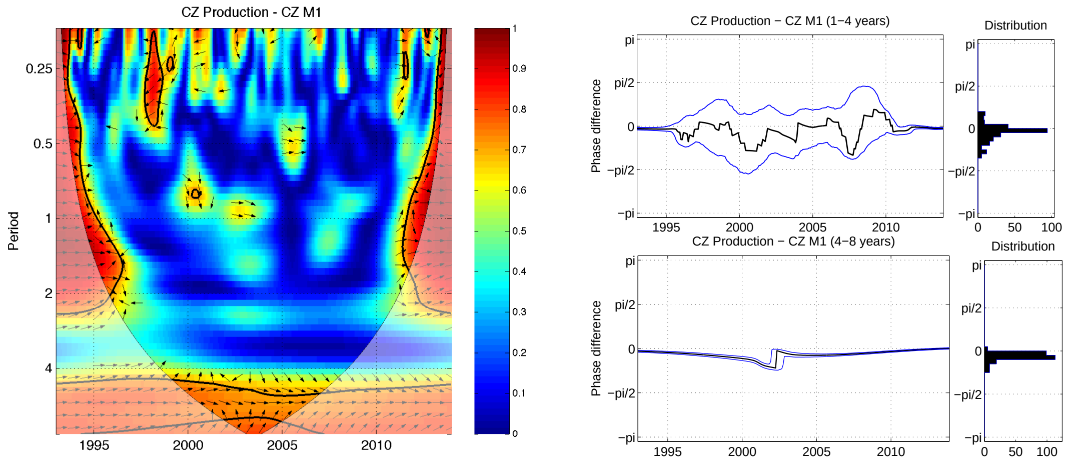


Figure A.38: The Czech Republic and money supply (M1); On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

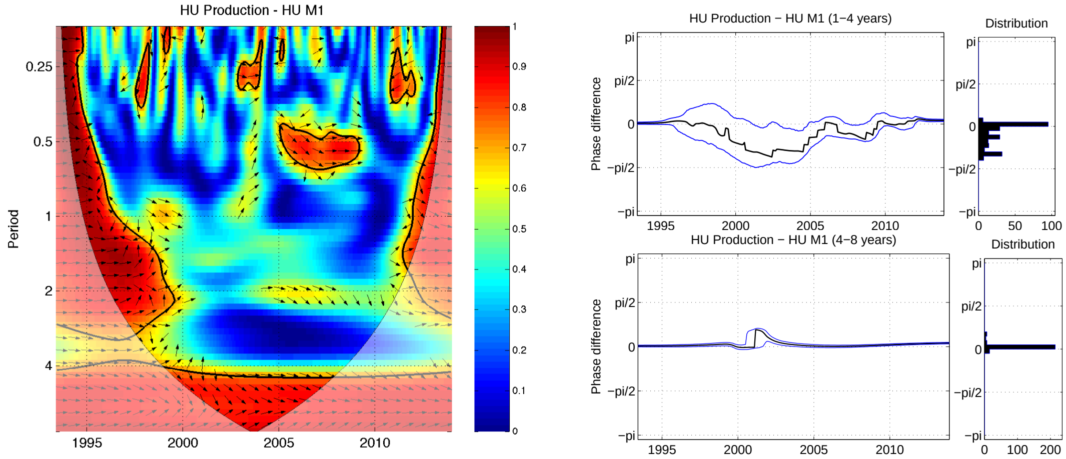


Figure A.39: Hungary production and money supply (M1); On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

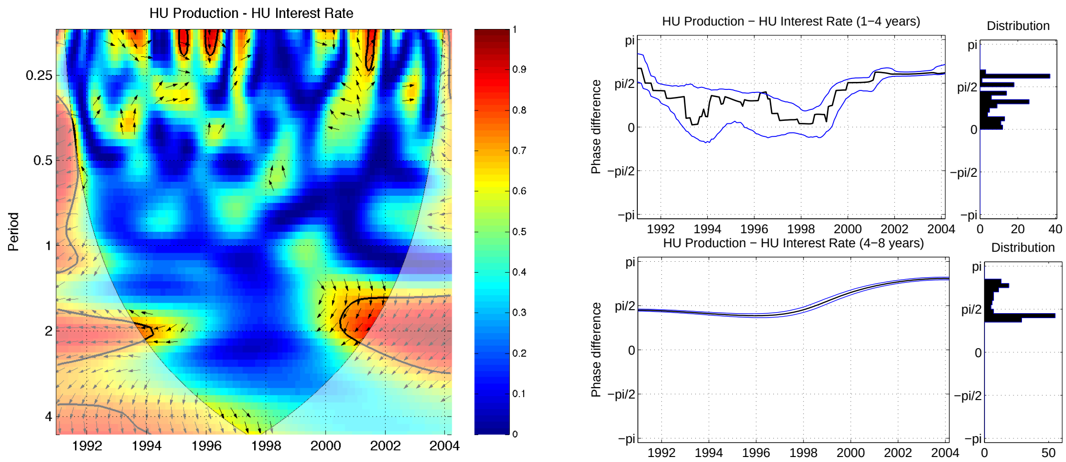


Figure A.40: Hungary production and 3-month interbank rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

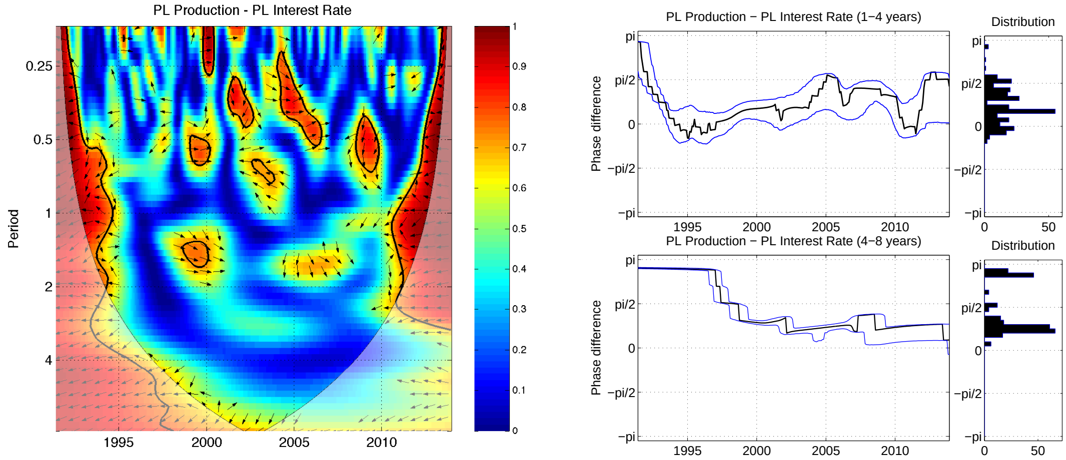


Figure A.41: Poland production and 3-month interbank rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

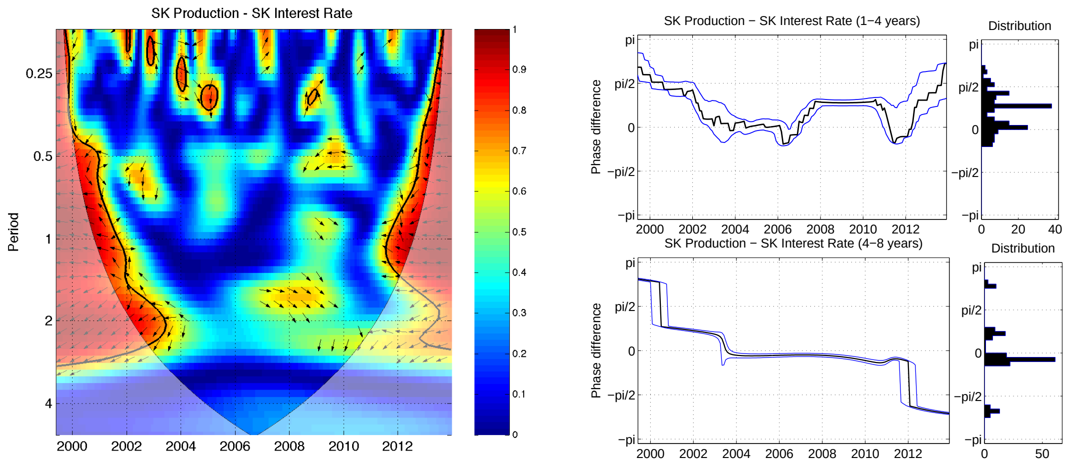


Figure A.42: Slovakia production and 3-month interbank rate; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

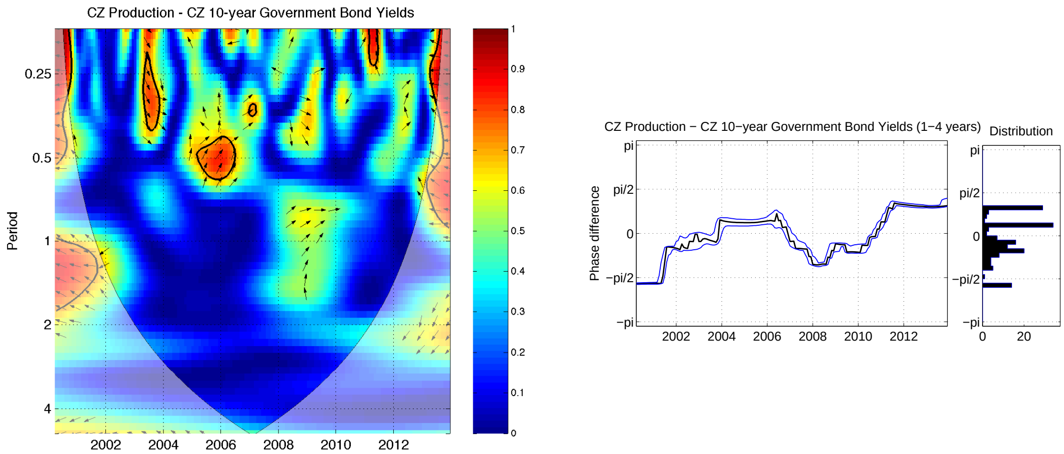


Figure A.43: The Czech Republic production and 10-year government bond yields; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

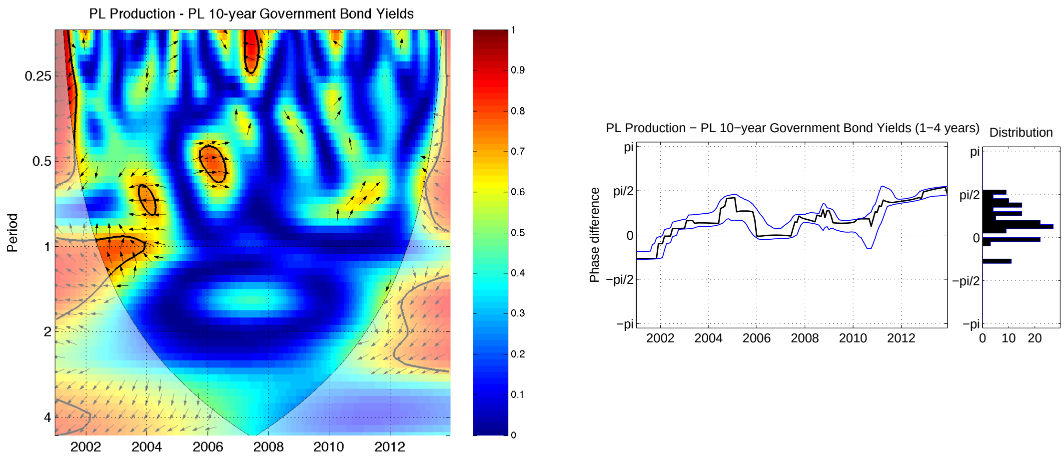


Figure A.44: Poland production and 10-year government bond yields; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

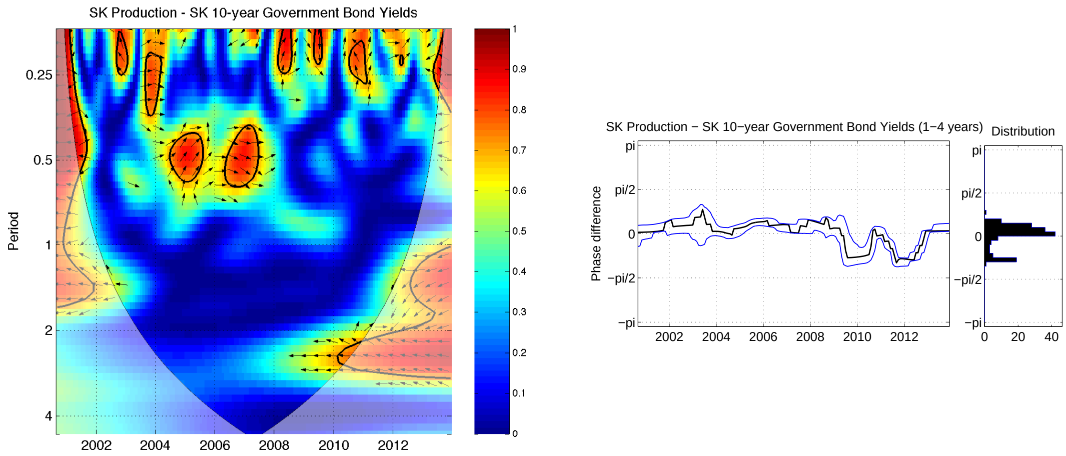


Figure A.45: Slovakia production and 10-year government bond yields; On the left: Wavelet coherency. On the right: Phase difference of two time series in specific period – blue line designates Monte Carlo based 90% confidence interval.

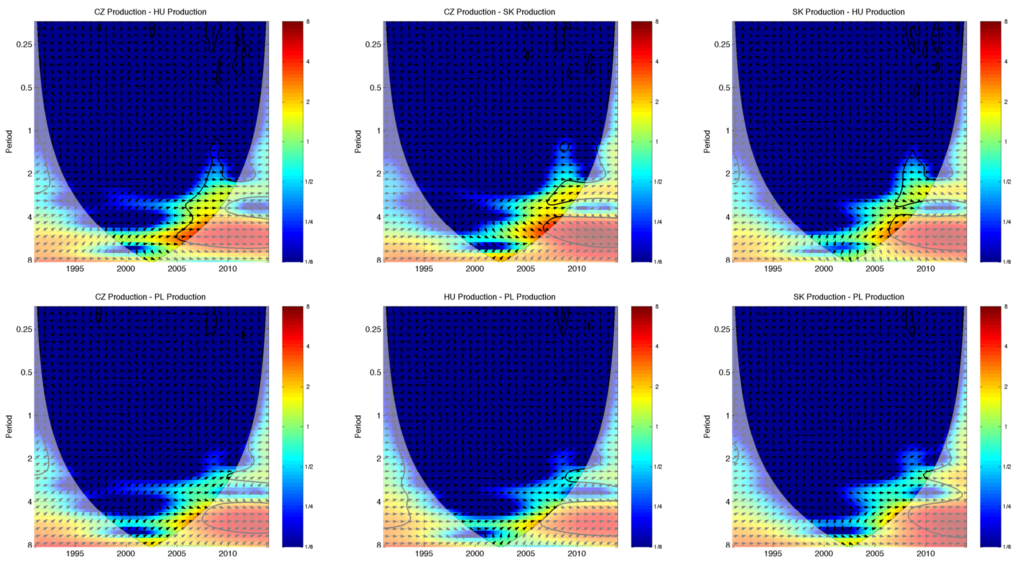


Figure A.46: Cross wavelet power: The Visegrad Four countries between each other.

R packages

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[3] William Constantine and Donald Percival. *wmtsa: Wavelet Methods for Time Series Analysis.* R package version 2.0-0. 2013.

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[10] Hadley Wickham. *scales: Scale functions for graphics.* R package version 0.2.3. 2012.

Master Thesis Proposal

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Proposed topic

Wavelet analysis of business cycles in the Visegrad Four

Topic characteristics

This thesis aims to study the real business cycles in the countries of Visegrád four. The main contribution of my thesis will be the application of the method of wavelet analysis on monthly data spanning from 1991 to 2013, which should result in discovery of the properties of business cycles in selected countries. It is my belief that the wavelet decomposition of the time series to time and frequency will broaden the understanding of stylized facts described in previous literature. Another aim of my thesis is to verify these stylized facts of business cycles for Visegrád countries discussed in this literature and potentially reveal additional characteristics arising from wavelet analysis while studying the dynamics changes in time and frequencies.

Hypotheses

1. The pro-cyclical behaviour of wages and money is present.
2. The unemployment rate behaves counter-cyclically to the output in the countries.
3. There is a low co-movement of the outputs of the Visegrád four countries.
4. Using time and frequency dimension of the cycles can reveal new evidence and understanding of the business cycles in the Visegrád four.

Methodology

There are three wavelet instruments I expect to use in order to uncover time-frequency relations that occur in selected time series. Firstly, to study the volatility of the time series, I will use the wavelet power spectrum. Thereby the time-frequency dynamics of the time series variance will be beneficial for a preliminary insight into the behavior of those series. Secondly, I will analyze the time-varying dynamics of correlations and thus the co-movement of variables by using the wavelet coherence. These correlations will provide additional information about cyclicalities of two series. Thirdly, the phase difference analysis will be used to find out whether time series data from other country is following another, or the other way around; in other words, the phase shifting will be studied.

Outline

1. Introduction
2. Literature review
3. Theoretical background
   (a) Wavelet methodology
4. Empirical analysis
   (a) Data analysis
   (b) Quantitative results
   (c) Comparison of stylized facts findings based on applied methods
5. Conclusion

Core bibliography

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