We study bipartite subgraphs of a random cubic graph in the thesis. We show, that an edge-maximum bipartite subgraph of a random cubic graph on n vertices has asymptotically almost surely less then $32 \cdot 0.9351n$ edges. We also show that the number of vertices of a vertex-maximum induced bipartite subgraph of a random cubic graph lies within interval [0.75n; 0.9082n]. To obtain the lower bound we design a randomized algorithm for finding a large induced bipartite subgraph of a random cubic graph. We discuss consequences of the results for graph homomorphisms, namely for Pentagon Conjecture posed by Nešetřil.