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Faculty of Social Sciences
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MASTER THESIS

Stock Price Bubbles: Identification and the Effects of Monetary Policy

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Declaration of Authorship

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Prague, May 16, 2014

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Abstract

This thesis studies bubbles in the U.S. stock market and how they are influenced by monetary policy pursued by the FED. Using Kalman filtering, the log-real price of S&P 500 is decomposed into a market-fundamentals component and a bubble component. The market-fundamentals component depends on the expected future dividends and the required rate of return, while the bubble component is treated as an unobserved state vector in the state-space model. The results suggest that, mainly in recent decades, the bubble has accounted for a substantial portion of S&P 500 price dynamics and might have played a significant role during major bull and bear markets. The innovation of this thesis is that it goes one step further and investigates the effects of monetary policy on both estimated components of S&P 500. For this purpose, the block-restriction VAR model is employed. The findings indicate that the decreasing interest rates have a significant short-term positive effect on the market-fundamentals component but not on the bubble. On the other hand, quantitative easing seems to have a positive effect on the bubble but not on the market-fundamentals component. Finally, the results suggest that the FED has not been successful at distinguishing between stock price movements due to fundamentals or the price misalignment.

JEL Classification	C22, C32, E43, E44, G12, G38
Keywords	bubbles, Granger causality, impulse response analysis, Kalman filter, monetary policy, S&P 500, state-space models, stock markets, VAR models

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Abstrakt

Tato práce se zabývá bublinami na americkém akciovém trhu a jejich souvislostí s měnovou politikou FEDu. Reálná hodnota indexu S&P 500 je rozdělena pomocí Kalmanova filtru na ,,fundamentální" a spekulativní část (bublinu). Fundamentální část hodnoty indexu závisí na očekávaných budoucích dividendách a požadované míře návratnosti, zatímco bublina je odhadnuta jako nepozorovaný vektor ve state-space modelu. Výsledky naznačují, že bubliny mohly hrát významnou roli během některých býčích a medvědích trhů a zapříčinit významnou část dynamiky indexu S&P 500 hlavně v posledních dekádách. Inovací této práce je, že jde o krok dále a studuje efekty monetární politiky na obě odhadnuté části indexu. Vektorový autoregresivní model s blokovými restrikcemi je použit k této analýze. Výsledky ukazují, že klesající úrokové sazby mají pozitivní krátkodobý vliv na fundamentální část indexu, ale nemají vliv na spekulativní část indexu. Na druhou stranu se zdá, že kvantitativní uvolňování pozitivně ovlivňuje odhadnutou bublinu, ale nemá vliv na fundamentální část indexu S&P 500. Výsledky této práce zároveň naznačují, že FED nebyl v minulosti úspěšný při rozlišování, zda byly pohyby cen akcií zapříčiněny změnou fundamentálních ukazatelů nebo bublinami.

Klasifikace JEL	C22, C32, E43, E44, G12, G38
Klíčová slova	akciové trhy, analýza funkcí odezvy, bubliny, Grangerova kauzalita, Kalmanův filtr, monetarní politika, S&P 500, state-space modely, VAR modely

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Acronyms

ADF Augmented Dickey-Fuller

CPI consumer price index

FED Federal Reserve System of the United States

KPSS Kwiatkowski-Phillip-Schmidt-Shin

S&P Standard & Poor's 500

VAR vector autoregression

Master Thesis Proposal

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Proposed topic	Stock Price Bubbles: Identification and the Effects of Monetary Policy

Topic characteristics Since 1980s, there has been a heated debate whether and into which extent are the asset prices governed by their fundamental value and which part of the prices can be attributed to speculative bubbles. Steep rises and sudden falls in both stock and housing prices that we could witness in past decades suggest that other components than just the fundamental value may be contained in asset prices. The golden 1920s followed by the stock exchange crash in 1929 resulting in the Great Depression, the so called IT-bubble of late 1990s, or the housing and stock prices boom resulting in the sub-prime crisis that hit in 2008 are the most famous examples of periods when bubbles could play a substantial role.

The purpose of most central banks including the US Federal Reserve System is price stability. This thesis aims to investigate whether Federal Reserve's monetary policy is effective in its pursuit of stable prices in terms of being able to influence the non-fundamental part of asset prices that I refer to as a bubble. Specifically, I will analyze the transmission from US short term interest rates to the extracted bubble part of S&P 500 Index.

The crucial part of the thesis will consist of determining the non-fundamental (or bubble) part of S&P 500 Index. There have been a number of theoretical models developed to explain how asset prices including a bubble can evolve and behave. The choice of the econometric method suitable for estimating the bubbles depends primarily on the underlying theoretical model which is believed to generate the prices. For this purpose, I will employ the same procedure as in Wu (1997), who incorporates a speculative rational bubble process into the standard linear rational expectations model of stock price determination. Following Wu (1997), I will jointly estimate the parametric bubble process, the stock-price equation, and the dividend process using Kalman filtering technique.

Having extracted this time series that one can believe contains the bubble part of S&P 500 Index, I will use the VAR model proposed by Sims (1980) in order to analyze the transmission from monetary policy interest rates to the estimated bubbles. In order to be able to take an aggregate picture of the transmission mechanism, other US market variables will be included in the VAR estimation. The dataset will also include output, inflation, and exchange rate. The dataset will be obtained from S&P database, Thomson Datastream, and from Shiller (2000).

There is a wide variety of literature on modeling and estimating asset price bubbles and also on monetary policy and its effect on asset prices. However, to my best knowledge, there has not been a study attempting to extract the non-fundamental part from stock prices and investigating how it is affected by monetary policy. Therefore, I believe that my thesis will be useful for the current research.

Hypotheses

1. H0: A bubble component can be detected in the time series of S&P 500 Index.
2. H0: The bubble component will be positive during boom periods and negative during market turmoils.
3. H0: There is no transmission between the short term exchange rates set by FED and the estimated asset price bubble. A: The transmission can be detected

Methodology There will be two consecutive steps leading to answering my research question. The first step will be to estimate the bubble process from the values of S&P 500 Index and dividends of stocks contained in the index. For this purpose, I will follow Wuâ€™s (1997) methodology and I will jointly estimate the parametric bubble process, the stock-price equation, and the dividend process using Kalman filtering technique. In the next step, I will use the VAR model proposed by Sims (1980) in order to analyze the transmission from monetary policy interest rates to the estimated bubbles.

Outline

1. Introduction
2. Asset Price Bubbles Theory
3. Model
4. Data
5. Empirical Results
6. Conclusion

Core bibliography

1. Refet S. Gurkaynak. Econometric Tests of Asset Price Bubbles: Taking Stock; Finance and Economics Discussion Series 2005-04, Board of Governors of the Federal Reserve System (U.S.)., 2005.
2. William C. Hunter & George G. Kaufman & Michael Pomerleano (ed.). Asset Price Bubbles: The Implications for Monetary, Regulatory, and International Policies; MIT Press Books, The MIT Press, edition 1, volume 1, number 0262582538, October 2005.
3. Evžen Kočenda & Alexandr Černý. Elements of Time Series Econometrics: An Applied Approach; volume 33. Karolinum Press, Charles University, Prague, 2007.
4. Luboš Komárek & Ivana Kubicová. The Classification and Identification of Asset Price Bubbles; Czech Journal of Economics and Finance (Finance a uver), Charles University Prague, Faculty of Social Sciences, vol. 61(1), pages 34-48, January 2011.
5. Christopher A. Sims. Macroeconomics and Reality; Econometrica, 48(1): 1-48, January 1980.
6. Robert J. Shiller. Irrational Exuberance; Princeton University Press, 2000.
7. Yangru Wu. Rational Bubbles in the Stock Market: Accounting for the U.S. Stock-Price Volatility; Economic Inquiry, Western Economic Association International, vol. 35(2), pages 309-19, April 1997.

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Chapter 1

Introduction

The importance of financial markets for the global economy as well as for our everyday life has surged over the past decades. Periods of massive growth and sudden sharp falls in stock markets can threaten numerous aspects of general economic environment and can have disastrous consequences for functioning of the global financial system.

Over the past century, the U.S. stock market experienced a number of severe swings that heavily influenced the course of the global economy. Among the most prominent are the bull market of the 1920s followed by the Great Depression or the post-war boom of the 1950s and 60s with the subsequent bear market of the 1970s. The more recent swings include the so-called IT-bubble of the late 1990s followed by its sudden meltdown, only to start the next upswing in 2003. This was followed by the sub-prime debt financial crisis of 2008 and 2009.

All the above mentioned periods are characterized by the extreme increase in volatility of stock prices that, according to many economists, cannot be attributed to market fundamentals. For example, Shiller (1981) argued that over the past century, the U.S. stock prices had been five to thirteen times more volatile than what could be justified by news about the expected dividends. Although Shiller's methodology was criticized, there has been a lot of independent research concluding that the variability of stock prices is too large to be explained by the changes in the present value of the expected future dividends (see for example LeRoy & Porter (1981), West (1988a), Campbell & Shiller (1988b) or LeRoy & Parke (1992)).

Economists, realizing the failure of the simple present-value model[1] in ex-

[1] The simple present-value model refers to the linear discounted-dividends model.

plaining the stock price volatility, have devoted a substantial effort to searching for an alternative model. As a result of this endeavor, two leading approaches of how to amend the simple present-value model have emerged. The first approach is to allow for a variable discount rate, while maintaining the notion that the stock price is only determined by the discounted value of the expected future dividends. However, it has been shown that the variable discount rate can only marginally explain the stock price volatility. See, for example, Campbell & Shiller (1988a), Campbell & Shiller (1988b) or West (1988b).

The second approach allows for the stock price deviation from the so-called fundamental price[2] by incorporating a bubble component. Although researchers have been more successful at explaining volatility of stock prices using the theory of asset price bubbles (see for example Wu (1997), Bhar & Hamori (2005, p. 164 -187), or Al-Anaswah & Wilfling (2011)), empirical tests for stock market bubbles yield mixed results and there is not a general consensus even about the underlying theory. Refer to Gürkaynak (2005) for a summary of the controversy about detecting asset price bubbles.

The opinions on how monetary policy should react to asset price bubbles differ among researchers as well. Even if the identification problem is set aside, the proposed monetary policy response ranges from not reacting to bubbles at all (Bernanke & Gertler (1999) and Bernanke & Gertler (2001)), through indirectly responding via changing the inflation target (Orphanides (2010)), to actively correcting the price misalignment (Cecchetti *et al.* (2000)).

The first objective of this thesis is to review the theory about asset price bubbles and the related empirical results. Subsequently, keeping in mind that the bubble solution is only one of the plausible explanations for the failure of the simple present-value model, Kalman filtering is applied in order to estimate the stochastic bubble component in the U.S. stock market. Finally, the thesis investigates how monetary policy, pursued by the FED, reacts to and influences the estimated bubble.

The remainder of the work is divided as follows: the second part describes the theory related to asset price bubbles and provides a review of previous empirical research on this topic. The third part derives the model that is used for estimating the bubble and explains the methodology used in this thesis. The fourth part describes the data, the fifth part presents the empirical results, and the last part concludes.

[2] The fundamental price is usually defined as the present value of the expected future dividends.

Chapter 2

Literature Review

2.1 Theory of Asset Price Bubbles

Allan Greenspan, then a Federal Reserve Board chairman, used the term "irrational exuberance" in his speech given on 5 December 1996 suggesting what he thought had driven stock prices. Greenspan's comment would probably not have been that well remembered if it had not been followed by immediate slumps in stock markets worldwide and had not provoked such a strong reaction in financial circles. Global stock markets dropped precipitously. In Japan, the Nikkei index dropped 3.2%; in Hong Kong, the Hang Seng lost 2.9%; and in Germany, the DAX plummeted 4%. In London, the FT-SE 100 index was down 4% at one point during the day, and in the United States, the Dow Jones Industrial Average fell by 2.3% (Shiller (2005)). From the perspective of this thesis, what Allan Greenspan meant, and what was into some extent confirmed by the immediate reaction of the worldwide stock markets, was that the stock prices of the 1990s contained a bubble component.

The concept of asset price bubbles has been discussed almost since organized markets began. Bubbles are typically associated with dramatic asset price increases followed by a collapse. Researchers define the term bubble differently, but the common element in definitions of a bubble is a deviation of a price of an asset from what could be justified by "fundamentals"[1] (Kindleberger & Aliber (2011)).

Famous early examples of periods where bubbles might have been present include the Dutch tulip mania in the 17th century and the South Sea share

[1] In the context of this thesis, the fundamentals are all economic factors and variables that determine the expected future stock dividends and the discount rate.

price bubble in the 18th century. In the first case, the price of tulip bulbs rocketed between November 1636 and January 1637 only to suddenly collapse in February 1637 and by 1639, the price had fallen to around 0.005% of its peak value (Cuthbertson & Nitzsche (2005)).

The most prominent characteristics of both periods was the extreme price appreciation, which, however, is not a sufficient condition for a bubble. The two events are cited in literature because of pure speculative price appreciation without any reasonable economic foundation. This is another necessary symptom of a bubble.

The "roaring" 1920s and the subsequent crash in 1929 that preceded the Great Depression, the rise and the subsequent fall of the dollar spot FX-rate between 1982 and 1985, or the so-called IT-bubble of 1997 - 2000, have also been interpreted in terms of bubbles.

Since asset prices affect the real allocation in the economy, asset mispricing ultimately leads to misallocation of scarce resources. Hence, it is important to understand the circumstances under which these prices can deviate from their fundamental value. Bubbles have long intrigued economists and have led to several strands of models, empirical tests, and experimental studies.

The next section aims to summarize the theory related to asset price bubbles including different models and the inherent controversy related to the concept. As the empirical part of this thesis studies bubbles in stock prices, the theory below will be presented from the perspective of stock price bubbles. Nevertheless, the theory is also applicable to other asset classes.

2.1.1 Different Models for Bubbles

Several types of asset price bubbles have been specified in academic literature. The classification is based primarily on how they are thought to originate and develop. The models can be broadly divided into four groups. The first two groups of models are based on rational expectations, but differ in their assumption whether all investors have the same information or not. The third group of models builds on the interaction between rational an non-rational (behavioral) investors. The last group of models assumes heterogeneous believes of the traders.

Rational Speculative Bubbles under Symmetric Information

Asset prices contain a rational speculative bubble if agents are willing to pay for the stock more than what is justified by the value of the discounted stream of expected dividends, because they expect to be able to sell it at an even higher price in the future. An important feature of a speculative bubble is that the resulting high price is still an equilibrium price and the pricing of the equity is still rational, because there are no arbitrage opportunities.

The theoretical concept of bubbles in settings in which all agents have rational expectations and share the same information was first introduced by Tirole (1982). A necessary condition for this type of a bubble to exist is that the economy in which the bubble occurs cannot be Pareto efficient.[2] A bubble would make the seller of the bubble asset better off, which, in the Pareto efficient economy, would have to make the buyer worse off. Given that the agents are rational, no one would be willing to buy the bubble assets.

The bubble is introduced by expressing this period's asset price in terms of expectations about the next period.[3]

$$P_t = \frac{E_t\left[P_{t+1} + D_{t+1}\right]}{1 + R_{t+1}}, \tag{2.1}$$

where R_{t+1} denotes the discount rate for the period from time t to $t+1$. P_t stands for the price of the asset at time t. D_{t+1} is the dividend paid at time $t{+}1$. E_t is the mathematical expectation conditional on the information available at time t.

Solving the above equation forward and using the law of iterated expectations, it is straightforward to see that the equilibrium price of an asset is given by the discounted future stream of the expected dividends between time t and T plus the discounted expected value of the asset at time T.

For assets with finite maturity, the price of an asset after its maturity is zero. Hence, the price of the asset at time t is unique and does not allow for occurrence of rational speculative bubbles. On the other hand, for the securities with infinite maturity, the solution to the equation (2.1) is as follows (assuming a constant required rate of return):

[2] Pareto efficiency, or Pareto optimality, is a state of allocation of resources in which it is impossible to make any one individual better off without making at least one individual worse off.

[3] Formal step-by-step derivation of the rational bubble based on the logarithmic approximation of the present value model is provided in section 3.1.

$$P_t = E_t\left[\sum_{\tau=1}^{\infty}\frac{D_{t+\tau}}{(1+R)^{\tau}}\right] + \lim_{\tau\to\infty} E_t\left[\frac{P_{t+\tau}}{(1+R)^{\tau}}\right]. \tag{2.2}$$

Therefore, for the securities with infinite maturity, the price P_t only coincides with the future expected discounted future dividend stream if the so-called transversality condition $lim_{\tau\to\infty}E_t\left[\frac{P_{t+\tau}}{(1+R)^{\tau}}\right] = 0$ holds. Without imposing the transversality condition, there are many possible prices that solve the above expectational equation. A general solution to equation (2.2) then takes the following form:

$$P_t = E_t\left[\sum_{\tau=1}^{\infty}\frac{D_{t+\tau}}{(1+R)^{\tau}}\right] + B_t = P_t^f + B_t, \tag{2.3}$$

where B_t must satisfy the following condition

$$B_t = E_t\left[\frac{B_{t+1}}{(1+R)}\right] \tag{2.4}$$

and is referred to as the speculative rational bubble.[4] Equation (2.4) highlights that the bubble component B_t has to grow in expectations exactly at a rate equal to R.

Financial literature provides a number of different specifications of rational bubbles that satisfy equation (2.4). For example Blanchard & Watson (1983) defined a bubble that persists in each period only with probability π and bursts with probability $(1-\pi)$. If the bubble persists, it has to grow in expectation with a rate $(1+R)/\pi$. This faster bubble growth rate that is conditional on not bursting, is necessary to achieve the expected growth rate of R. Later, Froot & Obstfeld (1992) introduced the concept of "intrinsic" bubbles. This type of a bubble depends deterministically on aggregate dividends that are assumed to follow a stochastic process. Froot & Obstfeld (1992) argued that the explanatory potential of intrinsic bubbles lies in their ability to model very persistent deviations in asset prices and to explain overreactions to changes in fundamentals.

The condition that any rational bubble has to grow at an expected rate of R eliminates some cases, in which a rational bubble cannot exist. For instance, a positive bubble cannot emerge if its size has an upper limit. Therefore, the bubble cannot exist in the prices of commodities with close substitutes.

[4]For the proof showing why this condition must hold (for the case of the logarithmic approximation of equation (2.2)), refer to section 3.1.

The commodity with a bubble would become so expensive that it would be substituted with some other good. Also, under the free disposal assumption, a negative bubble on an asset cannot arise since the bubble would imply that the asset price has to become negative in expectation at some point of time. However, this observation together with equation (2.4) implies that once the rational bubble bursts, it cannot re-emerge. Therefore, rational bubbles can never emerge within the asset pricing model. They must be already present when the asset starts trading.

This feature of rational bubbles was criticized e.g. in Weil (1990), who argues, on theoretical grounds, that it is possible for assets to be undervalued. Weil (1990) introduces a bubble that may lead to an increase in interest rates. This increase is reflected in the discount rate used in the asset pricing model and thus decreases the fundamental price of an asset. Hence, a positive rational bubble may in fact decrease the overall price of an asset.

Another approach on how to allow for price decreasing bubbles is to use a logarithmic approximation of the asset pricing formula (2.1). Under the logarithmic specification, the logarithmic bubble component can become negative at any point of time t because it can never result in a negative price of the asset. Moreover, a bubble defined in this manner can oscillate from negative to positive continuously. This model specification was used e.g. in Wu (1995), Wu (1997), Kizys & Pierdzioch (2011) or Kim & Min (2011) and will be used in the empirical part of this thesis. The logarithmic approximation of the asset pricing formula (2.1) is described in detail in section 3.1.

Asymmetric Information Rational Bubbles

As the name suggests, this type of a bubble occurs in settings in which investors have different information, but all the market players are still rational. Unlike in the symmetric information case, it does not need to be commonly known whether the bubble is present or not. However, even if everyone knows that a certain asset price exceeds its fundamental value, an asymmetric information bubble can still occur. It is sufficient when some agents do not know that all the other agents/investors also know this fact.

Allen *et al.* (1993) state several necessary conditions for finite bubbles of this type to occur. First, the prices cannot be fully revealing. Secondly, the short selling must be constrained under at least some circumstances in some future periods in order for finite bubbles to persist. Lastly, it cannot be commonly known that the initial allocation is Pareto efficient because then no trades would take place.

An example of such an environment is studied by Allen & Gorton (1993) who introduced the concept of churning bubbles. Fund managers invest on behalf of their clients. Their trades are not motivated by the news about the fundamental value of an asset but rather by the desire to seize a portion of profit at the expense of their clients. As a result, assets can trade at prices which do not reflect their fundamental value and bubbles can exist. Furthermore, fund managers with limited liability might trade bubble assets since they only participate on the potential upside of a trade, not on the downside risk.

Bubbles Due to Limited Arbitrage

This type of a bubble can arise when rational investors interact with behavioral investors, who can be influenced by psychological biases. Under the efficient market hypothesis, rational arbitrageurs should undo any mispricing created by the non-rational investors. However, the literature concerning limits to arbitrage lists three types of risks preventing rational arbitrageurs from fully correcting the price misalignment.

First, shorting a bubble asset might be risky since a potential future shift in fundamentals may reverse the initial overpricing. Therefore, risk aversion limits the reaction of rational arbitrageurs to overpricing. This type of risk is known as the *fundamental risk*.

Second, rational traders face *noise trader risk* (De Long *et al.* (1990)). Short selling a bubble asset may be risky even if there is no fundamental risk. Noise traders may temporarily deviate the asset price even further from its fundamental value. Rational traders with short horizons are then less aggressive in correcting the price misalignment because they also care about short-term price changes. This is especially the case of fund managers who have to deal with outflow of funds when the portfolio under their management suffers from short-term losses. This can force them to unwind their positions exactly when the mispricing is the largest (Shleifer & Vishny (1995)).

The third type of risk is the *synchronization risk* introduced by Abreu & Brunnermeier (2003). A single trader cannot typically burst the bubble alone, a coordination among traders is required. Each rational trader faces the following trade-off: If they attack the bubble too early, they prevent themselves from realizing a higher potential profit. If they start short-selling too late, the mispricing may no longer be present. Each trader is trying to forecast when other rational traders start selling against the bubble. This is difficult because traders become sequentially aware of the bubble and they do not know what position in the queue they have. This lack of knowledge prevents rational investors from forecasting when the bubble will burst. Therefore, they cannot start the backward induction that would lead to bursting the bubble immediately. As such, even finite horizon bubbles can persist. Unlike the other limits to arbitrage models, the model of Abreu & Brunnermeier (2003) assumes that the traders prefer riding the bubble to attacking it. Brunnermeier & Nagel (2004) argue that there is supportive evidence in favor of this type of a bubble. They provide an example of technology stocks between 1998 and 2000. Hedge funds invested largely in these stocks, driving the prices even higher, even though hedge funds are among the most sophisticated investors and, according to the efficient market hypothesis, should be the price-correcting force.

Heterogeneous Beliefs Bubbles

Bubbles can also emerge when investors have heterogeneous beliefs and face short-sale constraints. This can lead to overpricing as pessimists cannot offset the price increasing pressures created by the optimists (Miller (1977)).

2.2 Empirical Testing for Asset Price Bubbles

As the empirical part of this thesis investigates rational bubbles in stock prices, this section will focus on reviews of the empirical research related mainly to them. An empirical survey of an alternative strand of models of 'irrational' bubbles can be found in Vissing-Jorgensen (2004).

Existing empirical literature is by no means unanimous as to the existence of rational bubbles in asset prices. Gürkaynak (2005) states that *"For each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble."* He argues that rejections of the simple present value model, that are interpreted by some as evidence of bubbles, can result from adopting oversimplifying assumptions about the fundamentals.

Most of the tests surveyed below reject the standard simple present-value model. Although the rejection of the null hypothesis is also consistent with other specifications not including the bubble, these tests provide enough evidence that the simple present-value model is inconsistent with the data. This motivates further research about this phenomena.

2.2.1 Variance Bounds Tests

The first tests for rational bubbles are the variance bound tests introduced by Shiller (1981) and LeRoy & Porter (1981). These tests were not originally designed for testing the presence of rational bubbles but were later interpreted in this fashion. The logic of these tests lies in imposing an upper limit on the variance of stock prices as it would be if the prices satisfied the standard present-value model. To demonstrate the principle of these tests, for the sake of simplicity consider Shiller's approach.[5]

The null hypothesis of the test is that the stock prices satisfy the standard present-value equation:

$$P_t = \sum_{i=1}^{\infty} \frac{E_t\left[D_{t+i}\right]}{(1+R)^i}. \tag{2.5}$$

The ex-post rational price can be expressed as the present value of the actual realized dividends:

$$P_t^* = \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+R)^i}. \tag{2.6}$$

Under rational expectations, the difference between the actual price and the ex-post rational price is an unforecastable zero-mean variable. Denoting this difference ε_t, the ex-post rational price at time t can be expressed as:

$$P_t^* = \sum_{i=1}^{\infty} \frac{E_t\left[D_{t+i}\right] + \varepsilon_i}{(1+R)^i} = P_t + \sum_{i=1}^{\infty} \frac{\varepsilon_i}{(1+R)^i}. \tag{2.7}$$

Variance of the ex-post rational price series would then be an upper bound of the variance of the observed price series:

$$Var(P_t^*) = Var(P_t) + \frac{\frac{1}{(1+R)^2}}{1 - \frac{1}{(1+R)^2}} Var(\varepsilon_t) \geq Var(P_t). \tag{2.8}$$

In practice, the problem of these tests is that the ex-post rational price is never observed as the infinite sum of the realized discounted dividends. Instead, it has to be approximated by assuming a terminal value of the ex-post rational price. Shiller (1981) used the sample average of a detrended real price as the

[5] Shiller's test only generates point estimates of variances, whereas LeRoy and Porter construct estimates of variances including their standard errors, so statistical significance can be tested.

approximated terminal value to show that actual price volatility exceeds the variance bound. He used his result as a mere critique of the present value model without attributing it to the presence of bubbles. However, other authors, for example, Tirole (1985) and Blanchard & Watson (1983), suggested that Shiller's results may be due to the presence of bubbles.

Shiller's approach earned criticism especially for using the mean price as the terminal ex-post rational price and for not using a sufficiently large sample (see e.g. Flavin (1983)). Other authors (e.g. Kleidon (1986)) showed that Shiller's tests would reject the null hypothesis for the data constructed from the net present value model if non-stationary time series are used.

Later, Campbell & Shiller (1988a) and Campbell & Shiller (1988b) derive a log-linear approximation of the dividend-price ratio and estimate a VAR system, allowing for time-varying discount rates. They find that even after relaxing the constant rate of return assumptions, there is still a substantial unexplained variance in the dividend-price ratio. Campbell and Shiller, however, do not make any conclusion about the presence of bubbles.

Cochrane (1992), on the other hand, looked for a discount rate process that would explain the observed volatility in the dividend-price ratio without incorporating the bubble. He found a process that satisfies the conditions imposed by the simple present-value model and at the same time fits the data. Therefore, he concluded that bubbles are not required to explain the volatility in the dividend-price ratio.

2.2.2 West's Two-Step Tests

West (1987) proposes a test that explicitly states a bubble in the alternative hypothesis. In the first step, West estimates the discount rate from the observed prices and dividends by exploiting equation (2.1) in the following way:

$$P_t = \frac{1}{1+R}\left(P_{t+1} + D_{t+1}\right) + u_t, \tag{2.9}$$

where u_t is $\frac{1}{1+R}\left(P_{t+1} + D_{t+1} - E_t\left[P_{t+1} + D_{t+1}\right]\right)$. The estimation is done using an instrumental variables regression (with D_t being the instrument) and is independent of the presence of a rational bubble.

The second step is characterizing the dividend process. West performs the test under both assumptions that dividends are stationary and non-stationary and estimates an AR(p) process for the dividends. To see the logic of West's test, assume that dividends are stationary and follow the following AR(1) process:[6]

$$D_t = \alpha D_{t-1} + \varepsilon_t, \tag{2.10}$$

where ε_t is the *iid* noise term. Estimating α, one can express the expected discounted value of future dividends as $V_t = \frac{\alpha}{1+R-\alpha} D_t$. Hence, under the null hypothesis $P_t = V_t$, by regressing P_t on D_t, one gets another estimate of $\frac{\alpha}{1+R-\alpha}$. In the final step, West uses a Hausman-specification test and strongly rejects the equality between the two estimates on the U.S. stock market data.

The two obvious issues with West's tests are that the dividends do not have to follow an AR(p) process and the discount rate may be time varying. When he allows for time-varying discount rates, his tests do not reject the null. West's methodology was also criticized in Dezhbakhsh & Demirguc-Kunt (1990) due to the usage of a Hausman specification test. The authors argue that the test tends to reject the null too often in small samples. They propose different tests with better small sample properties and find no evidence of bubbles.

2.2.3 Integration & Co-Integration Based Tests

Diba & Grossman (1988) note the fact that the rational bubble has to grow in expectations and cannot pop and restart. This makes the bubble process non-stationary regardless of how many differences are taken. A natural way of testing for bubbles is then checking whether the stock price is more explosive than the dividend process. Diba and Grossman test this hypothesis using unit root and co-integration tests and conclude that the null hypothesis cannot be rejected. The approach of Diba and Grossman was challenged by Evans (1991) who introduced the concept of periodically collapsing bubbles. His example of a periodically collapsing bubble is a process that never pops to zero but the bubble can collapse to a small non-zero value and then continue increasing. A process defined in this way still satisfies equation (2.4). Evans shows that with the increasing probability of a collapse of the bubble, the tests proposed by Diba and Grossman fail to detect bubbles. Therefore, Evans concludes that failing to reject the no-bubbles hypothesis with the integration/co-integration based tests is not proof that bubbles are not present in the data.

[6] In reality, dividends are often non-stationary, but the principle of the test is the same.

2.2.4 Intrinsic Bubbles

Froot & Obstfeld (1992) suggested a bubble that is determined by the level of dividends. Existence of such a bubble would make stock prices more sensitive to dividend innovations and could explain the excess volatility of stock prices compared to the dividends. They check for bubbles by exploiting the different behavior of the price-dividend ratio in the absence and presence of bubbles. Their results signal the presence of an intrinsic bubble in the stock-market data. However, the authors conclude that it may also merely show that the assumption about the simple present-value model is incorrect.

2.2.5 Bubbles as an Unobserved Variable

The econometric tests for bubbles discussed above assume very little about the structure of the bubble process. Many of them are tests of the simple present-value model against an unspecified alternative, with a bubble being only one of all possible alternatives. These tests do not produce explicit estimates of the bubble time series.

However, there is a strand of literature aiming to estimate the bubble component in asset prices explicitly. Burmeister & Wall (1982), testing rational expectations in price levels during the German hyperinflation, employed a Kalman filtering algorithm in order to estimate deviations from rational expectations model. Later, Wu (1995) used the Kalman filtering technique to estimate and test for stochastic bubbles in the exchange rate markets. Finally, applying Kalman filtering to the log-approximated model for stock prices introduced by Campbell & Shiller (1988b), Wu (1997) estimates and tests for stochastic bubbles in the U.S. stock market.

Wu (1997) treats the bubble as a deviation from the simple present-value model, which can be estimated as an unobserved variable through Kalman filtering. This method produces both point estimates of the bubble component and estimates of the corresponding standard errors. Therefore, the method can be used for testing in which of the periods the bubble was statistically significant in the data as well as it produces the time series of the estimated bubble component.

Using data from 1871 to 1991, Wu finds that the estimated bubble component accounts for a substantial portion of the U.S. stock prices. However, comparing the point estimates with their standard errors, he only finds a significant (positive) bubble during 1960s. Finally, Wu compares the errors of the price predictions generated by his model to the ones obtained through the simple present-value model and the intrinsic bubbles model introduced by Froot & Obstfeld (1992). Using the in-sample root mean square error and the mean absolute error, he concludes that his model does a considerably better job in fitting the data than both of the alternatively specified models.

Wu's methodology was used by Bhar & Hamori (2005) to study linkages between bubbles in global stock markets or by Kizys & Pierdzioch (2011) to study international spillovers of speculative bubbles in the CEE countries during the 2008 financial crisis. Kim & Min (2011) use Wu's approach to study housing price bubbles in Korea.

Al-Anaswah & Wilfling (2011) and Lammerding *et al.* (2012) amend Wu's model by incorporating Markov-switching into the assumed bubble process to detect speculative bubbles in international stock and crude oil prices, respectively. Robust evidence for the existence of speculative bubbles in stock prices as well as recent oil price dynamics is found.

Wu's methodology suffers from similar pitfalls as the tests for bubbles mentioned above. The methodology assumes that the discount rate is constant, the dividends follow an ARIMA$(h, 1, 0)$ process and the bubble component follows an ARIMA$(1, 1, 0)$ process. This is quite restrictive, yet it is in line with other tests for rational asset price bubbles.

Acknowledging the above mentioned issues, Wu's methodology is used in this thesis due to its ability to generate explicit estimates of the time series of the bubble component. This enables further investigation of the estimated bubble component. However, the results presented in this thesis are conditional upon validity of all the assumptions of the model. The author admits that there are alternative explanations of the dynamics of the stock prices.

The next chapter describes the methodology used in this thesis and provides a detailed derivation of the model used for estimating the bubble component.

Chapter 3

Model Specification and Estimation Strategy

This chapter first describes the model that is used for estimating the bubble component of stock prices (section 3.1). Secondly, section 3.2 presents the model for investigating the relation between the estimated bubble and the monetary policy.

3.1 Step 1 - Estimating the Bubble

This section combines different literature sources and the author's own calculations in order to provide a detailed step-by-step derivation and justification of the bubble estimation strategy, which, to the author's best knowledge, has not been published in a comprehensive manner before.

The model specification follows Wu (1997). However, the specification used in Wu (1997) is adjusted so that it reflects the standard timing conventions used in the current finance literature.[1]

In line with the rational bubbles theory, the model assumes that the bubble grows at the discount rate. However, the approximated logarithmic specifica-

[1] Wu (1997) starts from the following formula:

$$R = \frac{E_t\left[P_{t+1} + D_t\right]}{P_t} - 1,$$

where R is the constant required rate of return, E_t is the mathematical expectation conditional on information available at time t, P_t is the price of the stock at time t and D_t is the dividend paid at itme t. This model specification implies that dividends paid at time t are discounted into the present value (also at time t) by a discount rate $R > 0$. This is contra-intuitive and this thesis follows the current finance literature and uses D_{t+1} instead of D_t.

tion of the model is used to allow for a negative bubble component. Under this specification, the bubble can restart and collapse continuously and oscillate from positive to negative. The fact that the model is expressed in natural logarithms ensures that the bubble component, with possibly negative values, does not contradict the theory presented in chapter 2.

3.1.1 Log-Linear Approximation of the Standard Present-Value Model

The starting point is the standard definition of the return on a stock given by the following formula:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1, \tag{3.1}$$

or equivalently

$$P_t = \frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}}. \tag{3.2}$$

Here R_{t+1} denotes the return on the stock held from time t to time $t+1$. P_t stands for the price of the stock at the end of period t, or equivalently an ex-dividend price. Owning the stock from time t to time t+1 gives one a claim to next period's dividend D_{t+1} but not to this period's dividend D_t.

The specification in levels given by equation (3.1) has two obvious drawbacks for econometric modeling. First, this specification gives a positive probability to a negative price of a stock in the future if returns are assumed to be stochastic. Second, the specification is not linear and therefore not easily estimable by standard econometric methods. In order to tackle the two problems raised above, the log-linear approximation of equation (3.1), as suggested by Campbell & Shiller (1988b), is introduced. The log-linear approximation starts with the definition of the log return on stock, r_{t+1}:

$$r_{t+1} \equiv \log\left(1 + R_t\right). \tag{3.3}$$

Using (3.1), (3.3), and the convention that logs of variables are denoted by lowercase letters, we have

$$\begin{aligned} r_{t+1} &= \log\left(P_{t+1} + D_{t+1}\right) - \log\left(P_t\right) \\ &= p_{t+1} - p_t + \log\left(1 + \exp\left(d_{t+1} - p_{t+1}\right)\right). \end{aligned} \tag{3.4}$$

The last term on the right-hand side of (3.4) is a non-linear function of the log dividend-price ratio, $f\left(d_{t+1}-p_{t+1}\right)$, which can be approximated around its mean using a first-order Taylor expansion:

$$f\left(x_{t+1}\right) \approx f\left(\bar{x}\right)+f'\left(\bar{x}\right)\left(x_{t+1}-\bar{x}\right). \tag{3.5}$$

By substituting (3.4) into (3.5), we obtain

$$r_{t+1} \approx \xi_{t+1} \equiv \kappa+\rho p_{t+1}+(1-\rho) d_{t+1}-p_t, \tag{3.6}$$

where ξ_{t+1} is the approximated return and ρ and κ are constant terms defined by

$$\rho \equiv 1/\left(1+\exp\left(\overline{d-p}\right)\right), \tag{3.7}$$

where $\overline{d-p}$ is the average log dividend-price ratio, and

$$\kappa \equiv -\log(\rho)-(1-\rho)\log(1/\rho-1). \tag{3.8}$$

Emprically, in the U.S. data the log stock price gets a weight of ρ close to but below one, while the log dividend gets a weight of $1-\rho$ close to zero (Campbell *et al.* (1997)). This is intuitive because the dividend is, on average, much smaller than the stock price. Hence, a proportional change in the dividend has a much smaller effect than the same proportional change in the stock price.

Obviously, the approximation holds exactly if the log dividend-price ratio is constant, because then the last term on the right-hand side of (3.4) is also constant. If the variation in the log dividend-price ratio is small, then the approximation (3.6) will be accurate. Campbell *et al.* (1997) and Campbell & Shiller (1988b) evaluated the accuracy of the approximation (3.6) both theoretically and empirically on the U.S. data. They concluded that the approximation is very precise in terms of capturing the dynamics of stock prices but misstates the mean stock return. However, the error created by neglecting the higher-order terms in the Taylor expansion of equation (3.4) is small and almost constant. The constant approximation error does not affect any results presented in this thesis, since no restrictions on the means of the data are tested.

In order to proceed further, it is possible to express p_t from equation (3.6) to obtain:

$$p_t=\kappa+\rho p_{t+1}+(1-\rho) d_{t+1}-\xi_{t+1}. \tag{3.9}$$

Equation (3.9) holds ex post, but it also holds in expectations. Taking expectations and noting that $E_t[p_t] = p_t$ because p_t is known at time t, one obtains:

$$p_t = \kappa + \rho E_t[p_{t+1}] + (1-\rho)E_t[d_{t+1}] - E_t[\xi_{t+1}]. \tag{3.10}$$

Assuming that the expected rate of return ξ_t is constant and equal to the required rate of return ξ, we obtain the following approximation of the standard linear rational expectations model for stock price determination:

$$p_t = \kappa + \rho E_t[p_{t+1}] + (1-\rho)E_t[d_{t+1}] - \xi. \tag{3.11}$$

Equation (3.11)[2] is often used as a starting point of many researchers studying rational asset price bubbles. See e.g. Balke & Wohar (2009), Al-Anaswah & Wilfling (2011), or Kim & Min (2011).

3.1.2 Intuition behind the Log-Linear Approximation

Above, the approximation (3.11) was justified rigorously using a first-order Taylor expansion of the equation (3.4) and the assumption of the constant required rate of return. However, it is also possible to show intuitively why the approximation is reasonable. The intuition is based on adopting two simplifying assumptions:

- Assumption about the constant dividend-price ratio
- Assumption about the constant dividend growth.

That is,

$$d_t - d_{t-1} = \Delta d_t \equiv g \tag{3.12}$$

and

$$d_t - p_t \equiv \delta. \tag{3.13}$$

Equations (3.12) and (3.13) imply that the stock price grows at the same constant rate as the dividends. Moreover, the ratio of the stock price to the sum of the stock price and the dividend is also constant and equal to ρ defined by (3.7).[3]

[2] with different timing of dividends as discussed above

[3] To see this, consider:

$$\begin{aligned} p_{t+1} - p_t &= p_{t+1} - p_t + d_t - d_t = p_{t+1} + \delta - d_t \\ &= p_{t+1} + d_{t+1} - d_{t+1} + \delta - d_t = -\delta + \delta + g = g \end{aligned}$$

Under these assumptions, the log stock return r_{t+1} defined by (3.3) is equal to ξ_{t+1}, defined by (3.6), and is also constant. To show this, consider an alternative representation of equivalence (3.6):

$$\begin{aligned} \xi_{t+1} &= \kappa + \rho\left(p_{t+1} - d_{t+1}\right) + d_{t+1} - p_t \\ &= \kappa - \rho\delta + d_t - d_{t-1} - p_t + d_{t-1} \\ &= \kappa + (1-\rho)\,\delta + g \end{aligned} \tag{3.14}$$

Equation (3.14) implies that under the assumptions given by (3.12) and (3.13), $\xi_{t+1} \equiv \xi$ is constant. To further show that ξ is equal to r_{t+1}, note that (3.7) and (3.13) imply:

$$\delta = \log\left(1/\rho - 1\right). \tag{3.15}$$

Substituting (3.15) into the definition of κ (3.8), we have:

$$\kappa = -\log\left(\rho\right) - \left(1-\rho\right)\delta. \tag{3.16}$$

Substituting (3.16) into (3.14), we obtain:

$$\begin{aligned} \xi &= -\log\left(\rho\right) + g \\ &= \log\left(\frac{P_{t+1} + D_{t+1}}{P_{t+1}}\right) + \log\left(\frac{P_{t+1}}{P_t}\right) \\ &= \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \\ &= r_{t+1}, \end{aligned} \tag{3.17}$$

where the last equivalence follows from definitions (3.1) and (3.3). This shows that under assumptions (3.12) and (3.13), ξ is constant and exactly equal to r_{t+1}. Replicating steps in equation (3.10), we again arrive at equation (3.11).

3.1.3 Solution to the Present-Value Model

Solving equation (3.11) for p_t by forward iteration and using the law of iterated expectations $E_t\left[E_{t+m}\left[d_{t+m+n}\right]\right] = E_t\left[d_{t+m+n}\right]$ yields the following:

---

and

$$\frac{P_t}{P_t + D_t} = \frac{1}{1 + \frac{D_t}{P_t}} = \frac{1}{1 + \exp\left(\delta\right)} = \rho,$$

where the last equivalence follows from the definition of ρ (3.7).

$$\begin{aligned}
p_t &= \kappa - \xi + \rho E_t\left[p_{t+1}\right] + (1-\rho)\, E_t\left[d_{t+1}\right] \\
&= \kappa - \xi + \rho E_t\left[\{\kappa - \xi + \rho E_{t+1}\left[p_{t+2}\right] + (1-\rho)\, E_{t+1}\left[d_{t+2}\right]\}\right] + (1-\rho)\, E_t\left[d_{t+1}\right] \\
&= \kappa - \xi + \rho\left\{\kappa - \xi + \rho E_t\left[p_{t+2}\right] + (1-\rho)\, E_t\left[d_{t+2}\right]\right\} + (1-\rho)\, E_t\left[d_{t+1}\right] \\
&= ... \\
&= \frac{(\kappa-\xi)\left(1-\rho^i\right)}{1-\rho} + \rho^i E_t\left[p_{t+i}\right] + (1-\rho)\sum_{j=0}^{i-1}\rho^j E_t\left[d_{t+j+1}\right], \quad \text{for} \quad i = 1, 2, ...
\end{aligned} \tag{3.18}$$

Now, let $i \to \infty$. If the transversality condition $\lim_{i\to\infty}\rho^i E_t\left[p_{t+i}\right] = 0$ holds, we would have the unique no-bubble solution for the stock price which is referred to as the market-fundamental solution:

$$p_t = p_t^f = \frac{(\kappa-\xi)}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\rho^j E_t\left[d_{t+j+1}\right]. \tag{3.19}$$

However, if the transversality condition is violated, then equation (3.19) is only a particular solution to equation (3.11). The general solution to equation (3.11) then takes the following form:

$$p_t = \frac{\kappa-\xi}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\left(\rho^j E_t\left[d_{t+j+1}\right]\right) + b_t = p_t^f + b_t, \tag{3.20}$$

where b_t is referred to as a rational speculative bubble. In order for equation (3.20) to be a solution to equation (3.11), b_t must satisfy the following equation:

$$E_t[b_{t+i}] = \frac{1}{\rho^i} b_t, \quad \text{for} \quad i = 1, 2, ... \tag{3.21}$$

To show why b_t must satisfy equation (3.21), consider expressing equation (3.20) for p_{t+1} instead of p_t and then substituting it into equation (3.11). We obtain

$$p_t = \kappa - \xi + \rho E_t\left[\frac{\kappa-\xi}{1-\rho} + (1-\rho)\sum_{j=0}^{\infty}\left(\rho^j E_{t+1}\left[d_{t+j+2}\right]\right) + b_{t+1}\right] + (1-\rho)\, E_t\left[d_{t+1}\right]. \tag{3.22}$$

Using the the law of iterated expectations $E_t\left[E_{t+1}\left[d_{t+j+2}\right]\right] = E_t\left[d_{t+j+2}\right]$, we get

$$\begin{aligned} p_t &= \kappa - \xi + \rho \left[\frac{\kappa - \xi}{1-\rho} + (1-\rho) \sum_{j=0}^{\infty} \left(\rho^j E_t \left[d_{t+j+2} \right] \right) + E_t \left[b_{t+1} \right] \right] + (1-\rho) E_t \left[d_{t+1} \right] \\ &= \sum_{j=0}^{\infty} \rho^j (\kappa - \xi) + (1-\rho) \sum_{j=0}^{\infty} \left(\rho^j E_t \left[d_{t+j+1} \right] \right) + \rho E_t \left[b_{t+1} \right] \\ &= \frac{\kappa - \xi}{1-\rho} + (1-\rho) \sum_{j=0}^{\infty} \left(\rho^j E_t \left[d_{t+j+1} \right] \right) + \rho E_t \left[b_{t+1} \right] \\ &= p_t^f + \rho E_t \left[b_{t+1} \right]. \end{aligned} \tag{3.23}$$

However, this is a contradiction since (3.23) and (3.20) cannot, in general, both be solutions to (3.20). These two solutions are equivalent only if

$$E_t[b_{t+1}] = \frac{1}{\rho} b_t, \tag{3.24}$$

which solving by forward iteration for $i = 1, 2, ...$ yields exactly equation (3.21).

To be able to estimate the model using standard econometric methods, this thesis follows Wu (1997) and assumes that the bubble process $\{b_t\}$ is linear and can be described by the following equation:

$$b_t = \frac{1}{\rho} b_{t-1} + \eta_t, \tag{3.25}$$

where the innovation term η_t is assumed to be serially uncorrelated with zero mean and a constant variance equal to σ_η^2.

Since the log stock prices and dividends usually appear not to be stationary[4], the model can be specified in its difference form. Taking the first difference of equation (3.20) yields:

$$\Delta p_t = (1-\rho) \sum_{j=0}^{\infty} \rho^j \left(E_t \left[d_{t+j+1} \right] - E_{t-1} \left[d_{t+j} \right] \right) + \Delta b_t = \Delta p_t^f + \Delta b_t. \tag{3.26}$$

In order to obtain a parsimonious specification of the model, this thesis follows Wu (1997) and assumes that the log dividend process entering equation (3.26) contains a unit root and can be approximated by an ARIMA$(h, 1, 0)$ process as follows:

[4]This assumption will be tested in the data.

$$\Delta d_t = \mu + \sum_{j=1}^{h} \phi_j \Delta d_{t-j} + \varepsilon_t, \tag{3.27}$$

where ε_t is assumed to be serially uncorrelated with zero mean and a constant variance equal to σ_ε^2. The autoregressive order h in (3.27) is to be determined from the data using information criteria.[5] Additionally, the dividend innovation term ε_t is assumed to be uncorrelated with the bubble innovation term η_τ for all t and τ.

3.1.4 Companion Form of the Model

In what follows, it will be convenient to express the model in its companion form. Defining the $(h \times 1)$ vectors

$$\boldsymbol{Y_t} = (\Delta d_t, \Delta d_{t-1}, ..., \Delta d_{t-h+1})', \;\; \boldsymbol{u} = (\mu, 0, ..., 0)', \quad \boldsymbol{\nu_t} = (\varepsilon, 0, ..., 0),$$

and the $(h \times h)$ matrix

$$\boldsymbol{A} = \begin{pmatrix} \phi_1 & \phi_2 & ... & \phi_{h-1} & \phi_h \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & 0 \end{pmatrix},$$

we may express equation (3.27) in the following form:

$$\boldsymbol{Y_t} = \boldsymbol{u} + \boldsymbol{A}\boldsymbol{Y_{t-1}} + \boldsymbol{\nu_t}. \tag{3.28}$$

Furthermore, defining the $(1 \times h)$ vector

$$\boldsymbol{g} = (1, 0, ..., 0)$$

and using the definitions of $\boldsymbol{Y_t}$, $\boldsymbol{u}$, and $\boldsymbol{\nu_t}$, we may write:

$$d_t = \Delta d_t + d_{t-1} = \boldsymbol{g}\boldsymbol{Y_t} + d_{t-1},$$

$$d_{t+j+1} = \boldsymbol{g}\boldsymbol{Y_{t+j+1}} + d_{t+j} = \boldsymbol{g}\sum_{k=1}^{j+1} (\boldsymbol{Y_{t+k}}) + d_t,$$

[5] Methodology described in appendix A.

$$E_t\left[d_{t+j+1}\right] = \boldsymbol{g}\sum_{k=1}^{j+1}\left(E_t\left[\mathbf{Y_{t+k}}\right]\right) + d_t, \tag{3.29}$$

and

$$\begin{aligned} E_t\left[\mathbf{Y_{t+k}}\right] &= E_t\left[\boldsymbol{u} + \boldsymbol{A}\mathbf{Y_{t+k-1}} + \boldsymbol{\nu_{t+k}}\right] \\ &= \boldsymbol{u} + \boldsymbol{A}E_t\left[\mathbf{Y_{t+k-1}}\right] \\ &= \ldots \\ &= \boldsymbol{u}\sum_{l=1}^{k}\left(\boldsymbol{A}^l\right) + \boldsymbol{A}^k\mathbf{Y_t}. \end{aligned} \tag{3.30}$$

Similarly,

$$E_{t-1}\left[d_{t+j}\right] = \boldsymbol{g}\sum_{k=1}^{j+1}\left(E_{t-1}\left[\mathbf{Y_{t+k-1}}\right]\right) + d_{t-1}, \tag{3.31}$$

and

$$E_{t-1}\left[\mathbf{Y_{t+k-1}}\right] = \boldsymbol{u}\sum_{l=1}^{k}\left(\boldsymbol{A}^l\right) + \boldsymbol{A}^k\mathbf{Y_{t-1}}. \tag{3.32}$$

Now it is possible to plug the right-hand sides of equations (3.30) and (3.32) into equations (3.29) and (3.31),respectively, and express the following:

$$\begin{aligned} E_t\left[d_{t+j+1}\right] - E_{t-1}\left[d_{t+j}\right] &= \boldsymbol{g}\left\{\sum_{k=1}^{j+1}\left(E_t\left[\mathbf{Y_{t+k}}\right]\right) - \sum_{k=1}^{j+1}\left(E_{t-1}\left[\mathbf{Y_{t+k-1}}\right]\right)\right\} + \Delta d_t \\ &= \boldsymbol{g}\left\{\sum_{k=1}^{j+1}\left[\boldsymbol{u}\sum_{l=1}^{k}\left(\boldsymbol{A}^l\right) + \boldsymbol{A}^k\mathbf{Y_t} - \boldsymbol{u}\sum_{l=1}^{k}\left(\boldsymbol{A}^l\right) - \boldsymbol{A}^k\mathbf{Y_{t-1}}\right]\right\} + \Delta d_t \\ &= \boldsymbol{g}\left(\sum_{k=1}^{j+1}\boldsymbol{A}^k\Delta\mathbf{Y_t}\right) + \Delta d_t \\ &= \boldsymbol{g}\left(\boldsymbol{A}\sum_{k=0}^{j}\boldsymbol{A}^k\Delta\mathbf{Y_t}\right) + \Delta d_t \\ &= \boldsymbol{g}\boldsymbol{A}\left(\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\left(\boldsymbol{I} - \boldsymbol{A}^{j+1}\right)\Delta\mathbf{Y_t} + \Delta d_t, \end{aligned} \tag{3.33}$$

where $\boldsymbol{I}$ is the identity matrix of the dimension $(h \times h)$ and the last line of (3.33) holds, provided that $\left|\lambda_i^A\right| < 1$ for each eigenvalue λ_i^A of $\boldsymbol{A}$.[6]

If we now substitute equation (3.33) for the term inside of the summation in equation (3.26), we get:

[6] This condition is trivially satisfied by the definition of $\boldsymbol{A}$ and the fact that ϕ_1 to ϕ_h are all smaller than one in absolute values.

$$
\begin{aligned}
\Delta p_t &= (1-\rho)\sum_{j=0}^{\infty}\left\{\rho^j\left[\boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{A}^{j+1}\right)\Delta\boldsymbol{Y_t}+\Delta d_t\right]\right\}+\Delta b_t \\
&= \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}(1-\rho)\left(\sum_{j=0}^{\infty}\rho^j\boldsymbol{I}-\sum_{j=0}^{\infty}\rho^j\boldsymbol{A}^{j+1}\right)\Delta\boldsymbol{Y_t}+\frac{(1-\rho)\,\Delta d_t}{(1-\rho)}+\Delta b_t \\
&= \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}(1-\rho)\left[\left(\boldsymbol{I}-\rho\boldsymbol{I}\right)^{-1}-\boldsymbol{A}\left(\boldsymbol{I}-\rho\boldsymbol{A}\right)^{-1}\right]\Delta\boldsymbol{Y_t}+\Delta d_t+\Delta b_t \\
&= \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}(1-\rho)\left[\boldsymbol{I}\frac{1}{1-\rho}-\boldsymbol{A}\left(\boldsymbol{I}-\rho\boldsymbol{A}\right)^{-1}\right]\Delta\boldsymbol{Y_t}+\Delta d_t+\Delta b_t \\
&= \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}\left[\boldsymbol{I}-(1-\rho)\,\boldsymbol{A}\left(\boldsymbol{I}-\rho\boldsymbol{A}\right)^{-1}\right]\Delta\boldsymbol{Y_t}+\Delta d_t+\Delta b_t,
\end{aligned}
\tag{3.34}
$$

where the transition from the second to the third line works, provided that $\left|\lambda_i^{\rho A}\right| < 1$ for each eigenvalue $\lambda_i^{\rho A}$ of $\rho\boldsymbol{A}$, which follows from the fact that eigenvalues of $\boldsymbol{A}$ fulfill this condition. Eigenvalues of $\rho\boldsymbol{I}$ are smaller than 1 in absolute value trivially since $\rho < 1$. Defining a $(1 \times h)$ vector[7]

$$
\boldsymbol{M} = \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}\left[\boldsymbol{I}-(1-\rho)\,\boldsymbol{A}\left(\boldsymbol{I}-\rho\boldsymbol{A}\right)^{-1}\right], \tag{3.35}
$$

we can write the equation (3.34) in the following form:

$$
\Delta p_t = \Delta d_t + \boldsymbol{M}\Delta\boldsymbol{Y_t} + \Delta b_t. \tag{3.36}
$$

When estimating the stock-price equation (3.36), a difficulty arises because the bubble component is not observed. This fact suggests expressing the present-value model in a state-space form and using the Kalman filter to estimate the bubble component.

3.1.5 State-Space Representation

The present-value model consisting of the stock price equation (3.36), the parametric bubble process (3.25), and the dividend process (3.28) belongs to the class of dynamic linear models and can be expressed in a state-space form. The state-space form representation presented in this follows Bhar & Hamori (2005, p. 164 - 187).

Let $\boldsymbol{z_t}$ denote an $(l \times 1)$ vector of output variables observed at time t that

[7] This is where the model specification used in this thesis differs from the specification proposed by Wu (1997). Wu, using different timing conventions, arrives at a different definition of the vector $\boldsymbol{M}$. His definition of M is as follows: $\boldsymbol{M} = \boldsymbol{gA}\left(\boldsymbol{I}-\boldsymbol{A}\right)^{-1}\left[\boldsymbol{I}-(1-\rho)\left(\boldsymbol{I}-\rho\boldsymbol{A}\right)^{-1}\right]$

can be described in terms of a possibly unobserved $(n \times 1)$ state vector $\boldsymbol{s_t}$ and an $(m \times 1)$ vector of input variables $\boldsymbol{g_t}$. The general representation of a dynamic linear model can then be written as follows:

$$\boldsymbol{s_t} = \boldsymbol{c} + \boldsymbol{F}\boldsymbol{s_{t-1}} + \boldsymbol{v_t} \tag{3.37}$$

$$\boldsymbol{z_t} = \boldsymbol{H}\boldsymbol{s_t} + \boldsymbol{D}\boldsymbol{g_t} + \boldsymbol{w_t}, \tag{3.38}$$

where $\boldsymbol{F}$, $\boldsymbol{H}$ and $\boldsymbol{D}$ are the matrices of parameters of dimensions $(n \times n)$, $(l \times n)$, and $(l \times m)$ respectively, $\boldsymbol{c}$ is an $(n \times 1)$ vector of parameters, and v_t and w_t are, respectively, $(n \times 1)$ and $(l \times 1)$ vectors of disturbances. The disturbance terms, v_t and w_t, are assumed to be serially and mutually uncorrelated and homoskedastic:

$$E\left(\boldsymbol{v_t}\boldsymbol{v'_\tau}\right) = \begin{cases} \boldsymbol{\Omega_v} & \text{for } t = \tau \\ \boldsymbol{0} & \text{otherwise,} \end{cases}$$

$$E\left(\boldsymbol{w_t}\boldsymbol{w'_\tau}\right) = \begin{cases} \boldsymbol{\Omega_w} & \text{for } t = \tau \\ \boldsymbol{0} & \text{otherwise,} \end{cases}$$

$$E\left(\boldsymbol{v_t}\boldsymbol{w'_\tau}\right) = 0 \quad \text{for all } t \text{ and } \tau,$$

where $\boldsymbol{\Omega_v}$ and $\boldsymbol{\Omega_w}$ are $(n \times n)$ and $(l \times l)$ matrices, respectively.

Equation (3.37) is called the *state equation* and equation (3.38) is called the *measurement equation* of the dynamic system.

In order to express the model in the state-space form given by (3.37) and (3.38), it should be noted that equation (3.36) can be written in the following manner:

$$\Delta p_t = \Delta d_t + \begin{pmatrix} m_1 & m_2 & ... & m_h \end{pmatrix} \Delta \boldsymbol{Y_t} + \Delta b_t,$$

or

$$\Delta p_t = \begin{pmatrix} (1+m_1) & (m_2-m_1) & \ldots & (m_h-m_{h-1}) & -m_h & 1 & -1 \end{pmatrix} \begin{pmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \ldots \\ \Delta d_{t-h} \\ b_t \\ b_{t-1} \end{pmatrix}, \tag{3.39}$$

where m_i is the i^{th} component of vector $\boldsymbol{M}$, as defined in (3.35).

Now, the model can be expressed a the state-space form by adopting the following notations:

$$\begin{aligned}
\boldsymbol{s_t} &= (\Delta d_t,\ \Delta d_{t-1}, \ldots,\ \Delta d_{t-h},\ b_t,\ b_{t-1})',\\
\boldsymbol{z_t} &= (\Delta p_t,\ \Delta d_t)',\\
\boldsymbol{g_t} &= \boldsymbol{0},\\
\boldsymbol{v_t} &= (\varepsilon_t,\ 0, \ldots,\ 0,\ \eta_t,\ 0,)',\\
\boldsymbol{w_t} &= \boldsymbol{0},\\
\boldsymbol{c} &= (\mu,\ 0,\ \ldots,\ 0)',\\
\boldsymbol{F} &= \begin{pmatrix}
\phi_1 & \phi_2 & \ldots & \phi_{h-1} & \phi_h & 0 & 0 & 0\\
1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0\\
0 & 1 & \ldots & 0 & 0 & 0 & 0 & 0\\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\\
0 & 0 & \ldots & 0 & 1 & 0 & 0 & 0\\
0 & 0 & \ldots & 0 & 0 & 0 & \frac{1}{\rho} & 0\\
0 & 0 & \ldots & 0 & 0 & 0 & 1 & 0
\end{pmatrix},\\
\boldsymbol{H} &= \begin{pmatrix}
(1+m_1) & (m_2-m_1) & \ldots & (m_h-m_{h-1}) & -m_h & 1 & -1\\
1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},\\
\boldsymbol{D} &= \boldsymbol{0}.
\end{aligned}$$

Given the above notations and the assumptions imposed on ε_t and η_t, we have

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim iid \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \right], \tag{3.40}$$

$$\boldsymbol{\Omega_v} = \begin{pmatrix} \sigma_\varepsilon^2 & 0 & ... & 0 \\ 0 & ... & ... & 0 \\ ... & ... & ... & ... \\ 0 & ... & \sigma_\eta^2 & 0 \\ 0 & ... & ... & 0 \end{pmatrix}, \tag{3.41}$$

and

$$\boldsymbol{\Omega_w} = \boldsymbol{0}. \tag{3.42}$$

In the state-space representation given by equations (3.37) and (3.38), the bubble is treated as an unobserved state variable. There are $h+3$ state equations and two measurement equations. The first state equation represents the dividend process (3.28) and the second to last state equation represents the bubble process (3.25). The remaining $h+1$ state equations are mere identities, which can be seen as assigning first lags of already defined state variables to newly defined state variables and thus representing the dynamics of the system. The first measurement equation represents the price equation (3.36) and the second one is again an identity. The second measurement equation can be understood as a connection between the system of the state equations on one hand and the measurement price equation on the other hand (Δd_t on the right-and side of the identity can be seen as a state variable entering the system of measurement equations through the identity and being assigned to the measurement variable Δd_t on the left-hand side of the identity).

3.1.6 Kalman Filtering

Suppose that we have a general dynamic linear model as specified by equations (3.37) and (3.38). The estimation of unobserved variables contained in the model can be obtained recursively by the Kalman filter, as described in Bhar & Hamori (2005, p. 83 - 103). Given the model specification, the state vector $\boldsymbol{s_t}$ is not observed completely and must be estimated. Suppose for the moment that $\boldsymbol{c}$, $\boldsymbol{F}$, $\boldsymbol{H}$, $\boldsymbol{D}$, $\boldsymbol{\Omega_v}$, and $\boldsymbol{\Omega_w}$ are known. Let $\boldsymbol{\hat{s}_{t,\tau}}$ denote the best linear estimate of $\boldsymbol{s_t}$ given the model and all observed data up to time τ. Given initial conditions, $\boldsymbol{s_1}$ and $\boldsymbol{P_1}$, $\boldsymbol{\hat{s}_{t,\tau}}$ and its associated covariance matrix $\boldsymbol{\hat{P}_{t,\tau}}$ can be obtained recursively through solving the following equations:

$$\begin{aligned}\hat{\boldsymbol{s}}_{t+1,t} &= \boldsymbol{c} + \boldsymbol{F}\hat{\boldsymbol{s}}_{t,t},\\ \boldsymbol{P}_{t+1,t} &= \boldsymbol{F}\boldsymbol{P}_{t,t}\boldsymbol{F}' + \boldsymbol{\Omega}_{\boldsymbol{v}},\\ \boldsymbol{K}_{t+1} &= \boldsymbol{P}_{t+1,t}\boldsymbol{H}'\left[\boldsymbol{H}\boldsymbol{P}_{t+1,t}\boldsymbol{H}' + \boldsymbol{\Omega}_{\boldsymbol{w}}\right]^{-1},\\ \hat{\boldsymbol{s}}_{t+1,t+1} &= \hat{\boldsymbol{s}}_{t+1,t} + \boldsymbol{K}_{t+1}\left[\boldsymbol{z}_{t+1} - \boldsymbol{H}\hat{\boldsymbol{s}}_{t+1,t} - \boldsymbol{D}\boldsymbol{g}_{t+1}\right],\\ \boldsymbol{P}_{t+1,t+1} &= \left[\boldsymbol{I} - \boldsymbol{K}_{t+1}\boldsymbol{H}\right]\boldsymbol{P}_{t+1,t},\end{aligned} \tag{3.43}$$

where

$$\boldsymbol{P}_{t+1,t} = E\left[\left(\boldsymbol{s}_{t+1} - \hat{\boldsymbol{s}}_{t+1,t}\right)\left(\boldsymbol{s}_{t+1} - \hat{\boldsymbol{s}}_{t+1,t}\right)'\right]$$

and

$$\boldsymbol{P}_{t+1,t+1} = E\left[\left(\boldsymbol{s}_{t+1} - \hat{\boldsymbol{s}}_{t+1,t+1}\right)\left(\boldsymbol{s}_{t+1} - \hat{\boldsymbol{s}}_{t+1,t+1}\right)'\right]$$

are the error covariance matrices, and $1 \leq t \leq T$.

The system of equations (3.43) forms the Kalman filter and is computed recursively forward. In the case of the current model, the system (3.43) simplifies, as $\boldsymbol{g_t}$, $\boldsymbol{w_t}$, $\boldsymbol{D}$, and $\boldsymbol{\Omega_w}$ are assumed to equal $\mathbf{0}$.[8]

A more efficient estimate of the state vector and its error covariance matrix can be obtained by using all information up to time T through the following full-sample smoother:

$$\begin{aligned}\hat{\boldsymbol{s}}_{t,T} &= \hat{\boldsymbol{s}}_{t,t} + \boldsymbol{J}_t\left(\hat{\boldsymbol{s}}_{t+1,T} - \hat{\boldsymbol{s}}_{t+1,t}\right),\\ \boldsymbol{P}_{t,T} &= \boldsymbol{P}_{t,t} + \boldsymbol{J}_t\left(\boldsymbol{P}_{t+1,T} - \boldsymbol{P}_{t+1,t}\right)\boldsymbol{J}_t',\\ \boldsymbol{J}_t &= \boldsymbol{P}_{t,t}\boldsymbol{F}'\boldsymbol{P}_{t+1,t}^{-1}, \quad t = T-1, T-2, ..., 1.\end{aligned} \tag{3.44}$$

This smoother is run backwards recursively.

The model parameters $\boldsymbol{c}$, $\boldsymbol{F}$, $\boldsymbol{H}$, $\boldsymbol{D}$, $\boldsymbol{\Omega_v}$, and $\boldsymbol{\Omega_w}$ are estimated by maximum likelihood. Let $\boldsymbol{\beta}$ denote the parameter vector and $L\left(\boldsymbol{\beta} \mid \boldsymbol{g}, \boldsymbol{z}\right)$ be the log-likelihood function given observations on the input vector $\boldsymbol{g}$ and output vector $\boldsymbol{z}$. The log-likelihood function is constructed as follows:

[8] Note that due to the definition of ρ, not all the eigenvalues of $\boldsymbol{F}$ are expected to lie inside of the unit circle. However, as argued in Hamilton (1994, p. 378 - 379), the Kalman filter can still be calculated. The consequence of some eigenvalues of matrix F lying outside of the unit circle is a greater uncertainty about the true value of $\boldsymbol{s_1}$, resulting in higher standard errors of the estimates of $\boldsymbol{s_t}$

$$L(\boldsymbol{\beta} \mid \boldsymbol{g}, \boldsymbol{z}) = \frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\{\log[det(\boldsymbol{H}\boldsymbol{P_{t,t-1}}\boldsymbol{H}' + \boldsymbol{\Omega_w})]\} - \\ - \frac{1}{2}\sum_{t=1}^{T}\left[\boldsymbol{\hat{w}'_{t,t-1}}(\boldsymbol{H}\boldsymbol{P_{t,t-1}}\boldsymbol{H}' + \boldsymbol{\Omega_w})^{-1}\boldsymbol{\hat{w}_{t,t-1}}\right], \tag{3.45}$$

where the innovation, $\boldsymbol{\hat{w}_{t,t-1}}$, and the error covariance matrix, $\boldsymbol{P_{t,t-1}}$, are both implicit functions of the unknown parameter vector, $\boldsymbol{\beta}$, and are evaluated using the Kalman filter. Once the maximum likelihood estimate of $\boldsymbol{\beta}$ is obtained, the smoothed estimates of the state vector and its error covariances can be produced through the Kalman filter and the full-sample smoother.

The procedure described in this section directly produces an estimate of the time series b_t. Having estimated this component of the log stock price p_t, one can substitute it in equation (3.20) and also obtain estimates of the market-fundamentals component p_t^f. The two estimated components of log stock prices, p_t^f and b_t, can now be studied further.

3.2 Step 2 - Investigating the Relation Between Monetary Policy and the Estimated Bubble

The vector autoregression (VAR) model suggested by Sims (1980) is employed to study linkages between the estimated bubble and fundamental components of stock prices and the macroeconomic variables. The general specification of the structural p^{th} order VAR is :

$$\boldsymbol{B_0}\boldsymbol{y_t} = \boldsymbol{c_0} + \boldsymbol{B_1}\boldsymbol{y_{t-1}} + ... + \boldsymbol{B_p}\boldsymbol{y_{t-p}} + \boldsymbol{\varepsilon_t}, \tag{3.46}$$

where $\boldsymbol{y_t}$ is an $(m \times 1)$ vector of observations, $\boldsymbol{c_0}$ is an $(m \times 1)$ vector of constants, $\boldsymbol{B_i}$ is an $(m \times m)$ matrix of parameters (for every $i = 0, ..., p$), and $\boldsymbol{\varepsilon_t}$ is an $(m \times 1)$ vector of structural disturbances or shocks. The main diagonal terms of the B_0 matrix (the coefficients on the i^{th} variable in the i^{th} equation) are scaled to 1. Every disturbance term of $\boldsymbol{\varepsilon_t}$ has zero mean, is serially uncorrelated, and $var(\boldsymbol{\varepsilon_t}) = \boldsymbol{\Lambda}$. $\boldsymbol{\Lambda}$ is a diagonal matrix, where the diagonal elements are the variances of the structural disturbances.

Aiming to capture the complex full picture of the economy, various macroeconomic variables enter the model. Following Havránek *et al.* (2010), beside the estimated bubble and the fundamental stock price components, the vector of observations $\boldsymbol{y_t}$ consists of transformations of a measure of the economy output, a measure of the aggregate price level, and the short-term interest rate. Havránek *et al.* (2010) also included a measure of the exchange rate in the vector of observations, which is omitted in this thesis. The reason is that this thesis studies he U.S. data and the U.S. are perceived as a largely closed economy. In addition to the above mentioned variables, a measure of monetary base is included as an endogenous variable. The reason for considering the monetary base is that it captures the the quantitative easing, which has recently become an important tool of the FED's monetary policy. Finally, a transformation of the oil price is included as an exogenous variable. The oil price is believed to play a significant role in the U.S. economy (see e.g. Balke *et al.* (2002)) and in the context of this thesis, it is used as a proxy for exogenous shocks to the economy.

The estimated bubble and fundamental components of stock prices entering the VAR system are based on the log-approximated representation of the present-value model and therefore are in logarithms by design. In order to ensure consistency and stability of the VAR system, the variables entering the model are transformed in the following way:

- Logarithm of a measure of the economy output is filtered using the Hodrick-Prescott filter and the cyclical component is used.[9]
- A measure of the aggregate price level is transformed into a series of log differences of price levels, $\log(\textit{price level}_t) - \log(\textit{price level}_{t-1})$, which is a logarithmic approximation of a percentage inflation .
- The short-term interest rate is used without any transformation.
- A measure of monetary base is used in log differences, $\log(MB_t)-\log(MB_{t-1})$.
- The oil price is used in its log differences, $\log(\textit{oil price}_t)-\log(\textit{oil price}_{t-1})$.
- The estimated bubble and fundamental components of stock prices enter the model in first differences as their nature is already logarithmic by design of the model.

The structural VAR defined by equation (3.46) cannot be estimated due to the endogeneity of dependent variables. However, the structural VAR can

[9] For details on the Hodrick-Prescott filter, refer to appendix A.

be transformed into the reduced-form VAR by pre-multiplying equation (3.46) by ${\boldsymbol{B_0}}^{-1}$ to obtain the identity matrix associated with $\boldsymbol{y_t}$. To estimate the coefficients of the reduced-form VAR, a simple OLS can be used. In order to determine the order p of the reduced-form VAR, information criteria are used.[10]

It is, however, obvious that not all the coefficients of the structural form can be recovered from the reduced-form coefficient estimates. In order to recover the parameters of the structural equation (3.46) from the estimated parameters of the reduced-form VAR, certain restrictions on some of the structural parameters must be imposed. For this purpose, this thesis uses the Cholesky Decomposition.

3.2.1 Cholesky Decomposition

The formal derivation of the Cholesky decomposition is presented in Hamilton (1994, p. 91). In practice, the Cholesky decomposition imposes a recursive casual restrictions from the top to the bottom variables in the vector of observations but not in reverse order. In other words, the first variable in the VAR is only affected contemporaneously by the shock to itself. The second variable in the VAR is affected contemporaneously by the shocks to the first variable and the shock to itself, and so on. Therefore, after the decomposition is applied, the matrix associated with $\boldsymbol{y_t}$ on the left hand side of the structural VAR equation is a lower triangular matrix with diagonal terms equal to 1. These restrictions enable recovering the coefficients of the structural-form VAR after the reduced-form VAR is estimated. They also enable tracking the impact of shocks any variable on other variables in the system by analyzing the impulse response functions.

The ordering of variables in the vector of observations is crucial for the Cholesky decomposition and is done based on the intuition behind the economic mechanisms governing the behavior of the variables. The variables are ordered in the following manner: a transformation of a measure of economic activity, a transformation of the price level, a transformation of the monetary base, the short-term interest rate, and transformations of the stock-price related variables. This ordering reflects the intuition about the speed of reaction of each variable to changing economic and financial conditions with the stock-price related variables being the fastest and the measure of economic output reacting the slowest. What remains is to determine the ordering of the bubble

[10] Information criteria are described in appendix A.

and the fundamental components of stock prices. Here the intuition is that the fundamental component of stock prices should reflect some underlying "fundamental" conditions and therefore react slower than the bubble component, which does not reflect any fundamentals and therefore is less prone to being delayed.

The concept of the Granger causality, introduced by Granger (1969), and the impulse response function analysis are used for the purpose of investigating the dynamics between the estimated bubble and the macroeconomic variables.

3.2.2 Granger Causality Testing

The idea behind the concept of the Granger causality is very simple, we say that y_i Granger causes y_j, if lagged values of y_j have any explanatory power on the current values of y_i. The Granger causality, therefore, can be studied using the reduced form VAR without imposing any contemporaneous restrictions on the variables. To test the null hypothesis of y_j not Granger causing y_i, the F-test can be used to test the joint hypothesis of no explanatory power of any of lagged values of y_j.[11] Note that the concept of the Granger causality only represents a statistical linkage and does not tell us anything about the underlying causal structure.

3.2.3 Impulse Response Functions Analysis

In order to be able to construct various impulse response functions, the complete specification of the system must be known and the parameters of the structural form VAR must be estimated. This means that we need to apply the Cholesky decomposition and impose the restrictions described above. After having estimated all the parameters of the structural VAR, the impulse response functions can be constructed.

The impulse response function, $IRF(m, i, j)$, gives the m^{th}-period response of y_j to a one-standard-deviation shock in ε_i, for $m = 0, 1, 2, ...$ Suppose that the structural VAR is given by the equation (3.46) and that $\varepsilon_{i,t}$ has a variance equal to σ_i^2. Consider a sequence of shocks, $\{\bar{\varepsilon}_{i,t}\}_{t=1}^{\infty}$ and let the series for $y_{j,t}$ generated by the system be given by $\{\bar{y}_{j,t}\}_{t=1}^{\infty}$. Now consider an alternative

[11] For the description of the F-test mechanism, see appendix A

series of shocks such that

$$\tilde{\varepsilon}_{i,t} = \begin{cases} \bar{\varepsilon}_{i,t} + \sigma_i & \text{for } t = \tau \\ \bar{\varepsilon}_{i,t} & \text{otherwise.} \end{cases}$$

Then, the $IRF(m,i,j)$ is defined as

$$IRF\,(m,i,j) = \tilde{y}_{j,\tau+m} - \bar{y}_{j,\tau+m},$$

provided that the rest of the system stays fixed.

Chapter 4

Data

This thesis employs U.S. financial and economy data from various sources in order to obtain a comprehensive data-set. Description of the data can be again divided into two sections: the data used for the stock bubble identification and estimation (section 4.1) and the data used for studying the effect of monetary policy on the estimated bubble and market-fundamentals components of stock prices (section 4.2).

4.1 Data Used for Stock Bubble Identification and Estimation

The S&P index is used as a benchmark for the U.S. stock prices. The index is designed to reflect the U.S. equity market and, through the market, the U.S. economy. The S&P focuses on the large-cap sector of the market; however, since it includes a significant portion of the total value of the market, it also represents the market. Companies in the S&P are considered leading companies in leading industries. As of February 2014, the unadjusted market capitalization of the S&P was more than USD 4.6 billion.[1]

The data on the real S&P price and the corresponding real dividend per share are employed.[2] Real price of S&P is the nominal value of S&P deflated by the consumer price index (CPI). Real dividends per share are the nominal dividends per share of S&P deflated by the CPI. The data used for estimating the bubble are annual observations. The sample has 144 observations and

[1] Source: S&P DOW JONES INDICES: S&P U.S. Indices Methodology, February 2014. Available at http://www.standardandpoors.com/.

[2] The data is available here: http://www.multpl.com.

covers years 1871 to 2014. The historical data on S&P price, S&P dividends, and the CPI is provided by Robert Shiller and is the same as used in his book *Irrational Exuberance* (2005).[3]

Real S&P Characteristics

The U.S. stock market experienced several significant swings in the studied period. The first major swing in the 20th century was connected to the World War I when the S&P lost roughly 60% in real terms between 1912 and 1921. This was followed by the bull market of the 1920s when the index gained around 290%. The market crashed in late 1929, causing the index to plummet by more than 60% by 1933. The market recovered by 1936 but The beginning of World War II again meant a serious slump when the index fell by more than 50% between 1937 and 1942 and did not recover until 1950s. The postwar boom of 1950s and early 1960s pushed the index up by more than 300% between 1950 and 1965. This upswing was followed by a crisis caused by the breakdown of the Bretton Woods system and the oil price shocks. S&P lost roughly 50% between 1973 and 1975. After a decade of stagnation, the index pushed to a 15-year unprecedented boom between 1985 and 2000 gaining more than 420%. The end of this period is also known as the IT-bubble. The correction after 2000 meant a decrease in the real value of S&P by almost 50% by 2003 just to rise by another roughly 50% by mid-2007. The period between late 2007 and 2009 is known as the sub-prime mortgage crisis and meant a real fall in S&P by almost 50%. In 2014, we seem to be in another boom period and the S&P is in real terms almost reaching its value from 2000 when the IT-bubble peaked.[4]

The real S&P dividends-per-share time series followed a similar path as the

[3] Although quarterly and monthly data on S&P price is available and employed by researchers (see e.g. Al-Anaswah & Wilfling (2011) or Balke & Wohar (2009)), the specification of the model and the nature of dividend payments suggest that using yearly data is a more appropriate approach. Since different companies included in the S&P pay dividends on different dates and with different frequency (quarterly, annually, semi-annually), employing higher-frequency data on dividends would lead to seasonality problems. Moreover, the higher-frequency data provided by Robert Shiller and employed e.g. in Al-Anaswah & Wilfling (2011) or Balke & Wohar (2009) are only overlapping moving windows of the past 12-month dividends. By employing this type of data, the researchers significantly smooth the time series of the realized dividend payments. The data on the dividend payment at time t then includes 3/4 or 11/12 of the data on the dividend payment at time $t-1$, for quarterly and monthly data respectively. When estimating the model, this approach would lead to underestimation of the parameters explaining the connection between the S&P price and S&P dividends.

[4] All the above mentioned market swings lasted for several years, in some cases even more than a decade. Therefore, the unavailability of an appropriate higher-frequency data on S&P dividends does not seem as a hurdle for the bubble identification.

S&P price; however with a considerably lower mean growth rate and variability. Exactly this difference in the properties of both time series gives the motivation for and is attempted to be explained in this thesis.

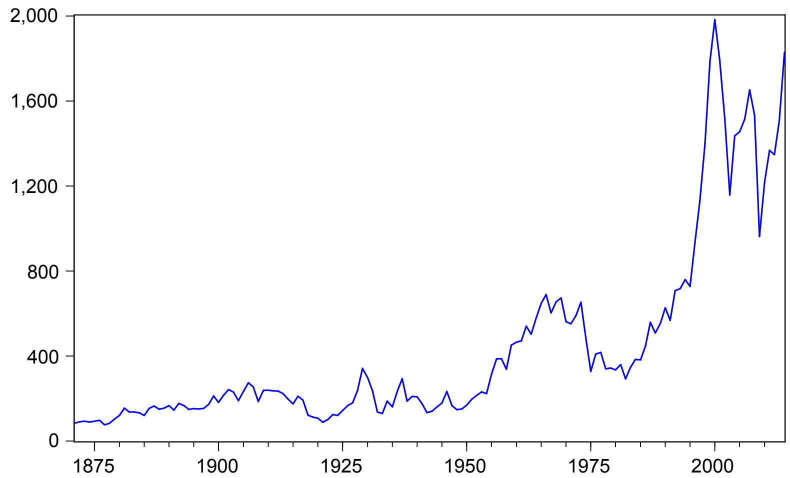
Historical values of S&P and the corresponding dividends per share are presented on figures 4.1 and 4.2. Both figures are in the real terms and correspond to the 2014 price level given by the CPI. Basic descriptive statistics of the time series of S&P and its dividends per share is provided in table 4.1:

Table 4.1: Real S&P Price and Dividend per Share Descriptive Statistics

	Mean	Min (year)	Max (year)
S&P	434.75	76.19 (1877)	1982.83 (2000)
Dividend per share	13.48	4.83 (1878)	35.53 (2014)
	Std. dev.	**Mean log diff.**	**Std. dev. of log diff.**
S&P	442.28	0.022	0.177
Dividend per share	6.63	0.014	0.112

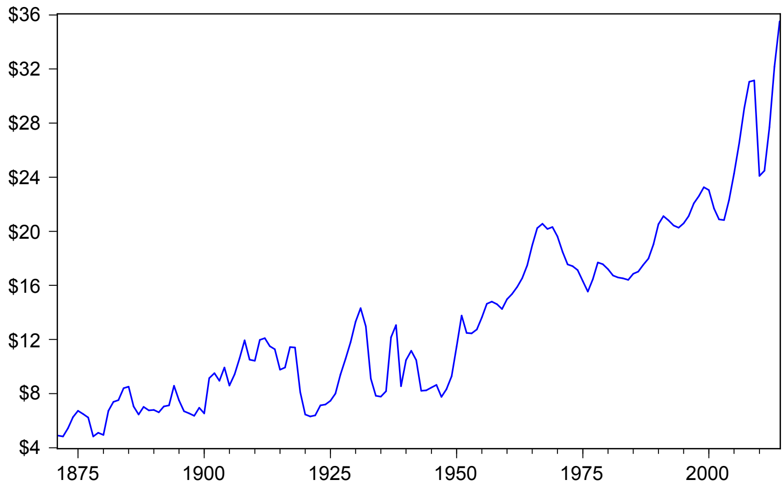
Note: Log difference of variable X_t is defined as $\log(X_t) - \log(X_{t-1})$ and is used as an approximation for percentage change in X_t. *Source:* Author's computations, Eviews.

Figure 4.1: Real S&P Price 1871 - 2014



Source: http://www.multpl.com

Figure 4.2: Real S&P Dividends per Share 1871 - 2014



Source: http://www.multpl.com

4.2 Data Used for Studying the Effects of Monetary Policy

Two sets of data enter the model to study the effects of monetary policy on the estimated bubble and market-fundamentals components of the real S&P. The first set of variables includes the bubble and the fundamental components of S&P estimated in the previous step. The second set consists of the data on the FED's monetary policy and general macroeconomic indicators, attempting to capture the full dynamics of the U.S. economy.

The estimated components of S&P are yearly data ranging from 1871 to 2014. The complete set of macroeconomic and financial variables is only available starting from 1959. Therefore, only the last 56 elements of the estimated S&P bubble and fundamental components are used to study the relation between these components and the macroeconomic variables.

On the other hand, the macroeconomic data is available on a quarterly basis. Quarterly data is often employed by researchers when studying relations among macroeconomic and financial variables, as some of the patterns may remain unrevealed if an annual data was used. In order to make use of all the available information, this thesis employs quarterly macroeconomic data as well. To address the mismatch between the frequency of the estimated data on S&P components and the macroeconomic data as well as to maintain as much information as possible, the estimated time series of the S&P bubble and fundamental components are linearly interpolated.

The set of macroeconomic and financial data consists of the real U.S. GDP, the U.S. CPI, the real U.S. monetary base, the federal funds rate, and the real crude oil price. The historical data is obtained from the Federal Reserve Economic Data (FRED) database maintained by the Research division of the Federal Reserve Bank of St. Louis.[5]

GDP

The real U.S. GDP is the inflation and seasonally adjusted value of the goods and services produced by labor and property located in the United States and is denoted in the Chained 2009 dollars.[6] The period being studied witnessed two major recessions in 1973 - 1975 and 2008 - 2009. These recessions are attributed to the events related to the oil-price shocks and the break-down of the Bretton Woods system in the earlier case and the sub-prime mortgage crisis in the latter.Mild and short-lived recessions also occurred in 1960, 1970, 1980, 1982, 1990, and 2001. The evolution of the real U.S. GDP is depicted on figure 4.3.

The analysis in this thesis uses the cyclical component of the log real U.S. GDP obtained through the application of the Hodrick-Prescott filter[7] instead of the absolute value of the real GDP. The filtering is applied to the entire period from Q1 1947 to Q4 2013, in which the data on GDP is available. The filtered cyclical component is then studied within the period of Q1 1959 - Q4 2013, for which the complete set of macroeconomic and financial variables is available. Figure 4.4 depicts the results of the filtering. Note that all the above mentioned periods of the U.S. recessions are captured by the filtering.

CPI

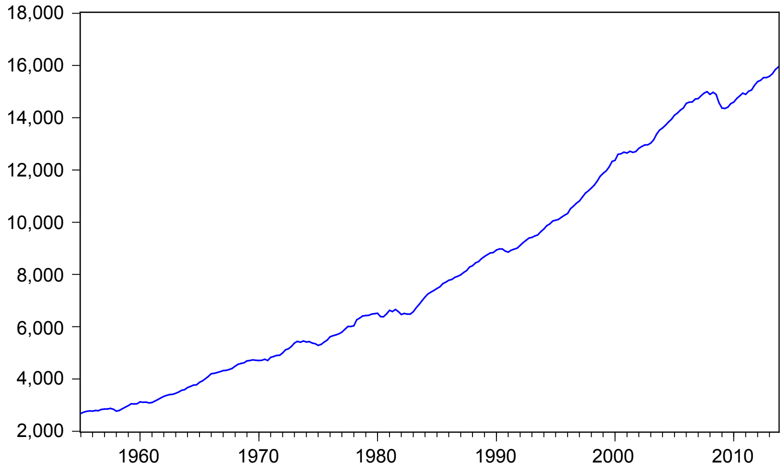
The U.S. CPI is measured by the Chained Consumer Price Index for All Urban Consumers (C-CPI-U), which is the measure of the average monthly change

[5] Available at http://research.stlouisfed.org/fred2.

[6] Chained dollars is a method of adjusting the real dollar amounts for inflation over time, so as to allow a comparison of figures from different years. The difference between chained dollars and the previous measure, constant dollars, is that while the latter is weighed by a constant basket of goods and services, chained dollars are weighted by a basket that changes from year to year so as to more accurately reflect spending (Mark McCracken, Definition of Chained dollars TeachMeFinance.com).

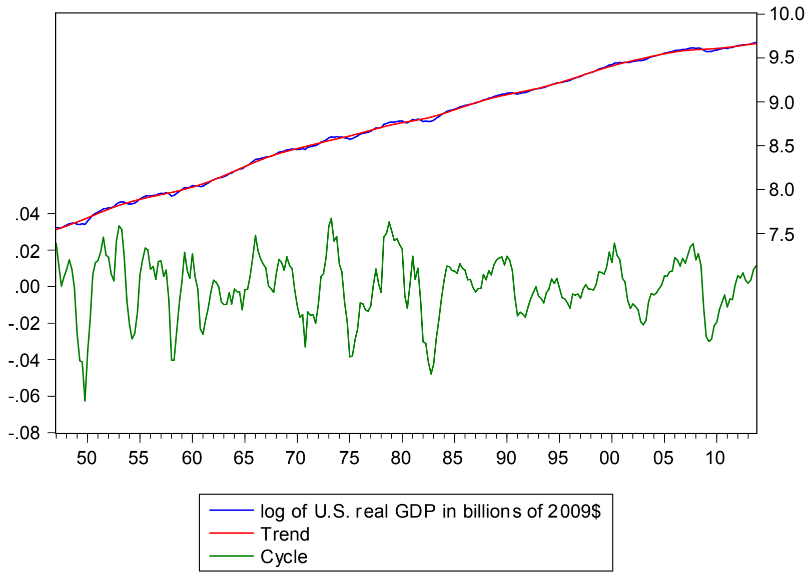
[7] The smoothing factor λ is set to 1600. For description of the Hodrick-Prescott filtering, see appendix A

Figure 4.3: Real U.S. GDP 1959 - 2013 (Billions of Chained 2009 USD)



Source: research.stlouisfed.org

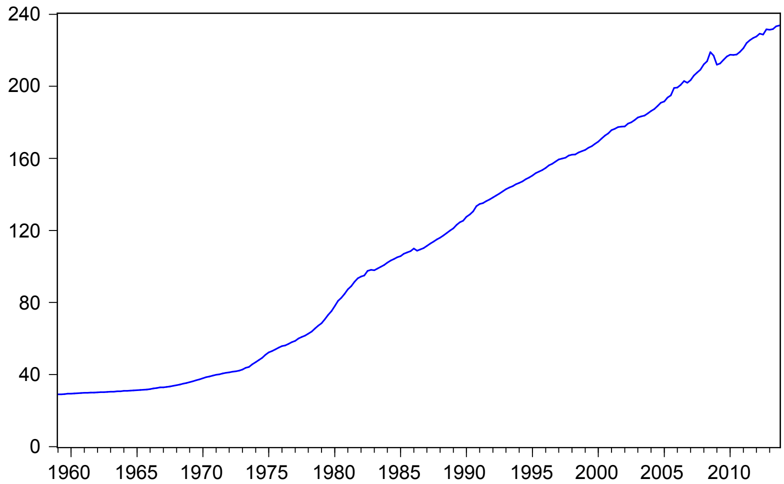
Figure 4.4: Log Real U.S. GDP 1947 - 2013 (Billions of Chained 2009 USD), Trend and the Cycle



Source: research.stlouisfed.org

in the price for goods and services paid by urban consumers between any two time periods[8]. The base level of the index is the August 1983 price level that is scaled to 100. This measure is seasonally adjusted. Figure 4.5 documents the evolution of the U.S. CPI in the studied period. The corresponding log-approximation of percentage inflation rate, defined as $\log(CPI_t) - log(CPI_{t-1})$, is depicted on figure 4.6.

Figure 4.5: U.S. CPI 1959 - 2013 (August 1983 = 100)



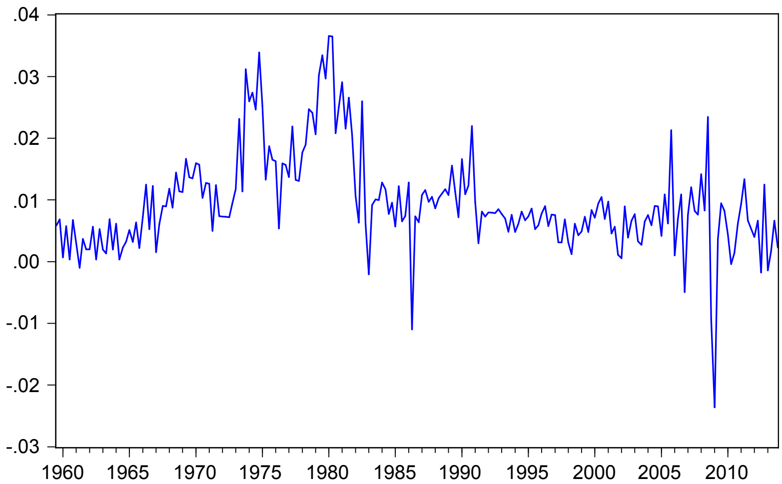
Source: research.stlouisfed.org

Monetary Base

The U.S. monetary base and the federal funds rate are used in order to reflect the FED's pursuit of monetary policy. The monetary base directly reflects the quantitative easing pursued by the FED, as the vast majority of the monetary base is sourced from the securities purchased by the FED (Mishkin (2007)). The FED started using quantitative easing heavily after the U.S. economy collapsed in connection with the sub-prime mortgage crisis. The FED held between USD 700 billion and USD 800 billion of Treasury notes on its balance sheet before the recession. In Q4 2008, the FED started buying USD 600 billion in mortgage-backed securities and kept the increased purchases until June 2010, when the accounting value of the mortgage-backed securities and Treasury notes held by the FED peaked at USD 2.1 trillion. In Q4 2010, the FED announced a second round of quantitative easing, buying USD 600 billion of Treasury securities by

[8] Bureau of Economic Analysis. "CPI Detailed Report", 2013.

Figure 4.6: U.S. Inflation rate 1959 - 2013



Source: research.stlouisfed.org

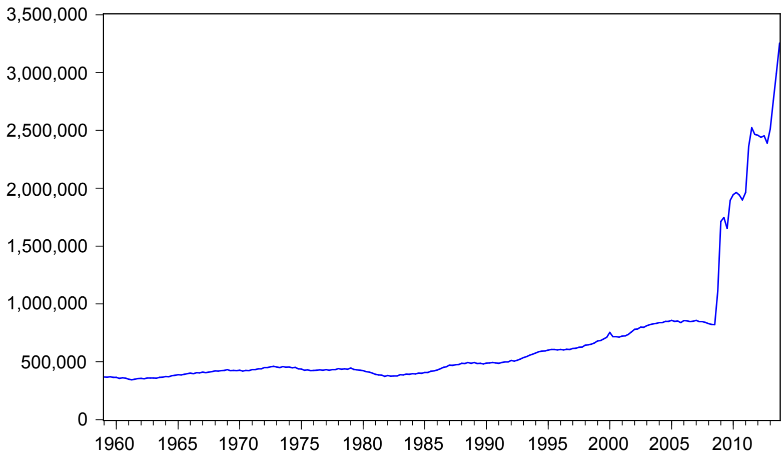
the end of the second quarter of 2011. The third round was announced in Q3 2012 and subsequently escalated in Q4 2012 when the FED announced an open-ended bond purchasing program of agency mortgage-backed securities in the value of USD 85 million per month.[9] The periods of FED's quantitative easing are precisely reflected in the evolution of the real monetary base, as presented on figure 4.7. Figure 4.8 depicts the evolution of the log differences of the real monetary base, $\log(MB_t) - \log(MB_{t-1})$, which are used for the analysis in this thesis.

Federal Funds Rate

The federal funds rate is the interest rate at which depository institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight. The rate that the borrowing institution pays to the lending institution is determined between the two banks; the weighted average rate for all of these types of negotiations is called the effective federal funds rate. The effective federal funds rate is essentially determined by the market but is influenced by the FED through open market operations in order to reach the federal funds rate target. The Federal Open Market Committee (FOMC) meets eight times a year to determine the federal funds target rate. As previously stated, this rate influences the effective federal funds rate through open market oper-

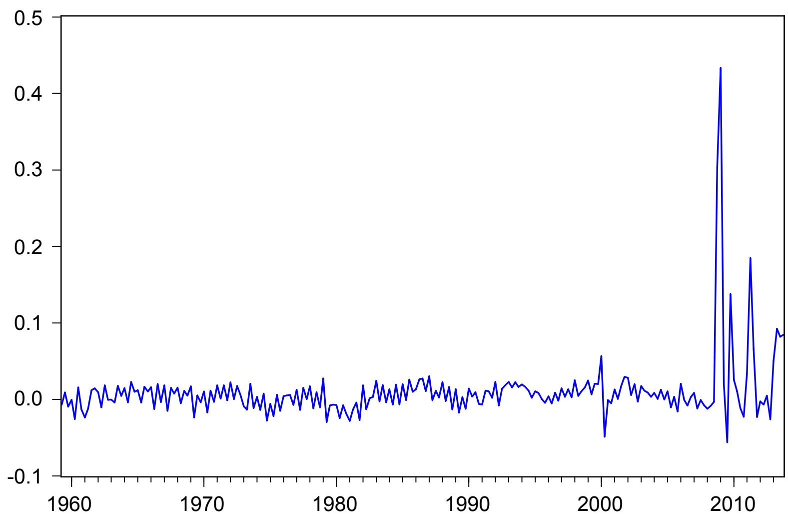
[9] http://www.investmentsolutions.co.za/Clients%20Newsletter/July%202013.

Figure 4.7: U.S. Real Monetary Base 1959 - 2013 (Millions of Chained 2009 USD)



Source: research.stlouisfed.org

Figure 4.8: U.S. Real Monetary Base 1959 - 2013 (Logarithmic Differences)

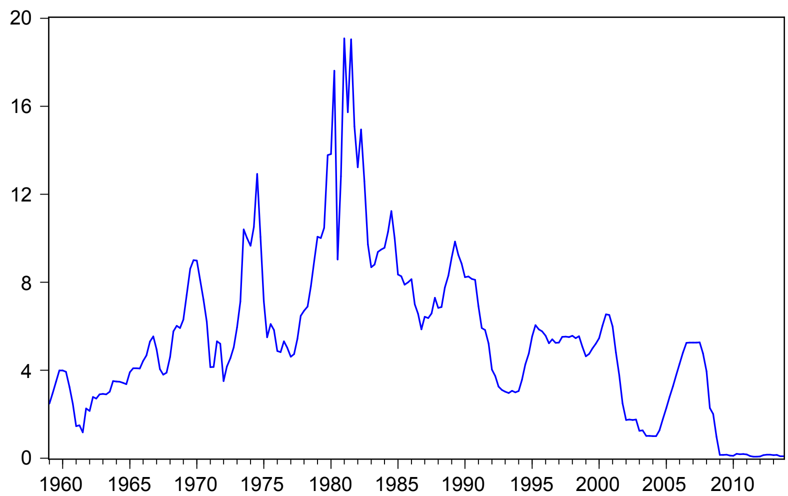


Source: research.stlouisfed.org

ations or by buying and selling of government bonds (government debt). The federal funds rate is the central interest rate in the U.S. financial market and it influences other interest rates including longer-term interest rates such as mortgages, loans, and savings.[10]

The evolution of the federal funds rate is depicted on figure 4.9. The figure reveals that the federal funds rate decreased substantially every time the U.S. economy went through recession. This reflects the reaction of FED's monetary policy to the market turmoils. Moreover, the figure shows that since 2008, the federal funds rate has been almost at zero, which, in the combination of the massive quantitative easing described above, represents an unprecedented economic environment that we are witnessing nowadays.

Figure 4.9: U.S. Federal Funds Rate 1959 - 2013 (in % p.a.)



Source: research.stlouisfed.org

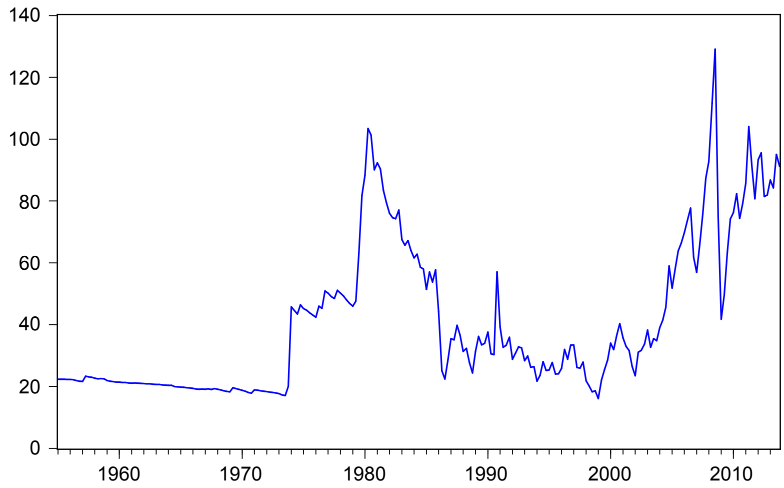
Crude Oil Price

The last variable entering the model is the real crude oil price. The oil price is believed to have an important effect on the U.S. economy (see e.g. Balke *et al.* (2002)) and is used as a proxy for exogenous shocks. Moreover, the U.S. recessions of 1973 - 1975 and 1980 are attributed to extremely growing oil prices. Figure 4.10 presents the evolution of the real oil price from 1953 to 2013. Interestingly, the figure shows that the upsurge in the real oil price

[10] Reference: Board of Governors of the Federal Reserve System. "Monetary Policy". http://www.federalreserve.gov/monetarypolicy/default.htm

before 2008, followed with the subsequent sudden collapse, might have been even more extreme than the development of the real oil price in 1970s.

Figure 4.10: Real Crude Oil Price 1959 - 2013
(Chained 2009 USD per barrel)



Source: research.stlouisfed.org

Chapter 5

Empirical Results

This chapter first discusses the estimation results of the dynamic linear model using the annual U.S. stock market data. After that, the linkages between the FED's monetary policy and the estimated bubble and fundamental components of the stock prices are investigated.

5.1 Estimating the Bubble Component

5.1.1 Dividend Process Estimation

Section 3.1 argued that the log dividends are assumed to be non-stationary and their first differences are assumed to be stationary. To validate this assumption, the Augmented Dickey-Fuller (ADF) and the KPSS tests were used.[1] When testing the levels of log dividends, the ADF test does not reject the null hypothesis of the unit root at any conventional significance level and the KPSS test strongly rejects the null hypothesis of stationarity. This suggests that the log dividends are integrated of at least order one. Proceeding to test for higher-order integration, both tests are applied to the first differences of log dividends. In this case, the ADF test strongly rejects the presence of a unit root in the data and the KPSS test does not reject the null hypothesis of stationarity. Therefore, the appropriateness of this assumption is confirmed. The results are summarized in table 5.1.

Section 3.1 also assumes that the log-dividend process follows an ARIMA$(h, 1, 0)$ process as specified in (3.27). To determine the autoregressive order h, the process is estimated using the OLS method for different choices of h. The Akaike

[1] Both tests are described in appendix A.

Table 5.1: Unit-root Tests for Real Dividends

Series	ADF	KPSS
Log Dividend - Level	-1.037 (0.739)	1.441 ***
Log Dividend - First Differences	-9.141 (0.000)	0.085 (X)

Notes:
The table reports the t-Statistics for the ADF test and the LM-statistics for the KPSS test. In the case of the ADF test, the lag selection is automatically determined based on the AIC criterion. The corresponding p-values of the test are shown in brackets.
In the case of the KPSS test, *** denotes significance at 1% level, (X) means no significance.
Source: Author's computations, Eviews.

information criterion and the Schwartz information criterion are computed.[2] Both criteria are minimized when $h = 2$. Thus, the model is estimated under the presumption that the log dividends follow an ARIMA$(2, 1, 0)$ process. Table 5.2 presents the estimation results of the parameters of the log-dividend process (with notations as specified by equation (3.27)).

Table 5.2: Least Square Estimation of the Differenced Log-Real Dividends

Coefficient	Point Estimate	Standard Error	P-Value
μ	0.013	0.009	0.172
ϕ_1	0.212	0.083	0.012
ϕ_2	-0.195	0.084	0.021

Source: Author's computations, Eviews.

The assumption about the dividends following an ARIMA$(h, 1, 0)$ process is quite restrictive and the model specification does not allow for MA, ARCH or GARCH terms, for example. On the other hand, the fact that companies tend to smooth dividends was documented (see e.g. Leary & Michaely (2011)) and so it is reasonable to expect dividends to follow an AR process. It would be more difficult to justify the presence of other terms in the log-dividend process.

Testing the residuals of the estimated ARIMA$(2, 1, 0)$ log-dividend process for autocorrelation does not indicate the presence of remaining autocorrelation in the residuals. The Ljung-Box Q-test (applied to the first 36 lags) rejects the null at a 10% significance level. For the detailed results of the test, see table B.1 and in appendix B. However, due to the restrictive options for the dividend process specification, a remaining autocorrelation may be present in

[2] Both criteria are described in appendix A.

the squared residuals, which may cause the model to be over parametrized (Kočenda & Černý (2007)).

5.1.2 Kalman Filtering Results

The stock-price equation (3.36), the dividend process (3.27), and the bubble process (3.25) are estimated jointly using the maximum likelihood and the Kalman filter. Table 5.3 presents estimates of the six unknown coefficients of the model.

Table 5.3: Maximum likelihood Estimates of the Complete State-Space Model

Coefficient	Point Estimate	Standard Error	P-Value
μ	0.013	0.011	0.254
ϕ_1	0.297	0.068	0.000
ϕ_2	-0.213	0.005	0.001
ρ	1.111	0.053	0.000
σ_ϵ	0.109	0.005	0.000
σ_η	0.188	0.012	0.000

Source: Author's computations, Eviews.

Except for the constant in the dividend process, the parameters are very precisely estimated. The values of coefficients of the estimated dividend process are close to those obtained from estimating the univariate dividend process.

However, the parameter ρ is estimated slightly above one, which contradicts the rational bubble theory. According to the theory, the parameter should be close to but below one. The estimate of ρ is consistent even if different subsamples are used for the estimation. Therefore, the estimated bubble component, if found significant, would indicate the failure of the standard present-value model but in a different way than what would be consistent with the rational bubbles theory.

This result is also contradictory with the previous findings of Wu (1997) and Bhar & Hamori (2005, p. 164 - 187), who found the coefficient ρ smaller than one using the real U.S. S&P data. However, both of these studies define the vector $\boldsymbol{M}$ (equation (3.35)) in a different way than it is defined in this thesis.[3]

Kizys & Pierdzioch (2011) arrived to the coefficient ρ slightly above one, using the CEE monthly data from 1995 to 2008. Their solution to the problem was to transform the coefficient in the following way, $\rho = 1/(1 + exp(-k)$, in order to make sure that $0 < \rho < 1$. However, by performing this transformation on the U.S. data that is employed in this thesis, the system becomes unstable and very sensitive to the initial values of the parameters that enter the algorithm. Hence, the results from this transformation are not used.

Therefore, the bubble process is estimated by the Kalman filter and the subsequent smoothing, with parameters specified in table 5.2. This is accompanied with acknowledging that the estimated bubble process reflects the deviation of stock prices from their fundamentals but is not consistent with the rational bubbles theory. Existence of non-explosive bubbles could be, however, justified by asymmetric information and behavioral finance models.

Regarding the residuals from the bubble estimation, a serial correlation is still present in them (see the Q-test results in table B.2 in appendix B). This suggest that a more sophisticated specification might be needed to fully describe the data generating process of the bubble component. However, the empirical literature attempting to explicitly estimate the bubble component typically does not report any residual diagnostics of the model (see e.g. Wu (1995), Wu (1997), Bhar & Hamori (2005, p. 164 - 187), or Kizys & Pierdzioch (2011)). Therefore, following the standard practice, this thesis assumes that the estimated bubble process is a sufficiently close approximation of the real process.

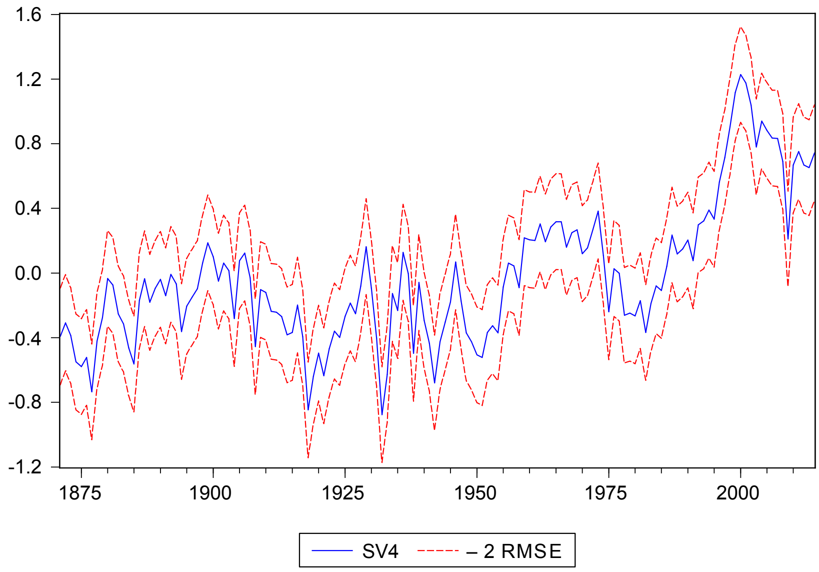
5.1.3 Estimated Bubble Component

The estimated bubble component of the log real S&P price is plotted in Figure 5.1. The figure presents point estimates of the bubble and the corresponding two-standard-errors interval.

A visual inspection of the plot reveals that the time series of the estimated bubble fluctuates greatly. The time series also reflects most of the major swings

[3] In Wu (1997) the definition of the vector $\boldsymbol{M}$ is based on a different specification of the present-value model, which is not in line with the conventions used in the current finance literature (see section 3.1). Bhar & Hamori (2005) use the same model specification as Wu but they arrive at a completely different definition of the vector $\boldsymbol{M}$ that is oversimplified and not in line with the formal step-by-step derivation of $\boldsymbol{M}$ as presented in section 3.1.

Figure 5.1: Estimated Bubble Component



Source: Author's computations, Eviews

in the U.S. stock market that were described in section 4.1. It slumped significantly during the World War I and recovered during 1920s. The bubble peaked in 1929. This was followed by a sharp fall and the bubble reached its trough in 1932. After the recovery in 1936, it collapsed again several times in the period before and during World War II. After that, the bubble had an increasing trend during the post-war boom in the 1950s. It remained positive throughout the 1960s and until 1973 when it reached its highest value, up to that point. In 1973, it collapsed again in connection with the breakdown of the Bretton Woods and the oil-price shocks. The bubble started recovering in 1982 and grew almost continuously for 18 years before it reached its absolute sample peak in 2000. The evolution after 2000 is interesting because despite the growth in the U.S. stock market between 2005 and 2007, the bubble component did not grow. Most recently, the bubble collapsed during 2008 and 2009 and recovered by 2011 to stagnate until 2014.

From the statistical point of view, the bubble was significantly positive in 1962, 1965, 1966, and 1973. A significant positive bubble emerged again in 1992 and it remained significantly positive all the way until 2014, with 2009 being the only exception. Interestingly, the bubble component was not significantly positive in late 1920s.

A significant negative bubble component was present throughout 1870s, then five more times before World War I. After World War I, the bubble stayed significantly negative until 1925. A significant negative bubble was present during the Great Depression in 1931 - 1933, during most years of World War II, and then also in the period period between 1948 and 1954. The last time the estimated bubble component was significantly negative was 1982.

In order to proceed from the approximated log representation of the real S&P price back to the real terms, note that it is possible to take exponential of equation (3.20) and express the real S&P price in the following way:

$$P_t = P_t^f B_t^q. \tag{5.1}$$

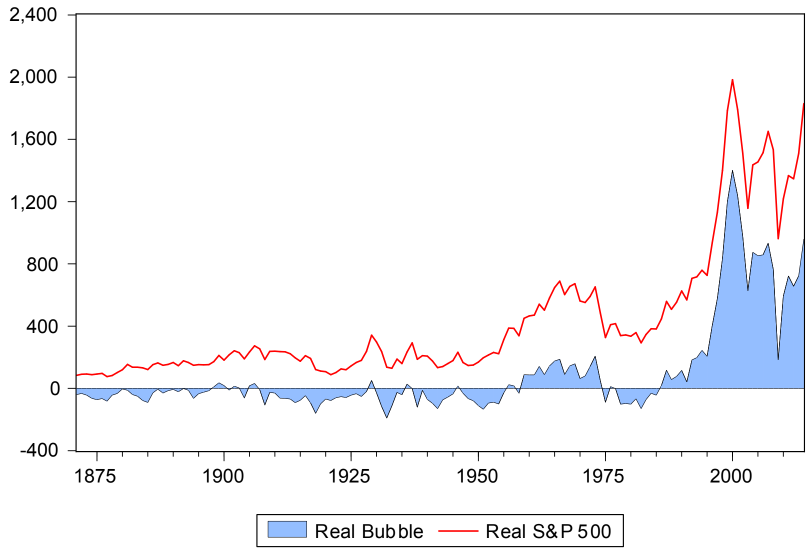
Now the real values of the estimated bubble component and the market fundamentals component can be extracted from the estimation results. In this sense, B_t^q acts as a coefficient that multiplies the market-fundamentals component in order to obtain the real S&P price. The actual bubble component, B_t, can be retrieved from the real S&P price and the market-fundamentals component trivially as $B_t = P_t - P_t^f$.

Figure 5.2 combines the time series of the estimated bubble component and the real S&P price. It is evident that the estimated bubble accounts for a substantial part of the real S&P price and bears most of its variability. The figure suggests that the bubble had a predominantly price decreasing effect in the period until mid 1950s and then between 1975 and 1985. On the other hand, the bubble component has increased enormously in recent decades, driving the real S&P price high above any levels that could be justified by market fundamentals.

Similar conclusions can be drawn from figure 5.3 which shows the situation from the perspective of the market-fundamentals component. The figure documents that the evolution of the market-fundamentals component was considerably less volatile than the evolution of the real S&P price. The figure also reveals that in many cases the market-fundamentals component followed a similar pattern as the real S&P but its development was substantially milder and slightly delayed. Furthermore, the growth in the fundamental component in the 1990s was very weak compared to the unprecedented increase in the real S&P in the same period.

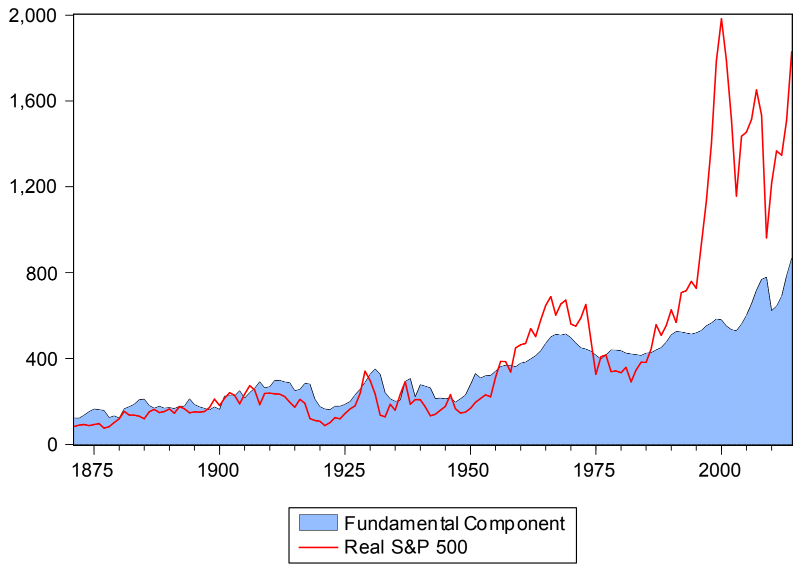
Figure 5.4 presents the time series of the ratio of the estimated bubble component to the real S&P price. For the sake of comparison, the estimated bubble component is depicted in the same graph. The figure reveals that the

Figure 5.2: Real S&P Price and Its Estimated Bubble Component



Source: Author's computations, Eviews

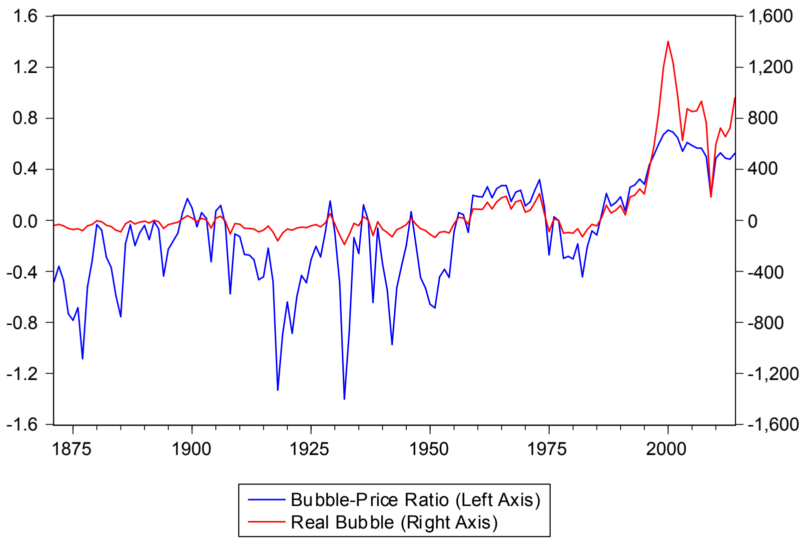
Figure 5.3: Real S&P Price and Its Estimated Fundamental Component



Source: Author's computations, Eviews

ratio ranges approximately between -140% and +71%. It increases during the major bull markets and decreases during the bear market periods. Moreover, the periods of increases and decreases in both time series correspond. This suggests that the bubble component (in the periods in which found significant) has been contributing to the market swings substantially more than the fundamental component.

Figure 5.4: Ratio of the Bubble Component to the Real S&P Price



Source: Author's computations, Eviews

5.2 Investigating Relations Between Monetary Policy and the Estimated S&P Components

The estimated bubble and fundamental components are now studied in the context of the macroeconomic and monetary variables described in section 4.2.[4] Both estimated components of the real S&P price have annual frequency. Therefore, they are interpolated in order to match the macroeconomic and monetary data that is available on a quarterly basis. The complete set of variables is then used for estimation of the VAR model. The log difference of the real oil price enters the model as an exogenous variable.

[4] For the purpose of the VAR analysis, both S&P components are used in their log representations.

5.2.1 Model Calibration

The time series used in the VAR analysis were transformed, as described in section 3.2, in order to ensure stability of the model. The ADF test and the KPSS test suggest that all the time series entering the model are stationary, except for the log differences of the real monetary base. Testing the log differences of the real monetary base yields mixed results due to the unprecedented recent periods of quantitative easing. However, as Lütkepohl (2007) argues, stationarity or strong co-integration of the series in a VAR model is not necessary as long as the system is stable as a whole. Therefore, in order not to lose information, the series of the log-differenced real monetary base enters the VAR model without being differenced again. For the sake of brevity, the results of the stationarity tests are not reported in this thesis but they are available upon request.

The appropriate number of lags included in the VAR model is determined based on both the Akaike information criterion and the Schwartz information criterion. It turns out that both criteria are minimized when the number of lags is equal to two. The system is then estimated jointly using the OLS method. The constant term is not considered. The detailed estimation results are reported in appendix B in table B.3.

5.2.2 Granger Causality

The first step in the analysis is to perform the Granger causality tests.[5] The aim of the analysis is to see if the variables reflecting the FED's monetary policy Granger cause the fundamental and the the bubble components of the real S&P price. The test results suggest that the federal funds rate does not Granger cause any of the S&P components. The real monetary base, on the other hand, Granger causes the bubble component but not the fundamental component of the real S&P price. The p-values of these tests are presented in table 5.4. The test results for the complete set of variables used in the VAR model can be found in appendix B in table B.6.

In the next step, the impulse responses are analyzed in order to confirm these initial results and make more specific conclusions about the effects of monetary policy.

[5] See appendix A for the details of the test procedure.

Table 5.4: p-Values of Selected Granger Causality Tests

Null hypothesis	Δb_t	Δp_t^f
FED funds rate does not Granger cause	0.174	0.447
Monetary base (log diff.) does not Granger cause	0.001	0.209

Source: Author's computations, Eviews.

5.2.3 Impulse Response Analysis

In order to describe the dynamics of the system, the impulse response functions are analyzed. The first observation is that the system is clearly stable. All the impulse responses diminish over the time and converge to zero. The full matrix of the impulse response functions is presented in figures B.4 - B.7 in appendix B.

The focus here is on studying the impulse responses of the two estimated real S&P components to shocks in the federal funds rate and the real monetary base. A general observation is that the responses are mild and short-lived. This indicates that monetary policy does not have any long-term effects on the real asset prices. Nevertheless, it is possible to identify certain significant impulse responses in the short to medium term.

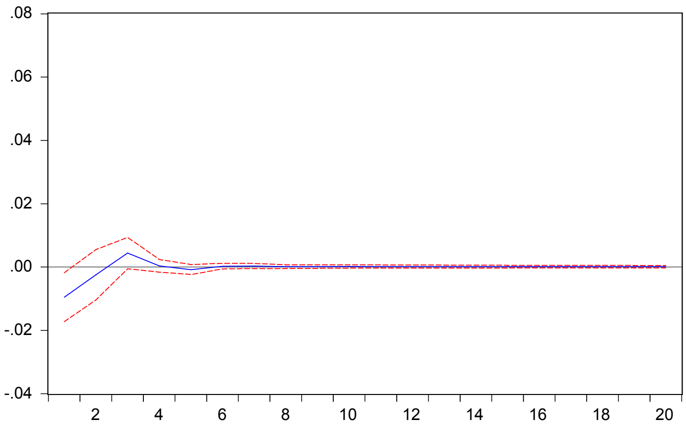
Shocks to the Federal Funds Rate

Figures 5.5 and 5.6 present the effects of the shocks to the federal funds rate. The impulse response of the S&P fundamental component is significantly negative in the first quarter after the shock. This result could reflect the following mechanism: a decrease in the nominal short-term interest rates translates into a decrease in the long-term interest rates and the discount rates that market participants use to value the expected future dividend streams. This pushes stock prices up. However, this effect is short-lived and becomes insignificant during the second quarter after the shock (which might be why this effect was not identified by the Granger causality analysis).

The response of the bubble to the federal funds rate is insignificant. This suggests that monetary policy pursued by influencing the nominal interest rates is not effective in correcting price misalignment on the stock markets. This result supports some conclusions that were made in previous research concerning the relation between monetary policy and asset prices. Bernanke & Gertler (1999), studying the U.S. and Japanese data, argued that *"it is nei-*

ther necessary nor desirable for monetary policy to respond to changes in asset prices". Bernanke & Gertler (2001) studied the relation between monetary policy and the stock price bubbles on theoretical grounds. The authors used an augmented version of the standard dynamic new-Keynesian, which assumes a non-fundamental component in stock prices. They concluded that the losses resulting from reacting to asset price bubbles outweigh the benefits. The conclusion that interest rate policy should not react directly to asset prices was also reached by Goodfriend (2003) (published in Hunter *et al.* (2005)). Goodfriend studied this topic in connection with an outbreak of inflation, a profit squeeze, and a productivity growth. More recently, Orphanides (2010), studying the financial crisis of 2008 and 2009, argued that central bankers should not use the interest rate policy to respond to emerging asset price misalignment, above and over what could be justified by pursuing the price stability objective.

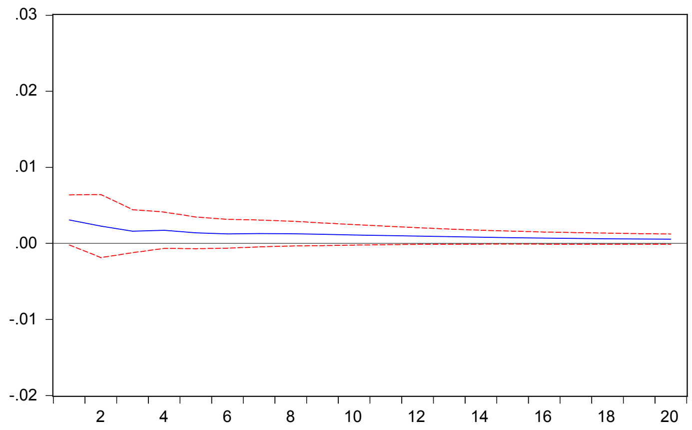
Figure 5.5: Effect of the FED Funds Rate Shocks on the Fundamental Component of S&P 500



Notes: Responses to Cholesky one standard deviation innovations +/- two standard errors. Time axis units - quarters.
Source: Author's computations, Eviews

The view presented above is, however, not shared unanimously in the finance literature. For example, Cecchetti *et al.* (2000) modeled the consequence of the central bank setting the short-term interest rate on the economy. The authors argue that *"monetary policy that pursues an inflation-targeting strategy should attempt to identify and respond to asset price misalignments"*. However, the authors admit that identifying asset price misalignments is difficult. The empirical analysis performed in this thesis provides evidence that, indeed, the FED has not been successful at distinguish between stock price movements due

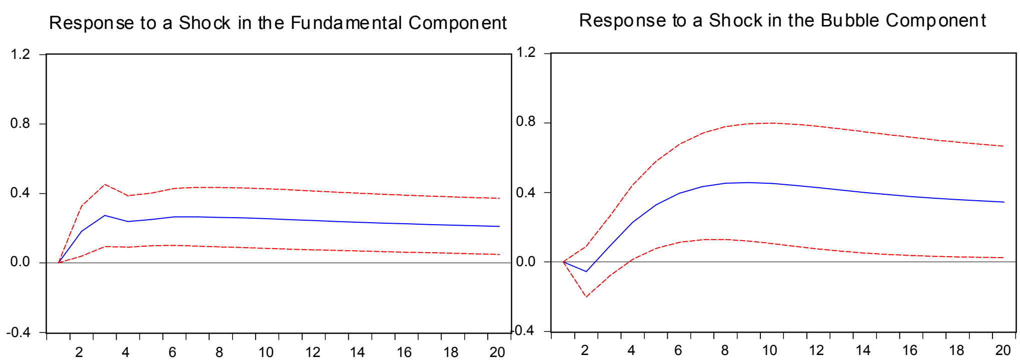
Figure 5.6: Effect of the FED Funds Rate Shocks on the Bubble Component of S&P 500



Notes: Responses to Cholesky one standard deviation innovations +/- two standard errors. Time axis units - quarters.
Source: Author's computations, Eviews

to the change in fundamentals and speculative price movements due to the bubble. Figure 5.7 shows that in the estimated VAR system, the federal funds rate reacts positively to both a shock in the estimated fundamental component and a shock in the estimated bubble component. Moreover, both impulse responses follow similar patterns. They quickly become significantly positive, are very persistent, and have a comparable magnitude.

Figure 5.7: Response of the FED Funds Rate to Shocks in the S&P 500 Components

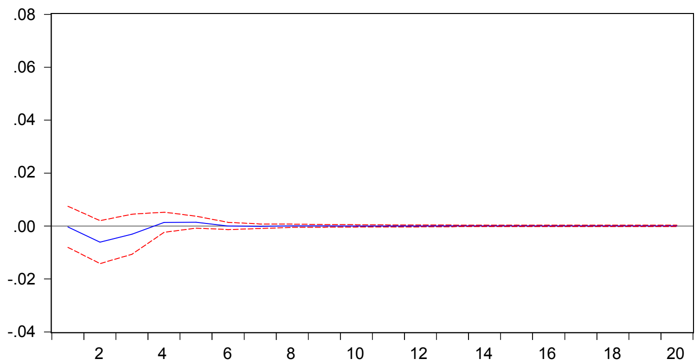


Notes: Responses to Cholesky one standard deviation innovations +/- two standard errors. Time axis units - quarters.
Source: Author's computations, Eviews

Shocks to the Real Monetary Base

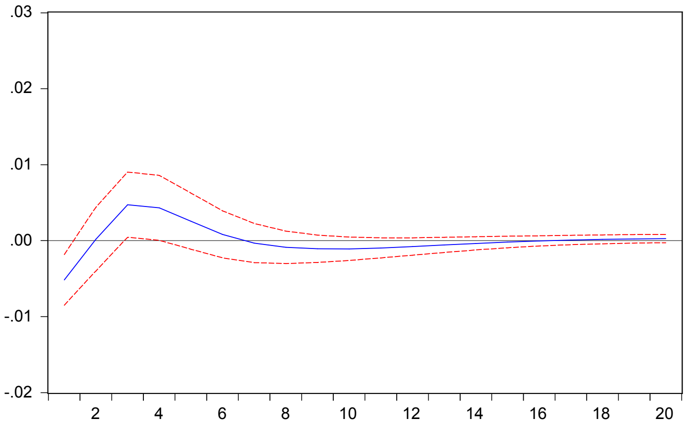
The effects of shocks to the monetary base are plotted on figures 5.8 and 5.9. The S&P fundamental component does not respond significantly to the shocks in the real monetary base, suggesting that quantitative easing does not affect the long-term expectations about future dividends. On the other hand, quantitative easing seems to evoke a significant response of the S&P bubble component. The response is significantly positive in the third and the fourth quarter after the shock but significantly negative in the first quarter after the shock. This suggests that the quantitative easing positively influences the size of the bubble in the medium term. The effect is not persistent and mitigates in the long term. In the first quarter after the shock, however, quantitative easing seems to have a weak negative effect on the bubble. This could be explained by certain market correction mechanism, as the market may initially overreact to the announcement of the FED about the planned quantitative easing. On the other hand, since the quantitative easing only occurred in the very last part of the observed sample, the results may not be robust. A further investigation into the effect of the FED's quantitative easing on the stock market bubbles would be needed.

Figure 5.8: Effect of Monetary Base Shocks on the Fundamental Component of S&P 500



Notes: Responses to Cholesky one standard deviation innovations +/- two standard errors. Time axis units - quarters.
Source: Author's computations, Eviews

Figure 5.9: Effect of Monetary Base Shocks on the Bubble Component of S&P 500



Notes: Responses to Cholesky one standard deviation innovations +/- two standard errors. Time axis units - quarters.
Source: Author's computations, Eviews

Chapter 6

Conclusion

This thesis studied bubbles in the U.S. stock market and how they react on the monetary policy pursued by the Federal Reserve System of the United States. The work tested and estimated the bubble component in the real S&P 500 price. The speculative bubble was estimated based on the theory of rational bubbles. The logarithmic approximation of the linear rational expectations model for stock prices was used to introduce the bubble. The bubble process, the dividend process, and the stock-price equation were all expressed in a state-space form and jointly estimated using the Kalman filtering technique. The logarithmic approximation of the model allows for both price increasing and price decreasing bubbles. The innovation here is that the model was adjusted in order to reflect the timing conventions of the linear rational expectations model, as understood in the current finance literature.

The author acknowledges that the bubble identification in this thesis is conditional upon several assumptions that were adopted in order to make the estimation feasible. The most important restrictions in the model are the assumed linear relations in all the estimated equations, the assumption about the constant required real rate of return, and the restrictive parametric specifications of the dividend process and the bubble process. Therefore, the bubble solution presented in this thesis is only one of several possible explanations for the failure of the standard discounted-dividends model.

The results of the estimation indicate that the bubble component of the real S&P 500 price has been significant in several major bull and bear markets over the past 143 years. Bubbles, therefore, might have played a role in past major U.S. stock market swings and have contributed to both stock market booms and stock market collapses. An interesting result found in this thesis is that during the stock market boom in the late 1920s, the bubble component did not play a significant role. However, it significantly contributed to the subsequent market crash resulting in the Great Depression. On the other hand, the data suggests that since 1992 until present day there has been a permanent significant positive bubble in the U.S. stock market, driving the real S&P price high above any levels that could be justified by market fundamentals, with 2009 being the only exception.

The estimated bubble component, however, contradicts the rational bubbles theory. Its data generating process does not seem to be explosive as stipulated by the definition of a rational bubble. Nevertheless, the existence of non-explosive bubbles could be justified by asymmetric information and behavioral finance models, which are also presented in this thesis.

In the second part of this thesis, a VAR model was estimated. The model combined the estimated S&P fundamental and bubble components with the standard macroeconomic and monetary indicators. The Granger causality tests and the impulse response functions were used in order to study the effects of the FED's monetary policy on the bubble and the fundamental component. The FED's monetary policy was divided into two main tools, short-term interest rate setting/influencing and quantitative easing. Effects of both tools were studied separately. To the author's best knowledge, effects of monetary policy on an explicitly estimated stock market bubble have not been studied in any published literature before.

Several principal conclusions can be made from the VAR analysis. First, the FED's monetary policy does not seem to have long-term effects on any of the S&P price components. Second, the monetary policy pursued by the FED by influencing the interest rates may have a short-term effect on the fundamental component of stock prices but it does not seem to have any effect on the price misalignment of the stocks. This result supports some conclusions from the related previous theoretical and empirical research. Thirdly, quantitative easing seems to have a significant positive short- to medium-term effect on the estimated bubble but not the fundamental component. Unfortunately, the robustness of the results concerning effects of quantitative easing suffer from the fact that the period when the FED has been using this tool is short compared to the entire data time span. The results seem to be driven mainly by the recent episodes of quantitative easing. Finally, the impulse response functions of the federal funds rate indicate that the FED has not been successful at distinguishing between price movements due to fundamentals and speculative price movements due to bubbles.

There are multiple possibilities for future research. It would be interesting to amend the model used for estimating the bubble with other explanatory variables that are believed to influence future dividends. These variables could consist of accounting performance indicators, like EBITDA, EBIT, or earnings. Another possible extension of this work would be adjusting the model in order to allow for more flexibility in its specification. Finally, adjusting the model so that it would be applicable to higher-frequency data could bring valuable results.

Bibliography

ABREU, D. & M. K. BRUNNERMEIER (2003): "Bubbles and Crashes." *Econometrica* **71(1)**: pp. 173–204.

AL-ANASWAH, N. & B. WILFLING (2011): "Identification of speculative bubbles using state-space models with Markov-switching." *Journal of Banking & Finance* **35(5)**: pp. 1073–1086.

ALLEN, F. & G. GORTON (1993): "Churning Bubbles." *Review of Economic Studies* **60(4)**: pp. 813–36.

ALLEN, F., S. MORRIS, & A. POSTLEWAITE (1993): "Finite Bubbles with Short Sale Constraints and Asymmetric Information." *Journal of Economic Theory* **61(2)**: pp. 206–229.

BALKE, N. S., S. P. BROWN, & M. K. YUCEL (2002): "Oil Price Shocks and the U.S. Economy: Where Does the Asymmetry Originate?" *The Energy Journal* **0(3)**: pp. 27–52.

BALKE, N. S. & M. E. WOHAR (2009): "Market fundamentals versus rational bubbles in stock prices: a Bayesian perspective." *Journal of Applied Econometrics* **24(1)**: pp. 35–75.

BERNANKE, B. & M. GERTLER (1999): "Monetary policy and Asset Pprice Volatility." *Economic Review* **Q(IV)**: pp. 17–51.

BERNANKE, B. S. & M. GERTLER (2001): "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review* **91(2)**: pp. 253–257.

BHAR, R. & S. HAMORI (2005): *Empirical Techniques in Finance.* Springer Finance. Springer.

Blanchard, O. J. & M. W. Watson (1983): "Bubbles, Rational Expectations and Financial Markets." *NBER Working Paper* **(945)**.

Box, G. E. P. & G. M. Ljung (1978): "On a measure of lack of fit in time series models." *Biometrika* **65(2)**: pp. 297–303.

Brunnermeier, M. K. & S. Nagel (2004): "Hedge Funds and the Technology Bubble." *The Journal of Finance* **59(5)**: pp. 2013–2040.

Burmeister, E. & K. D. Wall (1982): "Kalman filtering estimation of unobserved rational expectations with an application to the German hyperinflation." *Journal of Econometrics* **20(2)**: pp. 255 – 284.

Campbell, J., A. Lo, & A. MacKinlay (1997): *The econometrics of financial markets*. Princeton, NJ: Princeton Univ. Press, 2. print. with corrections edition.

Campbell, J. Y. & R. J. Shiller (1988a): "Stock Prices, Earnings, and Expected Dividends." *Journal of Finance* **43(3)**: pp. 661–76.

Campbell, J. Y. & R. J. Shiller (1988b): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies* **1(3)**: pp. 195–228.

Cecchetti, S. G., H. Genberg, J. Lipsky, & S. Wadhwani (2000): "Asset Prices and Central Bank Policy." *The Geneva Report on the World Economy No. 2* .

Cochrane, J. H. (1992): "Explaining the Variance of Price-Dividend Ratios." *Review of Financial Studies* **5(2)**: pp. 243–80.

Cuthbertson, K. & D. Nitzsche (2005): *Quantitative Financial Economics: Stocks, Bonds and Foreign Exchange*. Financial Economics and Quantitative Analysis Series. Wiley.

De Long, J. B., A. Shleifer, L. H. Summers, & R. J. Waldmann (1990): "Noise Trader Risk in Financial Markets." *Journal of Political Economy* **98(4)**: pp. 703–38.

Dezhbakhsh, H. & A. Demirguc-Kunt (1990): "On the Presence of Speculative Bubbles in Stock Prices." *Journal of Financial and Quantitative Analysis* **25(01)**: pp. 101–112.

Diba, B. T. & H. I. Grossman (1988): "Explosive Rational Bubbles in Stock Prices?" *American Economic Review* **78(3)**: pp. 520–30.

Dickey, D. A. & W. A. Fuller (1979): "Distribution of the Estimators for Autoregressive Time Series With a Unit Root." *Journal of the American Statistical Association* **74(366)**: pp. 427–431.

Evans, G. W. (1991): "Pitfalls in Testing for Explosive Bubbles in Asset Prices." *American Economic Review* **81(4)**: pp. 922–30.

Flavin, M. A. (1983): "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence." *Journal of Political Economy* **91(6)**: pp. 929–56.

Froot, K. A. & M. Obstfeld (1992): "Intrinsic Bubbles: The Case of Stock Prices." *NBER Working Papers* **(3091)**.

Goodfriend, M. (2003): "Interest rate policy should not react directly to asset prices." *Asset Price Bubbles: The Implications for Monetary, Regulatory, and International Policies* .

Granger, C. W. J. (1969): "Investigating Causal Relations by Econometric Models and Cross-spectral Methods." *Econometrica* **37(3)**: pp. 424–438.

Gürkaynak, R. S. (2005): "Econometric tests of asset price bubbles: taking stock." *Finance and Economics Discussion Series* **(2005-04)**.

Hamilton, J. (1994): *Time Series Analysis*. Princeton University Press.

Havránek, T., R. Horváth, & J. Matějů (2010): "Do Financial Variables Help Predict Macroeconomic Environment? The Case of the Czech Republic." *Working Papers* **(2010/06)**.

Hunter, W., G. Kaufman, & M. Pomerleano (2005): *Asset Price Bubbles: The Implications for Monetary, Regulatory, and International Policies*. MIT Press.

Kim, B. H. & H.-G. Min (2011): "Household lending, interest rates and housing price bubbles in Korea: Regime switching model and Kalman filter approach." *Economic Modelling* **28(3)**: pp. 1415–1423.

Kindleberger, C. & R. Aliber (2011): *Manias, Panics and Crashes: A History of Financial Crises, Sixth Edition*. Palgrave Macmillan.

Kizys, R. & C. Pierdzioch (2011): "The Financial Crisis and the Stock Markets of the CEE Countries." *Czech Journal of Economics and Finance (Finance a úvěr)* **61(2)**: pp. 153–172.

Kleidon, A. W. (1986): "Variance Bounds Tests and Stock Price Valuation Models." *Journal of Political Economy* **94(5)**: pp. 953–1001.

Kočenda, E. & A. Černý (2007): *Elements of Time Series Econometrics: An Applied Approach*, volume 33. Karolinum Press, Charles University, Prague.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, & Y. Shin (1992): "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?" *Journal of Econometrics* **54(1-3)**: pp. 159–178.

Lammerding, M., P. Stephan, M. Trede, & B. Wilfling (2012): "Speculative bubbles in recent oil price dynamics: Evidence from a Bayesian Markov-switching state-space approach." *CQE Working Papers 2312*, Center for Quantitative Economics (CQE), University of Muenster.

Leary, M. T. & R. Michaely (2011): "Determinants of Dividend Smoothing: Empirical Evidence." *Review of Financial Studies* **24(10)**: pp. 3197–3249.

LeRoy, S. F. & W. R. Parke (1992): "Stock Price Volatility: Tests Based on the Geometric Random Walk." *American Economic Review* **82(4)**: pp. 981–92.

LeRoy, S. F. & R. D. Porter (1981): "The Present-Value Relation: Tests Based on Implied Variance Bounds." *Econometrica* **49(3)**: pp. 555–74.

Lütkepohl, H. (2007): *New Introduction to Multiple Time Series Analysis*. New York.

Miller, E. M. (1977): "Risk, Uncertainty, and Divergence of Opinion." *Journal of Finance* **32(4)**: pp. 1151–68.

Mishkin, F. (2007): *The Economics of Money, Banking, and Financial Markets*. Addison-Wesley series in economics. Pearson Addison Wesley.

Orphanides, A. (2010): "Monetary Policy Lessons from the Crisis." *CEPR Discussion Papers 7891*, C.E.P.R. Discussion Papers.

Shiller, R. (2005): *Irrational Exuberance.* Broadway Books. Currency/Doubleday.

Shiller, R. J. (1981): "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* **71(3)**: pp. 421–36.

Shleifer, A. & R. W. Vishny (1995): "The Limits of Arbitrage." *NBER Working Papers* **(5167)**.

Sims, C. A. (1980): "Macroeconomics and Reality." *Econometrica* **48(1)**: pp. 1–48.

Tirole, J. (1982): "On the Possibility of Speculation under Rational Expectations." *Econometrica* **50(5)**: pp. 1163–81.

Tirole, J. (1985): "Asset Bubbles and Overlapping Generations." *Econometrica* **53(6)**: pp. 1499–1528.

Vissing-Jorgensen, A. (2004): *Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions.* NBER Chapters. National Bureau of Economic Research, Inc.

Weil, P. (1990): "On the Possibility of Price Decreasing Bubbles." *Econometrica* **58(6)**: pp. 1467–74.

West, K. D. (1987): "A Specification Test for Speculative Bubbles." *The Quarterly Journal of Economics* **102(3)**: pp. 553–80.

West, K. D. (1988a): "Dividend Innovations and Stock Price Volatility." *Econometrica* **56(1)**: pp. 37–61.

West, K. D. (1988b): "Dividend Innovations and Stock Price Volatility." *NBER Working Papers 1833*, National Bureau of Economic Research, Inc.

Wu, Y. (1995): "Are there rational bubbles in foreign exchange markets? Evidence from an alternative test." *Journal of International Money and Finance* **14(1)**: pp. 27–46.

Wu, Y. (1997): "Rational Bubbles in the Stock Market: Accounting for the U.S. Stock-Price Volatility." *Economic Inquiry* **35(2)**: pp. 309–19.

Appendix A

Elements of Time Series Econometrics Used in This Thesis

This appendix serves as an overview of the time series concepts and tests that were used in this thesis. The following time-series methodology overview draws mainly on Kočenda & Černý (2007) but it is described and illustrated in the context of this thesis.

A.1 Information Criteria

Information criteria are used in order to select the model that is the most parsimonious and satisfactorily captures the dynamics of the dividend process (3.27). For this purpose, the *Akaike information criterion (AIC)* and the *Schwartz information criterion (SIC)* are applied.

$$AIC = T \log SSR + 2n, \tag{A.1}$$

$$SIC = T \log SSR + n \log T, \tag{A.2}$$

where SSR is the *sum of squared residuals,* n is the number of explanatory variables, and T is the number of usable observations. Note that by adding more explanatory variables we lose usable observations, so to compare two models with different number of explanatory variables we have to adjust the overall number of observations we use. To select the best model, the value of information criteria is to be minimized. SIC will compared to AIC usually select more parsimonious model.

The multivariate generalizations of these criteria used to determine the number of lags in the VAR system (3.46) are given by:

$$AIC = Tlog|\Sigma| + 2m, \tag{A.3}$$

$$SIC = Tlog|\Sigma| + m \log T, \tag{A.4}$$

where $|\Sigma|$ is the determinant of the estimated variance-covariance matrix Σ of the residuals from the model and m is the total number of parameters estimated in the model.

A.2 Stationarity Testing

We say that a time series is *covariance stationary* if and only if:

1. $\mu_t = \mu_{t-s} = \mu < \infty$ for all t, s.
2. $var(y_t) = var(y_{t-s}) = \sigma^2 < \infty$ for all t, s.
3. $cov(y_t, y_{t-s}) = cov(y_{t-j}, y_{t-j-s}) < \infty$ for all t, s and j.

To test the stationarity of the dividend process and of the time series entering the VAR model, the ADF test and the KPSS test are employed.

A.2.1 Dickey-Fuller Tests

There are two similarly named tests widely used for testing the presence of unit root in time series assumed to be generated with $AR(p)$ processes. The first one was developed by Dickey & Fuller (1979) and can be applied only for data assumed to be generated with an $AR(1)$ process. The augmented version of this test, the augmented Dickey-Fuller test, is its extension for a general $AR(p)$ process. The augmented Dickey-Fuller test is based on testing the null hypothesis $\sum_{i=1}^{p} a_i = 1$, i.e. the time series contains a unit root, against the alternative $\sum_{i=1}^{p} a_i < 1$, which is a necessary condition for the stationarity of the generated time series. According to Kočenda & Černý (2007), the shortcoming of this test is its low power. This means that the test has a high chance of an error of the second type, in other words, the probability of not rejecting the false H_0 is high. That is why it is of use to employ another test to determine the stationarity correctly.

A.2.2 KPSS Test

This test owes its name to Kwiatkowski *et al.* (1992). In contrast with the ADF test, the null hypothesis of the KPSS tests is that the time series is stationary. Because of the different null hypotheses of both tests it is ideal to combine them when testing for stationarity. For detailed description of the test see again e.g. Kočenda & Černý (2007).

A.3 Ljung-Box Q-test

The Ljung-Box Q-test attributed to Box & Ljung (1978) is used to find autocorrelations in the first k lags of a time series, where k is arbitrary stated. The test is based on the Ljung-Box Q-statistic defined as:

$$Q = T(T+2)\sum_{i=1}^{k}\frac{\hat{\rho}_i{}^2}{T-i},$$

where $\hat{\rho}_i$ are elements of the sample autocorrelation function. Under the null hypothesis that all autocorrelations up to lag k are zero, the Q-statistic is χ^2 distributed with k degrees of freedom. This test is used for analyzing the residuals that should not contain autocorrelations if the ARIMA model is estimated correctly. This test is also often used to indirectly test, whether residuals from the estimated model can be *iid*. Obviously, when there is autocorrelation between residuals, they cannot be *iid*.

A.4 Hodrick-Prescott Filter

Hodrick-Prescott filter is used to separate the cyclical component of a time series from its trend. The filter was proposed already by Whittaker (1923). The method assumes that time series y_t is made of a trend component τ_t and a cyclical component c_t such that $y_t = \tau_t + c_t$. Given a positive smoothing factor λ, the trend component τ_t is obtained by solving the following:

$$\min_{\tau} = \left(\sum_{t=1}^{T}(y_t-\tau_t)^2 + \lambda\sum_{t=2}^{T-1}\left[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1)}\right]^2\right).$$

The higher λ, the smoother the filtered series is. The general practice is to use λ equal to 1600 for quarterly data.

A.5 F-Test Used in the Granger Causality Analysis

To test the null hypothesis that x_t does not Granger cause y_t, we estimate the reduced form of a VAR model that we want to test. This is called the unrestricted {VAR model. After that, we estimate the same model but without including any lags of x_t in the regression (the restricted model). A simple F-test is used to compare the two models and thus to assess the explanatory power of x_t. The F-statistic is given by:

$$F = \frac{(SSR_r - SSR_u)/p}{SSR_u/(T - Np - 1)},$$

where SSR_u and SSR_r stand for the sum of squared residuals of the unrestricted and restricted regressions, respectively, p is the number of restrictions (lags), T is the number of observations and N is the number of equations in the system. Under the null hypothesis, the F-statistics has an F-distribution with p and $T - Np - 1$ degrees of freedom.

Appendix B

Figures and Tables not Presented in the Main Text

Figure B.1: Ljung-Box Q-Test for the Dividend Process Residuals

Correlogram of DIVIDEND_PROCESS_RESID

Sample: 1871 2014
Included observations: 141

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.029	-0.029	0.1212	0.728
		2	-0.001	-0.002	0.1215	0.941
		3	-0.101	-0.101	1.6098	0.657
		4	-0.114	-0.122	3.5388	0.472
		5	-0.057	-0.068	4.0191	0.547
		6	-0.074	-0.094	4.8361	0.565
		7	0.020	-0.015	4.8934	0.673
		8	-0.117	-0.153	6.9795	0.539
		9	0.065	0.017	7.6222	0.573
		10	0.084	0.060	8.7024	0.561
		11	-0.043	-0.079	8.9830	0.623
		12	-0.120	-0.164	11.222	0.510
		13	0.118	0.120	13.415	0.416
		14	0.053	0.055	13.860	0.460
		15	-0.070	-0.107	14.642	0.477
		16	-0.113	-0.163	16.691	0.406
		17	-0.045	-0.029	17.021	0.453
		18	-0.109	-0.121	18.986	0.393
		19	0.058	-0.020	19.533	0.423
		20	0.164	0.085	23.999	0.242
		21	-0.147	-0.176	27.631	0.151
		22	-0.024	-0.099	27.726	0.185
		23	0.059	0.010	28.311	0.204
		24	0.119	0.076	30.737	0.162
		25	-0.024	-0.017	30.836	0.195
		26	0.023	-0.011	30.930	0.231
		27	0.092	0.085	32.436	0.216
		28	-0.011	0.048	32.459	0.256
		29	0.080	0.067	33.602	0.254
		30	-0.004	0.029	33.604	0.297
		31	-0.133	-0.041	36.833	0.217
		32	-0.139	-0.111	40.429	0.146
		33	0.041	-0.058	40.745	0.166
		34	0.032	0.020	40.939	0.192
		35	-0.104	-0.075	43.002	0.166
		36	0.089	0.041	44.508	0.156

Source: Author's computations, Eviews

Figure B.2: Ljung-Box Q-Test for the Bubble Process Residuals

Correlogram of BUBBLE_RESIDUALS

Date: 05/05/14 Time: 23:53
Sample: 1871 2014
Included observations: 143

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.032	-0.032	0.1535	0.695
		2	-0.181	-0.182	4.9822	0.083
		3	0.117	0.108	7.0255	0.071
		4	-0.022	-0.051	7.1009	0.131
		5	-0.109	-0.073	8.8781	0.114
		6	0.095	0.071	10.258	0.114
		7	0.139	0.125	13.223	0.067
		8	-0.069	-0.020	13.961	0.083
		9	-0.015	0.006	13.995	0.123
		10	0.226	0.199	21.977	0.015
		11	0.029	0.074	22.108	0.024
		12	-0.064	0.019	22.751	0.030
		13	0.026	-0.023	22.857	0.043
		14	0.088	0.093	24.115	0.044
		15	-0.132	-0.092	26.918	0.029
		16	-0.017	-0.032	26.966	0.042
		17	0.124	0.016	29.513	0.030
		18	-0.052	-0.031	29.965	0.038
		19	-0.035	-0.012	30.176	0.050
		20	0.147	0.056	33.799	0.028
		21	-0.068	-0.088	34.579	0.031
		22	-0.178	-0.128	40.037	0.011
		23	0.052	-0.007	40.509	0.013
		24	0.109	0.053	42.571	0.011
		25	-0.061	0.017	43.235	0.013
		26	-0.053	-0.063	43.731	0.016
		27	0.137	0.095	47.099	0.010
		28	-0.043	0.005	47.435	0.012
		29	-0.040	0.057	47.731	0.016
		30	0.153	0.084	52.045	0.008
		31	0.041	0.075	52.351	0.010
		32	-0.125	0.008	55.285	0.006
		33	0.031	0.036	55.466	0.008
		34	0.151	0.095	59.788	0.004
		35	-0.092	-0.034	61.397	0.004
		36	-0.013	0.021	61.429	0.005

Source: Author's computations, Eviews

Figure B.3: VAR System Estimation Results

Vector Autoregression Estimates

Vector Autoregression Estimates
Date: 05/05/14 Time: 23:39
Sample (adjusted): 1959Q4 2013Q4
Included observations: 217 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	LOG_REAL_G	D_LOG_CPI	D_LOG_MB_R	FED_FUNDS_	D_LOG_REAL	D_LOG_REAL
LOG_REAL_GDP_GAP_2	1.002392	0.123326	-0.372194	58.04492	-0.081608	-0.789427
	(0.07242)	(0.05024)	(0.38971)	(11.1553)	(0.62070)	(0.26154)
	[13.8420]	[2.45472]	[-0.95506]	[5.20337]	[-0.13148]	[-3.01835]
LOG_REAL_GDP_GAP_2	-0.112827	-0.085914	0.394003	-36.81062	0.167736	0.291851
	(0.07304)	(0.05067)	(0.39306)	(11.2511)	(0.62603)	(0.26379)
	[-1.54475]	[-1.69549]	[1.00241]	[-3.27172]	[0.26793]	[1.10637]
D_LOG_CPI(-1)	-0.010426	0.226105	1.303327	10.03922	0.475908	-0.587023
	(0.09178)	(0.06368)	(0.49392)	(14.1384)	(0.78669)	(0.33148)
	[-0.11360]	[3.55090]	[2.63873]	[0.71007]	[0.60495]	[-1.77089]
D_LOG_CPI(-2)	0.151250	0.412366	0.288666	24.70407	0.499760	-0.118838
	(0.09510)	(0.06598)	(0.51180)	(14.6501)	(0.81516)	(0.34348)
	[1.59036]	[6.24988]	[0.56402]	[1.68628]	[0.61309]	[-0.34598]
D_LOG_MB_REAL_2009	-0.008500	-0.004686	0.479920	1.019468	-0.157349	0.116014
	(0.01346)	(0.00934)	(0.07242)	(2.07307)	(0.11535)	(0.04860)
	[-0.63158]	[-0.50188]	[6.62668]	[0.49177]	[-1.36411]	[2.38689]
D_LOG_MB_REAL_2009	0.024581	0.025760	-0.124787	3.249062	-0.038201	0.061364
	(0.01407)	(0.00976)	(0.07570)	(2.16681)	(0.12057)	(0.05080)
	[1.74753]	[2.63968]	[-1.64851]	[1.49947]	[-0.31685]	[1.20790]
FED_FUNDS_RATE(-1)	-0.000246	0.000956	-0.004097	0.700286	-0.004433	0.000745
	(0.00046)	(0.00032)	(0.00246)	(0.07055)	(0.00393)	(0.00165)
	[-0.53773]	[3.01012]	[-1.66201]	[9.92543]	[-1.12918]	[0.45057]
FED_FUNDS_RATE(-2)	-0.000110	-0.000415	0.001742	0.225987	0.003158	0.000445
	(0.00045)	(0.00031)	(0.00241)	(0.06885)	(0.00383)	(0.00161)
	[-0.24527]	[-1.33826]	[0.72441]	[3.28212]	[0.82416]	[0.27536]
D_LOG_REAL_FUNDAM	0.019125	0.005344	-0.006430	3.164727	-0.177373	0.083230
	(0.00824)	(0.00572)	(0.04434)	(1.26934)	(0.07063)	(0.02976)
	[2.32094]	[0.93476]	[-0.14501]	[2.49320]	[-2.51136]	[2.79664]
D_LOG_REAL_FUNDAM	0.008455	0.003465	0.075212	2.337459	-0.245233	-0.014775
	(0.00822)	(0.00570)	(0.04423)	(1.26619)	(0.07045)	(0.02969)
	[1.02867]	[0.60767]	[1.70031]	[1.84606]	[-3.48080]	[-0.49770]
D_LOG_REAL_BUBBLE(-	0.032865	0.013553	0.004562	-2.339409	0.119574	0.751325
	(0.01968)	(0.01365)	(0.10592)	(3.03182)	(0.16870)	(0.07108)
	[1.66985]	[0.99260]	[0.04307]	[-0.77162]	[0.70881]	[10.5697]
D_LOG_REAL_BUBBLE(-	0.030841	-0.016827	0.013765	4.644619	0.002971	-0.074307
	(0.01853)	(0.01286)	(0.09973)	(2.85473)	(0.15884)	(0.06693)
	[1.66421]	[-1.30881]	[0.13803]	[1.62699]	[0.01870]	[-1.11021]
D_LOG_REAL_OIL_PRIC	-0.000820	0.020815	-0.076726	0.447884	-0.033366	0.001810
	(0.00347)	(0.00241)	(0.01867)	(0.53454)	(0.02974)	(0.01253)
	[-0.23631]	[8.64618]	[-4.10863]	[0.83788]	[-1.12182]	[0.14441]
R-squared	0.803380	0.667692	0.278528	0.918747	0.118589	0.651502
Adj. R-squared	0.791814	0.648145	0.236089	0.913967	0.066742	0.631002
Sum sq. resids	0.009738	0.004687	0.282009	231.0707	0.715398	0.127020

Source: Author's computations, Eviews

Figure B.4: Granger Causality Testing

Dependent variable: LOG_REAL_GDP_GAP_2009$_

Excluded	Chi-sq	df	Prob.
D_LOG_CPI	3.076641	2	0.2147
D_LOG_MB_R	3.118127	2	0.2103
FED_FUNDS	4.081029	2	0.1300
D_LOG_REAL	6.101060	2	0.0473
D_LOG_REAL	18.90189	2	0.0001
All	27.81345	10	0.0019

Dependent variable: D_LOG_CPI

Excluded	Chi-sq	df	Prob.
LOG_REAL_G	6.747730	2	0.0343
D_LOG_MB_R	7.779718	2	0.0204
FED_FUNDS_	23.41678	2	0.0000
D_LOG_REAL	1.159354	2	0.5601
D_LOG_REAL	1.722372	2	0.4227
All	56.19426	10	0.0000

Dependent variable: D_LOG_MB_REAL_2009$

Excluded	Chi-sq	df	Prob.
LOG_REAL_G	1.035747	2	0.5958
D_LOG_CPI	11.31078	2	0.0035
FED_FUNDS_	7.299381	2	0.0260
D_LOG_REAL	2.970786	2	0.2264
D_LOG_REAL	0.058413	2	0.9712
All	17.21460	10	0.0697

Dependent variable: FED_FUNDS_RATE

Excluded	Chi-sq	df	Prob.
LOG_REAL_G	33.04157	2	0.0000
D_LOG_CPI	5.839805	2	0.0539
D_LOG_MB_R	4.213663	2	0.1216
D_LOG_REAL	8.942955	2	0.0114
D_LOG_REAL	2.930377	2	0.2310
All	64.80157	10	0.0000

Dependent variable: D_LOG_REAL_FUNDAMENTAL

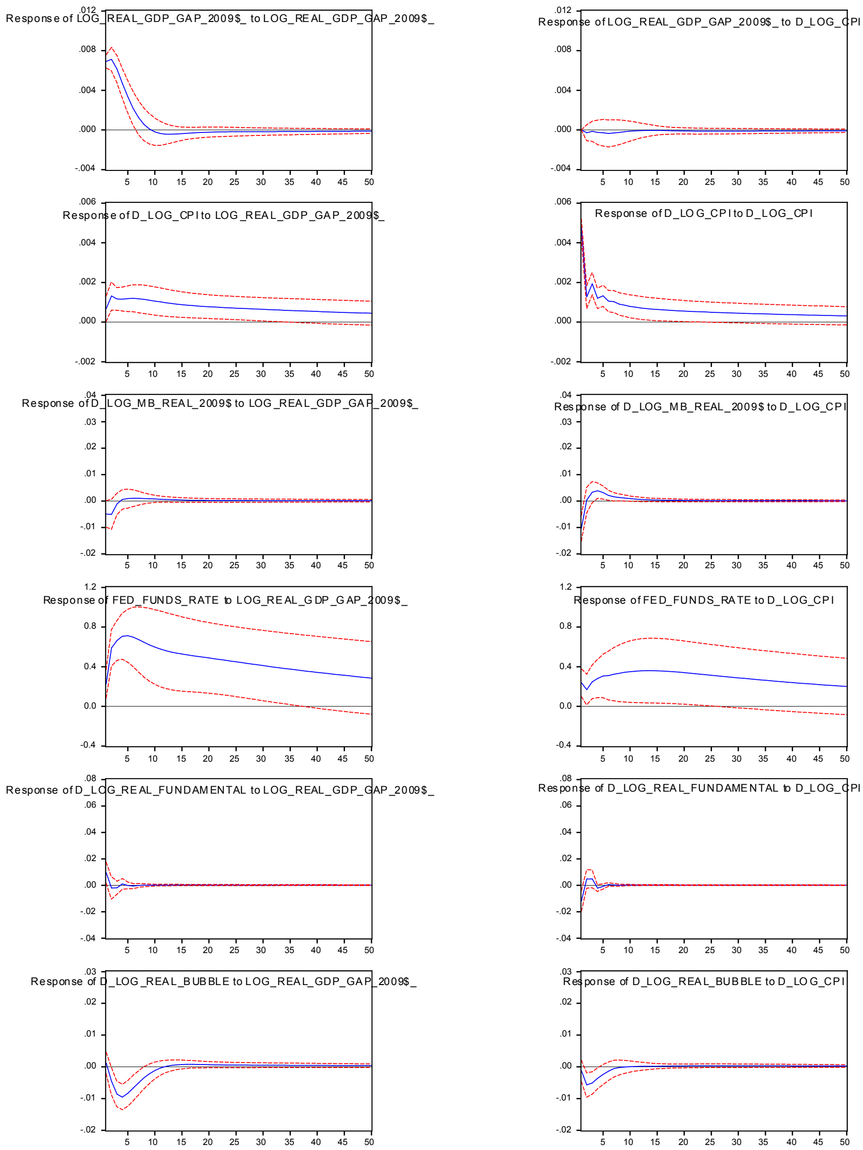
Excluded	Chi-sq	df	Prob.
LOG_REAL_G	0.111552	2	0.9458
D_LOG_CPI	1.425585	2	0.4903
D_LOG_MB_R	3.128176	2	0.2093
FED_FUNDS_	1.607221	2	0.4477
D_LOG_REAL	1.039583	2	0.5946
All	7.770128	10	0.6513

Dependent variable: D_LOG_REAL_BUBBLE

Excluded	Chi-sq	df	Prob.
LOG_REAL_G	18.06397	2	0.0001
D_LOG_CPI	4.991912	2	0.0824
D_LOG_MB_R	13.08783	2	0.0014
FED_FUNDS_	3.496588	2	0.1741
D_LOG_REAL	8.345992	2	0.0154
All	66.35530	10	0.0000

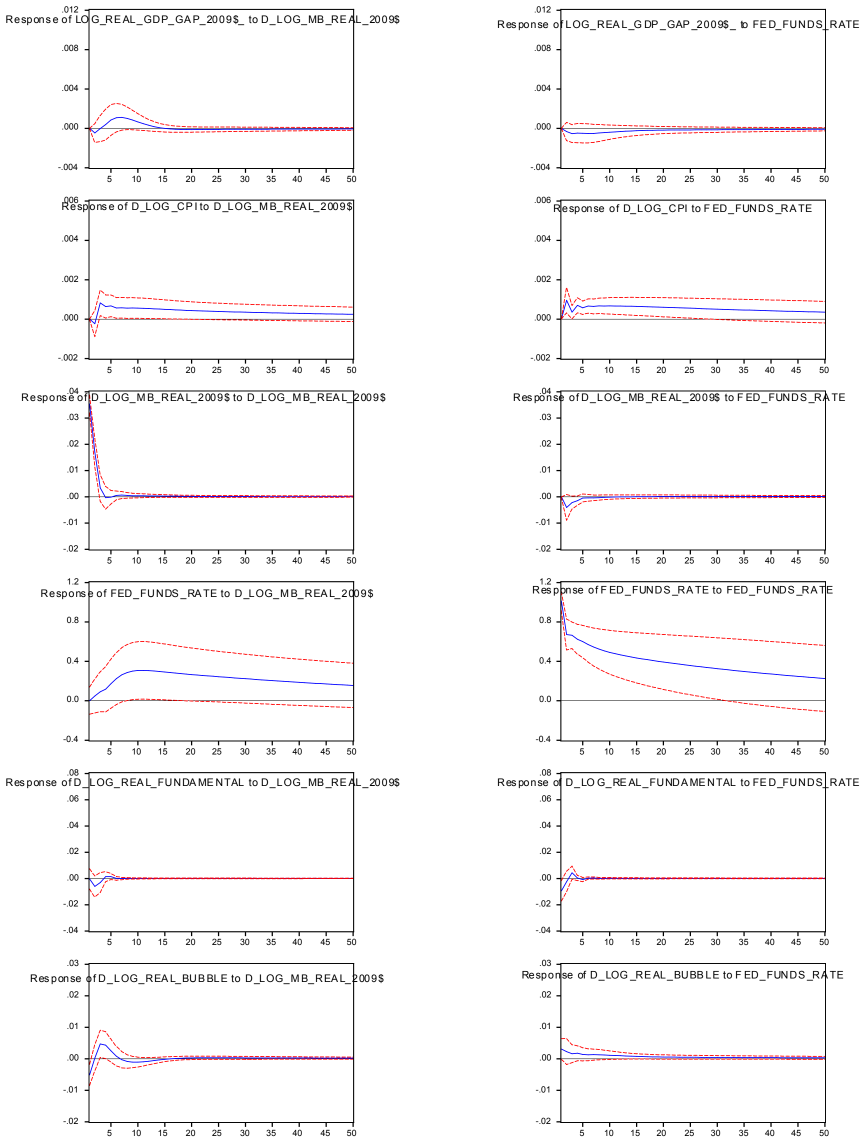
Source: Author's computations, Eviews

Figure B.5: Responses to Cholesky One S.D. Innovations +/- 2 S.E. (Part 1)



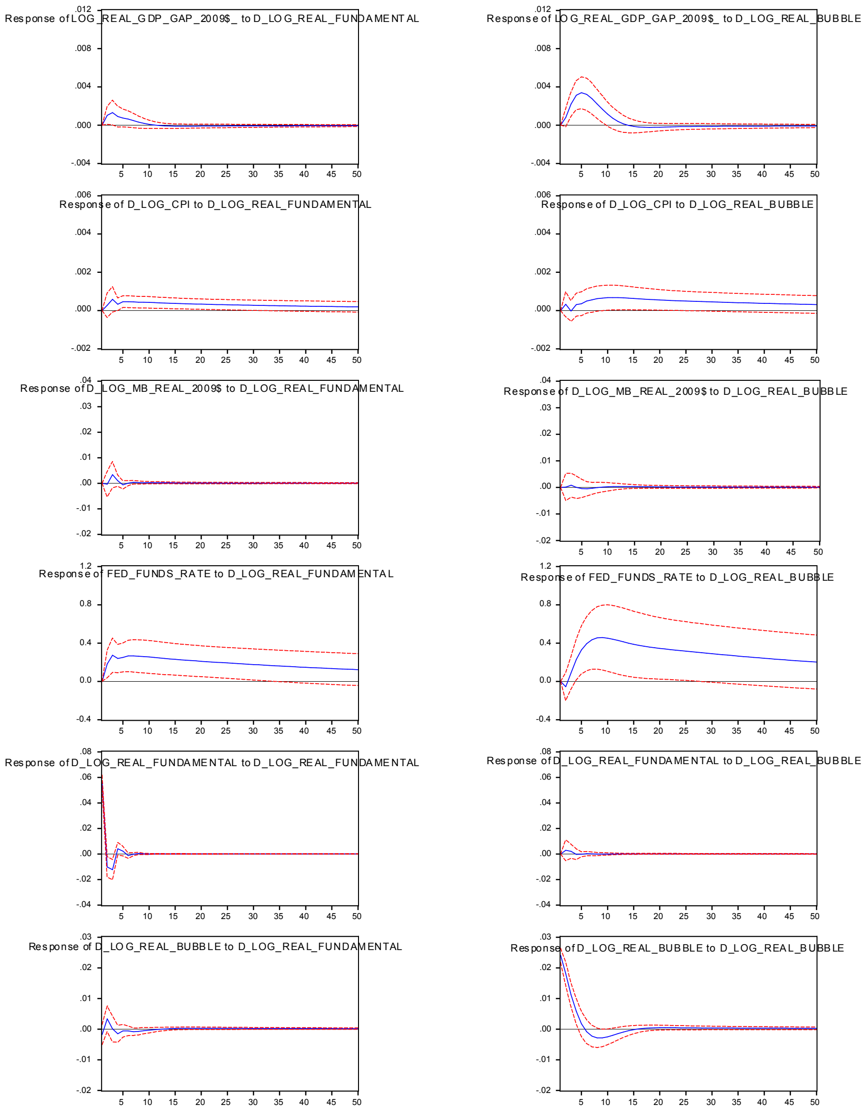
Source: Author's computations, Eviews

Figure B.6: Responses to Cholesky One S.D. Innovations +/- 2 S.E. (Part 2)



Source: Author's computations, Eviews

Figure B.7: Responses to Cholesky One S.D. Innovations +/- 2 S.E. (Part 3)



Source: Author's computations, Eviews