Charles University in Prague

Faculty of Social Sciences
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MASTER THESIS

Wealth inequality in dynamic stochastic general equilibrium models

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Declaration of Authorship

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Prague, May 15, 2014

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Here I would like to thank my diploma supervisor Josef Stráský for his valuable comments and help with writing my thesis.

Abstract

In my diploma thesis I propose a dynamic stochastic general equilibrium model to describe economic inequality. The model combines two approaches that were traditionally used to model inequality - first, it features two classes of agents that differ in their ownership of capital and second, each class consists of heterogeneous agents who are subject to uninsurable idiosyncratic shocks. This combination allows the two classes to behave in a fundamentally different way while maintaining the individual character of agents in the economy - a feature that has not been modeled before but which adequately describes the empirical reality. I show that the model with classical RBC structure and a single wage underestimates the observed inequality. When the wage differential is introduced through different taxation of the two classes, the model matches empirical inequality much better. Further I argue that the government can significantly reduce inequality at a relatively small cost in terms of output lost. Finally using Theil coefficient decomposition, I show how much of the total inequality is attributable to between-class and within-class inequalities.

JEL Classification	E10, E13, E21, E22, E24, E62, H23, C68
Keywords	DSGE, heterogenous agents, inequality, redistribution, perturbation methods

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Abstrakt

Ve své diplomové práci navrhuji model, který popisuje ekonomickou nerovnost. Model kombinuje dva přístupy, které byly tradičně používány k popisu nerovnosti - zaprvé obsahuje dvě třídy agentů, které se liší vlastnictvím kapitálu, a za druhé každá z těchto tříd je tvořena heterogenními agenty, kteří jsou vystaveni individuálním nepojistitelným šokům. Tato kombinace umožňuje, aby se jedna třída chovala velmi odlišně od druhé, a přitom zachovává individuální charakter jednotlivých agentů - tento prvek nebyl dosud v rámci DSGE literatury modelován, ačkoli adekvátně popisuje současnou realitu. V práci ukazuji, že model s klasickou RBC strukturou a jednotnou mzdou výrazně podhodnocuje pozorovanou nerovnost ve společnosti. Rozdílné mzdy lze v modelu zavést nepřímo pomocí odlišného zdanění jednotlivých tříd. Výsledky modelu pak lépe odpovídají empirickým zjištěním. Z modelu dále vyplývá, že vláda může významně snížit nerovnost za cenu relativně malého snížení celkového produktu. Nakonec pomocí dekompozice Theilova koeficientu ukazuji, kolik z celkové nerovnosti lze připsat nerovnosti uvnitř tříd a kolik nerovnosti mezi třídami.

Klasifikace JEL	E10, E13, E21, E22, E24, E62, H23, C68
Klíčová slova	DSGE, heterogenní agenti, nerovnost, přerozdělení, perturbační metody

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Acronyms

CEO Chief executive officer

CPS Current population survey

CRRA Constant relative risk aversion

DSGE Dynamic stochastic general equilibrium

F.O.C. First order condition

IRF Impulse response function

LAMP Limited asset market participation

OLG Overlapping generations

RBC Real business cycle

SIM Standard incomplete markets

US United States

Master Thesis Proposal

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Proposed Topic:

Wealth inequality in dynamic stochastic general equilibrium models

Topic Characteristics:

My thesis will focus on modeling inequality within the DSGE framework. I will construct a model which incorporates inequality as one of the exogenous variables and I will compare how the model performs vis-à-vis a model with no inequality. It will feature heterogeneous agents which differ in their ownership of capital in the economy. In my thesis, I will study how this underlying wealth inequality translates into income and consumption inequality and what are the possible impacts on other main macroeconomic variables (namely investment).

As such, my thesis will be an extension and generalization of a baseline model described by Kumhof and Ranciére (2010). In comparison to their model, I will allow the inequality parameter (possibly calibrated using Gini coefficient) to vary and I will study how the degree of inequality influences the model.

In addition, I will be interested in fiscal policy which tries to counter the possible negative effects of wealth inequality. I will compare the possible tax schemes based on their efficiency and discuss whether they can substitute the removal of the underlying inequality.

Hypotheses:

1. Wealth inequality has a negative impact on investment.
2. Uncertainty regarding wealth distribution leads to underinvestment.
3. Fiscal redistribution policy can reduce the negative effect of inequality but it cannot completely erase it.
4. Tax on capital is more efficient than tax on labor as inequality increases

Methodology:

For the purpose of my thesis I will use a standard DSGE modeling methods to construct and estimate the model. The economy will consist of two kinds of households – rich (capitalist) and poor ones, which will differ in the ownership of capital. This results in unequal distribution of capital income throughout the economy and creates disturbances in the economy.

The model will incorporate three sectors (parts of GDP) – consumption, investment and government. The model will be for a closed economy. The government will use redistribution policies to counter the negative effects of wealth inequality.

The thesis uses an equilibrium approach with one unique equilibrium, which is given by the estimation methodology which uses log-approximation around the steady state. The resulting system of differential equations will be solved via the Dynare toolkit in Matlab.

Outline:

1. Introduction
2. Literature review and discussion of inequality in general from both theoretical and empirical perspectives
3. Model description
4. Calibration and model solution
5. Presentation and discussion of results, graphical outcomes – simulated series, impulse responses
6. Discussion of alternatives, sensitivity analysis
7. Conclusion

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Author **Supervisor**

Chapter 1

Introduction

Inequality is on the rise. According to all empirical measures, inequality increased dramatically over the past 30 years all over the world. Especially in the case of US, inequality is almost a defining trait of its economy. From this very reason it has become critical to incorporate inequality into our models that try to describe the economy.

The traditional modelling framework of general stochastic equilibrium models that rely on a single representative agent is not able to include almost any kind of inequality, not to mention to describe how inequality actually arises or what are its causes and effects. Since one of the main purposes of DSGE models is to deliver policy implications for the government, the representative agent model must stay silent when asked the most critical question of our time - what should we do about inequality?

DSGE literature that does incorporate inequality can be divided into two streams - first, heterogeneous classes of agents and second, heterogeneous agents with idiosyncratic shocks. The former type of models divides society into several heterogenous classes and describes the behavior of each class separately. The classes themselves however are still defined as representative agents and the model cannot therefore reproduce inequality measures that rely on individual incomes such as the Gini coefficient. The latter type of models on the other hand completely abandons the representative agent framework and describes an economy where every agent faces idiosyncratic shocks (or has idiosyncratic preferences) which cannot be insured. This type of models is also known as incomplete markets models.

In my thesis I use the advantageous features of both types of models and construct a model with two heterogenous classes that differ in their ownership of capital (one class holds no capital) and each of those classes consists of heterogeneous agents who face idiosyncratic shocks to their employment. The model itself has a classical RBC nature with only real variables and no market imperfections.

Using this combined model it is possible to study to what degree can the empirically observed inequality be explained by real variables and parameters and how individual parameters contribute to inequality. The hypothesis is that with perfect competition on labor and capital markets and a single resulting wage and interest rate for all agents, the theoretical inequality will be much lower than its empirical counterpart. I further investigate, what are the modelled relations between income, consumption and wealth inequality and which of these is hardest to match with empirical evidence.

To provide policy recommendation for the government I introduce redistribution between the two classes and asses what are the costs in terms of lost output for reducing inequality. And finally using the decomposition of Theil inequality coefficient, I examine how much of the overall inequality can be attributed to within-class and between-class inequality and therefore what are the limits of the government's ability to reduce inequality using between-class redistribution.

The rest of the thesis is organized as follows: Chapter two reviews theoretical literature on inequality and summarizes what models were traditionally used to describe it. Chapter three surveys empirical literature on inequality and provides empirical evidence on rising inequality in the US. Chapter four describes the proposed DSGE model and its solution algorithm. Chapter five presents the results of the thesis and discusses possible policy implications and model extensions. Chapter six concludes.

Chapter 2

Review of theoretical literature on inequality

2.1 Types of inequality

When talking about inequality, one must carefully distinguish, what is the matter in question. As Amartya Sen (1995) puts it, the "inequality of what" is the central question with far reaching policy consequences. To reduce inequality always means to reduce one specific type.

It has become popular to distinguish the so called "basal" or fundamental inequality on one hand and income or wealth (in general) inequality on the other. The former is connected with the heterogeneous nature of human beings and encompasses things like gender, age, health status and education while the latter is usually the outcome of the former and includes wealth, income and consumption. As the lists suggest, the division is not made along the lines inborn/gained characteristics but is rather related to measurement and policy issues. It is easy to quantify income and consumption in terms of money and construct indices like the Gini coefficient (for the latest Gini report for the US see Kenworthy & Smeeding (2013)), but it is not so easy to measure and compare health status, skills and various human capacities in general.

In the DSGE modeling framework this distinction appears in the form of a division between exogenous and endogenous variables. The basal inequality enters the model on the input side - it influences model selection, types of variables used, assumptions on the utility function etc. Income inequality on the other hand is the outcome of the model, it is endogenous. The model itself is then merely a complex transmission mechanism which creates output inequality out of input inequality.

It is apparent that the basal inequality determines what possible outcome inequalities might arise in the model. When there is no inequality on the input side, as in the classical RBC representative agent model (King & Rebelo 2000), there can also be no inequality on the output side. The question what types of inequality should be admitted at the input side is therefore of crucial importance.

2.2 Heterogeneity in external shocks

We start the discussion of various types of heterogeneity with a class of models where agents (households) are ex ante identical but face uninsurable idiosyncratic shocks which in time create a distribution of incomes. This type of model is usually referred to as "Standard incomplete markets model (SIM)" (Heathcote *et al.* 2009b) or the "Bewley model" (Ljungqvist & Sargent 2004). This would be the model of choice for the classical liberal stream of thinking, because at the start, all individuals have equal opportunities (all people are equal) and the resulting inequality is just a result of the stochastic nature of the world. In most settings, there is also perfect mobility within the income distribution.[1] The idiosyncratic shocks are usually shocks to income or to hours worked, which translate into income. Clearly, the larger is the variance of the shocks, the wider is the resulting distribution of incomes. This is both an advantage and a disadvantage. The upside is that through different calibrations, we can get as close to the Gini coefficient as we want (Castaneda *et al.* 2003). But this also means that the model is somewhat arbitrary and lacks explanatory power. It also has quite straightforward policy implications - the the role of government would be to completely redistribute all incomes, i.e. to provide an ex post insurance, for which households would opt ex ante (due to risk aversion) but which they cannot achieve on their own ex post due to uninsurable idiosyncratic shocks. This implication actually closely mimics Rawls (1971) argument for redistributive justice (the veil of ignorance).

There are several extensions of this model which maintain the ex ante iden-

[1]The mobility depends on how the formation of capital is modeled. Capital can function as a personal buffer (self-insurance) against idiosyncratic shocks. Therefore if an individual receives positive shocks at the beginning and builds up his capital, he is likely to remain at the top of income distribution even after some time. The income mobility is thus reduced.

tity and add more sources (or channels) of heterogenous shocks. One possible line is heterogeneity in labor market, where we add a job fluctuation (some people are hired, some fired), which is usually modeled as idiosyncratic Markov chain. Postel-Vinay & Turon (2006) and Lise (2011) belong to this category.

Another important source of heterogeneity is health status. Even though it cannot be modeled per se, it can be introduced into the model in the form of events (diseases) which occur with some given probability and have various effects on income, employment or are associated with increased expenditures. The possibility of such shocks has a non-negligible impact on saving decisions (Nardi *et al.* 2006).

The overlapping generation models (OLG) represent a heterogeneity with respect to age (Huggett 1996). It is also possible to incorporate family structure into the model. For a detailed description of possible sources of heterogeneity and extensions of the basic SIM model, see Heathcote *et al.* (2009b).

2.3 Inequality in capability

Perhaps the greatest heterogeneity arises in differences in human capabilities. Amartya Sen (1999) characterizes all poverty as capability deprivation - the scope of what different people can do, can achieve, varies significantly. This means that people have various skills and traits ex ante and not only as an outcome of individual shocks. The idiosyncratic difference in skills and abilities in turn leads to difference in productivity and wage, which are arguably the most important factors contributing to income and wealth inequality (as I will discuss further in the result section of this thesis).

Different capabilities can also manifest as a denial of access of a part of the population to some key institution. Often this institution are financial markets, which means that the rich can take loans, buy stocks and bonds, while the poor must live only from their labor income. This assumption of limited asset market participation ("LAMP") leads to a distinction between two types of agents - Ricardian households who can smooth their consumption over time (and for whom the Ricardian equivalence holds) and the so called "rule-of-thumb" households who have no saving, no wealth and spend their entire income each period. Many models show that the LAMP assumption can generate a high degree of income and consumption inequality: Motta & Tirelli (2012), Swarbrick (2012), Guvenen (2006).

The LAMP assumption is likely to reflect reality as empirical estimates suggest that around 35 % to 50 % of households hold almost no wealth and live paycheck to paycheck (Mankiw 2000a, Forni *et al.* 2007). The disadvantage of the model is that the separation into Ricardian and Non-Ricardian households is exogenous, while in reality everyone has legally guaranteed equal access to financial markets. To actively participate in such markets is then an endogenous decision of households stemming from their preferences and financial options.

Other possibility of capability inequality is to allow households to choose whether to become entrepreneurs or workers. Being an entrepreneur yields additional income but is associated with higher risk. This model predicts substantial accumulation of wealth in the hands of entrepreneurs (Quadrini 2000).

One of the choices most influencing human capability is education. People choose different amounts of schooling (and various qualities) which then shapes their income profiles as well as almost all other aspects of their lives. Modelling education is a difficult task, but there has been some pioneering attempts trying to do so. Huggett *et al.* (2011) construct a model where agents in each period choose whether to work or study (accumulate human capital). Agents differ in their starting value of human capital, learning ability (speed at which they accumulate human capital) and wealth. Amount of schooling in this model is derived endogenously and depends on the starting value of human capital and learning ability. Simply put, smarter agents get more education and experience higher income once they are out of school. In real life however, even if a kid is smart, he may be denied education because its costs are prohibitively high (or there is just no education available for a given geographical region).

Heathcote *et al.* (2008) construct a model where individuals face idiosyncratic heterogeneous costs associated with education. Therefore if the cost is to high, individual chooses not to get higher education.[2]

2.4 Heterogeneity in preferences

It is often the case that inequality in capabilities arises as a result of heterogeneity in preferences rather than heterogeneity in external shocks or other circumstances. When we consider the fact that a majority of personal income is already determined before entering the labor market - almost 61 % (Huggett *et al.* 2011) - it seems rational to assume that people differ in their preferences

[2]This model is interesting also in many other ways, it features gender, marriage market, generation structure, retirement and many shocks and sectors. It is one of the most parsimonious models currently available.

which are set and constant throughout the entire life.[3] Therefore one person might work harder and save more than other person simply because he is more patient and values his free time less. For this reason it seems necessary to incorporate some kind of preference heterogeneity into the model.

The heterogeneity is easily added to a model that features discrete countable groups of individuals. For example the young and old in OLG models may have different preferences or males and females might differ in their risk aversion or any other parameter (see Heathcote *et al.* 2008). Another option is to assume heterogeneity in preferences at the individual level, creating a statistical distribution over some parameter. This is the approach taken by Krusell & Smith (1997). They impose a distribution over the time preference parameter and get large income inequality as a result. A similar approach is used in my thesis where a difference in time preference separates households into two classes, one of which holds assets while the other does not (thus creating the LAMP assumption mentioned in previous section).

It is important to note that the three above mentioned approaches[4] to inequality are not mutually exclusive, but they are rather complements and it is likely that the reality lies at the intersection of all three. Individuals differ in their initial amount of wealth (inheritance), skills and preferences, they choose different types of education and occupation and during their working life experience random shocks to their income, health, employment and family status. The optimal model which would aspire to mirror reality would have to include all these sources of inequality. In practice however, it is reasonable to include only some of them and focus on some specific variable of interest.

2.5 Inequality in outcomes

At the beginning of this chapter we divided all inequality into two types - input and output inequality and described various models which transform input inequalities into output inequalities. Since the centers of focus of each model are the output inequalities, which we can measure and compare across reality and models, it would be appropriate to discuss what these output (or outcome) inequalities are and how they are related to one another.

It is common to distinguish four types of outcomes - wealth, income, consumption and utility. The usual causal chain comes from wealth, which along

[3] At least they are modeled in such a way.

[4] I.e. the external shock inequality, capability inequality and preference inequality.

with labor generates income, which is used to purchase consumption, from which agents derive utility. Despite utility being the ultimate object of interest, it was traditionally understood that because utility is something subjective and unmeasurable we should turn our attention to some of its instruments (the other three variables). This cartesian assumption of subjectivity was recently challenged by a number of philosophers, most notably by Donald Davidson (2001), making the direct measurement of utility legitimate.

Since then the literature (both economic and psychological) on well-being and happiness has grown exponentially. It is therefore possible to quantify well-being inequality, using various methods and indices (for reference see Kahneman *et al.* 2003). But because the measurement process is far from being perfect and there is still a great degree of distrust in well-being economics, it is still more popular to focus on some intermediate inequality (income) and assume some further unspecified utility mapping over the instruments.

Consumption as a mediator between utility and income is somewhat neglected in the inequality literature. There are two reasons for this phenomenon: firstly consumption is much harder to measure than income (because there is no personal "consumption tax") and thus all available data comes from household surveys, and secondly consumption is so closely tied to income that it was deemed sufficient to measure income and assume some nicely behaving function between income and consumption. Recently several empirical studies suggested that income and consumption inequality are two very separate things. Krueger & Perri (2002) report that while in the last 30 years income inequality increased by 21 %, consumption inequality increased only by 10 %.[5] They also construct a model, where agents can insure themselves against income shocks and this leads to lower consumption inequality. In the standard incomplete markets model however, income inequality is always associated with consumption inequality. The relation between the two inequalities calls for further investigation.

Income inequality is also closely related to wealth inequality. Opposite to the case of consumption, wealth inequality is in reality usually much higher than income inequality. This is caused simply by the fact that wealth accumulates over time and is much more persistent than income. Furthermore a considerable part of wealth does not bear interest and sometimes even comes with a cost (maintenance for example) and therefore does not generate income difference.

[5] The particular numbers depend greatly on employed measurement methodology. Aguiar & Bils (2011) report an increase of 33 % and 17 % for income and consumption inequality, respectively, and argue that when we correct for measurement error, the difference between the two disappears entirely. They use U.S. consumption expenditure survey data.

In the modeling framework however, wealth usually means financial assets, bonds or capital in general, which is almost always interest bearing. This ambiguity in the notion of wealth, which is different in the data and in the models, is the reason why wealth inequality is rarely the focal variable and most models target income inequality instead.

2.6 Two types of models

All existing models in the current literature on inequality can be divided into two categories. The first type of models features heterogeneous agents who face uninsurable idiosyncratic shocks influencing their individual incomes. They are derived from the baseline income fluctuation "Bewley" model (Bewley 1977) and are also called incomplete market models. The result of these models is an income distribution which can be compared to the Gini coefficient. For a detailed overview of this stream of literature see for example Heathcote *et al.* (2009b) or Cagetti & De Nardi (2008).

The drawback of this type of models is that they allow heterogeneity only in stochastic distribution of parameters, shocks and preferences. There can be no fundamental dichotomy in the utility setup or in the access to various market products. It is impossible for example to deny access to financial markets to some types of households or to make other households stop working. The richest and poorest households are not different in any principal way. This also creates serious problems for the government, since it must distinguish individual households and the redistribution via taxation relies on perfect information. What we observe in reality is the opposite case - the government cannot allocate taxes individually but creates a uniform tax scheme with tax progression based on arbitrarily chosen levels of income.

The second category of models includes several types (or classes) of representative agents, who face class-specific aggregate shocks. The situation is usually modeled as a continuum of households of which a portion belongs to one class and the rest to the other. The division of households is often (but not always) done along the line Ricardian - Non-Ricardian. Inequality arises in the form of a fraction of incomes of the two classes. For reference see Motta & Tirelli (2012), Guvenen (2006) or a classical Gali *et al.* (2004).

In my thesis, I combine the two approaches thus creating inequality *between* and *within* different classes. This combination takes advantage of both types of models and offers large opportunities for modelling inequality more realistically. By maintaining the class structure, the model allows different classes of households to have radically different preferences. By introducing individual level heterogeneity into class level heterogeneity, it is possible that different types of households are able to insure themselves against different types of risk (with the rich having more insurance options than the poor). It also enables the shapes of distributions to differ significantly, for example the capitalist class will be able to produce extremely rich individuals, while the workers will be centered around the mean value. All of these features are impossible to model in either of the two approaches separately, making their combination a promising advancement.

Chapter 3

Review of empirical literature on inequality

The outcome inequality which we defined in the previous chapter includes wealth, income and consumption inequality. In this section, we will discuss how these can be measured and what are the empirical estimates for the US (we focus on US inequality because the calibration of the model proposed in this thesis is based on US stylized facts). There exists a vast plurality of inequality measures, we will however focus only on a select few, which are both most commonly used and which we are able to replicate using the model described in this thesis. And finally, we will also mention what are the causes behind rising inequality.

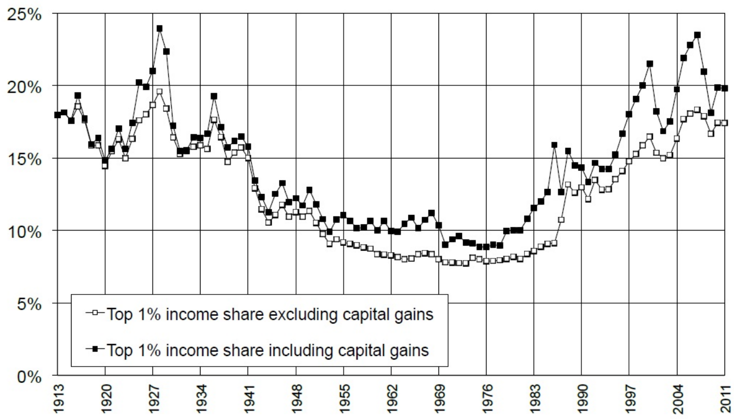
3.1 Shares of income, consumption and wealth

We start with simple measures. It is still very popular to compute various inequality shares and ratios - most commonly the shares of wealth, income or consumption of some part of population on total wealth, income or consumption. These shares are popular because they are easy to interpret and show inequality in a very straightforward way. Some of the most influential papers on inequality use these shares to show how much of economy's income and wealth belongs to top 1% best paid CEOs (Piketty & Saez 2006).

The following figure clearly depicts the evolution of the share of 1% top income percentile on total income over the past century:[6]

[6]Figure taken from Alvaredo *et al.* (2013), which is the most up-to-date paper by the same authors as cited above.

Figure 3.1: Top 1% income share in the United States

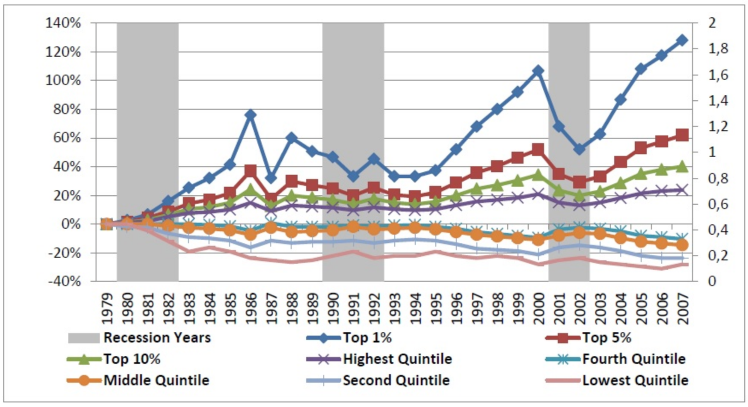


We can clearly see, that the evolution is U-shaped, reaching its minimum in late 70s and climbing to pre-war levels ever since. This is the only figure one needs when arguing about rising inequality in the United States.[7] It also directly contradicts Kuznets' original hypothesis that inequality has an inverse U shape in relation to wealth of a nation, meaning that it increases as a society gets industrialized and then decreases as the society moves to the services-based economy. The figure shows that over the last 30 years, the top 1% income share more than doubled even though the recent Great Depression somewhat decreased the rising trend.

The rise in top income shares is logically accompanied by a fall in bottom income shares as depicted on the following figure (taken from Kenworthy & Smeeding (2013)):

[7]USA generally counts among the countries with highest inequality in the world - for comparison, in most European countries, the income share of 1% richest households is half that of the US and is not U-shaped but decreases over time (Alvaredo *et al.* 2013).

Figure 3.2: Evolution of income shares in the US (1979 = 0)



The figure 3.2 presents percentage growths of various income shares since 1979, which illustrates two additional things: the income shares of everyone except the highest quintile (top 20%) have fallen and recessions seem to reduce income inequality. The current income shares of individual quintiles is summarized in the following table:[8]

Table 3.1: Income shares by quintiles

Quintile	Bottom 20%	Second 20%	Third 20%	Fourth 20%	Top 20%
Share	3.21%	8.75%	14.76%	22.35%	50.92%

Similar share ratios can be constructed for the other two variables of interest - consumption and wealth. Firstly, we will discuss consumption. The inequality in consumption is traditionally much smaller than income inequality, which is an empirical fact consistent with standard economic theory (propensity to save is increasing in income). In addition, consumption inequality also increases in time, but this increase is much smaller that the increase in income inequality (Krueger & Perri (2002), Heathcote *et al.* (2009a)). Since the data on consumption usually comes from consumption surveys while income data comes from income tax reports, it is important to establish a single measure for both inequalities so that they would be comparable. Krueger & Perri (2002) report that when we consider income and consumption for the same individuals, the trend in both inequalities is the same during expansions but diverges during recessions, in which consumption inequality decreases profoundly. The follow-

[8]Data: US census bureau, Consumer Expenditure Survey 2012 - post-tax income.

ing table presents the current consumption shares by individual quintiles:[9]

Table 3.2: Consumption shares by quintiles

Quintile	Bottom 20%	Second 20%	Third 20%	Fourth 20%	Top 20%
Share	8.6%	12.68%	16.71%	23.32%	38.63%

Among the three, wealth inequality is definitely the biggest. The shares of wealth are calculated based on US Survey of Consumer Finances and show a radical polarization of wealth ownership. The following table taken from Wolff (2011) illustrates the concentration of wealth (most recent year - 2007):

Table 3.3: Wealth shares by percentiles (net wealth)

Bottom 40%	Third 20%	Fourth 20%	Top 20%	Top 10%	Top 1%
0.2%	4%	10.9%	85%	73.1%	34.6%

The top 1% of richest households owns more than a third of total wealth in the economy and the top quintile owns practically all wealth in the US. On the other hand, bottom 40% have no assets at all. It should be noted that the net wealth measure includes also housing, whose prices plummeted during the recent crisis, so the share of bottom and especially middle quintiles is likely to be even lower now (as suggested further by the convergence of financial and net wealth Gini coefficients (see figure 3.5)). When we consider only financial wealth, then the bottom's 40% share is even negative, suggesting large indebtedness of poor households.

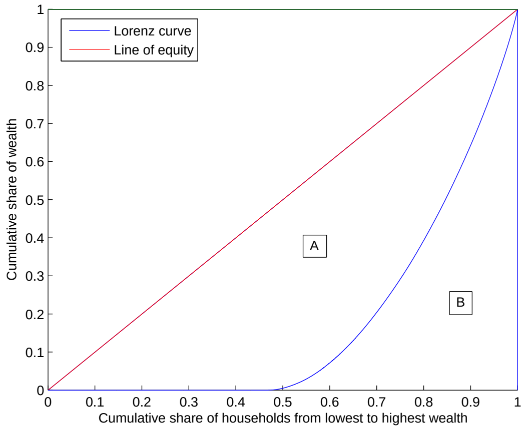
Even though the wealth inequality is already very high, it still rises over time. In the past 30 years, the wealth share of the top quintile increased by 4 percentage points, which suggests further concentration of capital among the wealthiest classes (Wolff 2011).

3.2 Gini coefficient

Perhaps the most common measure of inequality is the Gini coefficient. It measures relative inequality within a group (economy) and is derived from the Lorenz curve. The Lorenz curve plots the cumulative share of households in the economy against their cumulative income (or consumption and wealth) shares. The following figure shows an example of wealth Lorenz curve:

[9]Data: US census bureau, Consumer Expenditure Survey 2012 - using the same individuals as for income inequality.

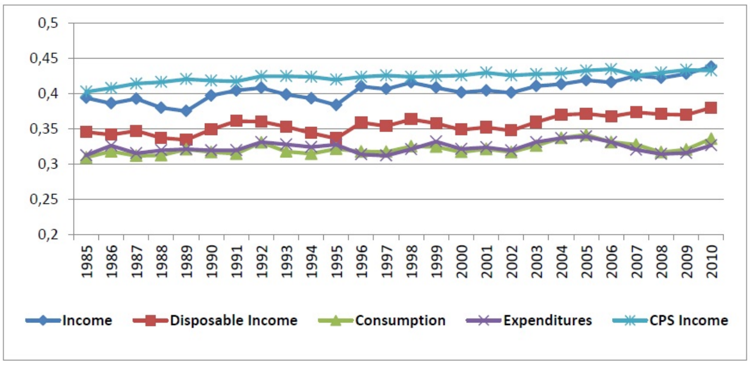
Figure 3.3: Lorenz curve (wealth)



The 45° degree line represents a line of equity - a situation where everyone holds the same wealth. The more curved the Lorenz curve is, the greater the wealth inequality. The Gini coefficient is defined as the area $\frac{A}{A+B}$. In the case of perfect equality, Gini is equal to zero (A=0) and in the case of perfect inequality (one household owns everything), it is equal to 1 (B=0).

Recently a series of Gini country reports has been published on the state of inequality in various countries. The following figure, taken from the paper on US (Kenworthy & Smeeding 2013), shows the evolution of income and consumption Gini coefficients since the 80s:

Figure 3.4: Gini coefficient for income and consumption



We can see that the Gini coefficient shows an increasing trend in all variables. The difference between the individual variables is caused by the inclusion/exclusion of medical care, education or income from food stamps along with other minor things (for a detailed description of the variables and the difference between them, see Kenworthy & Smeeding (2013) and Fisher *et al.* (2013)). The figure also confirms that consumption inequality is much lower than income inequality and it seems to increase less over time.

Similar trend applies also for the wealth Gini. The following figure presents the evolution of two wealth Gini coefficients:

Figure 3.5: Wealth Gini coefficients

The figure shows a huge increase in inequality during the 80s and another increase at the start of this century. The financial Gini is bigger than the net wealth Gini, which is caused by the house ownership of the poor and middle classes. During the recent crisis, much of this housing value evaporated, bringing the net wealth gini closer to its financial counterpart. Both Ginis are starting to get dangerously close to one, which suggests that increasingly larger shares of wealth are accumulating in the hands of increasingly smaller group of households.

3.3 Additional inequality measures

The differences between various percentiles of the income (wealth, consumption) distribution are accurately captured by the so called percentile income (wealth, consumption) ratios. They compare how much bigger is the income in the 90th or 50th percentile of income distribution compared to the income of the poor 10th percentile. This measure is also robust to large outliers at both sides of the distribution.

Meyer & Sullivan (2009) report that while income of top 90th percentile diverges from the income of the poor (and is already more than 10 times higher), consumption ratio remains fairly stable as depicted on the following two figures (taken from their paper):

Figure 3.6: Evolution of 90/10 percentile ratios

Figure 3.7: Income ratio

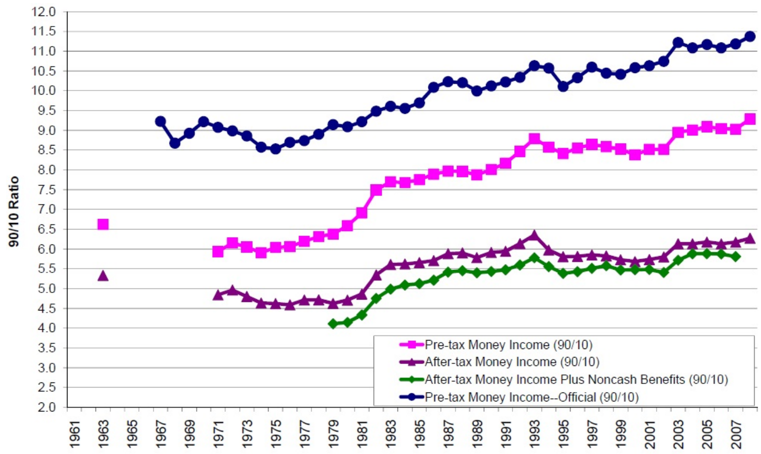
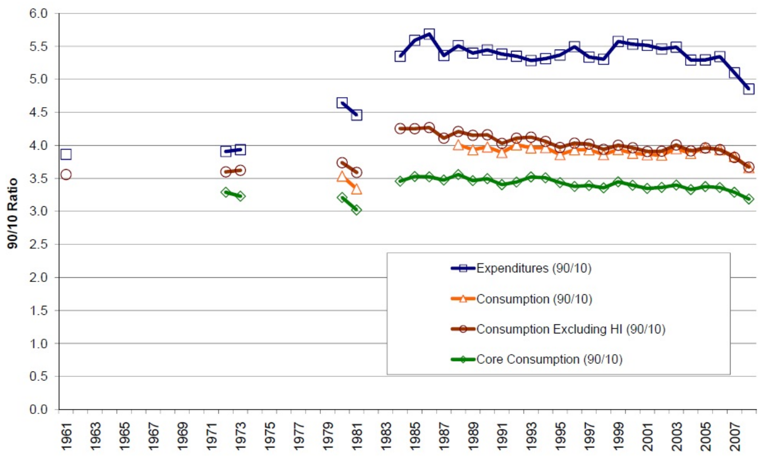


Figure 3.8: Consumption ratio



The reason for this development is that the propensity to save increases along with increasing income, which leads to higher accumulation of wealth (as reported above), while consumption remains more equitable. Another reason for this phenomenon might be the non-income benefits of the poorest 10%, which include food stamps, free healthcare or education.[10]

The figure further suggests that the increased inequality in consumption as reported by other measures is likely to be caused by the top several percentiles,

[10]Also the different definitions of consumption might play a role.

whose consumption and income are not captured by this ratio. Meyer & Sullivan (2009) also show that while the 90/50 income ratio increases over time, the 50/10 ratio remains stable, which means that especially the very rich are getting even richer, while the middle class and the poor do not diverge from each other.

The last inequality measure which we discuss is the Theil coefficient. It expresses relative entropy in income and ranges from 0 (no entropy = perfect equality) to 1 (perfect inequality).[11] The advantage over Gini and other inequality measures is its decomposability. According to the Theil coefficient, inequality in a group is equal to inequality within subgroups plus inequality between the subgroups. This feature allows us for example to assess the contribution of inequality of individual states in the US to the overall US inequality. The application of the coefficient is fairly technical and we will leave it to the result section of this thesis (5.4) along with its comparison to modelled inequality.

3.4 Causes of rising inequality

According to Kenworthy & Smeeding (2013), the consensus nowadays is that the technological change of the past several decades increased demand for highly skilled employees while at the same time there was no adequate increase in supply of skilled labor, which inevitably drove the wages of highly educated and skilled workers up. The wage differential and returns to education increased sharply (between 1979 and 2010, the wage differential between college graduate and high schooled graduate more than doubled).

Technological change not only increased wage differential, but also employment differential - low skilled workers experience unprecedented high levels of unemployment while people with university education have their jobs to a large degree secured. The labor market in the USA now resembles a winner-takes-it-all situation, especially after taking into account the rewards of top paid CEOs, which sky-rocketed in recent decades (Piketty & Saez 2006).

Further factors that might play a role are decreasing union membership, declining real value of minimum wage and the development of financial sector, which allowed much higher capital gains through leverage. Strangely enough, the role of the government plays only a minor role as taxes and transfers have

[11]In the case of a standardized Theil coefficient. Otherwise, the maximum value is equal to $log(n)$, where n is the size of the population.

stayed fairly constant since the 1970s (with the exception of the current crisis (Kenworthy & Smeeding 2013)).

Chapter 4

Model description

The model proposed in this thesis is a variation and extension of the classical RBC heterogeneous agents model with uninsurable idiosyncratic shocks and aggregate uncertainty. The model features three sectors - households, firms and government.

4.1 Households

The economy is composed of two classes of households. Each class consists of a continuum of heterogeneous agents who are subject to idiosyncratic employment shocks. The first class, from now on referred to as capitalists, forms 80% of population while the latter class called workers represents the remaining 20%.[12] The term capitalists simply means that the agents of this class are able to both accumulate capital holdings (or assets) and work, while workers get their income solely from labor and posses no assets. The name capitalists therefore does not refer to the top 5% richest as it sometime does but instead stands for the majority of agents who hold any amount of assets (however small this amount might be).

4.1.1 Capitalists' households

First, we will describe the capitalist part of the economy. This setup closely resembles the pioneering work of Krusell & Smith (1998) and its extensions Haan & Ocaktan (2009) and Preston & Roca (2007). Before formulating the optimization problem, it is necessary to clarify the notation, which features

[12] A thorough discussion of the shares of capitalists and workers in the economy can be found in section 4.6.

three sets of lower indices: the subscripts "c" and "w" stand for capitalists and workers and differentiates variables and parameters specific to the two classes; index "i" represent individuals and is used to specify which variables change across individual agents; index "t" labels the time parameter. The individual capitalist household's problem can be described as follows:

$$\max_{c_{c,i,t},\ a_{i,t+1}} \quad U_{c,i} = E_0 \sum_{t=0}^{\infty} \beta_c^t \left(\frac{c_{c,i,t}^{1-\gamma_c}}{1-\gamma_c} - P(a_{i,t+1}) \right) \tag{4.1}$$

where $c_{c,i,t}$ and $a_{i,t+1}$ represent streams of consumption and asset holdings for individual i. Note that the assets do not have a class subscript as only the capitalists hold assets rendering the index redundant. The parameters β and γ stand for the coefficients of time preference and risk aversion, respectively. The utility function is a classical CRRA function with respect to consumption. $P(a_{i,t+1})$ is the penalty function which restricts the individual debt holding by punishing households in terms of utility for holding too little capital. The inclusion of penalty function allows us to formulate the otherwise constrained optimisation problem as an unconstrained one.[13] I will use the particular specification of the penalty function described in Preston & Roca (2007):

$$P(a_{i,t+1}) = \frac{\phi}{(a_{i,t+1}+b)^2} \tag{4.2}$$

where b is the borrowing limit. Notice that when asset holding approaches the borrowing constraint (i.e. $-b$), the penalty function goes to infinity, harshly punishing the household in terms of utility. The coefficient ϕ is often referred to as barrier parameter and is calibrated in such a way so that the borrowing constraint is not violated while having small effect on optimal asset allocation.

Each period, capitalist households face the following budget constraint:

$$c_{c,i,t} + a_{i,t+1} = (1-\tau) r_t(k_t, l_t, z_t) a_{i,t} + w_t(k_t, l_t, z_t) e_{c,i,t} + (1-\delta) a_{i,t} \tag{4.3}$$

where r_t and w_t are interest rate and wage which depend on the aggregate capital, aggregate labor and productivity z_t. It is important to mention that all these aggregate variables do not depend on particular realisations of individual's asset holdings, but only on the cross-sectional average of the entire

[13] The problem with borrowing constraints in heterogeneous agents setting is that they are only occasionally binding (depending on the particular realisation of the idiosyncratic shock). The inclusion of the penalty function allows us to circumvent this problem.

population. This means that one individual cannot by his asset allocation influence the interest rate on his assets. Further δ is the depreciation rate of assets and $e_{c,i,t}$ is idiosyncratic employment opportunity which follows an exogenous continuous stochastic process.[14] This stochastic term is specified as an autoregressive process with steady state equal to μ_e (usually normalized to one) and adjustment coefficient ρ_e:

$$e_{c,i,t+1} = (1 - \rho_e)\mu_e + \rho_e e_{c,i,t} + \epsilon^e_{c,i,t+1} \tag{4.4}$$

where $\epsilon^e_{c,i,t}$ is normally distributed random variable with $\epsilon^e_{c,i,t} \sim N(0; \sigma^2_{c,e})$. This employment specification implies that households do not choose the amount of labor they wish to work. One might imagine the situation when people are employed in a full-time job where the actual amount of hours worked depends on the employer who may force the employee to work overtime or cut hours. Supply of labor is therefore set exogenously and does not depend on wage.

The employment shocks cannot be insured (therefore the model is labeled as "incomplete markets model") and create a significant variation in individuals' incomes. The only way to insure against these shocks is to accumulate enough assets so that the effect of idiosyncratic shock is diminished.

The aggregate productivity factor is also a continuous stochastic process with similar autoregressive nature (adjustment rate is given by ρ_z and steady state value of productivity by μ_z):

$$z_{t+1} = (1 - \rho_z)\mu_z + \rho_z z_t + \epsilon^z_{t+1} \tag{4.5}$$

where ϵ^z_t is normally distributed random variable with $\epsilon^z_t \sim N(0; \sigma^2_z)$ and $cov(\epsilon^z_t, \epsilon^e_{c,i,t}) = 0$. This also automatically implies $cov(e_{c,i,t}, z_t) = 0$ which means that capitalists' labor supply (and by extension also the factual amount of labor supplied) does not depend on the actual performance of the economy. Simply put, the capitalist class does not suffer from cyclical unemployment (contrary to the worker class).

Finally, the parameter τ represents the tax rate on interest gains imposed by the government.

[14]Contrary to the original Krusell & Smith (1998) paper where it was defined as a two state Markov chain. The advantage of Markov chains is that they more closely match the reality where agents are either employed on full-time or unemployed, which can itself create substantial inequality. The disadvantage is that Markov chains are hard to incorporate into a model with otherwise continuous variables.

4.1.2 Workers' households

The situation for workers is much simpler. As they do not posses any assets, they cannot choose between consumption and asset allocation and just consume their whole income. This is often called "rule-of-thumb" behavior.

Now it is necessary to discuss why workers hold no capital. In RBC type models, households are willing to supply unlimited amounts of capital at an interest rate that is given mainly by the time preference parameter. This automatically implies that when the two classes of households have different time preferences (which is the case as capitalists are more patient), one class ends up holding all the capital in the economy as it is willing to accept lower interest rate on their asset holdings. The situation complicates even further when we allow borrowing and lending between the two classes. In such case, workers would like to borrow assets from capitalists and eventually would accumulate large amounts of debt and spend all their labor income on interest payments. While this may to a degree resemble the actual condition of some poor households and can work in the short run, from a model perspective it is undesirable to allow it as this would lead to a Ponzi game with a degenerate steady state (or no steady state at all).

Therefore we do not allow workers to borrow assets. The separation into the two dichotomous classes thus arises endogenously as the result of the assumptions of RBC models and due to different time preferences.[15]

Having described the workers' situation, it is now possible to formulate their optimization problem[16] and behavior:

$$\max_{c_{w,i,t}} \quad U_{w,i} = E_0 \sum_{t=0}^{\infty} \beta_w^t \left(\frac{c_{w,i,t}^{1-\gamma_w}}{1-\gamma_w} \right) \tag{4.6}$$

where $c_{w,i,t}$ represents consumption for individual worker i. The parameters β and γ again stand for the coefficients of time preference and risk aversion, respectively.

[15]The problem of the classical RBC model is that it cannot accommodate a situation where two classes with distinct time preference parameters were to hold assets. Such situation would imply two distinct interest rates, which could coexist in one economy only if the capitals of both classes were not perfect substitutes or if there was a limit to the amount of assets one class could supply. Both options would complicate the model extensively and thus lie as possible future avenues for research beyond the scope of this thesis.

[16]Note that this is an optimization only *de iure*. Because workers do not choose between assets and consumption and simply consume their entire income, their behavior is *de facto* given exogenously by their budget constraint.

Each period, workers' households face the following budget constraint:

$$c_{w,i,t} = w_t(k_t, l_t, z_t)e_{w,i,t} + T_t \tag{4.7}$$

where w_t is the wage rate, which is the same for workers and capitalists as it depends solely on aggregate variables and labor of workers and capitalists is perfectly substitutable. This of course is not likely to be true in reality where we encounter a continuum of wages and individual people differ in their labor productivity. Nevertheless one wage rate simplifies the problem and allows modelling in a simple RBC framework.

$e_{w,i,t}$ is idiosyncratic employment opportunity for workers which again follows an exogenous continuous stochastic process:

$$e_{w,i,t+1} = (1 - \rho_e)\mu_e + \rho_e e_{w,i,t} + \rho_{ez}(z_t - \mu_z) + \epsilon^e_{w,i,t+1} \tag{4.8}$$

where μ_e is steady state labor, ρ_e is the adjustment coefficient and $\epsilon^e_{w,i,t}$ is normally distributed random variable with $\epsilon^e_{w,i,t} \sim N(0; \sigma^2_{w,e})$, ρ_{ez} is the sensitivity of employment to business cycle, z_t is aggregate productivity and μ_z its steady state. Even though it still holds that $cov(\epsilon^z_t, \epsilon^e_{w,i,t}) = 0$, the specification of idiosyncratic shock now implies that $cov(e_{w,i,t}, z_t) > 0$. Unlike capitalists, the employment of workers responds to business cycle fluctuation, creating cyclical unemployment (where unemployment is defined as negative deviation of employment from steady state).

Because labor income is the only source of income for workers, the employment shock is truly uninsurable and can create much more variation in consumption than in the case of capitalists' households.

And finally, T_t are lump-sum transfers from the government (also called "welfare").

4.2 Firms

The firm sector of the economy takes the form of a representative firm, which maximizes its profits. The firm borrows capital and labor from households in return for interest rate and wage and produces output. The firm's problem can be formulated in the following way:

$$\max_{l_t,\ k_t} \quad \Pi_t = y_t - r_t k_t - w_t l_t \tag{4.9}$$

where Π_t is profit, r_t and w_t are interest rate and wage, respectively, k_t is aggregate capital, l_t is aggregate labor and y_t is output, which is given by the Cobb-Douglas production function:

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha} \tag{4.10}$$

where z_t is aggregate productivity factor and α is the share of capital in output. It is important to clarify that even though the firms are modeled as a representative firm, they form a perfect competition of infinitesimally small firms. This ensures that interest rate and wage are given by marginal products of labor and capital. Given that the supply of labor is exogenous and vertical, the firms, if they formed a monopsony, could set the wage to zero and still produce the same amount of output. On the other hand, when firms form a perfect competition, wage is determined on the labor market and is given by the intersection of labor demand (marginal product of labor) and labor supply (exogenous process).

Also note that firms do not differentiate between labor supplied by workers and capitalists (they are perfect substitutes) which results in a single wage rate for the whole economy.

4.3 Government

The government sector in this economy collects taxes on capital income and redistributes them among the workers in the form of lump-sum transfers.[17] The government runs balanced budgets and faces the following budget constraint:

$$\lambda \tau r_t \int_0^1 a_{i,t} di = (1-\lambda) T_t \tag{4.11}$$

where λ is the share of capitalists in the economy, τ is the tax rate, r_t is the interest rate, $a_{i,t}$ are assets and T_t are lump-sum transfers per capita. It is clear that the government acts to reduce inequality. But given that the taxation is distortionary and reduces capital, the government faces an equity-efficiency trade-off.

[17] This is the government that tries to reduce inequality. Technically, the specification of the model also allows the opposite case, where the government taxes the workers (effectively reducing their labor income) and boosts the capital gains of capitalists. Such setup would perfectly resemble a Marxist universe where part of the worker's production (the so called "overproduction") is taken by capitalists.

It is obvious that the redistribution works on between-class basis. The government does not reduce the within-class inequality. If the government introduced within-class lump-sum taxation to erase inequality, the model would degenerate into a representative agent economy. Moreover, the government in reality is hardly able to do this.

Some recent literature suggests that a single flat tax rate may not be optimal for a government that tries to minimize inequality. Bohacek & Kejak (2005a) for example suggest a U-shape tax rate on individual income - such taxation encourages the cumulation of capital around the average level of capital. In my model however the government tries to minimize between-class rather than within-class inequality, therefore a single flat tax rate is unproblematic.

4.4 Aggregation and equilibrium

Aggregate variables in the economy are given by the following relations:[18]

$$k_t = \lambda \int_0^1 a_{i,t} di \tag{4.12}$$

$$l_t = \lambda \int_0^1 e_{c,i,t} di + (1-\lambda) \int_0^1 e_{w,i,t} di \tag{4.13}$$

$$c_t = \lambda \int_0^1 c_{c,i,t} di + (1-\lambda) \int_0^1 c_{w,i,t} di \tag{4.14}$$

$$y_t = \lambda \int_0^1 y_{c,i,t} di + (1-\lambda) \int_0^1 y_{w,i,t} di \tag{4.15}$$

Using equations (4.4) and (4.8) it is possible to simplify the equation for labor:

$$l_t = \mu_e + \frac{(1-\lambda)\rho_{ez}}{1-\rho_e}(z_t - \mu_z) \tag{4.16}$$

This follows from the fact that $\forall t : \int_0^1 \epsilon^e_{c,i,t} di \simeq \int_0^1 \epsilon^e_{w,i,t} di \simeq 0$.[19] In words, since all idiosyncratic employment shocks on average cancel out, aggregate labor is given by a constant plus the deviation of productivity from its steady state multiplied by some positive parameter. Therefore even the aggregate labor behaves pro-cyclically and is given by an exogenous stochastic process.

[18] The fact that the aggregate capital is equal to $\lambda \int_0^1 a_{i,t} di$ and not only $\int_0^1 a_{i,t} di$ is implied by the economy-wide equality of incomes $y_t = \lambda \int_0^1 y_{c,i,t} di + (1-\lambda) \int_0^1 y_{w,i,t} di$.

[19] The whole proof of equation (4.16) using the law of large numbers is given in Haan & Ocaktan (2009) (their proof is for single heterogeneous class without cyclical unemployment, the proof for two classes with cyclical unemployment is analogous).

The equilibrium of the economy is defined by a set of first order conditions stemming from capitalist households' and firms' optimization:

$$c_{c,i,t}^{-\gamma_c} = \frac{2\phi}{a_{i,t+1}^3} + \beta_c c_{c,i,t+1}^{-\gamma_c}(1 + (1-\tau)r_{t+1} - \delta) \tag{4.17}$$

$$r_t = \alpha z_t k_t^{\alpha-1} l_t^{1-\alpha} \tag{4.18}$$

$$w_t = (1-\alpha) z_t k_t^{\alpha} l_t^{-\alpha} \tag{4.19}$$

these three F.O.C.s along with budget constraints, stochastic processes and aggregation rules characterize the steady state and dynamic behavior of the economy. The situation however is more complicated because of the heterogeneous nature of the model. In order to forecast the evolution of wage and interest rate, households need to know how the stock of aggregate capital develops over time. Optimally, the households would like to forecast the development of the whole cross-sectional capital distribution, which would thus become a state variable. This however is not possible because the distribution is an infinite dimensional object. Therefore households use a boundedly rational behavior and parameterize the distribution by using only a few first moments of the distribution to forecast future prices (wages and interest rate). Krusell & Smith (1998) argue that it is sufficient for agents to use only the first moment of capital distribution - the mean - to construct the law of motion for capital (which would under optimal circumstances depend on all moments of capital distribution):

$$k_{t+1} = \zeta_0 + \zeta_1 k_t + \zeta_2 z_t \tag{4.20}$$

Note that the law of motion does not depend on any idiosyncratic shocks as they aggregate to zero. Using this law of motion, households are able to forecast future wage and interest rate and are able to make their optimal decisions. It would also be possible to include other moments of the capital distribution into the optimization, which is the approach considered for example in Haan & Ocaktan (2009) and Preston & Roca (2007). The number of moments that should be included is connected with the selection of the method for obtaining the law of motion for aggregate capital (that does not follow analytically from the model), which is the central question in all heterogeneous agents literature. Different solution methods are discussed in the next section.

4.5 Solution methods

The solution algorithm is usually composed of three distinct yet interconnected steps. First it is necessary to decide on the order of approximation of the solution, then on what method should be used to calculate individual policy function and finally on how to arrive at the aggregate law of motion.[20]

4.5.1 Order of approximation

The oldest solution method was developed in the original Krusell & Smith (1998) paper. Their key finding is that it is sufficient for households to use only the mean of capital distribution (first moment) to predict future prices - they call this property of the model the "approximate aggregation". They argue that the solution obtained using this method is very accurate (and therefore it is not necessary to include other moments), which is caused by the fact that "marginal propensity to save out of current wealth is almost completely independent of the levels of wealth and labor income, except at the very lowest levels of wealth. Furthermore, although some very poor agents have substantially different marginal savings propensities at any point in time, the fraction of total wealth held by these agent is always very small. Because it is so small, higher-order moments of the wealth distribution simply do not affect the accumulation pattern of total capital." (Krusell & Smith (1998), p. 870).

Similarly, Preston & Roca (2007) who do include second order moments show that the coefficients on these terms are very small and the improvement in accuracy is only around 2%. They argue that second order moments matter in the case when there are significant non-linearities present in the solution. However, the inclusion of second moments in combination with perturbation approach leads to rather nontrivial prerequisites on agents' rationality, which will be discussed further in this section.

4.5.2 Solution of individual policy functions

Once one chooses the desired number of moments to include, he then computes the individual policy functions using a guess specification of the aggregate law of motion (i.e. guess values for coefficients ζ_0, ζ_1 and ζ_2 in equation (4.20)).

[20] A comprehensive review of all possible existing solution methods is given in vol. 34 issue 1 of Journal of Economic Dynamics and Control (series of nine papers). Some modern approaches may not even include some of these steps or they can combine several solution methods.

Individual policy functions can be generally solved using two different methods - by projection or perturbation. The method chosen in this paper is the latter one, so I will only briefly mention the former and then describe the perturbation method in more detail.

Projection methods are based on numerical approximation of the policy function using common quadrature techniques. Their advantage is that they are general solution methods, can capture also the distributional aspect of the heterogeneous agents problem and can include considerable nonlinearities. For an overview of projection approaches see Judd (1992) or more recently a generalization of the method by Bohacek & Kejak (2005b). Thanks to these advantages they represent a growing field with much potential and a solution of my model with projection methods is a possible future extension. The main disadvantage is the absence of standardised computational software for projection methods.

Perturbation methods are a classical solution algorithm for DSGE models. They approximate the solution around a steady state of the economy using the Taylor expansion. For the approximation to be reasonably exact the underlying general solution has to be sufficiently linear, otherwise it can lead to little robustness and explosive solutions. Furthermore, perturbation methods do not allow more than one steady state. The upside is the presence of the Dynare package for Matlab which allows quick and simple solutions using perturbation. An integral part of the Taylor approximation is the choice of its order. This problem is logically connected to number of moments included in the aggregate law of motion for capital as they must be the same[21] (in fact, the order of Taylor expansion implies the number of moments included in the law of motion).

While the order of Taylor expansion in the solution of policy function is usually a technical issue, the number of moments included in the aggregate law of motion is a matter of agents' rationality. One must consider, what information is relevant for agents in their predictions of wage and interest rate. Note that the inclusion of second order terms[22] automatically means also the inclusion of cross-products and second powers like $a_{i,t}e_{c,i,t}$ and $a_{i,t}^2$ for which agents must also construct separate laws of motion (as described in Haan & Rendahl (2010)). And while it is reasonable to assume that agents in fact use the aggregate law of motion for capital to determine their wage next period,

[21] Except for the case when perturbation is combined with simulation as a method for deriving the aggregate law of motion.

[22] In combinations with explicit aggregation algorithm for computing the aggregate law of motion, which will be discussed in the next section.

it is questionable whether they also consider these second order terms with their laws of motion for the prediction of future prices (i.e. it is for example doubtful whether agents can possibly predict how the correlation of assets and employment will develop over time). For this reason, backed by the argument cited above by Krusell & Smith (1998) and the little influence of second order terms, I decided not to include second order terms and rely on the "approximate aggregation" property of the model.

4.5.3 Arriving at the aggregate law of motion

The last part of the solution is the derivation of the aggregate law of motion for capital using the individual policy functions. Still the most common method is the one presented in the original Krusell & Smith (1998) paper, which is based on simulation and regression. Using a starting distribution of assets and their corresponding policy rules, the method simulates the behavior of a large number of agents over a large number of time periods and then regresses the average asset holdings in period t on asset holding in period $t-1$ and on the aggregate productivity shock. This regression gives the coefficients of the aggregate law of motion of capital. Such process is necessarily an iterative procedure - using the new law of motion for aggregate capital, it then proceeds to compute new individual policy functions, which are then used for a new simulation and regression. The process iterates until there is no change between the new and the old laws of motion for aggregate capital. The disadvantage of this method is its computational intensity and the fact that it introduces sampling variance into the model.

Haan & Rendahl (2010) came with an alternative solution which is based on explicit aggregation of coefficients in the individual policy function and does not rely on simulation and regression. I will describe the usage of this approach for the specific case of my thesis (perturbation with first order Taylor approximation). The policy function for individual asset holding can be written in the following way:[23]

$$a_{i,t+1} = \theta_0 + \theta_1 a_{i,t} + \theta_2 e_{c,i,t} + \theta_3 z_t + \theta_4 k_t \tag{4.21}$$

This function determines the optimal allocation of individual assets for the next period based on variables in this period. Note that the coefficient θ_1 (along with

[23] I will use the notation of Haan & Ocaktan (2009) who consider a similar model.

other thetas) is the same for all possible values of $a_{i,t}$ which corresponds to the "approximate aggregation" property discussed earlier. To derive the aggregate law of motion for capital, we will integrate equation (4.21):

$$\int_0^1 a_{i,t+1} di = \theta_0 + \theta_1 \int_0^1 a_{i,t} di + \theta_2 \int_0^1 e_{c,i,t} di + \theta_3 z_t + \theta_4 k_t \tag{4.22}$$

now using $\lambda \int_0^1 a_{i,t+1} di = k_{t+1}$, $\lambda \int_0^1 a_{i,t} di = k_t$ and $\int_0^1 e_{c,i,t} di = \mu_e$ we can rewrite this equation as:

$$k_{t+1} = \underbrace{\lambda(\theta_0 + \theta_2 \mu_e)}_{=\zeta_0} + \underbrace{(\theta_1 + \lambda\theta_4)}_{=\zeta_1} k_t + \underbrace{\lambda\theta_3}_{=\zeta_2} z_t \tag{4.23}$$

which is identical to equation (4.20) and gives us the aggregate law of motion for capital. This approach is much easier and faster than the simulation and regression method, but doesn't come without a loss of generality. The explicit aggregation algorithm puts restrictions on individual policy function, which cannot contain any variable that does not aggregate to a variable present in the aggregate law of motion, which is why one cannot use second order Taylor expansion for individual rules and only first moments for aggregate rule (unlike in the regression and simulation approach). Similarly, the individual policy rules must contain variables in levels and not for example in logs.

With the new aggregate law of motion it is again necessary to iterate the process until the old and new coefficients of the aggregate law of motion converge. The main advantage of this approach in comparison to the original Krusell & Smith (1998) method is that we do not have to simulate the whole economy in each iteration and the iterative process is therefore much faster, albeit less general.

To conclude this section, the solution approach taken in this thesis is an iterative procedure with first order perturbation method that solves individual policy function and an explicit aggregation algorithm which derives the aggregate law of motion for capital.

4.6 Calibration

The calibration of the baseline model is standard among heterogeneous agents literature and follows Preston & Roca (2007) and Haan & Ocaktan (2009). This calibration matches the standard features of a large closed economy (US).

The following table summarizes the calibration:

Table 4.1: Baseline calibration

α	δ	β_c	β_w	γ_c	γ_w	μ_z	ρ_z	μ_e
0.36	0.025	0.98	0.95	2	5	1	0.75	1

ρ_e	ρ_{ez}	σ_z	$\sigma_{c,e}$	$\sigma_{w,e}$	ϕ	b	λ	τ
0.7	0.3	0.013	0.05	0.1	0.05	0	0.8	0

First of all, the share of workers in the economy is set to 20%. Empirical literature suggests even a much higher share of rule-of-thumb households (50% in US - Mankiw (2000b) or 40% for the Czech republic - Baxa & Adam (2012)). In the baseline model the 20% share is chosen for the simplicity of calculating the shares of the bottom 20% on income and consumption, which are the statistics reported annually by the US census bureau. Furthermore, the 20% is chosen to illustrate how the government can improve the conditions of the poorest quintile of households and how it is going to affect the aggregate output. A sensitivity analysis of the share of workers is performed.

The share of capital on output is approximately one third, which matches long-run US share of capital income in the economy. Similarly, the depreciation rate is 2,5% which along with other parameters implies 4.5% interest rate on capital. The question is, what is the target periodicity of this model. Traditionally RBC models were calibrated to match quarterly data, but nowadays a 4.5% interest rate is high even for yearly data in the situation with zero lower bound. Therefore we set the periodicity to be annual rather than quarterly.

The time preference parameters are different for the two classes, which leaves all capital in the possession of the more patient class (capitalists) therefore as long as $\beta_c > \beta_w$, the exact value of β_w is not important.

The coefficients of risk aversion are a subject of much controversy in the economic literature. Traditionally, risk aversion was calibrated as 1 (for example King & Rebelo (2000)), which implies logarithmic utility function that is easy to work with. Some more recent literature however suggests that the coefficient of risk aversion should be much higher, usually around 2 and even more (Meyer & Meyer (2005), Schechter (2007)). Another topic of interest is whether risk-aversion is correlated with individual wealth, and the coefficients of risk aversion should therefore be different for the two classes of households. Zhang *et al.* (2014) found that risk-aversion is hump-shaped in wealth with both the very poor and the very rich being risk averse. From these reasons, the coefficient of risk aversion for the capitalist class in the benchmark model is calibrated to 2, and for the workers to 5. A thorough sensitivity analysis is performed as well.

The stochastic processes for employment and productivity are standardized to a steady state equal to one and adjustment coefficients equal to 0.75 and 0.7 respectively. The higher volatility of workers' labor is given by larger standard deviation of their idiosyncratic employment shocks (0.1 compared to 0.05) and by the interaction term between employment and business cycle ($\rho_{ez} = 0.3$).

The barrier parameter is calibrated in accordance with Preston & Roca (2007) who use the same penalty function and is equal to 0.05.[24] The borrowing limit b is set to 0 to prevent households from holding negative amounts of assets. And finally, the tax rate in the benchmark model is set to zero with no redistribution from capitalists to workers.

[24] The steady state of capital is increasing in ϕ, but the effect is small as it should be.

Chapter 5

Results

This section presents the results of the model. First of all we will inspect the convergence of the parameters of individual and aggregate policy functions discussed in the previous section. To start the iteration algorithm, it is necessary to choose guess values for the aggregate law of motion, which must respect the steady state value of capital. The following functional form is therefore preferred with adjustment coefficients $\rho_{kk} = \rho_{kz} = 0.7$:[25]

$$k_{t+1} = (1 - \rho_{kk})\bar{k} + \rho_{kk} k_t + \rho_{kz}(z_t - \mu_z) \tag{5.1}$$

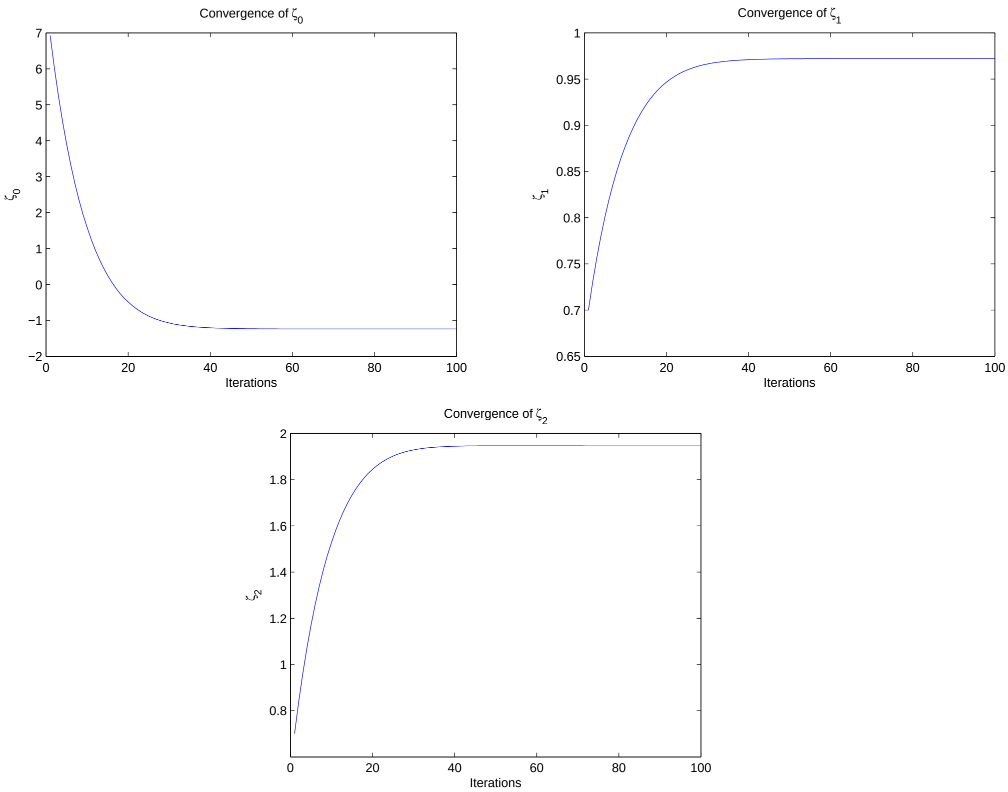
with $\bar{k}$ and μ_z being the steady states for capital and productivity. We can see that this form can be easily rearranged into equation (4.20). The speed of convergence is set to 10% (to prevent exploding solutions along the path of convergence) with the new coefficients of aggregate law motion being given by the following linear combinations:

$$\zeta_0^{NEW} = 0.9\zeta_0^{OLD} + 0.1\lambda(\theta_0 + \theta_2 \mu_e) \tag{5.2}$$
$$\zeta_1^{NEW} = 0.9\zeta_1^{OLD} + 0.1(\theta_1 + \lambda\theta_4) \tag{5.3}$$
$$\zeta_2^{NEW} = 0.9\zeta_2^{OLD} + 0.1\lambda\theta_3 \tag{5.4}$$

with thetas being the coefficients from individual law of motion (see equation (4.21)) and λ being as usual the share of capitalists in the economy. The following figures show the convergence of coefficients of aggregate law of motion:

[25] After convergence, the coefficients ρ_{kk} and ρ_{kz} are of course different.

Figure 5.1: Convergence of coefficients of aggregate law of motion



We can see that the convergence stabilizes roughly after 50 iterations and the coefficients are stable afterwards. The final law of motion after convergence for the benchmark model is given as:

$$k_{t+1} = -1.2397 + 0.9722k_t + 1.9463z_t \tag{5.5}$$

For completeness, the following table summarizes the steady state values of all variables:

Table 5.1: Steady state values

Assets (a)	Capital (k)	Consum. workers (c_w)	Consum. capitalists (c_c)
31.7861	25.4271	2.05157	2.69953

Labor (l)	Productivity (z)	Employment ($e_w = e_c$)	Interest rate (r)
1	1	1	0.04538

Wage (w)	Investment (i)	Income workers (y_w)	Income capitalists (y_c)
2.05157	0.794652	2.05157	3.49418

The rest of this section is organized as follows: firstly, we check the coherency of the model by inspecting the impulse response functions. Then we proceed to the various measures of inequality implied by the model and compare them to their empirical counterparts. Both parts include a sensitivity analysis and a discussion of the role of the government. And finally we discuss the policy implications and possible future extensions of the benchmark model.

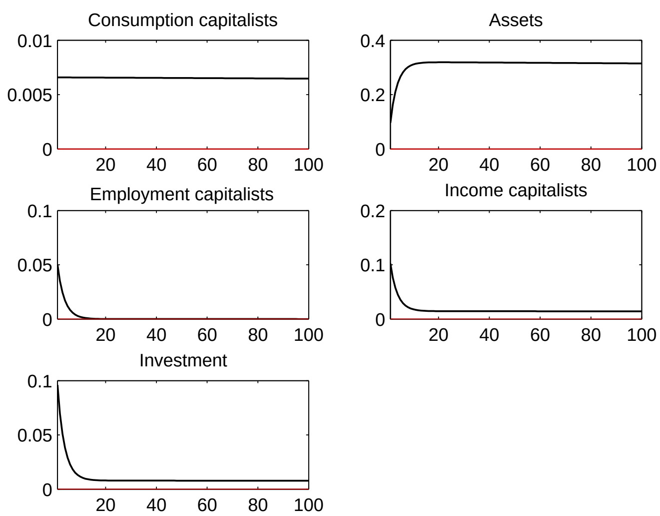
5.1 Impulse response functions

Impulse response functions allow us to examine the dynamics of the model by showing how the effects of individual exogenous shocks propagate through the economy. It should be noted that the impulse response functions resulting from this model are not designed to match their empirical (for example VAR) counterparts. Their purpose is to check the model inner consistency (that individual and aggregate functions are not self-contradictory) and to inspect whether the model behaves according to a standard RBC economic theory.

5.1.1 Idiosyncratic shocks

First we will present the set of idiosyncratic IRFs of capitalists' households (i.e. the shock to their employment):

Figure 5.2: IRFs - 1 s.d. positive shock to capitalists employment $\epsilon^{e}_{c,i,t}$



When a positive shock to $\epsilon^{e}_{c,i,t}$ hits a household, the employment (hours worked) of this household increases and then decays exponentially as implied by equation (4.4). The shock virtually disappear after ten periods. As employment increases, the household receives higher income through wages. Because the individual labor supply of one household is infinitesimally small, the increase in idiosyncratic employment does not imply a decrease in wage, which remains constant. Therefore the initial increase in income is twice higher than the increase in employment as the economy-wide wage rate in steady state is equal to 2.

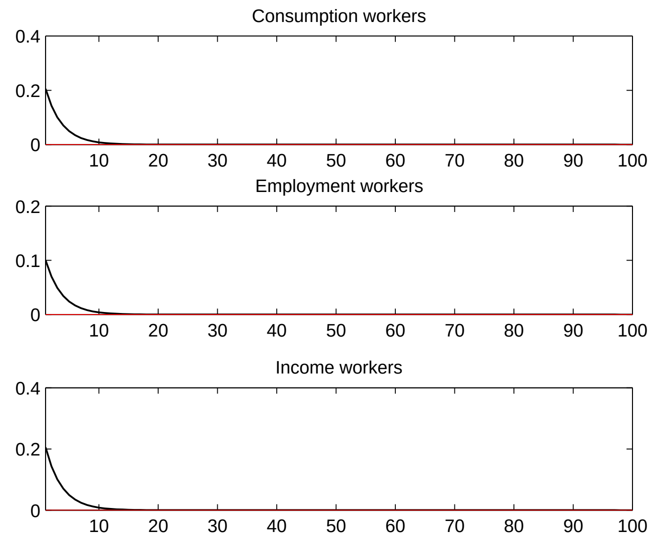
The increase in consumption and investment shows a permanent income behavior. Household wishes to perfectly smooth consumption which then jumps to a new steady state immediately after the shock. The rest of the additional labor income goes to investment by which the household eventually reaches a new higher level of individual asset holding. The higher income from assets is then used to fund the increased consumption spending even after the initial employment shock fades.

This permanent increase in all variables is enabled by the fact that the household can increase its asset holdings without decreasing their interest rate. In aggregate however, if the representative household increases its capital holding, interest rate goes down. From this reason, the permanent income behavior can only occur in the heterogeneous agent setup.

Because all idiosyncratic shocks aggregate to zero and a single shock has only infinitesimal effect, there is no response of aggregate variables to an increase in one household's employment.

The IRFs of workers are much simpler:

Figure 5.3: IRFs - 1 s.d. positive shock to workers employment $\epsilon^e_{w,i,t}$

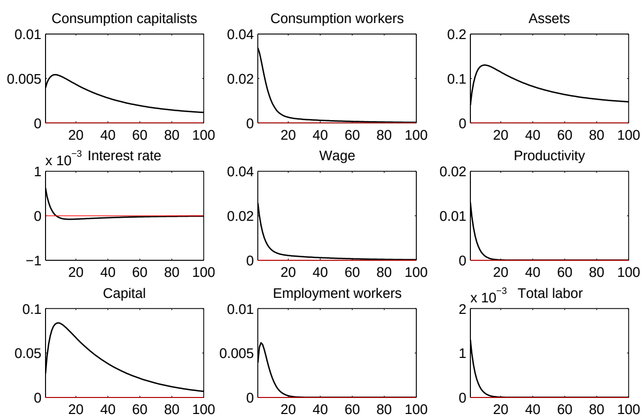


The whole employment shock translates through wage into income which is equal to consumption. Because workers cannot smooth consumption via asset holdings, their consumption perfectly matches their income and employment and they do not exhibit permanent income behavior.

5.1.2 Aggregate shocks

The dynamics of aggregate shock propagation are much more complex:

Figure 5.4: IRFs - 1 s.d. positive shock to productivity ϵ^z_t



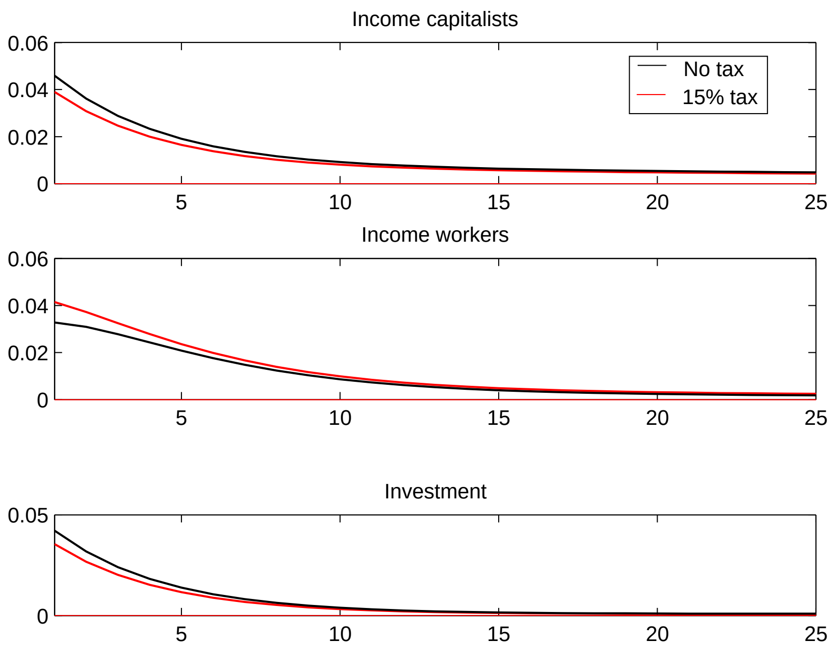
After the initial increase, productivity returns to its steady state at an exponential rate. This increase in productivity enhances the marginal products of capital and labor which leads to higher interest rate and wage. This increases income of both classes of households which must then decide how to spend this additional revenue. In the case of workers, the productivity shock stimulates their labor supply (which is pro-cyclical) and as a result, both their idiosyncratic employment and total labor in the economy go up.

While workers consume their entire additional income, capitalists split this revenue between consumption and investment. The jump in consumption is therefore much bigger in the case of workers than in the case of capitalists. With the increase in investment, capitalist households start to accumulate extra assets which along with decaying productivity eventually leads to the fall of interest rate even below the steady state value. The accumulation pattern of total capital closely resembles the one for individual assets. The key difference between the aggregate shock and idiosyncratic shock is that on the aggregate level, interest rate and wages adjust and households cannot accumulate assets infinitely without being punished by lower interest rate. As a result, there is no permanent income behavior of households in response to the increase in productivity and all variables return to their respective steady states.[26]

Generally speaking, the impulse response functions are consistent with a standard RBC economic theory. We will now inspect, how the dynamics of the model changes when we introduce government taxation. Under 15% tax rate on interest gains, one can already observe changes in the income paths of both types of households. The following figure presents the income and investment IRFs under different tax regimes:

[26] Although in the case of assets and capitalists' consumption the return can take very long time (more than 100 periods as depicted on the figure) because capitalist households are able to smooth consumption very well. Note that the interest rate is also sufficiently below its steady state even after 100 periods (consistently with larger capital), although this fact is not that visible from the figure.

Figure 5.5: IRFs with taxes - 1 s.d. positive shock to productivity ϵ_t^z



Since the taxation redistributes capital gains from capitalists to workers, the income path of workers is larger in magnitude when there are taxes present (in case of 15% tax rate the increases for capitalists and workers are actually identical), since they benefit from capitalists' accumulation of assets. This accumulation on the other hand is slightly depressed (increase in investment is lower), since it does not reward capitalists as much as in the no-tax scenario.

5.2 Poverty ratios

In this section, we will show how the model can describe and match the various poverty indices discussed in review of empirical literature. From the model design follows that the wealth index - the share of wealth held by the poorest 20% - is zero, which matches the actual empirical evidence (bottom 40% hold 0.2% of total wealth in the US). This result is robust to all possible calibrations of the model, as long as the discount factor (time preference) of workers is smaller than the one for the capitalists.

As for the income and consumption inequality, the situation is not as radical, as both classes work for the same wage and thus receive the same labor income. The inequality thus arises only from interest gains on asset holding. The following table summarizes the share of income and consumption of workers on total income and consumption for different calibration values:

Table 5.2: Share of workers' (bottom 20%) income and consumption in the economy

Calibration[27]	Share of income	Share of consumption
US real data 2012	3.21%	8.61%
Benchmark model	12.8%	15.97%
$\beta_c = 0.99$	12.8%	17.21%
$\beta_c = 0.975$	12.8%	15.56%
$\gamma_c = 1$	12.8%	15.96%
$\gamma_c = 4$	12.8%	15.96%
$\delta = 0.05$	12.8%	17.2%
$\delta = 0.01$	12.8%	14.52%
$\alpha = 0.4$	12%	15.39%
$\alpha = 0.5$	10%	13.8%
$\lambda = 0.6$	12.8%	15.97%

The first important observation is that the model is robust to different calibration values. The second observation is that in all cases, income inequality is bigger than consumption inequality. This is caused simply by the fact that while workers consume their whole income, capitalists must divide income into consumption and investment.

Further we can see that income inequality is unaffected by the values of time preference, depreciation, risk aversion or share of workers in population and is determined solely by the calibration of the production function. It is logical, since the production function divides the economy's income between labor and capital and the only difference between the two classes is the possession of capital. Therefore if labor gets 64% of income and capital gets 36% (in the benchmark case), then the bottom 10% share of population who do not hold any capital get only 6.4% share of total income, and the same applies for the next bottom 10%, giving the result for the bottom 20% equal to 12.8%. Note that the calibration of course affects the overall amount of capital and output in the economy, but not its distribution among the two classes.

As for the share of consumption, the situation is different. It can be said, that workers' share in total consumption fluctuates around 16%, but the calibration does have an effect. With increasing patience (β_c), capitalists are will-

[27] Changes are ceteris paribus with respect to the benchmark model.

ing to accept lower interest rate and accumulate more capital, which in turn increases the marginal product of labor thus increasing wages. As a result, even though the income distribution among the two classes remains unaffected, the capitalists' share of consumption decreases as they need to invest more to keep the larger amount of capital from depreciating.

The situation is the same for depreciation (δ) - when the depreciation is high, capitalists need to invest more and thus decrease their consumption share appropriately.

The risk aversion (γ_c) does not have much effect as it affects mainly the dynamics of the model and has little effect on the steady state value of capital and therefore on income and consumption distribution. This is a standard feature of RBC models. The same applies to the share of workers in the population ($1 - \lambda$) which has no effects on the income and consumption shares of the bottom 20% of population.[28]

When we compare the model results with empirical numbers for the US, we can say that the model severely underestimates the actual inequality. While in the US the income share of the bottom quintile is only 3.21%, even the most discriminatory calibration cannot create greater income inequality than 10% share. Furthermore the difference between consumption and income inequality is bigger than the baseline calibration could explain, hinting at some of the alternative calibrations with higher depreciation or time preference. The inability of the model to match the empirical data suggests that income inequality is caused mainly by factors which are not present in the model, most notably idiosyncratic productivity difference (and therefore wage differentials).

Next we will examine how can the government influence inequality through taxation and redistribution via transfers:

Table 5.3: Impact of taxation

Calibration	Share of income	Share of consumption	Output difference
US real data 2012	3.21%	8.61%	–
Benchmark model	12.8%	15.97%	0
$\tau = 0.05$	14.6%	17.99%	-2.86%
$\tau = 0.1$	16.4%	19.96%	-5.74%
$\tau = -0.1$	9.2%	11.77%	5.51%
$\tau = -0.2$	5.6%	7.35%	10.81%

First we will discuss the positive taxation schemes. Since the government im-

[28] Of course it would have an effect on the share of the 40% poorest - see the next section regarding the Gini coefficient.

poses taxes on capital interest gains, the taxation is distortionary as it changes the optimal amount of assets that households wish to hold. As a result the taxation decreases the capital accumulation in the economy which decreases the output. The government therefore faces the equity-efficiency trade-off.

The tradeoff is however much in favor of equity - already at 10% tax rate, the consumption inequality almost disappears and the total output is lower only by 5.74% than in the case without taxation. One main conclusion of the model therefore is that the government can substantially reduce inequality without large losses to output. Note that the government should not try to eliminate income inequality as it does not affect welfare (unlike consumption). Any tax rate higher than 10% is thus unnecessary and only hurts the economy.

The situation is much more interesting with negative taxation as it delivers results closer to the US empirical data. Already at -20% tax rate the modelled consumption inequality is quite close to its empirical counterpart (in fact, it is even larger).

Negative taxation reduces the income of workers and increases the interest gains on capital. Since all workers' income is labor income, negative taxation effectively reduces their wage and because households' labor supply is set exogenously, there is no difference between taxing wage income of workers and taking lump-sum taxes. Negative taxation in fact introduces artificially a wage differential between the two classes. Even though in reality the wage differential arises from differences in productivity,[29] we can already see from this artificial approximation that the model cannot match empirical inequality figures with a uniform wage setting across the economy. The extension of this model, which would feature explicitly labor market imperfections along with idiosyncratic labor productivity and wage differentials, is indeed warranted. Another interpretation of the negative tax is that it introduces a different taxation of labor and capital, with labor being taxed much more. This is in fact likely to be true in reality due to tax optimization of firms and general difficulty of collecting taxes on capital gains (compared to taxes on labor income). Therefore the model with negative taxation under this interpretation is not that far from reality.

[29] Unless we embrace the Marxist view in which productivity does not matter and all inequality is produced by the institutional setup in which capitalists control the government and artificially boosts their capital income while depressing the fair wage reward of workers - such situation would on the contrary be perfectly described by the model with negative taxation.

5.3 Gini coefficient

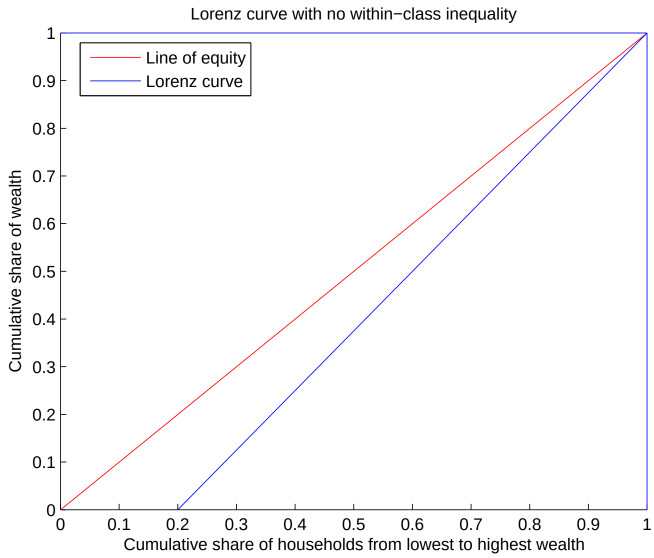
Poverty coefficients discussed in the previous section give a good measurement of between-class inequality. It however ignores the variability within the classes as well as economy-wide dispersion of assets, incomes and consumption. To capture this distribution and inequality, a number of measures is available. Still the most popular and widespread is the Gini coefficient. The Gini coefficient of income inequality for one time period t can be constructed in the following way:

$$G_t = \frac{1}{n}\left(n + 1 - 2\left(\frac{\sum_{i=1}^{n}(n+1-i)y_{i,t}}{\sum_{i=1}^{n} y_{i,t}}\right)\right) \tag{5.6}$$

where G_t is the value of Gini coefficient, n is the number of households in the economy and $y_{i,t}$ are incomes of households in the economy ordered from lowest to highest ($y_{i,t} \leq y_{i+1,t}$). Similar indices can be constructed for wealth and consumption inequality.

The computation of the Gini is based on simulating the economy using individual policy functions and idiosyncratic shocks. For simulation purposes we use 5000 households and set their initial wealth to be completely uniform, except for class differences. It means that for the benchmark economy, 4000 households start with assets (the steady state value of assets to be more specific) - these are the capitalists - and the remaining 1000 households start with zero assets - these are the workers. Since the distribution of initial wealth is uniform within the classes, the Lorenz curve of wealth inequality has the following shape:

Figure 5.6: Lorenz curve of initial wealth distribution

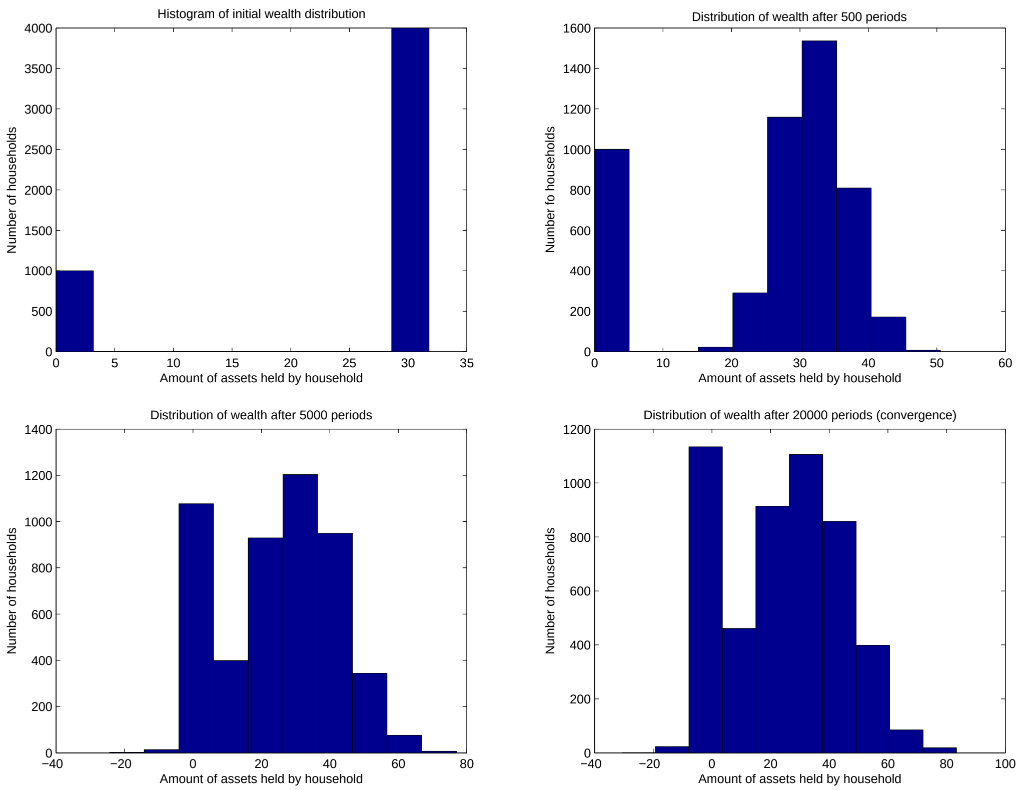


As can be seen from the figure, the first 20% poorest have no wealth at all and then the cumulative share of wealth increases linearly reflecting the uniform distribution of wealth among capitalists. From this simple Lorenz curve, we can read off the Gini coefficient directly and compute it as the area between the Lorenz curve and line of equity. Because the Gini coefficient is a relative measure of poverty, the actual value of assets the capitalists hold does not matter for the calculation of the index (i.e. the steady state value of assets does not play a role). The Gini coefficient for this initial case is 0.2, which is nowhere near the empirical estimate of 0.87 for the US.[30] Given the initial distribution, the simulation proceeds with shocking the economy via aggregate productivity and idiosyncratic labor shocks. This of course does not affect the distribution of wealth of workers, who simply absorb all shocks into their consumption (rule-of-thumb behavior) and their share in total wealth remains zero.

The following histograms represent the evolution of wealth distribution from the initial state to a steady state of wealth distribution:

[30]To get this number, the share of capitalists and workers would have to be reversed, which is also empirically unrealistic (even though not that much since the bottom 80% hold only 15% of total capital).

Figure 5.7: Evolution of wealth distribution

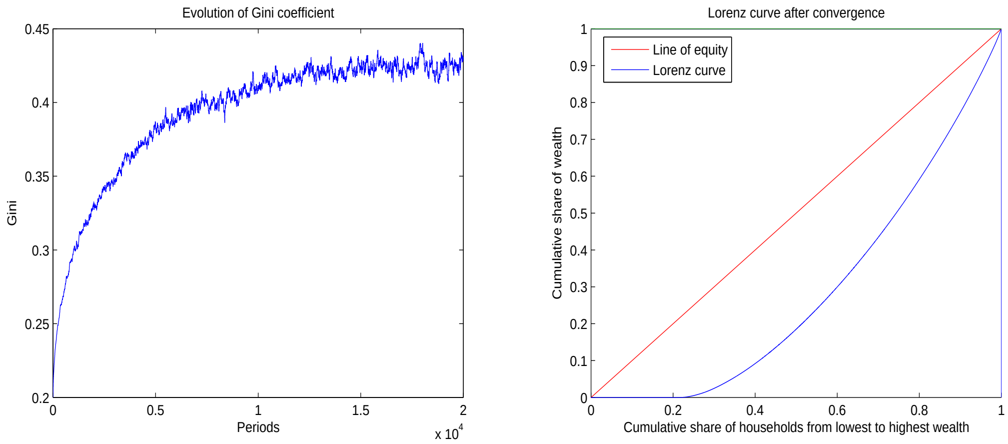


The first thing to notice is that the convergence from the initial state of equality takes a long time. Even after 500 periods, the distribution still evolves and it stabilizes after roughly 15000 periods (but already at 5000 periods, the distribution has the right shape). Naturally the wealth of capitalists is normally distributed with mean equal to the steady state of assets, which reflects the normal distribution of idiosyncratic labor shocks. This of course fails to match the heavy tailed distribution observed in the real world, since the maximum wealth reached by an individual in the model does not exceed 3 times the average wealth.

One can also notice that the distribution after convergence includes individuals with negative wealth which violates the borrowing constraint. Their number is small (around 1.5%) and their existence can be explained in the following way: firstly, the penalty function in capitalist households' utility function ensures only that the critical amount of households does not hold negative wealth and the economy does not collapse into a Ponzi scheme. Secondly, the individual policy functions are linear and therefore cannot significantly change the behavior of households at either side of wealth distribution.[31] The important thing is that the share of wealth, held by households whose decision rules are not optimal under the linear approximation, is small and therefore the approximate aggregation property discussed in section 4.5.1 holds.

To further illustrate the development of wealth distribution, the following two figures presents the evolution of Gini coefficient and the Lorenz curve[32] after convergence:

Figure 5.8: Lorenz curve and Gini of wealth distribution



The evolution of Gini clearly depicts the slow convergence starting at the value 0.2 and the stabilisation of the index at a value around 0.42. This value is only one half of the empirical estimate for the US suggesting (similarly as the poverty ratios) that the benchmark model drastically underestimates US inequality.[33] On the other hand, the Gini can significantly fluctuate over time, with prolonged periods above and under the steady state, which can capture the increase of empirical Gini observed in real data since the 80s.[34] Based on the model, the increase in Gini does not need to correspond to some structural break in the economy (as there are no structural breaks in DSGE), but can arise endogenously as a result of a combination of shocks in the economy and will fade out eventually, returning to its steady state value.

[31] Under linearity, households at the opposite sides of the wealth spectrum cannot have different propensities to save.

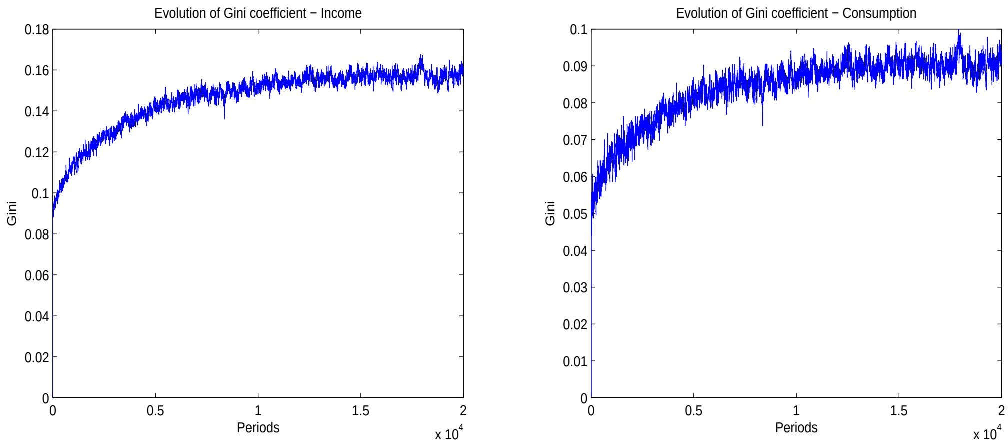
[32] For the purpose of Lorenz curve, the negative values of assets have been trimmed to zero.

[33] Furthermore when we consider only interest bearing wealth (as the model features only this kind of wealth), the difference is even higher - the Gini for financial wealth is 0.93 (Kenworthy & Smeeding 2013).

[34] In the last 10% of the sample (stable part of the distribution), the minimum and maximum of Gini were 0.0273 points apart while its empirical counterpart rose by 0.07 points in the period between 1983 and 2010 (or 0.03 between 1983 and 2004)(Wolff (2007)).

For a more complete picture of inequality in the benchmark model, the following two figures show the evolution of Ginis for income and consumption:

Figure 5.9: Gini for income and consumption



Similarly as in the case of wealth inequality, both of these indices underestimate empirical inequality measures of the US. To summarize, the following table presents the values of Gini coefficient for different calibrations and compares them with the observed data:

Table 5.4: Gini coefficients

Calibration	Wealth Gini	Income Gini	Consumption Gini
US real data[35]	0.87	0.38	0.33
Benchmark model	0.42	0.16	0.092
Stronger shocks[36]	0.65	0.24	0.103
More workers: $\lambda = 0.6$	0.56	0.22	0.125
Positive taxes: $\tau = 0.05$	0.39	0.126	0.066
Negative taxes: $\tau = -0.1$	0.41	0.195	0.133
Realistic scenario[37]	0.722	0.318	0.162

As can be seen from the table, the hardest thing to match with reality is the consumption Gini coefficient. Even in the calibration where wealth and income inequality are close to their empirical counterparts, the consumption

[35] Values taken from US country Gini report for year 2010 - Kenworthy & Smeeding (2013).

[36] The exact calibration is: $\sigma_z = 0.025$, $\sigma_{c,e} = 0.1$ and $\sigma_{w,e} = 0.15$, as well as $\beta = 0.99$ to boost the mean level of capital and prevent the borrowing constraint to be violated too often. Even still, at this scenario 10% of households hold negative amount of assets.

[37] This scenario tries to match the empirical Gini coefficients by combining stronger shock, higher share of workers and negative taxation (three above scenarios) - all of which are plausible and can be argued for (see sections 4.6 and 5.2 for the discussion of calibration).

Gini still remains at half the empirical estimate. To increase it, it would be necessary (within the RBC framework) to set the value of α to unrealistically large numbers.

Additional problems arise when we deviate from the benchmark calibration too much - firstly, the linearity assumption and approximate aggregation property become frail and implausible and the model calls for another solution algorithm, preferably the projection method which would also account for higher moments of wealth distribution and could model different propensities to save for different wealth groups. Secondly, there is a trade-off between matching the inequality (and wealth distribution) in the economy and matching the moments of real economic variables - where the benchmark (classical) calibration fits the moments more accurately than the calibration with higher inequality. This in fact calls for a different modeling framework (which would not have such a trade-off) rather than different calibration. And lastly, bigger shocks and higher inequality make the borrowing constraint not binding resulting in a nontrivial portion of population with negative assets. To approach this phenomenon properly, the model would need to incorporate an explicit banking sector.

5.4 Other income inequality measures

In this section we only briefly mention other two measures of inequality - Theil coefficient and income ratios.

The Theil coefficient is becoming a popular measure of income inequality. It represents information entropy or lack of diversity in the data. It is defined as follows:

$$T_t = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_{i,t}}{\overline{y_t}} log \frac{y_{i,t}}{\overline{y_t}}\right) \tag{5.7}$$

where T_t is the value of Theil coefficient, $y_{i,t}$ is income of individual i and $\overline{y_t}$ is the average income. The Theil coefficient yields values between 0 (complete equality = complete lack of diversity) and $log(n)$ (perfect inequality - one person gets all income in the economy). To be comparable across populations with different sizes, it is usually standardized by dividing the coefficient by $log(n)$ so that it fits between zero and one. From the definition, it is clear that the Theil coefficient is not suitable for wealth inequality, because logarithm cannot handle zero or negative values of wealth.[38] Therefore we will use it only for income inequality, for which it was designed and for which we have empirical estimates.

Similarly to other inequality measures, the Theil coefficient implied by the benchmark model underestimates real inequality. The standardized Theil for US is around 0.02 while the model suggests a value close to 0.004 - a five times lower value. The interesting thing about Theil coefficient is its decomposability into subgroups, which is not doable for example for the Gini coefficient. This allows us to tell how much of the inequality is caused by the inequality between workers and between capitalists. The decomposition can be written as follows:[39]

$$T_t = \underbrace{s_{w,t}T_{w,t}}_{Inequality\ of\ workers} + \underbrace{s_{c,t}T_{c,t}}_{Inequality\ of\ capitalists} + \underbrace{s_{w,t}log\frac{\overline{y_{w,t}}}{\overline{y_t}} + s_{c,t}log\frac{\overline{y_{c,t}}}{\overline{y_t}}}_{Between-class\ inequality} \quad (5.8)$$

where $s_{w,t}$ and $s_{c,t}$ are workers' and capitalists' shares of total income (see table 5.2), $T_{w,t}$ and $T_{c,t}$ are Theil coefficients for the two classes, $\overline{y_{w,t}}$ and $\overline{y_{c,t}}$ are mean incomes within the two classes and $\overline{y_t}$ is the mean income over the whole economy. The following table shows the contributions of the three decomposed inequality elements to the overall Theil coefficient:

Table 5.5: Theil index decomposition

Decomposed part:	Workers' ineq.	Capitalists' ineq.	Between-class ineq.
Contribution:	3.5 %	45 %	51.5 %

The decomposition clearly suggests that the inequality caused by idiosyncratic labor shocks to workers' employment cannot create significant long-term inequality (even though the shocks are stronger than in the case of capitalists).[40] On the other hand, if households can save parts of their additional income in the form of assets, this can create persistent inequality as it will yield individuals with high values of wealth whose income can stay above the average for very long periods of time. Even still, the most inequality comes from the class dichotomy itself, which simply reflects the different mean values of income

[38] Technically $\lim_{x \to 0} xlog(x) = 0$, therefore zero values of wealth are possible.

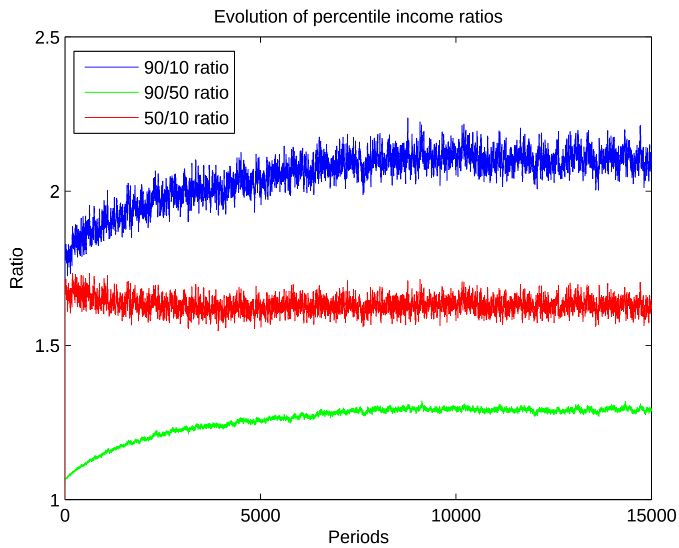
[39] For a thorough discussion of Theil coefficient and its decomposability, see for example Akita (2000).

[40] It would be interesting to compare this number to a model where employment follows a Markov chain rather than continuous stochastic process and can therefore create much bigger inequality even among the worker class. Unfortunately such comparison does not exist in the current literature.

of the two classes. The direct implication of this decomposition is that if the government were to decrease class inequality via taxation and transfers, only half of the actual inequality (as measured by Theil coefficient) would disappear. To mitigate inequality even further, the government would have to adopt some sort of progressive tax scheme or within-class redistribution (as suggested for example by Bohacek & Kejak (2005a)).

The last inequality measures we report are the 90/10, 50/10 and 90/50 percentile income ratios. They compare the incomes at the 90th, 50th an 10th percentile level, and are therefore not affected by the extremes on both sides of the income distribution. The following figure depicts the evolution of percentile income ratios (starting again at a point of complete within-class equality):

Figure 5.10: Evolution of percentile income ratios



The 90/10 and 90/50 ratios start the from same point and 90/50 ratio starts from the value 1, as in full within-class equality the 90th and 50th percentiles are same (mean income of capitalists). Then the ratios start to diverge, but stabilize rather quickly (compared to other inequality measures). Note that the 90/10 and 50/10 ratios are much more volatile since their denominator is small in value and more stable (see the Theil decomposition). The following table compares their steady state values with empirical data for the US[41]:

[41] Post-tax income as reported by Meyer & Sullivan (2009)

Table 5.6: Income percentile ratios

Ratio:	90/10	90/50	50/10
US reality:	6	2.2	2.8
Implied by model:	2.1	1.29	1.62

As can be seen in the table, the model performs the worst in the case of 90/10 ratio. This means that the income tails of the model economy are much lower than in the reality and the distribution of income is too narrow. The same conclusion, albeit in smaller scale, applies also to the other two ratios. The results of percentile ratios are therefore consistent with other inequality measures reported in previous sections and they all suggest that under standard RBC calibration, the model proposed in this thesis underestimates the actual inequality which is present in the US.

5.5 Policy implications

The model which I described in the sections above was purposefully designed in the RBC framework to make it simple and transparent and was meant to show in theory how it is possible to model inequality using heterogeneous classes and agents. As such, it was stylized to such a degree as to make it doubtful whether it can have any real-world policy implications. Therefore one has to treat the recommendations of this model with caution.

The role of the government in this model is fairly limited as it only collects taxes in the form of a distortionary capital gains tax and redistributes them among the workers in the form of transfers. The goal of the government is to reduce inequality. The model suggests that as long as the share of the poor in the economy is low, the government can significantly reduce inequality without any major impact on aggregate output. When the share of workers (the poor) is 20%, the government can virtually erase consumption inequality with a 10% tax rate and the output will drop only by 5.74%. Depending on government preferences, this may be viewed as an acceptable tradeoff between equity and efficiency.

Such redistribution can however erase only the a priori inequality stemming from the fact that workers own no capital. It cannot erase the inequality that arises over time due to idiosyncratic employment shocks. As the Theil decomposition suggests, the between-class inequality accounts for about 50%

of inequality in the economy and thus there is a limit to the government's ability of reducing inequality.

The government should be also careful not to try to reduce income inequality beyond the point where the consumption inequality is erased. As workers hold no capital, the capitalists alone are responsible for investment, keeping the capital in the economy from depreciating. In this situation, some income inequality is natural and has no effect on consumption and well-being of households.

5.6 Possible model extensions

There is a plurality of ways in which the benchmark model could be extended. First and foremost, to match the empirical data it is imperative to include idiosyncratic differences in productivity which would create wage differentials. As implied by the model, without differences in wage, the model cannot hope to accurately describe the observed inequality.

It would be also benefitial to transform the model into a New-Keynesian framework with prices, market imperfections and central bank. The addition of a foreign sector would further allow it to match the Czech economy (as well as most other European economies). The role of the government should be extended to include within-class redistribution and other taxation schemes (labor tax), to see which tax regime helps at reducing inequality the most.

The benchmark model features two classes only, but from a sociological point of view the society is usually stratified into three classes - the rich, the poor and the middle class, where all three are likely to behave differently and have their own sets of preferences. The advantage of the model is that it allows for this kind of distinction and the inclusion of a very rich class (which would hold much more assets than the "middle capitalist" class described in this model) should be unproblematic. With multiple classes, the model could also incorporate a banking sector through which the poor could borrow from the rich.

The currently employed solution method, which uses perturbation approach combined with the direct aggregation of coefficients in the policy function, limits the scope of inequality which can occur in the model. The imposed linearity does not allow agents with different wealth levels to have different propensities to save. The solution of the model using projection rather than perturbation would allow second order (and higher) terms to be included. Thanks to its generality, the projection method offers also other options for model extension like multiple steady states and is therefore starting to be the preferred method for solving heterogeneous agents models.

Finally, the model could use a Bayesian approach to calibration of parameters. The question remains, which features of the real economy should the calibration try to match - whether the moments of variables, the impulse responses or the inequality measures. This remains a puzzle and goes beyond the scope of this thesis.

Chapter 6

Conclusion

In this thesis I proposed a model that combines heterogenous classes and heterogenous agents with idiosyncratic shocks - the two types of models that were traditionally used to describe inequality using DSGE framework. My model features two classes that differ in their ownership of capital - the worker class who spends its entire income on consumption and holds no assets and the capitalist class who is able to smooth consumption through accumulation of assets.

I designed the model to have a classical RBC structure with real variables and perfect competition on labor and capital markets. As a result the model features a single wage and interest rate and the idiosyncracy is present only in employment. I showed that such a model significantly undershoots the actual inequality in the economy. Even in the most realistic scenario the model cannot generate higher wealth Gini coefficient than 0.72 while in reality we observe a value of 0.87 (for the US, on which the model is calibrated). The model also suggests that the classical parameters like time preference or depreciation play only a minor role in explaining inequality. Therefore the main reason for this undershooting is the absence of idiosyncratic productivity differences and wage differentials, which in reality are the key drivers of rising inequality.

To illustrate this fact I created an artificial wage differential between the two classes through government taxation and the results were indeed much closer to empirical data. It also proved that consumption inequality is harder to model than income and wealth inequality.

To investigate how much of the overall inequality is caused by within-class and between-class inequality, I used Theil coefficient decomposition and found that inequality among the worker class is almost negligible while inequality among the capitalist class and between-class inequality contribute roughly equally to the total inequality in the economy.

The government in this model plays a redistributive role and transfers funds from the richer class to the poorer one. It faces an equity-efficiency trade-off as reducing inequality comes at a price of reducing aggregate output (government uses distortionary tax on capital gains). I showed that when the share of workers in the economy is relatively small (20%), the government can completely eliminate between-class consumption inequality with a cost of reducing output only by 5.7%. However even a complete between-class redistribution will reduce the overall inequality only by one half, as suggested by the Theil coefficient decomposition.

The main contribution of the thesis is therefore theoretical as it shows how one can model heterogeneity more accurately via both a class structure and individual specificity - the two features that actually exist in reality and which have not been modelled within the DSGE framework before. Even though the proposed model fails to match the empirical evidence on inequality due to its simple RBC nature and limited size, its extensions that would feature imperfects markets, money and most importantly differences in productivity are likely to give a very realistic picture of today's economies and are therefore warranted.

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Appendix A

Matlab codes

This appendix presents the Matlab and Dynare codes that were used to solve and simulate the model.
The first Matlab program defines variables and solves the steady state of assets:

```
%%%%%%%%%%%%%%% Steady state solver %%%%%%%%%%%%%%%
  gamma  = 2;           % risk aversion
  delta  = 0.025;       % depreciation
  betta  = 0.98;        % time preference
  alfa   = 0.36;        % share of capital on output
  miz    = 1;           % steady state productivity
  roz    = 0.75;        % adjustment productivity
  sigmaz = 0.013;       % volatility productivity
  phi    = 0.05;        % barrier parameter
  rok    = 0.7;         % initial parameter for law of motion
  mie    = 1;           % steady state employment
  roe    = 0.7;         % adjustment employment
  sigmae1 = 0.05;       % volatility employment capitalists
  sigmae2 = 0.1;        % volatility employment workers
  roez   = 0.3;         % cyclicality of employment workers
  lambda = 0.8;         % share of capitalists
  tau    = 0;           % tax rate


pi = (1-alfa)*lambda^alfa + (1-tau)*alfa*lambda^(alfa-1);
syms a
```

```
solve(2*phi*((a^alfa)*pi - delta*a)^gamma +
+ (a^3)*(betta - betta*delta -1) +
+ betta*alfa*(1-tau)*(lambda^(alfa-1))*a^(alfa+2) == 0, a)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Choose the stable steady state (real, positive) and plug it into all subsequent programs where "ass" is required (assets steady state). The following program is the mother Matlab program that runs Dynare program in a loop (note1: run only the mother program and not the Dynare programs; note2: when in the following programs a line of code spans several lines of text, in Matlab, compress it to a single line):

```
%%%%%%%%%% Mother program %%%%%%%%%%%%%%%
ass = 31.7838916986589918973;      % st.st. assets
lambda = 0.8;                      % share of capitalists
kss = lambda*ass;                  % st.st capital
rok = 0.7;                         % guess value for law of motion
vZetaOld = ones(1,3);              % var. that stores old coefs.
vZetaOld(1) = (1-rok)*kss - rok;   %  - of aggregate law of motion
vZetaOld(2) = rok;
vZetaOld(3) = rok;
vZetaNew = ones(1,3);              % the same for new coefficients
convergence = ones(3,2,100);       % var. for convergence process
dLambda = 0.1;                     % speed of convergence
vTheta = ones(1,5);                % variable for coefs. in -
pZeta0 = vZetaOld(1);              % - individual law of motion
pZeta1 = vZetaOld(2);
pZeta2 = vZetaOld(3);
save InitParams.mat pZeta0 pZeta1 pZeta2; % saves coefs for Dynare

% The following solves the model iteratively using old coeffs of
% aggregate law  of motion and computes and stores the new ones:
for i = 1:100
dynare diplomkaNEWTAX1.mod noclearall;        % runs model (dynare)

% computes new coefs of aggregate law of motion out of
% individual coefs.
vZetaNew(1) = lambda*(vTheta(1) + vTheta(3));
```

```
vZetaNew(2) = vTheta(2) + lambda*vTheta(5);
vZetaNew(3) = lambda*vTheta(4);

% now store both new and old coefs of aggregate law of motion
convergence(1,1,i) = vZetaOld(1);
convergence(2,1,i) = vZetaOld(2);
convergence(3,1,i) = vZetaOld(3);
convergence(1,2,i) = vZetaNew(1);
convergence(2,2,i) = vZetaNew(2);
convergence(3,2,i) = vZetaNew(3);

% The new coefs become old coefs for the next iteration
vZetaOld = dLambda * vZetaNew + (1-dLambda) * vZetaOld;
pZeta0 = vZetaOld(1);
pZeta1 = vZetaOld(2);
pZeta2 = vZetaOld(3);
delete InitParams.mat;     % delete the old stored values
save InitParams.mat pZeta0 pZeta1 pZeta2; % store new ones
end  % end of the convergence loop

% And finally run the model with converged coefficients
dynare diplomkaNEWTAX2.mod noclearall;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The following Dynare code solves the model for the use of iterations - it has a compressed form and does not include all variables (the ones that are not necessary and are only a product of identities (i.e. investment, output...). The name of the file is "diplomkaNEWTAX1.mod":

```
// Model solver (compressed) -  diplomkaNEWTAX1.mod //
var cc cw a r w z k ec ew l;

varexo epsz epse1 epse2;

parameters gamma delta betta alfa miz roz sigmaz phi
cssc cssw ass wss rss kss rok mie roe roez sigmae1
sigmae2 pZeta0 pZeta1 pZeta2 lambda tau;
```

```
  gamma  = 2;           // risk aversion
  delta  = 0.025;       // depreciation
  betta  = 0.98;        // time preference
  alfa   = 0.36;        // share of capital on output
  miz    = 1;           // steady state productivity
  roz    = 0.75;        // adjustment productivity
  sigmaz = 0.013;       // volatility productivity
  phi    = 0.05;        // barrier parameter
  rok    = 0.7;         // initial parameter for law of motion
  mie    = 1;           // steady state employment
  roe    = 0.7;         // adjustment employment
  sigmae1 = 0.05;       // volatility employment capitalists
  sigmae2 = 0.1;        // volatility employment workers
  roez   = 0.3;         // cyclicality of employment workers
  lambda = 0.8;         // share of capitalists
  tau    = 0;           // tax rate

  ass =   31.783891698658;        // steady state assets
  kss = lambda*ass;               // steady state capital
  rss   = alfa*(kss)^(alfa-1);    // steady state interest rate
  wss   = (1-alfa)*(kss)^(alfa);  // steady state wage
  cssc  = (ass^alfa)*(1-tau*alfa) - delta*ass;
  // steady state consumption capitalists
  cssw  = wss +tau*rss*kss/(1-lambda);
  // steady state consumption workers

// load coefs for aggregate law of motion
load InitParams;
set_param_value('pZeta0',pZeta0);
set_param_value('pZeta1',pZeta1);
set_param_value('pZeta2',pZeta2);

model;
//(1) euler equation capitalists
cc^(-gamma) = phi*2/(a^3) +
+ betta*(cc(+1))^(-gamma)*(1 + (1-tau)*r(+1) - delta);
```

```
//(2) interest rate
r = alfa*z*((k(-1))^(alfa-1))*l^(1-alfa);

// (3) wage
w = (1-alfa)*z*((k(-1))^(alfa))*l^(-alfa);

// (4)budget constraint capitalists
cc + a  = (1-tau)*r*a(-1) + w*ec + (1-delta)*a(-1);

// (5)budget constraint workers
cw  =  w*ew + tau*r*k(-1)/(1-lambda);

// (6) aggregate shock
z = (1-roz)*miz + roz*z(-1) + epsz;

// (7) law of motion for capital
k = pZeta0 + pZeta1*k(-1) + pZeta2*z;

// (8) law of motion for labor
l = mie + ((1-lambda)*roez/(1-roe))*(z - 1);

// (9) idiosyncratic shock capitalists
ec = (1-roe)*mie + roe*ec(-1) + epse1;

// (10) idiosyncratic shock workers
ew = (1-roe)*mie + roe*ew(-1) + roez*(z-1) + epse2;

end;


initval;
  cc  = cssc; // consumption capitalists
  cw  = cssw; // consumption workers
  a   = ass;  // assets
  r   = rss;  // interest rate
  w   = wss;  // wage
  z   = 1;    // productivity
```

```
  k   = kss;  // capital
  ec  = 1;    // employment capitalists
  ew  = 1;    // employment workers
  l   = 1;    // total labor
end;

steady;

check;

shocks;
var epsz = sigmaz^2;
var epse1 = sigmae1^2;
var epse2 = sigmae2^2;
end;

stoch_simul(order=1,nocorr,noprint,nomoments,IRF=0);

// Now read the coefficients of individual law of motion
mPolicy = [oo_.dr.ys'; oo_.dr.ghx'; oo_.dr.ghu'];
mPolA = mPolicy(:,4);
mPolA(1) = mPolicy(1,3);

// Rearrange parameters
dTheta0 = mPolA(1)-mPolA(2)*mPolA(1)-mPolA(5)-mPolA(3)-
- mPolA(4)*mPolicy(1,7);
dTheta1 = mPolA(2);
dTheta2 = mPolA(5);
dTheta3 = mPolA(3);
dTheta4 = mPolA(4);

// Save parameters
vTheta = [dTheta0 dTheta1 dTheta2 dTheta3 dTheta4];
///////////////////////////////////////////////////
```

And finally the following program is for the model with all variables and retrieves the IRFs:

```
// Model solver full -  diplomkaNEWTAX2.mod //
var cc cw a r w z k ec ew l yc yw i;

varexo epsz epse1 epse2;

parameters gamma delta betta alfa miz roz sigmaz phi
cssc cssw ass wss rss kss rok mie roe roez sigmae1
sigmae2 pZeta0 pZeta1 pZeta2 lambda tau;

  gamma  = 2;          // risk aversion
  delta  = 0.025;      // depreciation
  betta  = 0.98;       // time preference
  alfa   = 0.36;       // share of capital on output
  miz    = 1;          // steady state productivity
  roz    = 0.75;       // adjustment productivity
  sigmaz = 0.013;      // volatility productivity
  phi    = 0.05;       // barrier parameter
  rok    = 0.7;        // initial parameter for law of motion
  mie    = 1;          // steady state employment
  roe    = 0.7;        // adjustment employment
  sigmae1 = 0.05;      // volatility employment capitalists
  sigmae2 = 0.1;       // volatility employment workers
  roez   = 0.3;        // cyclicality of employment workers
  lambda = 0.8;        // share of capitalists
  tau    = 0;          // tax rate

  ass =   31.783891698658;         // steady state assets
  kss = lambda*ass;                // steady state capital
  rss   = alfa*(kss)^(alfa-1);     // steady state interest rate
  wss   = (1-alfa)*(kss)^(alfa);   // steady state wage
  cssc  = (ass^alfa)*(1-tau*alfa) - delta*ass;
  // steady state consumption capitalists
  cssw  = wss +tau*rss*kss/(1-lambda);
  // steady state consumption workers
  ycss  = wss + (1-tau)*rss*ass;
  // steady state income capitalists
  ywss  = wss + tau*rss*kss/(1-lambda);
```

```
  // steady state income workers
  iss   = delta*ass; // investment

// load coefs for aggregate law of motion
load InitParams;
set_param_value('pZeta0',pZeta0);
set_param_value('pZeta1',pZeta1);
set_param_value('pZeta2',pZeta2);

model;
//(1) euler equation capitalists
cc^(-gamma) = phi*2/(a^3) +
+ betta*(cc(+1))^(-gamma)*(1 + (1-tau)*r(+1) - delta);

//(2) interest rate
r = alfa*z*((k(-1))^(alfa-1))*l^(1-alfa);

//(3) wage
w = (1-alfa)*z*((k(-1))^(alfa))*l^(-alfa);

// (4) budget constraint capitalists
cc + a  = (1-tau)*r*a(-1) + w*ec + (1-delta)*a(-1);

// (5) budget constraint workers
cw  =  w*ew + tau*r*k(-1)/(1-lambda);

// (6) aggregate shock
z = (1-roz)*miz + roz*z(-1) + epsz;

// (7) law of motion for capital
k = pZeta0 + pZeta1*k(-1) + pZeta2*z;

// (8) law of motion for labor
l = mie + ((1-lambda)*roez/(1-roe))*(z - 1);

// (9) idiosyncratic shock capitalists
ec = (1-roe)*mie + roe*ec(-1) + epse1;
```

```
// (10) idiosyncratic shock workers
ew = (1-roe)*mie + roe*ew(-1) + roez*(z-1) + epse2;

// (11) income capitalists
yc = (1-tau)*r*a(-1) + w*ec;

// (12) investment
i = yc - cc;

// (13) income workers
yw  =  w*ew + tau*r*k(-1)/(1-lambda);
end;


initval;
  cc  = cssc; // consumption capitalists
  cw  = cssw; // consumption workers
  a   = ass;  // assets
  r   = rss;  // interest rate
  w   = wss;  // wage
  z   = 1;    // productivity
  k   = kss;  // capital
  ec  = 1;    // employment capitalists
  ew  = 1;    // employment workers
  l   = 1;    // total labor
  yc  = ycss; // income capitalists
  yw  = ywss; // income workers
  i   = iss;  // investment

end;

steady;

check;

shocks;
```

```
var epsz = sigmaz^2;
var epse1 = sigmae1^2;
var epse2 = sigmae2^2;
end;


stoch_simul(periods=2100, order = 1,irf =100);
///////////////////////////////////////////////////
```

After running the mother program and getting the converged laws of motion, it is possible to simulate the economy na compute various inequality measures using the following program:

```
%% Economy simulator %%
  gamma  = 2;           % risk aversion
  delta  = 0.025;       % depreciation
  betta  = 0.98;        % time preference
  alfa   = 0.36;        % share of capital on output
  miz    = 1;           % steady state productivity
  roz    = 0.75;        % adjustment productivity
  sigmaz = 0.013;       % volatility productivity
  phi    = 0.05;        % barrier parameter
  rok    = 0.7;         % initial parameter for law of motion
  mie    = 1;           % steady state employment
  roe    = 0.7;         % adjustment employment
  sigmae1 = 0.05;       % volatility employment capitalists
  sigmae2 = 0.1;        % volatility employment workers
  roez   = 0.3;         % cyclicality of employment workers
  lambda = 0.8;         % share of capitalists
  tau    = 0;           % tax rate
  ass =   31.783891698658;        % steady state assets
  kss = lambda*ass;               % steady state capital

t = 15000;  %% number of periods
n = 5000;   %% number of households
eshocks1= normrnd(0,sigmae1,[lambda*n t]);  % shocks c.
eshocks2= normrnd(0,sigmae2,[(1-lambda)*n t]); % shocks w.
zshocks = normrnd(0,sigmaz,[1 t]); % productivity shocks
```

```
% Variables:
a = ones(t,n);
c = ones(t,n);
y = ones(t,n);
k = ones(1,t);
w = ones(1,t);
r = ones(1,t);
z = ones(1,t);
l = ones(1,t);
e = ones(t,n);

% Starting values of variables:
a(1:2,1:lambda*n) = ass;
a(1:t,(lambda*n+1):n)=0;
k(1:2) = lambda*ass;
c(1:lambda*n,1) = (ass^alfa)*(1-tau*alfa) - delta*ass;

% Simulation loop
for i = 2:(t-1)
    k(i) = lambda*mean(a(i,1:lambda*n));
    z(i) = (1-roz) + roz*z(i-1)+zshocks(i);
    l(i) = 1 + (1-lambda)*roez*(z(i)-1)/(1-roe);
    w(i) = (1-alfa)*z(i)*((k(i))^(alfa))*l(i)^(-alfa);
    r(i) = alfa*z(i)*((k(i))^(alfa-1))*l(i)^(1-alfa);
    for j = 1:lambda*n
        e(i,j) = (1-roe) + roe*e(i-1,j)+eshocks1(j,i);
        a(i+1,j) = vTheta(1) + vTheta(2)*a(i,j) +
        + vTheta(3)*e(i,j) + vTheta(4)*z(i) + vTheta(5)*k(i);
        c(i,j)   = (1-tau)*r(i)*a(i,j) + w(i)*e(i,j) +
        + (1-delta)*a(i,j) - a(i+1,j);
        y(i,j)   = (1-tau)*r(i)*a(i,j) + w(i)*e(i,j);
    end
    for m = 1:(1-lambda)*n
        e(i,m+lambda*n) = (1-roe) + roe*e(i-1,m+lambda*n) +
        + roez*(z(i)-1) + eshocks2(m,i);
        c(i,m+lambda*n) = w(i)*e(i,m+lambda*n) +
```

```
        + tau*r(i)*k(i)/(1-lambda);
        y(i,m+lambda*n) = c(i,m+lambda*n);
    end
end

% Sort variables for the Gini
asort = ones(t,n);
csort = ones(t,n);
ysort = ones(t,n);
for i = 1:(t-1)
    asort(i,:)=sort(a(i,:));
    csort(i,:)=sort(c(i,:));
    ysort(i,:)=sort(y(i,:));
end

% Compute wealth gini
giniA = ones(1,t);
for i = 1:(t-1)
    sum1 = 0;
    sum2 = 0;
    for j = 1:n
        sum1 = (n+1-j)*asort(i,j) + sum1;
        sum2 = asort(i,j) + sum2;
    end
    giniA(i) = (n+1-2*(sum1/sum2))/n;
end

% Compute consumption gini
giniC = ones(1,t);
for i = 1:(t-1)
    sum1 = 0;
    sum2 = 0;
    for j = 1:n
        sum1 = (n+1-j)*csort(i,j) + sum1;
        sum2 = csort(i,j) + sum2;
    end
    giniC(i) = (n+1-2*(sum1/sum2))/n;
```

```
end

% Compute income gini
giniY = ones(1,t);
for i = 1:(t-1)
    sum1 = 0;
    sum2 = 0;
    for j = 1:n
        sum1 = (n+1-j)*ysort(i,j) + sum1;
        sum2 = ysort(i,j) + sum2;
    end
    giniY(i) = (n+1-2*(sum1/sum2))/n;
end

% Compute Lorenz curve (lor)
sum = 0;
lor = ones(n);
atrim = asort(t-1,1:n);
atrim(1:350)=0;
for i = 1:n
    sum = atrim(i) + sum;
end
cum = 0;
ind = 1:n;
for i = 1:n
    cum = atrim(i)+cum;
    lor(i) = cum/sum;
end

% Compute theil coefficient
theilY = ones(1,t);
for i = 1:(t-1)
    ybar = mean(y(i,:));
    sum1 = 0;
    for j = 1:n
        sum1 = (y(i,j)/ybar)*log(y(i,j)/ybar)+sum1;
    end
```

```
    theilY(i) = sum1/n;
end

% Compute percentile income ratios
ratio1 = ones(1,t);
for i = 1:(t-1)
    ratio1(i) = ysort(i,0.9*n)/ysort(i,0.1*n);
end

ratio2 = ones(1,t);
for i = 1:(t-1)
    ratio2(i) = ysort(i,0.9*n)/ysort(i,0.5*n);
end

ratio3 = ones(1,t);
for i = 1:(t-1)
    ratio3(i) = ysort(i,0.5*n)/ysort(i,0.1*n);
end

% Compute theil coefficient decomposition
theilC = ones(1,t);
ytotalC = ones(1,t);
for i = 1:(t-1)
    ybarC = mean(y(i,1:lambda*n));
    sum1 = 0;
    sum2 = 0;
    for j = 1:lambda*n
        sum1 = (y(i,j)/ybarC)*log(y(i,j)/ybarC)+sum1;
        sum2 = y(i,j) + sum2;
    end
    ytotalC(i) = sum2;
    theilC(i) = sum1/(lambda*n);
end

theilW = ones(1,t);
ytotalW = ones(1,t);
for i = 1:(t-1)
```

```
    ybarW = mean(y(i,(1+lambda*n):n));
    sum1 = 0;
    sum2 = 0;
    for j = 1:(1-lambda)*n
        sum1 = (y(i,j+lambda*n)/ybarW)*log(y(i,j+
        +lambda*n)/ybarW)+sum1;
        sum2 = y(i,j+lambda*n) + sum2;
    end
    ytotalW(i) = sum2;
    theilW(i) = sum1/((1-lambda)*n);
end

ytotal = ones(1,t);
for i = 1:(t-1)
    sum1 = 0;
    for j = 1:n
        sum1 = y(i,j) + sum1;
    end
    ytotal(i)=sum1;
end

shareC = ones(1,t);
shareW = ones(1,t);
meanC = ones(1,t);
meanW = ones(1,t);
meantotal = ones(1,t);
theilCW = ones(1,t);
for i = 1:(t-1)
shareC(i) = ytotalC(i)/ytotal(i);
shareW(i) = ytotalW(i)/ytotal(i);
meanC(i) = mean(y(i,1:lambda*n));
meanW(i) = mean(y(i,(1+lambda*n):n));
meantotal(i) = mean(y(i,:));
theilCW(i) = shareC(i)*theilC(i) + shareW(i)*theilW(i) +
+ shareC(i)*log(meanC(i)/meantotal(i)) +
+ shareW(i)*log(meanW(i)/meantotal(i));
end
```