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Faculty of Social Sciences
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BACHELOR THESIS

Are More Liquid Stocks Also More Efficient?

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, May 15, 2014

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Signature

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Abstract

Liquidity and informational efficiency are closely watched features of financial markets. Together with stock exchange size effect, captured by market capitalization, this thesis examines the triple of relationships among these three stock market properties. Applying methods of sequences and reversals ratio test, autocorrelation coefficient test and variance ratio test provided us with 14 proxy measures of efficiency for each stock. Daily prices and volumes traded for period 2003 - 2013 of 206 stocks sampled from 22 stock exchanges were used. The same data were used for Amihud illiquidity measure. The positive relationship between stock efficiency and liquidity was not strongly supported neither rejected. It turned out that stock liquidity is very strongly positively dependent on size of stock exchange where is that particular stock listed. It was also concluded that there are more efficient stocks listed in larger stock exchanged.

Abstrakt

Likvidita a informační efektivita jsou pozorně sledovanými vlastnostmi finančních trhů. Společně s vlivem velikosti burzy, měřeným tržní kapitalizací, je cílem této práce prozkoumat trojici vzájemných vztahů. Aplikací metod testu založeném na bodu zvratu, testu autokorelace a testu podílů rozptylů jsme získali 14 zástupných měr efektivity pro každou akcii. Byly použity denní ceny a zobchodované objemy 206 akcií vybraných z 22 burz za období 2003 - 2013. Stejná data se využila pro získání Amihudovy míry nelikvidity. Kladný vztah mezi efektivitou akcí a jejich likviditou nebyl výrazně potvrzen ani vyvrácen. Výsledky ukázaly, že likvidita akcií je velmi silně kladně závislá na velikosti burzy, na které je akcie kótovaná. Stejně tak se došlo k závěru, že efektivnější akcie jsou kótovány na větších burzách.

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Bachelor Thesis Proposal

Author	Petr Kupka
Supervisor	PhDr. Ladislav Krištoufek Ph.D.
Proposed topic	Are More Liquid Stocks Also More Efficient?

Topic characteristics   On the one hand, efficient market hypothesis, based on prompt incorporating of information into the individual stock price, provides methods and tests allowing us to determine the intensiveness of the market efficiency. On the other hand, liquidity, a key part of empirical analysis of finance, influences conclusions in asset pricing and corporate finance management. It is measured through various proxy variables using high-frequency data. The ultimate goal of this thesis is to analyse the dependence between liquidity and efficiency of stocks. This dependence is assumed to be positive. Moreover, stock origin could have effect on properties of this relationship. This will be explored more deeply too.

Hypotheses

Relationship between stock liquidity and efficiency is positive.

There is significant influence of some stock origin feature on its liquidity.

There is significant influence of stock origin feature on its efficiency.

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5. Conclusion

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         Author   Supervisor

Chapter 1

Introduction

Liquidity and market efficiency are the key features of financial markets, yet little attention was dedicated to their mutual relationship and common determinants. Ability to trade large volume of financial asset within short time period at low costs is considered to be liquidity. Liquid markets are characterized by higher volumes traded, lower volatility and higher trading activity. There is wide range of viewpoints on what makes the market efficient. Nevertheless, market is assumed to be efficient if it reflects all available information in prices of financial assets traded. The speed and accuracy of incorporating of the information are the determinants of this market property. Together with effect of stock exchange size, purpose of this thesis is to examine the triple of relationships among these three stock market features.

Traders, regulators, exchange institutions and researchers devoted a lot of time and effort to gain better understanding of liquidity since it is a key concept in finance. According to Chordia *et al.* (2005), liquid asset would be denoted the one that is easily, quickly and at low cost exchangeable for money which is considered as the most liquid asset.

Liquidity is often characterized by the high level of trading activity. The more actively do traders participate in a market the lesser costs related to transactions and less affected is the asset's price. In work directly related to the liquidity and trading activity, Chordia *et al.* (2001) attempt to identify and explore their determinants and provide better understanding to the changes in these two variables. Approaching the topic from inventories paradigm, it is suggested that liquidity is driven by factors influencing risk of dealer's inventory, the short-term and long-term interest rates, default spreads, market volatility and movements in the market. Informed speculation paradigm adds impor-

tant international events and influential governmental announcements about state of economy and government's intention as a possible sources of change in liquidity and trading activity. Day of week and main holidays effects have been taken into consideration too. Observing time period from 1988 to 1998 of stock sampled from New York Stock Exchange and averaged each trading day, Chordia *et al.* (2001) concluded that equity market returns, market volatility and short-term interest rate influence significantly market liquidity and trading activity. Moreover, while Tuesdays are on average days with the largest trading activity and highest liquidity, Fridays could be described as the opposite. Among another interesting results belong the inverse relationship between down-market, variable describing movement of a market in both directions up and down, and both effective and quoted bid-ask spreads. In down-market the spreads are increasing substantially and vice versa.

These results are well in line with the results reported in Chordia *et al.* (2000). This earlier work aims to observe commonality in liquidity which can be described as less recognizable determinants of liquidity inducing correlated movements. The main point of this empirical work lies in confirmation that individual liquidity measures move synchronously even after accounting for the well-known determinants such as volume, volatility and returns.

From somewhat different perspective is approached later study of the relationships among liquidity, stock market returns and trading activity. Instead of measuring the trading activity by volume, it was preferred to use the imbalance between buying and selling orders. According to Chordia *et al.* (2002), daily order imbalances are strongly positively autocorrelated. At the same time, excess of either buy or sell orders reduce market-wide liquidity. In terms of New York Stock Exchange, it is possible to predict liquidity changes based on market returns due to the fact that down market days are accompanied by days of decreased liquidity. The opposite is true in case of up market days. Furthermore, investors are considered to be contrarian on average as buying orders prevails in periods when market falls while the selling activity is more common when market rises.

The above mentioned studies explored factors that allow market participants to anticipate changes in market liquidity. Following paper is dedicated to its relation to the market efficiency which is also very closely watched feature of financial markets. Any financial asset is said to be efficient if its price reflects all relevant information without any time lags. Chordia *et al.* (2008) states that there are three competing opinions regarding the relation between

liquidity and market efficiency.

The supportive attitude assumes limited risk-bearing capacity of market makers which, together with incoming order flows, causes price pressure resulting in deviation from fundamental value of an asset. Due to this, past order flows may allow agents who are attentively monitoring the market to predict short horizon returns. The faster the arbitrageurs take action the earlier will market adjust to its initial position. Thus, higher trading activity reduces predictability of returns which means higher market efficiency. On the other hand, market makers may fail to react to order flow properly. Some agents could decide to make a profit with trading on information about order flow. Such agents make asset prices reflect more the order flow while market liquidity could decrease as market makers face adverse selection problem. Finally, third hypothesis suppose that rational market makers identify order imbalances and adjust quotes promptly enough. This would mean that order flows serve no longer as an illiquidity instrument for predicting market returns. Observing comprehensive sample of all NYSE stock trader during period from 1993 to 2002, Chordia *et al.* (2008) concluded that return predictability is diminished in time of high market liquidity. This is consistent with hypothesis that increased activity in periods of higher liquidity implies rise in market efficiency. Although methods used in this study are different from those that are intended to be applied in this thesis, it is be very beneficial to be able to compare results of this thesis to the findings published in this empirical analysis.

This bachelor thesis is structured as follows. First part is dedicated to the review of empirical studies related to the market efficiency. It begins with random walk hypothesis and continues with papers providing evidence of price development predictability and review of weak form efficiency tests from stock exchanges worldwide. Then, methodology part is consisted of description of tools for evaluating stocks' efficiency and liquidity together with forming our hypotheses and stating expectations of our results. It is followed by a short price and volume time series description. Finally, empirical results of the three relationships are presented and compared to the findings provided in other empirical papers.

Chapter 2

Market Efficiency

2.1 Classic Taxonomy

Literature published on topic of market efficiency is divided by classic taxonomy, described in Fama (1970), [1] into three subsets according to relevant informational sets one by one presented in this section.

The weak form of the EMH asserts that based on obsolete information, assumed to be incorporated into historical prices, it is impossible to earn any additional profits after risk-adjustment. Therefore, the information set includes only the historical prices and returns. Assuming risk neutrality of market participants, this form of EMH is equivalent to the random walk hypothesis Fama (1965). Thus, market prices take a random and unpredictable path.

The semi-strong form of the EMH states that traders, provided with all publicly available information, are unable to make extra risk-adjusted profits. The reason is immediate incorporation of publicly announced information into market prices. Hence, investors do not gain any advantage by improving their predictions using this information. Apparently, semi-strong form includes weak form as well, as past prices are basically subset of a set composed of publicly available information. Tests for semi-strong form serve for ascertaining whether security prices reflect public information announcements quickly enough.

The strong form of the EMH augments the semi-strong form by the set of private undisclosed information. Therefore, the information set consists of publicly and privately known information both historical and current. According to the strong form of the EMH, investors cannot exceed the normal risk-adjusted

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[1]Fama notes that distinction between weak and strong form of tests was first suggested by Roberts (1967)

return using insiders' information as it leaks out quickly and, therefore this information is incorporated into prices as well. The strong form tests are dedicated to find out whether specific investors use private information for trading. However, most researches are devoted to weak and semi-strong forms of the EMH since researchers are hardly ever provided with private information.

In this thesis, we are interested only in the weak form of the efficient market hypothesis. Key empirical findings are presented in next part. However, we focus especially on random walk hypothesis as it is the one that is examined in the empirical part of this thesis.

2.2 Empirical Papers

Comparing the number of literature published on market liquidity to the vast quantity of literature written on market efficiency, it was difficult to choose the most influential papers to be reviewed in this section. During the last four decades, the efficient market hypothesis (EMH) earned remarkable empirical attention. The concept of market efficiency undoubtedly belongs among the most controversial and vivid topics of finance. Plenty of papers presenting both supporting or opposing evidences of the EMH validity have already been published. This section is devoted to empirical papers that has provided both evidence in favor of efficient market hypothesis and evidence of capital markets' inefficiencies. In first part, papers in support of efficient market hypothesis are reviewed realizing that plenty of empirical papers could be included there. It was decided to include those with larger impact and papers that provide understanding of market prices' random behavior. Papers mentioned in second part are just several of those published with intention to challenge efficient market hypothesis. These papers commonly describe cases in which it is possible to predict security prices development to certain extent. The third part rather summarizes more recent papers that investigate weak form of stock market efficiency in various stock exchanges.

2.2.1 Random Walk Hypothesis

In 1933, Alfred Cowles published an empirical paper on possibility to forecast prices of common stocks based on their historical price developments. Analyzing about 7500 recommendations of individual common stocks during the period from January, 1928 to June, 1932, it was found that more than half of

forecasts were unsuccessful. Conclusion of the Cowles (1933) is that following a forecast gains no significant advantage over purely random investment strategy.

Cowles & Jones (1937) focuses rather on stock return behavior. In this analysis, sequence of stock returns signs is compared to penny-tossing series as it is expected with probability one-half that tails will follow heads and vice versa. This paper tries to answer whether sequences and reversals ratio differs with the length of return. Observing twenty-seven different series from 1835 to 1935 with various returns from 20 minutes to 10 years, it was found out that for shorter units of time sequences are more frequent than reversals. However, it was later pointed out by Working (1960) that using monthly averages from data on daily and monthly basis will cause a positive correlation. Cowles (1960) admits that this drawback led him and Jones to erroneous conclusion that sequences largely excess reversals in shorter time periods.

As regards description of biases caused by temporal aggregation, Working (1960) provides additional evidence of spurious correlation in consecutive price changes caused by averaging random walk data over time. Working notices that the behavior of sampled data may differ from the original stochastic process he observed and sampled data from. He concluded that even presence of spurious correlation in data does not necessarily mean that price series do not follow random walk.

In Kendall (1953), it was attempted to find difference between long-term and short-term movements of economic time series. He assumed that the time series are influenced by certain causations. Kendall analyzed twenty-two mainly British industrial share price series on weekly basis. He intended to construct models conforming the share prices behavior. Nevertheless, there was not observed any systematic pattern in the data. In summary of results, Kendall states that price series behave randomly to the extent that it is impossible to fit them any systematic model. He revealed little serial correlation within the series, nevertheless, he expresses conviction that the negligible size of the correlation could hardly serve for any movement prediction. Similar belief is presented in Roberts (1959), less comprehensive paper focusing on both US indices and individual companies' stock data comparable to those used by Kendall. Roberts, however, perceive changes in stock prices behavior as a roulette with no memory of previous spins. So called "Chance Model" assumes independence and stable relative frequencies of outcomes. Thus, Roberts suggests model, the part that Kendall (1953) missed, but he provides no profound empirical analysis of the price series.

Analogy between stock prices behavior and behavior of large number of molecules is presented in Osborne (1959). In analysis of New York Stock Exchange prices, Osborne used methods of statistical mechanics. To him, changes in stock prices seemed to move randomly and in a manner similar to the Brownian motion.

The purpose of Fama (1965) was to carry out empirical test whether random-walk model describes stock price movements. Independence of successive price changes and uniformity of price changes distribution were the two basic assumptions that random walk model require. Daily returns adjusted of dividends and splits of thirty DJIA stock over period of five years seemed to fulfill both the serial independence and distribution uniformity. Moreover, Fama concluded that random-walk serves as adequate description of reality.

Papers reviewed above could be considered as EMH supportive ones. Despite the fact that these papers were elaborated with distinctive purposes, they together created strong believe in validity of security prices random movement. In the light of these evidences, technical analysis seemed to be much less powerful tool for predicting stock prices development.

2.2.2 Predictability of Price Development

Another test of random walk hypothesis brought Lo & MacKinlay (1989). Analysis of US stock indices' weekly returns during period from 1962 to 1985 provided the analysts with sufficient number of observations to be able to rely on law of large numbers. Analysis detected spurious correlation caused by incorporation of new information in stocks of larger companies earlier than to the stocks of the smaller ones while all stocks together formed one index. Lo and MacKinlay applied variance ratio test and rejected random walk hypothesis as variance of the price return sequence observed grew with time more than linearly as it is assumed to be in random walk model. Interestingly enough, they concluded that individual stocks' variance fits the random walk model well.

To document returns' variance of common stocks was the main intention of French & Roll (1986). Daily returns of stocks listed on New York and American Stock Exchanges over twenty year period from year 1963 were analyzed. French and Roll found out that during non-trading periods, usually weekends, are stock return variances significantly lower than those in time when the New York exchanges are open. Explanation to this phenomenon suggested by them is that it is more likely stock prices to be affected in trading-time as flow of

public information is more frequent. Also, they note that trading itself may evoke volatility. They observed that mispricing accounts for 4-8% of total daily return variance.

Fama & French (1988) tests hypothesis suggesting dividend yield as an important factor for longer horizon predictions. Empirical survey brought that D/P ratio, portion of market price of a stock payed to investors as a dividend, can explain more than 25% of the variance in terms of two year, three year and four year returns. They explained it by stating that more than linear growth of expected returns' variance is caused by persistence of expected returns. At the same time, unexpected movements in current prices influence negatively expected returns. According to the authors, these two facts together can result in temporary or mean-reverting components of prices.

In the same year, Poterba and Summers published analysis with consistent evidence. Short time periods stock returns are positively serially correlated whereas in long horizons it was discovered negative autocorrelation. This conclusion was reached using the variance ratio tests of market prices of US stocks and indices for period 1926-1986. Poterba & Summers (1988) provide evidence of mean reversion in the long stock prices series, however, they failed to reject random walk hypothesis as the significance level was not high enough. This findings are in favor of contrarian investment rule which is based on buying stock that have not performed well in the past while selling those that grew faster than was expected.

Long-term memory is a feature accompanied with high persistence which is characterized by autocorrelation of stock returns. If stock returns are auto-correlated, it increases predictability of particular financial asset and decreases its efficiency. Lo (1991) creates adjusted R/S test that is robust to short-term correlations documented in Lo & MacKinlay (1989). This tool is applied to US indices data with intention to detect possible non-random behavior of market prices. However, presence of long-term memory was not statistically significant at 5% level in any of examined periods. To the contrary of earlier evidence, Lo (1991, p. 1308) states: "I find little evidence of long-term memory in historical US stock market returns."

2.2.3   Weak Form Efficiency in Stock Exchanges

[p. 383]

Exploration of Korean Stock Exchange market index efficiency is purpose of

Ayadi & Pyun (1994). In this empirical analysis, variance ratios test developed by Lo and MacKinlay was applied. Variance ratio test deals with spurious correlation, pattern often present in data from stock markets of developing countries. According to authors, traditional tests of random walk are influenced by the spurious correlation which could lead to incorrect conclusion. Using close prices for five year period, it was found out that under homoskedasticity assumption random walk hypothesis would be rejected for all intervals. However, heteroskedasticity robust test statistic fails to reject the null hypothesis.

Recent testing of weak form efficiency, more accurately martingale model, by Kim & Shamsuddin (2008) is extensive empirical study of nine Asian markets: Hong Kong, Indonesia, Japan, Korea, Malaysia, Philippines, Singapore, Taiwan and Thailand. There was applied more methods of testing including variance ratio test and Chow-Denning test to be allowed to test wider range of holding periods. Authors brightly divided their dataset into two subperiods to be able to conduct analysis before and after the Asian crisis in 1997. This allowed them to find out whether regulatory measures concerning transparency of corporate governance affected markets after the the deep recession. Markets of Singapore and Thailand indicated improvement as they became more efficient after the crisis. However, Philippine, Malaysian and Indonesian stock markets have been denoted as insufficiently efficient ones in both periods 1990-1996 and 1998-2005. The rest of the countries (Hong Kong, Japan, Korea, Singapore, Taiwan) which are considered as developed or at least advanced among the emerging countries demonstrated efficiency. This paper also provides comprehensive comparison of Asian markets studies. Especially methodology, data used and above all results are presented. Findings stated in Kim & Shamsuddin (2008) are mostly consistent with results of other authors' studies.

As regards Russian stock market, Abrosimova *et al.* (2002) conducted a weak-form efficiency test of Russian Trading System index for period 1995-2001. For determining whether random walk hypothesis holds, it was used unit root, autocorrelation and variance ratio tests on daily, weekly and monthly time series. Abrosimova *et al.* (2002) found that for monthly data is RTS index fairly efficient while observing daily and weekly returns led to rejection of random behavior of the index. According to author, RTS is at least in terms of monthly returns efficient because it is composed of the largest, most liquid and carefully watched stocks by both global and Russian investors. Notice, please, that there is indirectly expected positive relationship between index's efficiency and liquidity of its component stocks.

Comparison of US stock markets and countries of Central and Eastern Europe was published in Diviš & Teplý (2005). Weak-form efficiency analysis of Czech PX 50, Hungarian BUX, Polish WIG and Slovak SAX indices is done relatively to the DJIA index which is considered to be highly efficient and serves in this paper as a benchmark. Applying variance ratio tests on weekly data from period 1993-2004 confirmed informational efficiency of DJIA index. The same approach led to conclusion that the random walk hypothesis could not be rejected for any of the indices under heteroskedasticity of errors assumption. However, there is noticeable improvement of test statistics for CEE indices in later subperiod from year 1998. According to Diviš & Teplý (2005), the reason is development and further stabilization of stock exchanges in studied countries. Besides the fact that DJIA is far more efficient, it is interesting to remark that corresponding test statistics of Polish WIG index are all more in favor of efficiency than test statistics of Czech PX 50 or Slovak SAX indices.

Similarly, Smith & Ryoo (2003) carried out variance ratio tests for various European indices including Greece, Hungary, Poland, Portugal and Turkey over time period from 1991 to 1998. Logarithm of daily market prices with holding periods of two, four, eight and sixteen days, in general, does not follow random walk in all countries except Turkey. Authors state that Istanbul market incorporates public information more promptly, thus it turns out to be more weak-form efficient. Moreover, they consider higher liquidity of Turkish market as very important factor in relation to its efficiency. Authors suggest topic of this relation to further investigation.

Chapter 3

Methodology

The methodology part begins with an ideas and papers shaping the efficient market hypothesis. The middle part is devoted to description of tools for testing stock efficiency and measuring stock liquidity. Finally, three main hypothesis of the empirical part are stated and expressed in testable way.

3.1 Conceptual Framework of Market Efficiency

Ideas shaping EMH could be found even before the beginning of the 20th century. However, Bachelier (1900) dissertation is regarded as the first considerable contribution to this theory. Nevertheless, his work was given no attention for more than fifty years. He was also aware of market prices random behavior. Result of empirical research made in 1930s, Cowles (1933), indicated stock market forecasters' incapability to forecast. Its continuation, Cowles (1944), added that professional investors cannot beat the market. Based on observation, Kendall (1953), Working (1934) and Roberts (1959) presented consistently that speculative price series may be well described by random walk.

First theorems proved by Samuelson (1965) and Fama (1965), assuming competitive markets and rationality of agents, showed that security prices follow random walks. Fama (1970, p. 383) reviews theory and empirical work on the efficient markets by stating:

> "A market in which prices always 'fully reflect' all available informations is called 'efficient'."

As regards possibility to test EMH defined in this manner, Fama expressed need for closer specification of term "fully reflect" as a price formatting process. In his later research, Fama (1991) pointed out precondition for the strong

version of the hypothesis. Basically, information and trading costs are always equal to zero. Precondition for weaker version is satisfied if prices reflect information to the extent that additional benefits of trading on information are lesser than the marginal costs. Assuming trading costs equal to zero is quite far from reality and rather serves as unattainable benchmark. However, its advantage lies in leaving issue of reasonable information and trading costs aside. In addition to Fama (1970), Malkiel (1992) defines efficient market as:

> "...market is said to be efficient with respect to some information set, Φ, if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set, Φ, implies that it is impossible to make economic profits by trading on the basis of Φ."

By this definition Malkiel suggests two ways of testing market efficiency. The first and more difficult to carry out in practice is based on measuring the response of market prices after revealing some set of information to investors. If no significant change in prices occurs, market could be considered as efficient with respect to this information set. As an alternative approach, observing the profits made by trading on certain set of information is proposed. In essence, if traders were able to consistently create profits for a long period of time then market fails to be efficient. Almost all researches on market efficiency are based on this idea (Campbell *et al.* 1997).

Assumptions According to these definitions, the efficient market hypothesis holds as long as following conditions are satisfied (Shleifer 2000):

(i) Many rational profit maximizing participants exist who actively trade, hence security prices are assessed rationally.

(ii) If some participants are irrational, their trades cancel each other or rational arbitrageurs eliminate their influence keeping price unaffected.

(iii) All information is available without any costs to all investors at approximately same time. Prompt reactions of market participants to the new information cause stock prices adjustment.

Let elaborate more on the third assumption. It consists of three parts: No costs of obtaining information, no cost of trading and availability of all information

to all market participants. Although, none of them would be met in the real market, these conditions are sufficient for market efficiency, but not necessary ones (Fama 1970). Fama explains that even an inhibition of the transaction flows by imposing costs of trading does not necessarily imply market inefficiency. As long as "sufficient numbers" of investors initiate transactions based on available information, market prices "fully reflect" this set of information.

Rationality of Agents   As regards rationality of market participants condition of the EMH, Shleifer (2000) divides it into tree progressively weaker assumptions.

As it was already stated in (i), investors are presumed to value the securities rationally. Such investors' evaluation of a security would not be different from its fundamental value. Once investors will be provided new information related to this security they would reassess its value and take appropriate action in the market. The strongest assumption of investors rationality is sufficient for proper absorption of information into the security price. However, perfect rationality of investors is not expected to be valid in practice.

If market participants lack rationality, it is assumed that transaction initiated by them are random and all they cancel each other leaving market price unaffected. In case of large number of irrational traders with uncorrelated strategies, their trades will probably result in no impact on market price. However, this argument is quite limited as it relies substantially on diversity of the irrational investors' strategies.

To the extend that investors trade irrationally and their transactions are correlated, there are arbitrageurs in the market who eliminate their effect on market price. Argument of arbitrageurs benefiting form irrational price adjustments developed by Friedman (1953), is very intuitive. When market price of a security appears to be substantially different from its fundamental value, rational profit maximizing trader would not hesitate to use this opportunity. Transactions initiated by arbitrageurs would lead security price to the rationally valued one. In fact, there are many arbitrageurs in the market competing with each other to earn profits, therefore it is reasonable to believe in quick and effective match of market and fundamental values. Friedman (1953) states further implication of arbitrage. Simply, irrational investors are involved in transactions with arbitrageurs who profit from such trading. The other side of the same coin is as follows. Presence of arbitrageurs means presence of irrational traders who lose their wealth. As they cannot lose money forever, they

are made to quit the market. Thus both, their influence on the market price and number of such market participants are eliminated by arbitrageurs.

Coherence and power of the theoretical arguments for the rational behaviour of a market seem to be infrangible. Irrationality of market participants is not binding for existence of efficient market as long as there is present "sufficient number" of rational investors. Nevertheless, Barberis & Thaler (2003) argue that at first glance Friedman's arguments appear to be valid, but it would not be perfectly correct after deep theoretical reconsideration. The drawback is seen in assertion that irrationality of market participants create opportunity for riskless profits. According to Barberis & Thaler (2003), widely mispriced asset is corrected with strategies which can be both risky and costly. Thus, rational market participants could be discouraged to invest in such asset. As a consequence, market price would deviate from fundamental value of the asset and the mispricing would persist.

3.2 Random Walk Tests

In this section, random walk models from theory of market efficiency and tests for determination whether this models suits the real market conditions well. For all types of random walk models with drift, two following properties are described:

$$E[P_t|P_0] = P_0 + \mu t$$

$$Var[P_t|P_0] = \sigma^2 t$$

This means that based on price from previous period P_0 we expect this period price P_1 to be the sum of P_0 and the drift term which expresses the average movement within the examined period. The second equation describes linear dependence of price's variance on time. These two properties are used for testing the random walk hypothesis in various forms. For proper description of test statistics construction, it was found inspiration in Campbell *et al.* (1997).

3.2.1 Random Walk I Model: IID Increments

The first random walk model is characterized with independent identically distributed increments (IID). Price behavior under this model suits to following

equation:

$$P_t = \mu + P_{t-1} + \epsilon_t, \qquad \epsilon_t \sim IID(0, \sigma^2)$$

where μ describes the expected value of the price difference commonly referred as drift, and ϵ_t denotes unobservable increment which is independent of increments observed in any other time. At the same time, ϵ_t identically distributed with mean μ and variance σ^2. Assuming ϵ_t independent means that there is no linear neither nonlinear function of any form that could be used to trace dependency within increments. This assumption will be relaxed in random walk III model which is presented below. Normality assumption for increments is desirable for easier computation. However, under the normality assumption there is small but still positive probability that price in certain time turns out to be negative. For this reason, it is preferable to use $p_t = log(P_t)$ which is transformation of natural logarithm of P_t. Thus, equation

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim IID \ \mathcal{N}(0, \sigma^2)$$

describes the random walk I model with all desirable properties.

Sequences and Reversals

Sequence and Reversals test serving for testing random walk I model was presented in Cowles & Jones (1937). Under the without drift conditions, sequences and reversals test is based on idea that stock price is equally probable to increase and decrease. Series of consecutive movements in the same direction is referred as sequence while movement in opposite direction than the preceding on is called reversal. CJ ratio is constructed by summing all sequences and all reversals and putting them into the ratio. The closer is the CJ ratio to one the more balanced is the price development and the higher probability that the security price will follow random walk I model. Applying this approach to the random walk with drift model is a little less straightforward.

Let p_t denotes natural logarithm of stock price in time t and follows random walk with drift with IID increments.

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim IID(0, \sigma^2)$$

First of all, construction of indicator for distinguishing the sign of return is

needed.

$$I_t = \begin{cases} 1 & \text{if } r_t \equiv p_t - p_{t-1} > 0 \\ 0 & \text{if } r_t \equiv p_t - p_{t-1} \leq 0. \end{cases}$$

I_t is an indicator whether p_t simply increased with value of 1 or decreased with value of 0. Moreover, I_t has not fair chance to be either 0 or 1 as it has to be taken into account effect of the drift.

$$I_t = \begin{cases} 1 & \text{w.p.} \quad \pi \\ 0 & \text{w.p.} \quad 1 - \pi, \end{cases}$$

where

$$\pi \equiv P(r_t > 0) \qquad r_t \sim IID(\mu, \sigma^2).$$

Having $n + 1$ observations of returns $r_1, ..., r_{n+1}$, total number of sequences N_s and reversals N_r may be computed using I_t.

$$Y_t \equiv I_t I_{t+1} + (1 - I_t)(1 - I_{t+1})$$

$$N_s \equiv \sum_{t=1}^{n} Y_t \qquad N_r \equiv n - N_s$$

In case of RW model without drift, it is equally likely that two consecutive returns are in the same direction as in the opposite one. CJ ration would be then expected to be equal to one.

$$\widehat{CJ} \equiv \frac{N_s}{N_r} = \frac{N_s/n}{N_r/n} = \frac{\widehat{\pi}_s}{1 - \widehat{\pi}_s} \xrightarrow{pr} \frac{\pi_s}{1 - \pi_s} = CJ = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

In general case of model with drift,

$$Y_t = \begin{cases} 1 & \text{w.p.} \quad \pi_s = \pi^2 + (1 - \pi)^2 \\ 0 & \text{w.p.} \quad 1 - \pi_s. \end{cases}$$

Presence of nonzero drift causes sequences more likely than reversals because either positive or negative drift is accompanied with certain trend.

$$CJ = \frac{\pi^2 + (1 - \pi)^2}{2\pi(1 - \pi)} \geq 1$$

To be able to test the random walk hypothesis distribution of $\widehat{CJ}$ test statistics has to be matched with appropriate distribution. Realize that Y_t is Bernouli random variable and N_s is sum of it. In other words, N_s is binomial random

variable which is approximately normal distribution with mean equal to $n\pi_s$.

$$Var[N_s] = n\pi_s(1 - \pi_s) + 2nCov[Y_t, Y_{t+1}]$$

$$= n\pi_s(1 - \pi_s) + 2(\pi^3 + (1 - \pi)^3 - \pi_s^2)$$

Therefore, the test statistics $\widehat{CJ}$ is computed as

$$\widehat{CJ} = \frac{N_s}{n - N_s},$$

and belongs asymptotically to normal distribution

$$\widehat{CJ} \sim \mathcal{N}\left(\frac{\pi_s}{1 - \pi_s}, \frac{n\pi_s(1 - \pi_s) + 2(\pi^3 + (1 - \pi)^3 - \pi_s^2)}{n(1 - \pi_s)^4}\right).$$

3.2.2   Random Walk II Model: Independent Increments

Random Walk II model is generalized case of random walk I model as it assumes independence of increments but assumption of identically distributed increments is no longer valid. This model deals with changes in external environment that influence the price creation. Economic, technological or institutional conditions in long history of stock exchanges' existence are not constant, thus distribution from which are increments observed varies across time too. But there is still not possible to find any relationship among increments that would allow us to predict its future development. Equation

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim INID$$

expresses well the random walk II model where INID means that increments ϵ_t are independent but not identically distributed random variables.

3.2.3   Random Walk III Model: Uncorrelated Increments

The third and the most general version of random walk models presented in this thesis is model where each increment is linearly unrelated to any of the other increments. Random Walk III model, however, does not constrain any nonlinear dependence of increments which is the relaxation of independence assumption. If $k, l \in N$ and $k \neq l$ then for increments of RW1 hold $E[\epsilon_k, \epsilon_l] = E[\epsilon_k]E[\epsilon_l] = 0$ following assumptions of RW2, $E[\epsilon_k, \epsilon_l] = E[\epsilon_k]E[\epsilon_l]$. While for increments in RW3 satisfy only $Cov[\epsilon_k, \epsilon_l] = 0$. Thus, random walk III model

is described by following equation:

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad Cov[\epsilon_k, \epsilon_l] = 0 \qquad k, l \in N; k \neq l$$

Nonidentical distribution of increments is advantage of RW II and RW III models because it enables ϵ_t's to infringe homoskedasticity assumption which is very binding and often not present in stock market data. Allowing for nonlinear dependence together with heteroskedasticity of increments simplifies testing of random walk hypothesis and correspond better to the real world conditions.

Autocorrelation Coefficients

Determination of correlation coefficient between stock's returns and their lags is the nature of the autocorrelation coefficients test. The lags are used for examination of linear dependence among more than just adjacent returns. Under assumption of covariance-stationary process r_t, definitions of autocovariance and autocorrelation functions are as follows. First step is definition of autocovariance function $\gamma(k)$:

$$\gamma(k) \equiv Cov[r_t, r_{t+k}]$$

Value of gamma simply depends on the distance between returns. From the covariance-stationarity assumption arise that this autocovariance function is independent of t. Autocorrelation function $\rho(k)$ uses prior autocovariance function's definition and assumption of covariance-stationarity.

$$\rho(k) \equiv \frac{Cov[r_t, r_{t+k}]}{\sqrt{Var[r_t]}\sqrt{Var[r_{t+k}]}} = \frac{Cov[r_t, r_{t+k}]}{Var[r_t]} = \frac{\gamma(k)}{\gamma(0)}$$

For a sample of T returns $\{r_t\}_{t=1}^T$, estimators of the coefficients are:

$$\hat{\gamma}(k) = \frac{1}{T} \sum_{t=1}^{T-k} (r_t - \bar{r}_T)(r_{t+k} - \bar{r}_T), \qquad 0 \leq k \leq T$$

$$\hat{\rho}(k) = \frac{\hat{\gamma}(k)}{\hat{\gamma}(0)}$$

$$\bar{r}_T = \frac{1}{T} \sum_{t=1}^{T}$$

Fuller (1976) states additional assumptions for data generating process of $\{r_t\}$ under which is vector of autocorrelation coefficient estimators asymptotically

multivariate normal. Thus for any $k, k \in N; k < T$ holds

$$\sqrt{T}\hat{\rho}(k) \sim \mathcal{N}(0, 1).$$

Variance Ratios

As it was already mentioned above, the random walk model implies that the variance of increments is linearly dependent on time. For example, quarterly stock returns conforming to the random walk hypothesis have triple the variance as monthly returns. Tests based on this implication are referred to as Variance Ratios tests. If the random walk hypothesis holds, the basic idea of Variance Ratios is:

$$VR(q) \equiv \frac{Var(r_1 + ... + r_q)}{qVar(r_1)} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho(k) \approx 1$$

With certain simplification, no matter what k is, $\rho(k)$ stays close to zero under the random walk hypothesis. Thus, the more the particular stock returns tend to deviate from the random walk the more Variance Ratio differs from 1. For purposes of Variance Ratios test they will be described both RW1 and RW3 version of this test. First, let consider the RW1 model:

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad \epsilon_t \sim IID\ \mathcal{N}(0, \sigma^2)$$

$$r_t = \mu + \epsilon_t$$

Having at disposal $nq + 1$ stock prices observation and taking their logarithms $p_t = log(P_t)$, definition of sample mean u and sample variance σ_a^2 estimators are as follows:

$$\hat{u} \equiv \frac{1}{nq}\sum_{k=1}^{nq}(p_k - p_{k-1}) = \frac{1}{nq}(p_{nq} - p_0)$$

$$\hat{\sigma_a}^2 \equiv \frac{1}{nq-1}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{u})^2.$$

Variance of summed q returns is after improvement of finite-sample properties estimated with

$$\hat{\sigma_c}^2(q) \equiv \frac{1}{m}\sum_{k=q}^{nq}(p_k - p_{k-q} - q\hat{u})^2,$$

where

$$m \equiv q(nq - q + 1)\left(1 - \frac{q}{nq}\right).$$

Therefore, variance ratio estimate is defined as follows:

$$\widehat{VR}(q) \equiv \frac{\hat{\sigma_c}^2(q)}{\hat{\sigma_a}^2}.$$

Nevertheless, $\widehat{VR}(q)$ is not unbiased estimator. Lo & MacKinlay (1989) documents experiment simulations which confirmed that $\widehat{VR}(q)$ is close to the asymptotic limits of finite-sample properties. Test statistic z is asymptotically standard normal under assumptions of RW1 model:

$$z(q) \equiv \frac{\sqrt{nq}(\widehat{VR}(q) - 1)}{\sqrt{\frac{2(2q-1)(q-1)}{3q}}} \sim \mathcal{N}(0, 1).$$

It would be naive to believe that variance of returns is constant over time. Change of variance over time would violate assumptions of RW1 model. For this purpose Lo & MacKinlay (1988) developed Variance Ratio version for RW3 model in which returns are supposed to be at least uncorrelated. At the same time, stock returns do not have to be all of the same distribution. Therefore, variance of returns is not necessarily the same for all returns and stable over time. Besides the assumption of linear independence of returns, Lo & MacKinlay (1989) adds conditions on the extend of heterogeneity and degree of dependence among returns. Moreover, sample of ϵ_t is required to be asymptotically uncorrelated. Let assume random walk III model:

$$p_t = \mu + p_{t-1} + \epsilon_t, \qquad Cov[\epsilon_k, \epsilon_l] = 0 \qquad k, l \in N; k \neq l$$

The heteroskecastic version of standardized variance ratios test statistic is defined as follows:

$$\Psi(q) = \frac{\sqrt{nq}(\widehat{VR}(q) - 1)}{\sqrt{\hat{\theta}}} \approx \mathcal{N}(0, 1),$$

where

$$\hat{\theta}(q) \equiv 4 \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right)^2 \hat{\delta}_k$$

is heteroskedasticity robust estimator of $\theta(q)$. Heteroskedasticity-consistent

estimator of δ_k is defined flowingly:

$$\hat{\delta}_k = \frac{nq \sum_{j=k+1}^{nq} (p_j - p_{j-1} - \hat{u})^2 (p_{j-k} - p_{j-k-1} - \hat{u})^2}{\left[\sum_{j=1}^{nq} (p_j - p_{j-1} - \hat{u})^2\right]^2}.$$

As assumptions for RW3 model are weaker and more suitable to the real market conditions, there is no doubt that the heteroskedasticity-consistent test statistic reflects better market properties of a particular stock.

3.3   Amihud Illiquidity Measure

As regards liquidity, any measure used should be able to consistently express the time and cost of exchanging financial asset for cash and vice versa. Simple but reliable illiquidity measure is presented in Amihud (2002). Expectably, it uses stock returns and volatility time series, however, its composition is extraordinarily easy. Also, it is suitable to apply Amihud illiquidity measure to daily data which are more accessible and complete by far. A comprehensive study of liquidity measures, including Amihud illiquidity measure, is presented in Goyenko *et al.* (2009). It tests the hypothesis whether low-frequency measures can provide good estimate of high-frequency transaction costs. Conclusion of this empirical paper was generally in favor of the hypothesis with some positive commentaries on properties of Amihud illiquidity measure. This measure is defined as follows:

$$ILL_k = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{|r_{ik}|}{volume_{ik}}\right),$$

where ILL_k is illiqidity of k-th stock, r_{ik} is return of k-th stock in i-th period and $volume_{ik}$ is volume of k-th stock in i-th period measured in currency of stock's denomination.

3.4   Hypotheses

This thesis aims to find comprehensive way to analyses relationships between three properties of a stock. Above stated methods allow us to test or measure somehow these properties and explore their mutual influence. Let explicitly state the hypothesis.

Market efficiency, representing the speed and ease of information incorporation process into the stock price, may improve as the trading activity increases. The more traders participate on evaluating of particular stock the higher probability that the stock implicitly carries all available information. Moreover, in situations when traders or investors are not willing to trade, the fundamental value of stocks may diverge from market price which is demonstration of inefficiency. Therefore, positive relationship between market efficiency and trading activity which is together with average trading volume hand in hand with market liquidity is expected. Let denote this hypothesis H_1.

The second as well as the third hypothesis relies on the fact that larger stock exchanges attracts more active traders. Even it may seem straightforward, it is unfortunately over the scope of this thesis to test it. Perhaps, every company prefers to be listed in stock exchange with high trading activity as it reduces costs of obtaining additional capital as it is stated in Jacoby *et al.* (2000) and Amihud & Mendelson (1986). However, company managers mind fees as well. Investors may be attracted by greater variety of stocks listed in larger stock exchanges but for sure they want to find buyer for every selling order as fast as possible and at the lowest cost. This should be more probable in the stock exchanges with larger number of traders. Simply, the more traders the earlier is the trade realized and the, thus the lower costs of trading captured by liquidity. Therefore, the second hypothesis suggests the positive dependence among the size of stock exchange where is a particular stock listed and liquidity of a stock. Let use H_2 notation for this hypothesis in following parts of this text.

Using the implication that larger stock exchanges serve for more market participants, reasoning behind the third hypothesis, e.i. positive relationship between size and market efficiency, implies from the reasoning for the first two hypothesis. Generally, it is expected that all the simple mutual relationships between informational efficiency, liquidity and size of a particular stock exchange will result to be significant but almost surely positive. The third will be represented by H_3.

Proxy Measures Description

Before making decision on the simple regression model specification, let explore proxies that were obtained from price and volume time series. It is important to realize that our proxies construction was based on time dimension aggregation. This mean that each proxy represents certain stock's property.

As regard proxies representing efficiency, all of them are test statistics to the standard normal which makes it difficult to interpret. Further transformation of such data could increase significance of certain relationship, nevertheless, it would be too compicated to describe such dependence properly. Using rationale of t-statistics that were obtained by Cowless and Jones ratio, autocorrelation coefficient and variance ratios, the further from zero the less probable for random walk hypothesis to be valid. Therefore, it was decided to take simple absolute value the t-statistics representing market efficiency. For summary table of t-statistics' absolute values observations see Table 3.1. Notice that we are provided with t-statistics based on various period lags. In Figure 3.1, it is depicted dependence of average absolute value of t-statistics on its lags. Decrease of the t-statistics represents decrease returns' predictability based on the returns observed in period of particular lag. It is not surprising that yesterday's returns allow investors to predict returns of today better than those obtained last week. For more detailed summary about market capitalization of stock exchanges, Amihud illiquidity, and stock efficiency proxies see Tables A.1 - A.4. [1]

Table 3.1: Proxy Measures Summary

Variable	AVG	Std. Dev.	Min	Max
illiquidity	11733.37	79679.28	0.232	1086000
cj	9.94	49.74	0.003	545.552
ar1	4.81	5.08	0.016	21.373
ar2	1.77	1.74	0.001	15.626
ar4	1.27	1.08	0.001	6.995
ar8	1.22	1.28	0.012	11.454
ar16	1.29	1.10	0.001	7.144
vr2	4.85	5.06	0.002	21.354
vr4	4.38	4.52	0.034	18.621
vr8	3.68	3.50	0.014	14.21
vr16	2.95	2.62	0.021	10.344
vrhr2	1.08	1.21	0.001	7.444
vrhr4	1.12	1.23	0.003	7.452
vrhr8	1.16	1.15	0.008	6.656
vrhr16	1.14	1.04	0.001	5.503

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[1]These tables are attached in Appendix

Figure 3.1: Dependence of Test Statistics on Lags
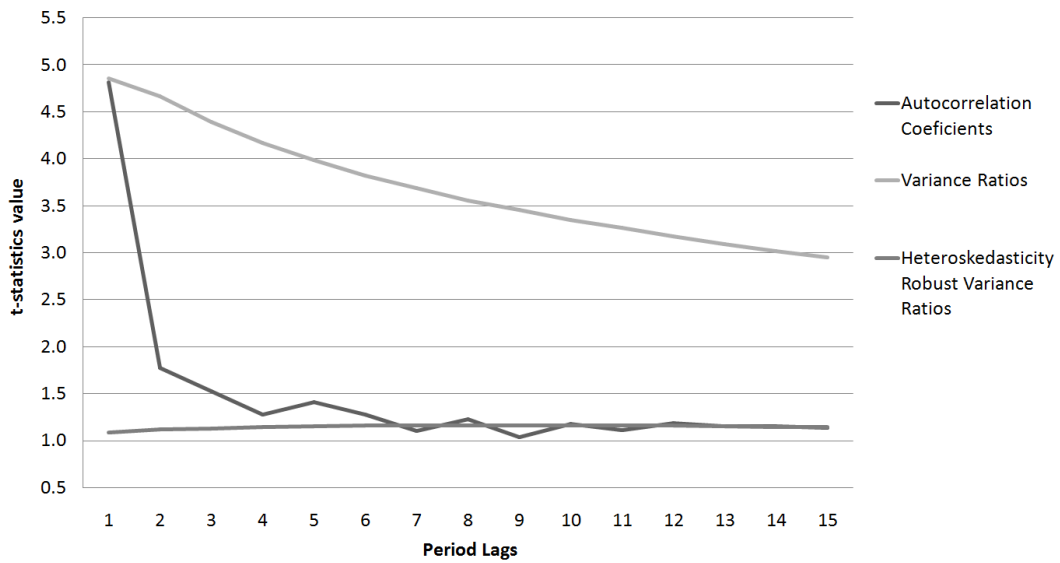
The Amihud illiquidity measure produced relatively small but diverse values. Illiquidity data were rescaled, concretely multiplied by 10^9, for easier treatment and reporting. In Table 3.1, it could be seen that difference between maximum and minimum values is relatively very large. Based on this observation, logarithmic transformation of the data obtained from Amihud illiquidity measure would be probably more suitable for our analysis.

Testable Models

In this part, ways to test the hypotheses are presented. Dependence between two variables will be explored by means of simple regression model. This mean should be sufficient for our purposes. Slope coefficient sign and significance will be the objects of our interest. However, it is important to select the appropriate forms of the models for this cross sectional part of our analysis.

For purpose of testing the H_1 hypothesis, stock liquidity and efficiency relationship, natural logarithm of Amihud illiquidity data is considered as an explanatory variable. As a dependent variables are chosen t-statistics from various tests of efficiency. All of them are in from standard normal distribution. There are 14 different models describing the relationship of our interest specified as follows:

$$CJ_i = \beta_0 + \beta_1 log(Illiquidity_i) + \epsilon_i \qquad i = 1, ..., n,$$

where CJ_i denotes sequence and reversals ratio test statistic of i-th stock.

$$AR_{q,i} = \beta_0 + \beta_1 log(Illiquidity_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 1, 2, 4, 8, 16$$

where $AR_{q,i}$ denotes autocorrelation coeficient test statistic with parameter q of i-th stock.

$$VR_{q,i} = \beta_0 + \beta_1 log(Illiquidity_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 2, 4, 8, 16$$

where $VR_{q,i}$ denotes variance ratios test statistic with parameter q of i-th stock.

$$VRHR_{q,i} = \beta_0 + \beta_1 log(Illiquidity_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 2, 4, 8, 16$$

where $VRHR_{q,i}$ denotes heteroskedasticity robust variance ratios test statistic with parameter q of i-th stock. In all equations, $Illiquidity_i$ is the Amihud illiquidity value assigned to i-th stock. Coefficients of these models will be estimated by method of least squares. Therefore, hypothesis H_1 is valid if β_1 is positive and significant. Positive β_1 implies that lower illiquidity values would result in smaller test statistics which represent higher efficiency. H_1 hypothesis will be treated separately for every single type of test statistic. It could be expected that both size and significance of β_1 coefficient will differ with a parameters of the test statistics.

The hypothesis assuming positive relationship between stock exchange size and its liquidity, H_2 hypothesis, is tested in similar manner as the H_1 hypothesis. There is, however, only one equation to be explored:

$$log(Illiquidity_i) = \beta_0 + \beta_1 log(MarketCap_i) + \epsilon_i \qquad i = 1, ..., n,$$

where $MarketCap_i$ is a market capitalization of a stock exchange where the i-th stock is listed and $Illiquidity_i$ is the Amihud illiquidity value assigned to i-th stock. To be in line with H_2 hypothesis, OLS estimate of slope coefficient β_1 has to be negative and significant. This would simply mean that in bigger stock exchanges are listed less illiquid stocks. There are used logarithmic transformations of both variables as they are very wild in scopes.

The third hypothesis, H_3, postulates the presence of more efficient stocks in stock exchanges with higher value of market capitalization. There are again 14 different proxies for the stock efficiency which divide analysis of this rela-

tionship.

$$CJ_i = \beta_0 + \beta_1 log(MarketCap_i) + \epsilon_i \qquad i = 1, ..., n,$$

where CJ_i denotes sequence and reversals ratio test statistic of i-th stock.

$$AR_{q,i} = \beta_0 + \beta_1 log(MarketCap_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 1, 2, 4, 8, 16$$

where $AR_{q,i}$ denotes autocorrelation coeficient test statistic with parameter q of i-th stock.

$$VR_{q,i} = \beta_0 + \beta_1 log(MarketCap_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 2, 4, 8, 16$$

where $VR_{q,i}$ denotes variance ratios test statistic with parameter q of i-th stock.

$$VRHR_{q,i} = \beta_0 + \beta_1 log(MarketCap_i) + \epsilon_i \qquad i = 1, ..., n \qquad q = 2, 4, 8, 16$$

where $VRHR_{q,i}$ denotes heteroskedasticity robust variance ratios test statistic with parameter q of i-th stock. As it was already stated, $MarketCap_i$ represents a market capitalization of a stock exchange where the i-th stock is listed. It is expected β_1 to be negative and significant in these equations to meet the H_3 hypothesis.

Nevertheless, autocorrelation coefficients and variance ratios with lower parameters, indicating the lags, are more important for our analysis. As test statistics express the possibility to forecast the stock price developement, even well predictable stocks would be considered as random walk cases if the prediction would be based on useless data from old history. On the other hand, if the stock would be strongly predictable, this feature could be recognizable based data with higher lags. This hold for H_1 as well.

Chapter 4

Data

Methods described above were applied to daily prices and volumes of common stocks for period from January, 2003 to December, 2013. The price series were adjusted of dividends and splits in the following way. In case of dividends, the amount paid out to the shareholders was added to the all periods after the dividend payout date. Splitting share in two or more identical shares was dealt with simple multiplication of the post dividend periods by the amount of newly created shares. Due to the fact that these data were obtained from the Yahoo Finance, they were already after the dividends and splits adjustments. It was chosen 25 stock exchanges of the highest size for last 15 years. But price and volume time series of stocks listed in some of the very large stock exchanges were not available or described at the Yahoo Finance. The largest 22 stock exchanges with accessible data were included in our dataset from which 206 stocks were sampled at random. Both price and volume series vary in length as the stock exchanges are based in various countries following different regulations, customs and bank holidays are set differently.

Average market capitalization for period 2003 - 2013 was chosen as an appropriate proxy variable for size of a stock exchange. It is not surprising that the largest three stock exchanges are NYSE, NASDAQ and London Stock Exchange. On the opposite side of the scale are New Zealand Exchange, Buenos Aires Stock Exchange and Wiena Stock Exchange. As regards market capitalization of stock exchanges, most of the data were complete and downloaded from World Federation of Exchanges website. Data for Borsa Italiana of period from 2009 to 2011 were provided at its official website. Due to the extension of the dataset, it is not desirable to report all stocks' returns, price developments, volumes and other features descriptive statistics. The stock exchanges level of

our data is further more reportable.

Table 4.1: Market Capitalization of Stock Exchanges Summary

	Stocks Sampled	Market Capitalization (billions of USD)							
		2003-2013 AVG	Min	Max	Std. Dev.	AVG growth rate	2007 YE	2008 YE	2008 vs 2007
NYSE	17	11275.24	9208.93	17949.88	2400.71	9%	15650.83	9208.93	-41%
NASDAQ	15	3157.26	2248.98	6084.97	981.63	14%	4013.65	2248.98	-44%
London Stock Exchange Group	10	2698.57	1868.15	4428.98	709.46	14%	3846.46	1868.15	-51%
Euronext	4	1765.08	2076.41	4222.68	681.57	12%	4222.68	2101.75	-50%
Hong Kong Stock Exchange	9	1275.82	714.60	3100.78	857.41	25%	2654.42	1328.77	-50%
Frankfurt Stock Exchange	10	1188.95	1079.03	2105.20	342.00	14%	2105.20	1110.58	-47%
Toronto Stock Exchange	10	1172.50	910.23	2186.55	467.45	18%	2186.55	1033.45	-53%
SIX Swiss Exchange	9	848.93	727.10	1540.70	236.52	10%	1274.66	880.33	-31%
Australian Stock Exchange	10	743.43	585.53	1454.49	313.18	18%	1298.32	683.87	-47%
Bombay Stock Exchange	8	542.65	279.09	1819.10	500.22	29%	1819.10	647.20	-64%
Korea Exchange	10	535.67	298.25	1234.55	328.31	24%	1122.61	470.80	-58%
BM&F Bovespa	7	530.51	234.56	1545.57	462.06	33%	1369.71	591.97	-57%
National Exchange of India	8	516.04	252.89	1660.10	477.17	30%	1660.10	600.28	-64%
Taiwan Stock Exchange	10	450.79	356.71	822.71	165.22	16%	663.72	356.71	-46%
Shenzhen Stock Exchange	10	380.23	115.66	1452.15	510.63	47%	784.52	353.43	-55%
Borsa Italiana	6	322.64	256.89	1072.53	296.46	-3%	1072.53	522.09	-51%
Singapore Exchange	9	312.04	148.50	765.08	218.06	26%	539.18	264.97	-51%
Mexican Exchange	9	243.24	122.53	526.02	136.81	19%	397.72	234.05	-41%
Indonesia Stock Exchange	8	137.58	54.66	428.22	140.52	35%	211.69	98.76	-53%
Wiena Stock Exchange	8	80.54	56.52	236.45	52.30	22%	236.45	76.29	-68%
Buenos Aires Stock Exchange	11	44.38	34.25	63.91	9.23	17%	57.07	39.85	-30%
New Zealand Exchange	8	35.76	24.21	65.96	10.87	14%	47.49	24.21	-49%

First of all, see simple data summary for market capitalization in Table 4.1. Notice that over the examined period five stock exchanges fell to their bottom in year 2008, the most severe year for financial markets in that crisis. Moreover seven stock exchanges were not able to reach the pre-crises level of market capitalization till the end of year 2013. Among those seven, Wiena Stock Exchange, Bombay Stock Exchange and National Stock Exchange suffered from the strongest declines, all of them over 60%. On the other side, Shenzen Stock Exchange and Indonesia Stock Exchange, the fastest growing ones, were able to reach quadruple of the 2008 year end market capitalization level. From this point of view, our sample represents growing stock exchanges as well as developed ones. Turning back to the largest stock exchange, NYSE, it creates almost 40% of total market capitalization of our sample. Also, due to the fact that the smallest, New Zealand Exchange, is more 300 times smaller than NYSE, market capitalization data will be used through logarithmic transformation. This step makes the market capitalization data more balanced which could be seen in Figure 4.1.

Figure 4.1: Market Capitalization of Stock Exchanges
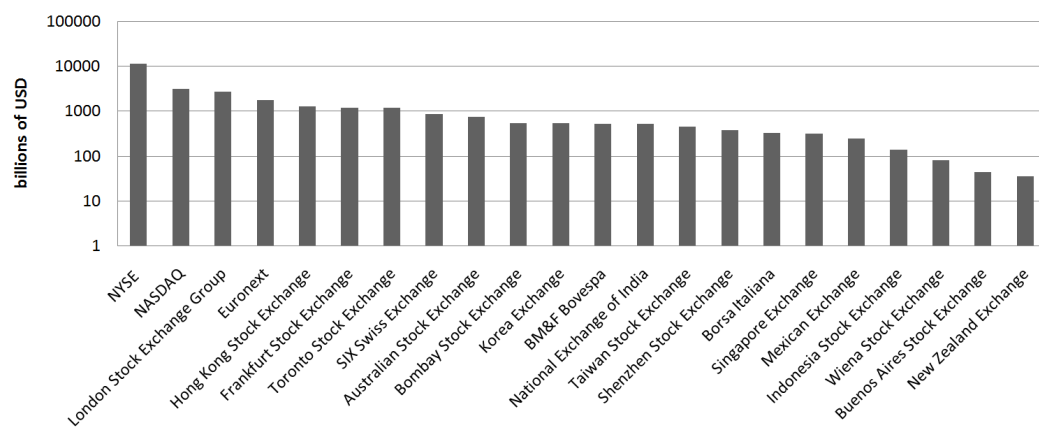
Chapter 5

Results

This section is divided into four parts. First, we will focus on results of the main relationship between stock efficiency and stock liquidity. Then, findings of the dependence between stock exchange size and stock liquidity are presented. Followed by empirical results related to the relation between stock efficiency and stock exchange market capitalization are stated. Finally, our results are discussed with respect to other empirical papers.

5.1 Stock Efficiency and Stock Liquidity

Results of the main relationship, stock liquidity and informational efficiency, are presented in Table 5.1. This table shows properties of possible dependence between natural logarithm of Amihud illiquidity, as an explanatory variable, and fourteen dependent variables from fourteen simple linear regressions. For every single regression, we are provided with six numbers: slope coefficient, standard error of the slope coefficient, t-statistic of the slope coefficient, heteroskedasticity robust standard error of the slope coefficient, heteroskedasticity robust t-statistic of the slope coefficient and coefficient of determination. Moreover, there is also significance level indication for confidence level of our H_1 hypothesis. Heteroskedasticity robust standard error and heteroskedasticity robust t-statistic of the slope coefficient that are written in grey are seen as less important. This is due to insignificant presence of heteroskedasticity in disturbances from the related regression model. Results of White test for heteroskedasticity of disturbances, presented in White (1980), are recored in Table 5.2. Tables describing the other two relationships are structured in the same way, thus it is refrained from repeating this description.

Table 5.1: Dependence of Efficiency Proxies on Amihud Illiquidity

Dependent Variables	Explanatory Variable Log(Illiquidity)		Dependent Variables	Explanatory Variable Log(Illiquidity)	
	Slope Coefficient			Slope Coefficient	
	(Standard Errors)	(Het. Rob. Std. Err.)		(Standard Errors)	(Het. Rob. Std. Err.)
	[t-statistic]	[Het. Rob. t-statistic]		[t-statistic]	[Het. Rob. t-statistic]
	Coefficient of Determination (R^2)			Coefficient of Determination (R^2)	
CJ	0.072		VR4	0.089	
	(0.985)	(0.782)		(0.089)	(0.084)
	[0.07]	[0.09]		[1.00]	[1.07]
	0.000			0.005	
AR1	0.165		VR8	0.029	
	(0.1)	(0.096)		(0.069)	(0.066)
	[1.65]*	[1.71]*		[0.41]	[0.44]
	0.013			0.001	
AR2	-0.038		VR16	-0.007	
	(0.034)	(0.029)		(0.052)	(0.05)
	[-1.09]	[-1.3]		[-0.14]	[-0.15]
	0.006			0.000	
AR4	0.010		VRHR2	0.012	
	(0.022)	(0.019)		(0.024)	(0.018)
	[0.44]	[0.51]		[0.48]	[0.64]
	0.001			0.001	
AR8	-0.014		VRHR4	0.001	
	(0.025)	(0.02)		(0.024)	(0.018)
	[-0.54]	[-0.68]		[0.05]	[0.07]
	0.001			0.000	
AR16	0.003		VRHR8	-0.012	
	(0.022)	(0.019)		(0.023)	(0.017)
	[0.14]	[0.16]		[-0.54]	[-0.72]
	0.000			0.001	
VR2	0.161		VRHR16	-0.020	
	(0.098)	(0.096)		(0.021)	(0.016)
	[1.65]*	[1.67]*		[-0.98]	[-1.27]
	0.013			0.005	

*   We do not reject the H_1 hypothesis with 90% confidence

Grey color of some heteroskedasticity robust standard errors and t-statistics denotes insignificant heteroskedasticity of disturbances

Table 5.2: White Test for Efficiency Proxies and Amihud Illiquidity

	Dependent Variables						
Explanatory Var.	CJ	AR1	AR2	AR4	AR8	AR16	VR2
LogIlliquidity	0.14	1.45	0.62	1.42	0.76	0.83	1.36

	Dependent Variables						
Explanatory Var.	VR4	VR8	VR16	VRHR2	VRHR4	VRHR8	VRHR16
LogIlliquidity	1.06	1.12	0.81	2.01	2.38*	2.79*	3.09**

**   Heteroskedasticity of disturbances with 95% confidence
*   Heteroskedasticity of disturbances with 90% confidence

Regarding the sequence and reversal ratio test statistics, they turned out

to be unpredictable by our dependent variable at all. Very similarly could be commented all other dependent variables but autocorrelation coefficient and variance ratio test statistics both with lag of one period. Both autocorrelation coefficient and variance ratio are influenced by the illiquidity with 90% confidence. The 90% confidence level is reached also when computing heteroskedasticity robust standard errors. Using White test, however, there was no significant evidence of heteroskedastic disturbances in these two regressions. Thus, they are not taken into account. In both of these cases with significant explanatory variables, the relationship was estimated to be positive. Thus, H_1 hypothesis holds at least for autocorrelation coefficient and variance ratio test statistics with one period lag. These efficiency proxies allow us interpret the slope coefficient. Additional increase of Amihud Illiquidity by 1% makes $AR1$ and $VR2$ test statistics increase by approximately 0.0016. Dependence of $AR1$ and $VR2$ on LogIlliquidity is depicted as a scatter plot in Figures A.1 and Figures A.2 attached in Appendix.

5.2 Stock Liquidity and Market Size

Dependence among stock liquidity and size of a stock exchange where is a particular stock listed is plotted in Figure 5.1 on decadic log scale. Stocks are ordered from the most liquid to the least liquid one. Such ordering reveals negative dependence between illiquidity and stock exchange market capitalization.

Figure 5.1: Market Capitalization and Amihud Illiquidity Scatter Plot
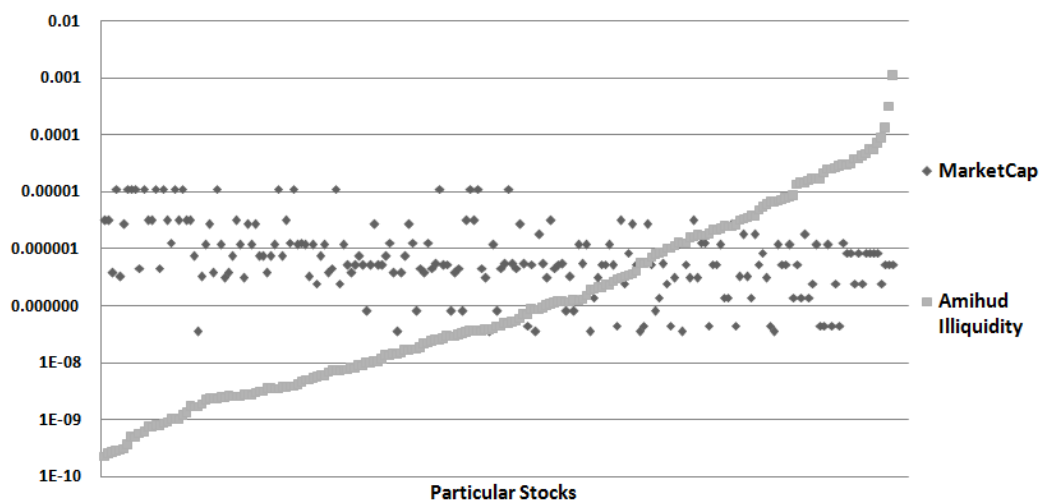
Nevertheless, Figure 5.1 is just illustration of the relationship described in Table 5.3. The same structure of a table as in previous subsection shows very strong dependence between logarithm of market capitalization and logarithm of Amihud illiquidity. Slope coefficient which is estimated to be -0.936 is strongly significant with standard error of 0.157 which is higher than heteroskedasticity robust standard error by 0.03. Overall, these two variables are related with 99% confidence in negative manner. This means that our H_2 hypothesis holds and is supported with strong significance. On average, this slope estimate could be interpreted as the elasticity between Amihud illiquidity and stock market capitalization. Therefore, one percent increase in market capitalization is connected to 0.936 % decrease in market capitalization. White test results, stated in Table 5.4, discovered strong presence of heteroskedasticity in our data. Thus, we may take in consideration also heteroskedasticity robust test statistic which indicate much stronger significance. Scatter plot of this relationship could be find in Appendix as Figure A.3.

Table 5.3: Dependence of Amihud Illiquidity on Market Size

Dependent Variables	Explanatory Variable Log(MarketCap)	
	Slope Coefficient	
	(Standard Errors)	(Het. Rob. Std. Err.)
	[t-statistic]	[Het. Rob. t-statistic]
	Coefficient of Determination (R^2)	
Log(Illiquidity)	-0.936	
	(0.157)	(0.127)
	[-5.97]***	[-7.37]***
	0.149	

*** We do not reject the H_2 hypothesis with 99% confidence

Table 5.4: White Test for Amihud Illiquidity and Market Size

	Dependent Variable
Explanatory Var.	LogIlliquidity
LogMarketCap	4.63**

** Heteroskedasticity of disturbances with 95% confidence

5.3 Stock Efficiency and Market Size

Dependence between efficiency test statistics and stock market capitalization is presented in Table 5.5. Sequence and reversals ratio test statistics are well described by the explanatory variable market capitalization. But autocorrelation

coefficient and variance ratios test statistics turned out to have more significant relationship to the stock exchange market capitalization. The heteroskedasticity robust standard errors provide more than 99% confidence in favor of our H_3 hypothesis. They can be used because heteroskedastic disturbances are present in our regression model. In addition to that, testing the significance of variance ratio with parameter equal to four with heteroskedasticity robust standard errors fits the H_3 hypothesis too. All of the significant relationships were estimated with negative sign which is in line with our expectation and hypothesis. Nevertheless, there is insufficient similarity in size of the significant effects, thus its overall interpretation could be misleading and incorrect. Figures A.4 - A.6 depicts the CJ, $AR1$ and $VR2$ dependences on $LogMarketCap$. This scatter plot could be also find in Appendix.

Table 5.5: Dependence of Efficiency Proxies on Market Size

Dependent Variables	Explanatory Variable Log(MarketCap)		Dependent Variables	Explanatory Variable Log(MarketCap)	
	Slope Coefficient			Slope Coefficient	
	(Standard Errors)	(Het. Rob. Std. Err.)		(Standard Errors)	(Het. Rob. Std. Err.)
	[t-statistic]	[Het. Rob. t-statistic]		[t-statistic]	[Het. Rob. t-statistic]
	Coefficient of Determination (R^2)			Coefficient of Determination (R^2)	
CJ	-4.124		VR4	-0.325	
	(2.374)	(2.052)		(0.216)	(0.168)
	[-1.74]*	[-2.01]*		[-1.5]	[-1.93]*
	0.015			0.011	
AR1	-0.486		VR8	-0.144	
	(0.242)	(0.186)		(0.168)	(0.135)
	[-2.01]**	[-2.62]***		[-0.85]	[-1.06]
	0.019			0.004	
AR2	0.107		VR16	-0.076	
	(0.083)	(0.07)		(0.126)	(0.105)
	[1.28]	[1.53]		[-0.61]	[-0.73]
	0.008			0.002	
AR4	0.024		VRHR2	-0.091	
	(0.052)	(0.056)		(0.058)	(0.039)
	[0.46]	[0.43]		[-1.57]	[-2.33]**
	0.001			0.012	
AR8	0.052		VRHR4	-0.059	
	(0.061)	(0.077)		(0.059)	(0.041)
	[0.84]	[0.67]		[-1.01]	[-1.46]
	0.003			0.005	
AR16	0.091		VRHR8	-0.013	
	(0.053)	(0.059)		(0.055)	(0.04)
	[1.72*]	[1.53]		[-0.23]	[-0.32]
	0.014			0.000	
VR2	-0.494		VRHR16	0.008	
	(0.241)	(0.185)		(0.05)	(0.037)
	[-2.05]**	[-2.67]***		[0.16]	[0.22]
	0.020			0.000	

***   We do not reject the H_3 hypothesis with 99% confidence
**   We do not reject the H_3 hypothesis with 95% confidence
*   We do not reject the H_3 hypothesis with 90% confidence
Grey color of some heteroskedasticity robust standard errors and t-statistics denotes insignificant heteroskedasticity of disturbances

Table 5.6: White Test for Efficiency Proxies and Market Size

Explanatory Var.	Dependent Variables						
	CJ	AR1	AR2	AR4	AR8	AR16	VR2
LogMarketCap	0.81	3.56**	0.28	0.27	1.34	2.37*	3.52**

Explanatory Var.	Dependent Variables						
	VR4	VR8	VR16	VRHR2	VRHR4	VRHR8	VRHR16
LogMarketCap	3.67**	3.64**	3.95**	2.38*	2.41*	2.56*	2.81*

**   Heteroskedasticity of disturbances with 95% confidence
*   Heteroskedasticity of disturbances with 90% confidence

5.4 Discussion

The only influential paper investigating liquidity and market efficiency is Chordia *et al.* (2008). Although this topic was partly approached with different methodology, the main message was in support of our H_1 hypothesis. Our results were also partly in favor of significant and positive dependence among stock market liquidity and stock efficiency but they are not considered as very strong ones. Perhaps, application of different liquidity measures or developing different proxy for stock efficiency would lead in more significant findings. Extension of the methods used for assessing this relationship could be challenge for the next empirical analysis.

There was little literature published on the relationship between stock exchange size and liquidity of a corresponding stock. Even simple comparison of stock exchanges from the liquidity point of view would allow us to discuss this connection. It is very unfortunate as it was the most significant and according to the hypothesis stated from the three relations we examined.

Predictability of stock prices behavior was mainly examined on developed stock markets, such as New York Stock Exchange, NASDAQ or London Stock Exchange. Nevertheless, Diviš & Teplý (2005) compared stock exchanges indices from Central and Eastern Europe to the DJIA index from United States. In results, it is presented significantly higher efficiency of DJIA compared to the all the rest indices which were also from significantly smaller stock exchanges. Similar findings, but in terms of Asian markets, were stated in Kim & Shamsuddin (2008). Stock exchanges with higher average market capitalization indicated to incorporate public information faster than the smaller ones. Both of these papers conform the H_3 hypothesis as well as our overall findings. Result of relationship between market size and stock efficiency was expectable regarding the papers already published which assess differences in efficiency. The larger stock exchanges usually demonstrated higher informational efficiency.

Chapter 6

Conclusion

The goal of this thesis was to examine the triple of relationships among three stock market features: stock liquidity, informational efficiency of a stock and stock exchange size where a particular stock is listed. Although, we focused on relationship between stock liquidity and its efficiency, analysis of the two additional relationships brought interesting results too. Results of the relationships observed were at least partly in support of our hypotheses, the uniqueness of this thesis is in the unusual method of assessing the stock efficiency and scope of the data explored.

It was observed daily prices and volumes of 206 stocks listed in 22 stock exchanges for period from January, 2003 to December, 2013. From this time series, it was created 14 various proxies of market efficiency and Amihud illiquidity measure. Together with stock exchange market capitalization data we were able to examine the three relationships.

First of all, it should be clearly stated that the methods applied to our data did not strongly support neither reject the dependence among liquidity and efficiency of a particular stock. There were more stock efficiency proxies independent of Amihud illiquidity than those that indicated significance of the relationship. The dependent ones, however, were consistent concerning the size and positive sign the effect, which is in line with our hypothesis.

The stock market size and stock liquidity relation was surprisingly very strongly significant. It was observed confidence of 99% in favor of our hypothesis that the effect will be strong and positive. Cohesion between these to stock features is recognizable by naked eye. Base on this results, our second hypothesis is almost surely valid which is with respect to the strength of the confidence very bright point of this thesis.

Finally, the stock exchange size and stock efficiency relationship results were well expectable as it was stated in discussion. There were five stock efficiency proxies considered as significant dependent on market capitalization representing the stock exchange size. The autocorrelation coefficient and variance ratio with one period lag test statistics remained predictable even for 99% confidence threshold. All the individual test statistics which were under strong influence of the stock exchange size effect showed positive dependence between these two properties. Therefore the larger stock exchange the more efficient stock are listed in it.

Different perspective to these three relationships could bring extension of the liquidity measurement methods. Amihud illiquidity is assumed to be simple but reliable tool, nevertheless, greater variety of liquidity measures, especially the high frequency ones, would allow us to asses liquidity more completely. Such empirical analysis upheld by greater sample of stock exchanges would provide useful comparison.

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Appendix A

Figures

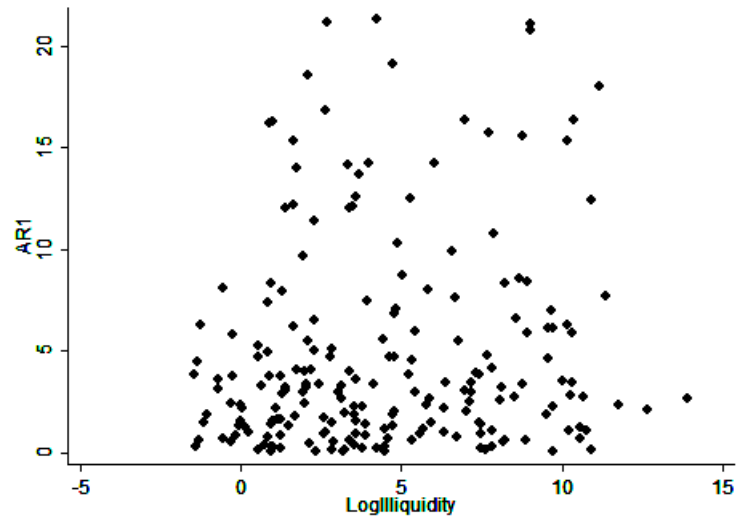Figure A.1: Scatter plot of AR1 and LogIlliquidity relationship

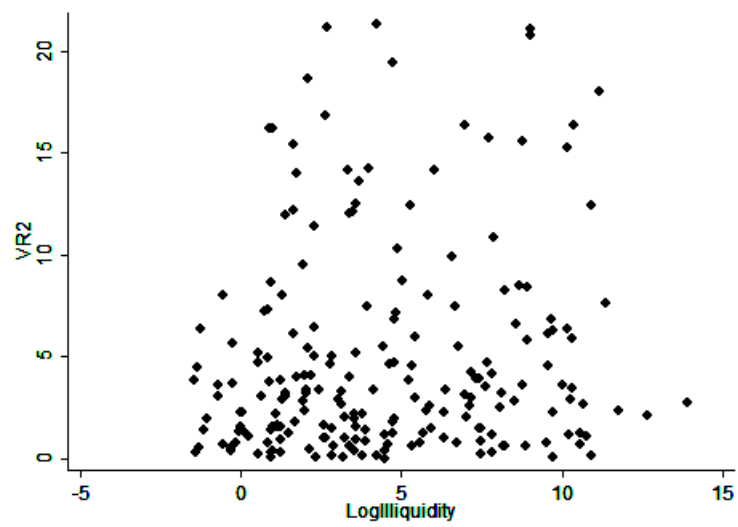Figure A.2: Scatter plot of VR2 and LogIlliquidity relationship

Figure A.3: Scatter plot of LogIlliquidity and LogMarketCap relationship
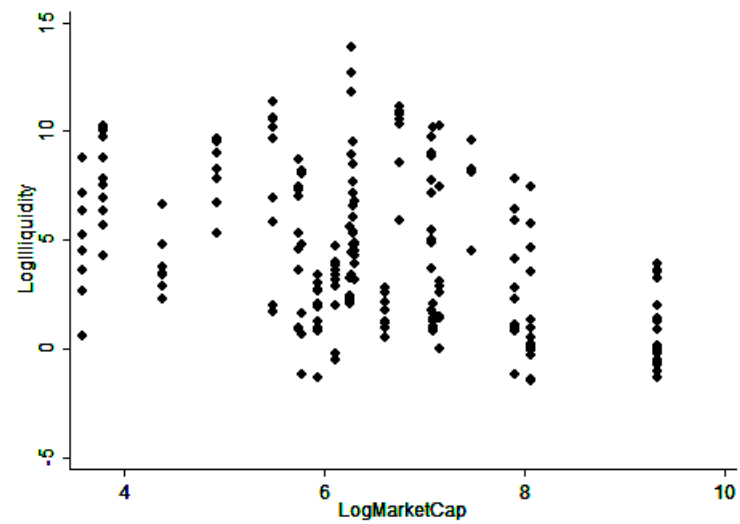

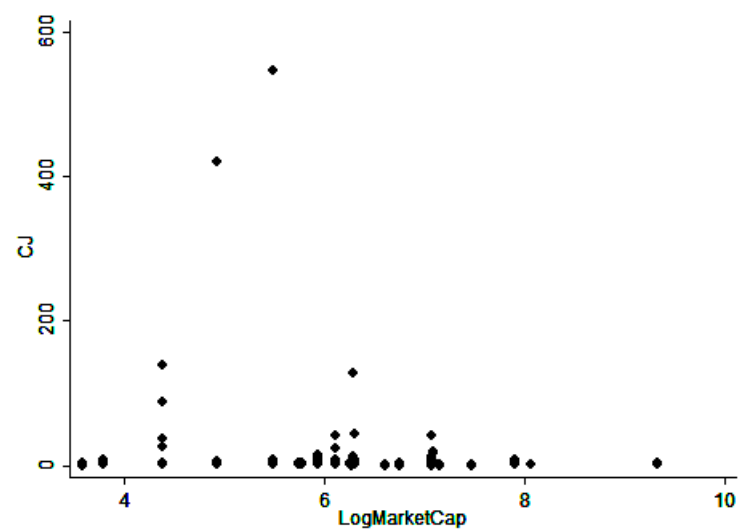Figure A.4: Scatter plot of CJ and LogMarketCap relationship

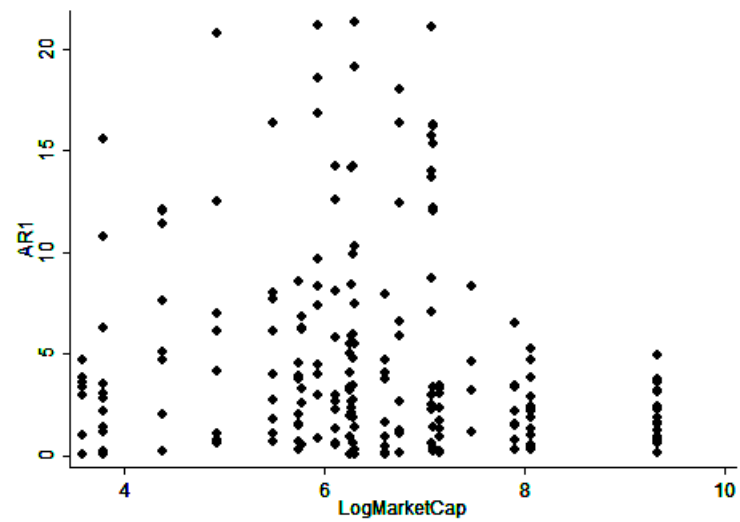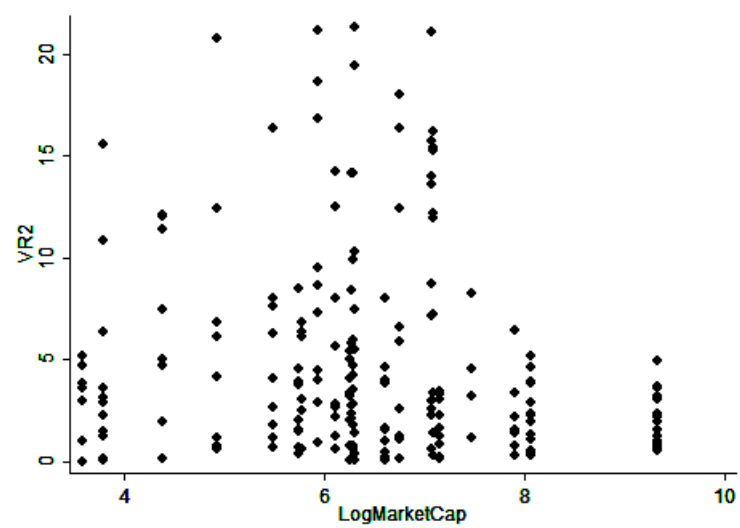Figure A.5: Scatter plot of AR1 and LogMarketCap relationship

Figure A.6: Scatter plot of VR2 and LogMarketCap relationship

Appendix B

Tables

Table B.1: Summary of Data Used for Testing I

	Stocks	Market Cap. (billions of USD) 2003-2013 AVG	Amihud Illiq. (multiplied by 10^9) Stock AVG (Std. Dev.)	CJ t-statistic Stock AVG (Std. Dev.) [AVG p-value]	Autocorrelation Ratio t-statistics				
					par. = 1	par. = 2	par. = 4	par. = 8	par. = 16
					Stock Average (Standard Deviation) [Average p-value]				
NYSE	17	11275.24	9.91 (15.35)	1.52 (0.91) [0.26]	1.97 (1.31) [0.22]	2.00 (1.48) [0.25]	1.17 (1.16) [0.45]	1.46 (1.62) [0.4]	1.92 (1.76) [0.29]
NASDAQ	15	3157.26	144.25 (412.97)	1.34 (0.43) [0.22]	2.21 (1.57) [0.23]	1.25 (0.97) [0.37]	1.26 (0.82) [0.33]	1.29 (0.59) [0.27]	1.07 (0.82) [0.42]
London Stock Exchange Group	10	2698.57	354.64 (737.02)	2.70 (2.23) [0.1]	2.25 (1.7) [0.18]	1.28 (0.77) [0.32]	1.14 (0.84) [0.4]	1.16 (0.72) [0.35]	1.25 (0.91) [0.36]
Euronext	4	1765.08	5384.33 (5342.2)	0.33 (0.24) [0.75]	4.32 (2.61) [0.06]	1.91 (1.34) [0.27]	2.30 (1.31) [0.21]	1.12 (0.84) [0.4]	1.20 (0.79) [0.36]
Hong Kong Stock Exchange	9	1275.82	3647.00 (9096.83)	0.91 (0.46) [0.41]	1.81 (1.22) [0.27]	2.00 (1.4) [0.2]	1.49 (1.25) [0.37]	0.59 (0.44) [0.59]	1.37 (0.71) [0.27]
Frankfurt Stock Exchange	10	1188.95	2543.55 (7618.82)	9.94 (7.05) [0.03]	9.27 (6.69) [0.18]	2.87 (4.29) [0.19]	1.34 (0.86) [0.31]	1.52 (1.34) [0.36]	0.99 (0.87) [0.46]
Toronto Stock Exchange	10	1172.50	3548.78 (5135.46)	8.51 (11.73) [0.19]	8.88 (6.6) [0.06]	2.53 (1.75) [0.18]	1.16 (1.05) [0.44]	1.08 (0.57) [0.35]	0.94 (0.56) [0.42]
SIX Swiss Exchange	9	848.93	36730.44 (21586.33)	1.80 (1.03) [0.19]	7.16 (6.46) [0.16]	2.42 (2.77) [0.22]	1.54 (1.05) [0.28]	1.10 (0.9) [0.43]	1.77 (1.12) [0.26]
Australian Stock Exchange	10	743.43	6.17 (4.71)	0.93 (0.61) [0.43]	2.51 (2.42) [0.31]	2.16 (1.06) [0.15]	1.07 (0.7) [0.38]	1.38 (1.17) [0.36]	0.70 (0.48) [0.53]
Bombay Stock Exchange	8	542.65	176.98 (269.72)	7.94 (13.37) [0.09]	8.17 (7.75) [0.24]	1.87 (0.89) [0.16]	1.70 (2.15) [0.43]	1.69 (0.9) [0.2]	1.00 (0.88) [0.46]
Korea Exchange	10	535.67	2575.30 (3955.15)	15.30 (37.22) [0.22]	4.55 (4.24) [0.16]	1.38 (1.39) [0.39]	1.45 (0.7) [0.23]	0.83 (0.42) [0.45]	2.09 (1.48) [0.23]

Table B.2: Summary of Data Used for Testing II

	Stocks	Variance Ratios t-statistics				VR Hererosk. Robust t-statistics			
		par. = 2	par. = 4	par. = 8	par. = 16	par. = 2	par. = 4	par. = 8	par. = 16
		Stock Average				Stock Average			
		(Standard Deviation)				(Standard Deviation)			
		[Average p-value]				[Average p-value]			
NYSE	17	2.00	2.32	2.46	2.06	0.58	0.73	0.91	0.93
		(1.26)	(1.53)	(1.4)	(1.1)	(0.41)	(0.51)	(0.56)	(0.53)
		[0.19]	[0.18]	[0.17]	[0.17]	[0.59]	[0.52]	[0.43]	[0.42]
NASDAQ	15	2.07	1.52	1.72	1.58	0.58	0.50	0.69	0.77
		(1.63)	(1.36)	(1.24)	(1.09)	(0.52)	(0.45)	(0.48)	(0.56)
		[0.28]	[0.37]	[0.28]	[0.28]	[0.62]	[0.65]	[0.54]	[0.51]
London Stock Exchange Group	10	2.22	1.89	1.86	1.83	0.61	0.63	0.75	0.89
		(1.7)	(1.49)	(1.5)	(1.14)	(0.51)	(0.54)	(0.64)	(0.51)
		[0.18]	[0.29]	[0.34]	[0.24]	[0.59]	[0.59]	[0.54]	[0.43]
Euronext	4	4.30	4.87	3.49	2.22	1.02	1.35	1.29	1.13
		(2.59)	(2.29)	(1.76)	(1.67)	(0.99)	(1.1)	(1.02)	(0.94)
		[0.06]	[0]	[0.01]	[0.27]	[0.49]	[0.36]	[0.37]	[0.43]
Hong Kong Stock Exchange	9	1.79	2.11	1.93	1.68	0.43	0.63	0.70	0.68
		(1.22)	(1.02)	(1.29)	(1.13)	(0.41)	(0.42)	(0.6)	(0.52)
		[0.28]	[0.15]	[0.24]	[0.27]	[0.7]	[0.57]	[0.56]	[0.55]
Frankfurt Stock Exchange	10	9.95	8.97	7.56	5.97	2.15	2.12	2.26	2.25
		(6.02)	(5.58)	(3.94)	(2.68)	(1.42)	(1.34)	(1.22)	(1.15)
		[0.09]	[0.08]	[0.01]	[0.02]	[0.23]	[0.23]	[0.15]	[0.13]
Toronto Stock Exchange	10	8.87	8.37	6.79	5.26	1.36	1.51	1.57	1.56
		(6.59)	(5.77)	(4.44)	(3.05)	(0.86)	(0.97)	(1.05)	(0.9)
		[0.06]	[0.09]	[0.11]	[0.1]	[0.29]	[0.27]	[0.28]	[0.25]
SIX Swiss Exchange	9	7.14	6.01	5.58	4.16	0.77	0.77	0.92	0.85
		(6.46)	(5.22)	(3.89)	(3.09)	(0.64)	(0.58)	(0.55)	(0.58)
		[0.16]	[0.16]	[0.09]	[0.17]	[0.53]	[0.51]	[0.42]	[0.47]
Australian Stock Exchange	10	2.53	3.55	3.43	3.09	0.71	1.20	1.44	1.54
		(2.42)	(2.34)	(1.57)	(1.15)	(0.79)	(0.91)	(0.78)	(0.67)
		[0.3]	[0.07]	[0.03]	[0.02]	[0.6]	[0.35]	[0.23]	[0.18]
Bombay Stock Exchange	8	8.24	7.18	5.55	4.27	2.41	2.47	2.24	2.02
		(7.79)	(6.7)	(5.1)	(3.58)	(2.77)	(2.71)	(2.38)	(1.94)
		[0.23]	[0.22]	[0.19]	[0.16]	[0.38]	[0.37]	[0.34]	[0.32]
Korea Exchange	10	4.85	4.60	3.47	2.45	1.19	1.30	1.17	1.00
		(4.05)	(3.33)	(2.72)	(1.93)	(1.05)	(1.02)	(0.95)	(0.8)
		[0.1]	[0.03]	[0.15]	[0.16]	[0.4]	[0.33]	[0.39]	[0.43]

Table B.3: Summary of Data Used for Testing III

	Stocks	Market Cap. (billions of USD) 2003-2013 AVG	Amihud Illiq. (multiplied by 10^9) Stock AVG (Std. Dev.)	CJ t-statistic Stock AVG (Std. Dev.) [AVG p-value]	Autocorrelation Ratio t-statistics				
					par. = 1	par. = 2	par. = 4 Stock Average (Standard Deviation) [Average p-value]	par. = 8	par. = 16
BM&F Bovespa	7	530.51	221156.31 (369270.79)	1.18 (0.92) [0.4]	5.87 (4.01) [0.01]	1.78 (1.27) [0.23]	1.48 (0.77) [0.25]	0.93 (0.28) [0.37]	1.23 (0.66) [0.31]
National Exchange of India	8	516.04	44.02 (85.59)	1.76 (1.06) [0.23]	3.00 (1.79) [0.17]	1.42 (1.25) [0.36]	1.26 (0.83) [0.34]	1.57 (1.27) [0.33]	1.00 (0.54) [0.38]
Taiwan Stock Exchange	10	450.79	32.62 (31.27)	9.31 (12.11) [0.02]	5.09 (4.73) [0.14]	1.66 (0.82) [0.2]	0.80 (0.59) [0.5]	1.09 (0.68) [0.38]	1.08 (0.62) [0.36]
Shenzhen Stock Exchange	10	380.23	10.32 (9.04)	5.57 (4.18) [0.04]	9.44 (6.73) [0.04]	1.57 (1.74) [0.41]	1.21 (1.04) [0.4]	1.25 (1) [0.38]	2.15 (1.42) [0.21]
Borsa Italiana	6	322.64	1149.23 (1585.06)	2.10 (1.16) [0.13]	4.28 (2.3) [0.1]	2.01 (0.96) [0.13]	1.34 (0.64) [0.27]	0.77 (0.37) [0.47]	0.62 (0.44) [0.57]
Singapore Exchange	9	312.04	1174.95 (1790.1)	1.26 (0.62) [0.27]	2.97 (2.43) [0.17]	2.03 (1.23) [0.16]	1.92 (1.12) [0.23]	1.79 (1.35) [0.29]	1.74 (1.08) [0.23]
Mexican Exchange	9	243.24	23254.30 (26884.49)	62.64 (170.74) [0.2]	5.39 (4.66) [0.09]	1.64 (1.41) [0.34]	0.96 (0.58) [0.41]	1.37 (1.2) [0.39]	1.51 (1.42) [0.35]
Indonesia Stock Exchange	8	137.58	5994.63 (5699.86)	54.39 (138.09) [0.13]	6.63 (6.55) [0.16]	1.20 (0.91) [0.37]	0.91 (0.7) [0.46]	1.39 (0.97) [0.32]	1.25 (0.78) [0.34]
Wiena Stock Exchange	8	80.54	145.01 (246.36)	37.35 (46.7) [0.03]	6.90 (4.36) [0.11]	1.91 (2.62) [0.55]	1.24 (1.16) [0.44]	2.49 (3.84) [0.52]	0.69 (0.69) [0.59]
Buenos Aires Stock Exchange	11	44.38	9514.87 (10622.15)	3.77 (3.25) [0.03]	4.28 (4.63) [0.2]	1.20 (0.89) [0.38]	1.21 (1.66) [0.53]	0.48 (0.32) [0.65]	1.07 (0.84) [0.42]
New Zealand Exchange	8	35.76	1086.53 (2104.07)	1.57 (0.84) [0.24]	2.56 (1.56) [0.2]	1.05 (0.85) [0.43]	0.98 (0.71) [0.43]	0.45 (0.42) [0.68]	0.97 (0.5) [0.39]

Table B.4: Summary of Data Used for Testing IV

	Stocks	Variance Ratios t-statistics				VR Hererosk. Robust t-statistics			
		par. = 2	par. = 4	par. = 8	par. = 16	par. = 2	par. = 4	par. = 8	par. = 16
		Stock Average (Standard Deviation) [Average p-value]				Stock Average (Standard Deviation) [Average p-value]			
BM&F Bovespa	7	5.85	5.32	4.11	3.28	1.41	1.47	1.41	1.41
		(4.01)	(4.16)	(3.25)	(2.37)	(1.09)	(1.31)	(1.27)	(1.16)
		[0.01]	[0.08]	[0.14]	[0.14]	[0.32]	[0.34]	[0.37]	[0.35]
National Exchange of India	8	3.00	2.52	1.70	1.65	0.80	0.74	0.61	0.69
		(1.82)	(2.24)	(2.12)	(1.64)	(0.52)	(0.61)	(0.71)	(0.65)
		[0.18]	[0.27]	[0.45]	[0.36]	[0.48]	[0.54]	[0.64]	[0.57]
Taiwan Stock Exchange	10	5.07	4.73	3.71	2.70	0.82	0.90	0.83	0.72
		(4.72)	(4.57)	(3.61)	(2.76)	(0.91)	(1)	(0.96)	(0.88)
		[0.13]	[0.11]	[0.16]	[0.3]	[0.56]	[0.54]	[0.57]	[0.62]
Shenzhen Stock Exchange	10	9.44	8.06	6.13	4.67	2.41	2.35	2.12	1.87
		(6.73)	(6.27)	(4.84)	(3.44)	(2.01)	(2.07)	(1.87)	(1.56)
		[0.04]	[0.06]	[0.11]	[0.1]	[0.26]	[0.29]	[0.31]	[0.31]
Borsa Italiana	6	4.24	4.15	3.95	3.87	1.57	1.63	1.69	1.79
		(2.32)	(2.54)	(2.55)	(3)	(1.31)	(1.34)	(1.28)	(1.45)
		[0.1]	[0.08]	[0.06]	[0.12]	[0.36]	[0.34]	[0.28]	[0.28]
Singapore Exchange	9	2.98	1.88	1.96	1.52	0.87	0.63	0.75	0.70
		(2.4)	(1.69)	(1.52)	(1.18)	(0.79)	(0.62)	(0.67)	(0.58)
		[0.17]	[0.27]	[0.25]	[0.33]	[0.51]	[0.61]	[0.55]	[0.55]
Mexican Exchange	9	5.39	4.34	3.61	2.97	1.41	1.24	1.40	1.40
		(4.66)	(4.22)	(2.99)	(2.09)	(1.08)	(1.08)	(1.07)	(1.14)
		[0.09]	[0.19]	[0.19]	[0.1]	[0.37]	[0.43]	[0.37]	[0.35]
Indonesia Stock Exchange	8	6.60	5.30	4.01	3.31	1.19	1.15	1.10	1.18
		(6.53)	(5.5)	(4.01)	(2.66)	(1.12)	(1.19)	(1.14)	(1.02)
		[0.15]	[0.27]	[0.24]	[0.18]	[0.45]	[0.47]	[0.48]	[0.42]
Wiena Stock Exchange	8	6.85	6.49	5.45	4.42	0.40	0.47	0.52	0.58
		(4.36)	(4.46)	(3.63)	(3.11)	(0.32)	(0.41)	(0.44)	(0.48)
		[0.12]	[0.13]	[0.11]	[0.17]	[0.7]	[0.67]	[0.64]	[0.61]
Buenos Aires Stock Exchange	11	4.31	3.41	2.64	1.94	1.03	0.93	0.90	0.85
		(4.62)	(3.93)	(3.26)	(2.38)	(0.96)	(0.98)	(0.98)	(0.85)
		[0.2]	[0.29]	[0.33]	[0.35]	[0.48]	[0.53]	[0.55]	[0.54]
New Zealand Exchange	8	2.78	2.52	1.99	1.42	1.05	1.08	0.98	0.81
		(1.79)	(1.71)	(1.54)	(1.27)	(0.74)	(0.72)	(0.63)	(0.65)
		[0.21]	[0.25]	[0.25]	[0.39]	[0.41]	[0.39]	[0.41]	[0.5]