

Report on the Thesis by Mr. Mahmoud Attya Mohamed Gad.

Title of the thesis:

Optimization Problems under (max,min)-Linear Constraints and Some Related Topics.

Author: Mahmoud Gad

The author studies in his thesis one class of optimization problems, which from the classical point of view are in general non-differentiable and nonconvex. The special structure of the problems makes possible to propose computationally effective algorithms for finding global optimal solutions of the problems. The problems are formulated on algebraic structures, in which the operations addition and multiplication from the classical linear algebra are replaced by the operations „maximum“ and „minimum“. This enables to introduce a series of special algebraic structures given by a triplet $\langle S, \max, \min \rangle$, which differ by the set S , on which the operations \max and \min are defined. Set S is e.g. equal to a subset of real numbers extended appropriately with infinite elements $\infty, -\infty$. The optimization problems are then formulated on a cartesian product of a finite number of sets S . Such structures appeared in the literature under various names as e.g. extremal algebra, path algebra, fuzzy algebra, (max,min)-algebra and others.

In the introductory Chapters 1, 2, the author describes the present state of art in the discipline and some results from the literature concerning optimization problems with (max,+)-linear constraints. A systematic study of problems on (max,min)-algebra, which is the content of the author's thesis, has not yet appeared in the literature.

The main part of the thesis is the study of properties and algorithms for optimization problems formulated on S^n , where $S = [-\infty, \infty)$ with appropriately extended operations \max and \min . The objective function of the studied problems is max-separable, i.e. it is a function of n variables, which has the form $f(x_1, \dots, x_n) = \max_j f_j(x_j)$. The set of feasible solutions is described by systems of equations and/or inequalities, in which so called (max,min)-linear functions occur, i.e. functions of the form $g(x) = \max_j (\min(c_j, x_j))$, where c_j are given real numbers. The author brings also some operations research problems, where the obtained theoretical results may be applied as e.g. reliability of transportation networks or some machine-time problems.

In Chapter 3, the author studies properties of systems of (max,min)-linear inequalities and optimization problems with (max, min)-linear inequality constraints and proposes effective algorithms for their solutions.

Chapter 4 is devoted to systems of (max,min)-linear equations. Using the derived properties of such systems, the author proposes an effective algorithm for solving optimization problems under (max, min)-linear equation constraints.

Since the operation „max“ replacing classical addition is only a semi-group operation, it is not possible to transfer variables in equations and inequalities from

one side to the other. Therefore it is a substantial difference whether we consider problems with equations or inequalities with variables on one side of the relation (so called one sided equations and inequalities) and problems with relations containing variables on both sides. Problems in preceding Chapters 3, 4 had variables on one side of equations and/or inequalities. Problems with the two sided equations and inequalities are treated in the next two chapters. Chapter 5 studies problems with systems of two sided equations and Chapter 6 is devoted to problems with systems of two sided inequalities. In both chapters the problems have on both sides of the equality or inequality relations (max,min)-linear functions of n variables and the objective functions are continuous max-separable functions.

Chapter 7 is devoted to problems, in which the set of feasible solutions described by a system of (max, min)-linear equations is empty. The author proposes a parametric approach, which makes possible to find the closest vector of the right hand sides of the given unsolvable problem, with which is the given system solvable. Such approach is important especially in operations research applications mentioned in the introductory part of the thesis.

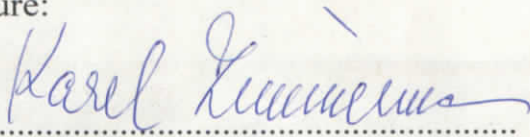
Chapter 8 describes some extensions and generalizations of the problems studied in Chapters 3,4, which unify the studies of (max,+)- and (max,min)-linear systems and make possible to solve some problems beyond the (max,min)-linearity as well as beyond the algebraic structures considered. Let us note that problems considered in the last two chapters of the thesis can be further developed and become a subject of further research.

Conclusion.

The present thesis by Mr. Mahmoud Gad contains new mathematical results partially published in scientific mathematical journals. Some of the results were also presented at the international conferences and appeared in the corresponding Proceedings. The author had to study a relatively wide scientific literature, with which he was not familiar before beginning his doctoral studies. I can therefore recommend submitting the presented thesis by Mr. Mahmoud Gad for defense at the corresponding defense commission at the Faculty of Mathematics and Physics of the Charles University in Prague.

Prague, October 21, 2014.

Signature:



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Prof. Karel Zimmermann, DrSc