Title: Optimization Problems under $(\max, \min)$-Linear Constraints and Some Related Topics.

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Abstract: Problems on algebraic structures, in which pairs of operations such as $(\max, +)$ or $(\max, \min)$ replace addition and multiplication of the classical linear algebra have appeared in the literature approximately since the sixties of the last century. The first publications on these algebraic structures appeared by Shimbel [37] who applied these ideas to communication networks, Cunninghame-Green [12, 13], Vorobjov [40] and Gidffer [18] applied these algebraic structures to problems of machine-time scheduling. A systematic theory of such algebraic structures was published probable for the first time in [14]. In recently appeared book [4] the readers can find latest results concerning theory and algorithms for $(\max, +)$-linear systems of equations and inequalities. Since operation max replacing addition in no more a group, but a semigroup opperation, it is a substantial difference between solving systems with variables on one side and systems with variables occuring on both sides of the equations. The former systems will be called "one-sided" and the latter systems "two-sided". The aim of the thesis is to provide a unifying survey of some recent author's results concerning the investigation of $(\max, \min)$-linear equation and inequality systems and some optimization problems under $(\max, \min)$-linear constraints. Besides, we propose some generalizations to non-linear systems, which unify in one model the $(\max, +)$- and $(\max, \min)$-linear problekms and extend the results beyond $(\max, +)$- and $(\max, \min)$-linear structures. Further a special problem called "incorrectly posed problem" is intrduced and effective methods for its solution are proposed for $(\max, \min)$-linear and non-linear equation systems are considered. We bring also some motivated examples from the area of operations research as well as illustrating numerical examples.

The first subject of this thesis is the investigation of properties of systems of $(\max, \min)$-linear equations and inequalities. The second subject of this thesis is solving optimization problems subject to $(\max, \min)$-linear equation and

inequality constraints. The thrird subject of research in the present thesis is the investigation of so called incorrectly posed one-sided $(\max, \min)$-linear systems of equations. The fourth part of the thesis is devoted to a generalization of the research of $(\max, \min)$-linear problems on some types of max-separable nonlinear problems.

Keywords: Optimization Problems, $(\max, \min)$-Linear Constraints, Nonconvex Optimization.