Review of Janjagava Dissertation, July 1, 2014.

Chapter 1: Duopoly, competition, escape dynamics, and non-cooperative collusion

In his 1999 monograph "The conquest of American inflation," Tom Sargent used the notion of a self-confirming equilibrium, together with adaptive learning via the Kalman filter, to model the behavior of policy makers. He found that his model could explain long run inflation bias (a Nash outcome, similar in nature to the Kydland Prescott result) as well as pronounced episodes of low inflation (a Ramsey outcome, similar to a commitment policy) – he called these episodes "escapes." While Sargent's work was largely numerical in nature, in a technical tour-de-force, Sargent's student, Noah Williams, used the theory of large deviations to obtain an analytic characterization escapes. The literature on escapes and their applications has developed somewhat since these papers were published, though progress has been more limited than I expected, due in part, I think, to the complicated technical aspects of the analysis.

In this first Chapter, Batlome Janjgava uses the notions of self-confirming equilibria and escape dynamics to model the equilibrium behavior of firms in a duopolistic setting. The firms do not know the true values of the parameters specifying the (linear) demand structure; instead, they believe the parameters may be time-varying (they are not) and use the Kalman filter to obtain recursive estimates of them, just as Sargent did in his monograph. Given their estimates of the price/quantity relationship (i.e. the demand structure), each firm supplies quantity up to a random shock to maximize profit – see equation (1.7). Note that a given firm does not act strategically; rather, its beliefs (estimates obtained via the Kalman filter) capture the behavior of the other firm.

A self-confirming equilibrium (SCE) of the model is determined as a set of beliefs which impart orthogonality of the (forecast model) regressors and the corresponding forecast errors. In a SCE, the firms' beliefs are coincident and fixed: the only variation in their actions come from the random shocks to their supply decisions, and since these shocks are independent, the firms' actions are uncorrelated. In this sense, the firms are not "cooperating," and, in fact, it is straightforward to show that an SCE corresponds to the usual "non-cooperative" Nash equilibrium of the rational model.

While the SCE is stable under adaptive learning (so that a decreasing gain algorithm would impart local convergence), an occasional sequence of supply-decision shocks may lead to changes in the firms' beliefs and subsequent actions. These altered actions lead to new data that appear to reenforce the firms' changed beliefs, and this process – this escape of the Nash outcome – continues. Along an escape, the firms' beliefs and actions are altered in the same way, which correlates their actions and leads the economy to the cooperative outcome. Once at the cooperative outcome, agents' beliefs stabilize and their actions again become uncorrelated, and the economy (slowly) returns to the SCE.

The intuition just provided explains the escape behavior found by the author in his numerical work – see, for example, Figure 1.1. However, the author goes beyond identifying and intuiting simulated time-series. Using analytic techniques developed by Kolyuzhnov, Bogomolova, and Slobodyan, the author characterizes mean escape times and directions, as well as detailing the dynamics of price wars and cooperation.

Janjgava's model provides a very nice analysis of price wars in a duopolistic environment. It is particularly interesting to find that collusion is fragile not only to demand shocks – a well-known result from the literature – but also to model uncertainty and the endogenous evolution of beliefs. I anticipate a version of this paper will be published in a high-quality journal.

Chapter 2: Free entry and social efficient under unknown demand parameters

While under certain conditions increased competition is socially beneficial, when markets are imperfect and when firms pay fixed costs, free entry may be welfare reducing: the equilibrium number of firms may exceed the socially optimal level. This result, know as the excess entry theorem, *turns on the business-stealing effect:* free entry reduces per-firm production to inefficient levels. The free entry theorem is famously emphasized by Mankiw and Whinston (MW) (1986), and may be used to promote a policy of limiting competition in certain industries.

Imperfect competition provides the opportunity for collusion, which is generally welfare reducing. While explicit collusion can be assumed away via appropriate institutional structure (anti-trust law), implicit collusion is less easily managed. To the extent an increase in the number of firms makes implicit collusion is more difficult to trigger and support, it is natural to think that free entry may preclude collusion, and in this way, be welfare improving. This *competition effect* is notably absent the MW analysis, and may overturn their policy prescription. It is also the focus of this chapter of the dissertation.

Janjgava builds on his Chapter 1 effort to produce a model of oligopolistic competition. The principal modeling distinction is the generalization of the duopoly model to allow for N firms. As before, firms use Bayesian updating to learn about the demand parameters. There is a unique self-confirming equilibrium (SCE) in the model corresponding to the non-cooperative Nash outcome; also, the Bayesian updating algorithm leads to periodic escapes from the SCE: the economy is rapidly driven toward the collusive (cooperative) outcome before slowing returning to the SCE.

Whether and when escapes occur depends upon the dynamically estimated conjectural variations parameter ρ, which measures the degree of collusion implicit in the beliefs of agents. When ρ crosses the "interdependence threshold" ρ^*, their beliefs are driven away from the SCE values, and an escape toward collusion occurs. In Proposition 5 of the Chapter, Janjgava establishes the important result that ρ^* is increasing in the number of firms. In this way, more firms reduces the probability that implicit collusion will be triggered; further, if the number of firms is larger enough, collusion may not be possible.

In Section 5, Janjgava addresses the policy implications of Proposition 5. He notes that if the socially optimal number of firms is large enough then collusion is not possible and may thus be ignored. In this case, the excess entry theorem and the policy prescriptions of MW obtain. However, if the socially optimal number of firms is not high enough to prevent collusion, then the possibility of collusion must be considered by the planner. This makes the planner's problem quite difficult and only approachable numerically. Janjgava finds that if fixed costs are relatively low then free entry may not lead to excessive entry, and thus the policy prescriptions of MW may be overturned.

This is a very nicely written chapter, quite polished. Janjgava's model provides a natural avenue through which to address the potential for collusion-costs when considering free-entry restrictions.

Chapter 3: Cooperation in the prisoner's dilemma under belief-based learning

A Nash equilibrium (of a game) has the appealing property that players (agents) have no incentive to change their strategies. While a natural equilibrium concept, an immediate question arises: how do players learn to implicitly coordinate their actions in a manner consistent with the Nash equilibrium? Even under full information, this question is non-trivially answered, with various notions of refinement and stability available. And when agents do not have full information, for example regarding opponents' payoff functions, the question of equilibrium attainment gains further immediacy.

Within the context of an infinitely-repeated simultaneous-move game, several possible learning mechanisms have been considered, differentiated both by the information assumed available to the agents and by the decision algorithm given the acquisition of new information. Under beliefs-based (BB) learning, agents know the strategy space and their own payoff matrix, and observe the actions of opponents. They form beliefs about their opponents' strategies, and choose their actions optimally given these beliefs. Agents' beliefs are then updated via a Bayesian learning algorithm. The principal result is that the asymptotic outcome is Nash.

An alternative mechanism is aspiration-based (AB) learning. Here an agent only knows his own strategy space and observes his realized payoff. He sticks with a strategy as long as this realized payoff meets a particular criterion or aspiration, for example that this payoff is not less than the average of the realized payoffs across agents. In contrast to BB-learning, it can be shown, in a repeated prisoner's dilemma (PD) game, that AB-learning can only support the cooperation outcome.

In this final chapter, Janjgava proposes a third learning mechanism that is similar in spirit to the adaptive learning algorithms studied in the first two chapters. He allows his agents the same information as is provided in AB-learning, but assumes a distinct decision-algorithm. His agents estimate a model of their own payoff function and choose the optimal (mixed) strategy against their expected payoff as defined by the estimated model. Janjgava finds that, like AB-learning, only cooperation is supported in the long run.

Section 3.2 provides an ambitious theoretical development of the proposed learning mechanism (which really needs a name) in a general environment. Section 3.3 then applies the theory to a very simple PD environment, and it is within this environment that precise results are obtained. While some fleshing-out of results and further discussion of implications will be needed for this paper to be ready for submission to a high-quality journal, the work of this chapter provides, in my view, a natural alternative to the learning mechanisms available in the game-theory literature as well as convincing initial results that are suggestive of this new learning mechanism's importance.

Summary and conclusion

This unusually well-written dissertation provides numerous significant, original results, including those that, in my view, will merit publication in high-quality journals. I find that this dissertation warrants a defense.