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MASTER THESIS

Modelling Conditional Quantiles of CEE Stock Market Returns

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature and that the thesis has not been used for obtaining another title.

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Prague, May 14, 2015

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Abstract

Correctly specified models to forecast returns of indices are important for investors to minimize risk on financial markets. This thesis focuses on conditional Value at Risk modeling, employing flexible quantile regression framework and hence avoiding the assumption on the return distribution. We apply semi-parametric linear quantile regression (LQR) models with realized variance and also models with positive and negative semivariance which allows for direct modelling of the quantiles. Four European stock price indices are taken into account: Czech PX, Hungarian BUX, German DAX and London FTSE 100. The objective is to investigate how the use of realized variance influence the VaR accuracy and the correlation between the Central & Eastern and Western European indices. The main contribution is application of the LQR models for modelling of conditional quantiles and comparison of the correlation between European indices with use of the realized measures. Our results show that linear quantile regression models on one-step-ahead forecast provide better fit and more accurate modelling than classical VaR model with assumption of normally distributed returns. Therefore LQR models with realized variance can be used as accurate tool for investors. Moreover we show that diversification benefits are decreasing over time.

JEL Classification C52, C53, G10, G15, G17,
Keywords VaR,high-frequency data, economic forecast, conditional quantiles, quantile regression

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Abstrakt

Správne definované modely na predpovedanie výnosov indexov sú dôležité pre investorov, kvôli minimalizovaniu rizika na finančných trhoch. Táto práca sa zameriava na podmiené modelovanie Value at Risk, ktorá využíva rámec flexibilnej kvantilovej regresie, a tým sa môže vyhnúť predpokladu o normálne rozdelených výnosoch. Aplikujeme semiparametrickú lineárnu regresiu kvantilov (LQR) s realizovaným rozptylom a tiež model s pozitívnou a negatívnou semivarianciou, ktorá umožňuje priame modelovanie kvantilov. Do úvahy berieme ceny štyroch európskych akciových indexov: českého PX, maďarského BUX, nemeckého DAX a londýnskeho FTSE 100. Naším cieľom je zistiť, ako použitie realizovaných rozptylov ovplyvňuje presnosť VaR a koreláciu medzi strednou a východnou Európou so západoeuróskymi indexmi. Hlavným prínosom práce je aplikácia modelov LQR pre modelovanie podmienených kvantilov a porovnanie korelácie medzi európskymi indexmi s využitím realizovaných mier. Naše výsledky ukazujú, že pri jednokrokovej prognóze lineárny kvantilový regresný model poskytuje lepšie odhady a taktiež presnejšie predpovede ako klasický VaR model s predpokladom normálne distribuovaných výnosov. Z tohoto dôvodu, LQR modely s realizovanou varianciou môžu byť použité ako presné nástroje pre investorov. Naviac ukážeme, že prínosy z diverzifikácie klesajú v čase.

Klasifikácia JEL	C52, C53, G10, G15, G17,
Kľúčové slová	VaR, vysokofrekvenčné dáta, ekonomická predpoveď, podmienené kvantily, kvantilová regresia

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Topic characteristics

Constantly present uncertainty is a typical sign of financial markets that split investors into two groups - on risk lovers and risk haters. In One form or another, analyses of stock market returns are a part of their daily work. Over the last years, there is an ongoing discussion about how to find the best volatility estimators by using the high-frequency (intra-daily) information. Studies have shown that estimators of volatility based on the intra-day data have improved the ability to measure financial market volatility.

Intra-day information also allows us to estimate numerous financial decisions, but most estimation requires assumption of normal distribution, e.g. Value at Risk (VaR). VaR is unstable when losses are not normally distributed, because loss distributions tend to have fat-tail not allow modelling upper and lower quantiles independently. To avoid these asymmetries without constructing a parametric distribution, quantile regression can be used. Conditional quantile regression methodology was already used by Zikes and Barunik (2013), who introduced conditional quantile models to avoid making restrictive assumptions on the dynamics of the conditional distributions on S&P 500 and WTI Crude Oil futures contracts. We will follow their approach by applying their model on CEE stock market return.

Most of studies are focused on measurement of entire realized volatility and on elimination of any information that could be contained by both, positive and negative intra-day returns. That is why Barndorff-Nielsen, Kinnebrock and Shephard (2010) came with “realized semivariance”, decomposing usual realized variance into components of positive intra-day returns and negative intra-day returns. Patton, Sheppard (2011) measured realized semivariance on S&P 500 and other 105 individual stocks and they had shown that volatility is strongly related to the volatility of past negative returns than to one with positive returns. We will apply their finding on our data set.

Moreover, realized semivariance models can serve as risk management tool for investors trading on stock markets. However for asset pricing and portfolio allocation is not only important to understand quantified movements on domestic market, but also to connect and correlate with movements of other markets. Christoffersen, Errunza, Jacobs and Hugues (2010) show that correlations have been significantly trending upward studied markets. This is the reason, why we will try to detect correlation between our CEE markets and German DAX Index.

Hypotheses

1. Linear quantile regression performs reasonably well and explains volatility movements when applied on given dataset.
2. Negative realized semivariance is more important for a future volatility then positive realized semivariance.
3. Due to globalization, the importance of an international portfolio diversification is decreasing.

Methodology

To verify our hypotheses, we are going to use 5 minutes intra-day stock returns from Central and Eastern European stock markets returns. This data is going to be used to model our returns, when we will follow Zikes and Barunik (2013) in use of linear semiparametric model for quantiles of future returns proposed by Koenker & Bassett (1978).

To decompose variance on positive and negative semivariances, we will follow Zikes and Barunik (2013) and Patton, Sheppard (2011), who shows that quadratic variation process contains from two parts – integrated variance and jump variation.

DAX index formed by 30 German companies traded on Frankfurt Stock Exchange is going to be used for verification of our last hypothesis. We will use this index as the exogenous variable in the model. Coefficient of this exogenous variable will show the correlation between our markets, which may explain the possibility of international portfolio diversification.

Expected Contribution

I will conduct conditional quantile modelling of CEE Stock markets. Results will show that conditional quantile modelling explaining movements of stock markets. Also I will provide empirical evidence that decomposing of realized variance into positive and negative components has important role in the decision making, i.e. VaR analysis. Moreover, the results of diversification part will provide proof of decreasing importance of portfolio diversification through international markets in Europe.

Outline

1. Introduction
2. Theoretical background
3. Data
4. Empirical analysis
5. Conclusion

Core bibliography

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Chapter 1

Introduction

Over the past two decades, financial markets experienced several main events which radically influenced investors' way of thinking. Asian financial crisis in 1997 which was just a start, followed by IT "dot-com" bubble in 2000 and mainly recent Global financial crisis in 2008 increased the uncertainty on the markets. Due to this uncertainty, investors want to understand full dynamics of return of their portfolio.

However, most of the investors analyze data on daily basis, which can exclude numerous important events occurring within the day. Fortunately, thanks to technology innovations and increased computational power we are now able to use so called "high-frequency" data which capture financial information on the markets several times a day. The frequency of data can differ, as we are now able to get the prices in the size of the smallest tick at the specific stock index. Already in 1998 Andersen and Bollerslev came with an idea of employing high-frequency data for the construction of volatility measurement through cumulative squared intraday returns. With the use of the intra-day variance, we can improve modeling of future returns.

Therefore, over the past years numerous economists started to use high-frequency data. One of the applications was the specification of the "ideal" frequency, as with increasing frequency of the data, the microstructure noise occurs due to bid-ask spread. Bandi and Russell (2008) found that the biasness of the noise can be diminished by filtering on 5-minute intraday data. Due to this different effect of positive and negative news, Barndorff-Nielsen, Kinnebrock and Shephard (2010) proposed semivariance estimator to confirm the statement of Kahneman and Tversky (1982) and Wells, Hobfoll and Lavin (1999) that downside variance is more informative than upside one. Later on, Patton and Sheppard (2013) confirmed that division of variance can significantly improve future volatility forecasts.

One of the most common applications is estimation of investor's risk by Value at Risk metric, which measures the exposure to market risk. On the one hand, VaR is the most widely used measure of the risk, as its main advantage is simplicity; however on the other hand, it has several disadvantages – as the assumption about the normally distributed returns is required during calculation. This is the case of RiskMetrics, developed by J.P. Morgan and Reuters (1996). To avoid this normality assumption, quantile regression can be used. In our work, we follow Zikes and Barunik (2014) in their linear semiparametric modeling approach for quantile regression to forecast VaR over the quantiles.

Moreover, as financial markets became more integrated at the 90s, correlation increased. This rise can be seen in deregulation of capital markets and change in movement of capital between developed countries. Christoffersen, Errunza, Jacobs and Hugues (2012), Bekaert, Hodrick and Zhang (2009), Evans and McMillan (2009) confirmed that the effect of international diversification is slowly disappearing. We believe that this trend is similar for the European markets. Hence we measure the correlation with dynamic conditional correlation (DCC) GARCH introduced by Engle and Sheppard (2001). Later we use realized measures to get more precise results using quantile correlation (QR) structure of Campbell, Koedijk and Kofman(2000).

In our work we focus on performance of linear quantile regressions, its ability to model stock market data across different quantiles. The results of the analysis are then used to calculate Value at Risk of our portfolio. Moreover, we analyze not only the "whole" variance, but also both positive and negative semivariance as these components contain different information with various significance. Also due to the globalization of the markets, authors assume that importance of the diversification of portfolio between markets is decreasing and our empirical analysis shows the proof of this hypothesis on European data.

In the analysis, we use 5-minute high-frequency data since January 1, 2008 till October 13, 2014 of Prague Stock Exchange, Budapest Stock Exchange, Deutsche Boerse AG and Financial Times Stock Exchange indices, representing European Stock market. To the best of our knowledge, this study presents the primary results for this region with usage of high-frequency data.

Our results for four stock indices show that linear quantile regression with realized measures improved the accuracy of the forecasts on the one-step-ahead period in comparison to classical model with assumption of normally distributed returns. Quantile regression models mostly performed well based on backtesting methods. These results are consistent with Zikes and Barunik (2014) findings on S&P 500 index in United States. Moreover, we estimated that the diversification is slightly decreasing over time. By using realized measures to calculate the correlation between the CEE and Western European stock indices, we found that QR shows smaller correlation in comparison to DCC model.

The rest of the thesis is structured as follows. Theoretical background of realized measures, linear quantile regression model, Value at Risk and diversification concepts are described in the Chapter 2. Chapter 3 describes data and Chapter 4 is dedicated to empirical application. Chapter 5 concludes.

Chapter 2

Theoretical background

2.1. Realized Measures

During investment decisions, investors experience problems connected with distribution of returns. Most of the models assume that the portfolio has Gaussian distribution, which is not true most of the time (Bucley, Saunders and Seco, 2008). Zikes and Barunik (2014) showed that this normality can be avoided by quantile regression with usage of high-frequency data, which does not rely on parametric assumptions.

Andersen and Bollerslev (1998) came with an idea about usage of high-frequency data for the construction of volatility measurement through cumulative squared intraday returns. They found that proposed volatility measures provide radical improvement in comparison with daily data. For estimation they used exchange rates of Deutschemark – U.S. Dollar and Japanese Yen – U.S. Dollar and found that even when daily volatility models perform quite well, while explaining most of variability of volatility factor, the models with realized volatility are more precise.

2.1.1. Realized Variance

To start with realized volatility concept, firstly we create logarithmic returns

$$r_{t,i} = ln(P_{t,i}) - ln(P_{t,i-1}) \tag{2.1}$$

where $i = 1, 2, ..., m$ is the intra-day interval and P_t denotes the stock price in the interval i of day t. These returns evolve continuously through time

$$r_{t,i} = \mu_t dt + \sigma_t dW_t \tag{2.2}$$

where μ_t denote the drift, σ_t volatility and W_t is standard Brownian motion. As $dt \to 0$ the drift between observations is getting close to zero. However the drift and volatility does not need to be constant in continuous time interval, which can result into

$$r_t = \int_{t-1}^{t} \mu_s ds + \int_{t-1}^{t} \sigma_s dW_s \tag{2.3}$$

Following Andersen and Bollerslev (1998), the realized variance is sum of intra-day returns. Realized variance (RV) is then specified as

$$RV_{t,M} = \sum_{i=0}^{M-1} r_{t,i}^2 = \sum_{i=0}^{M-1} (p_{t,i+1} - p_{t,i})^2 \tag{2.4}$$

where $RV_{t,M}$ is realized variance in the day t with M intraday returns.

Realized volatility for assets can be then constructed by simply taking squared root of realized variance

$$RVOL_{t,M} = \sqrt{RV}_{t,M} \tag{2.5}$$

In general, realized volatility can be represented as a standard deviation. Let us assume arbitrage-free logarithmic return with normal distribution, which is consisting from predictable drift and standard deviation $r_t = \mu_t + \sigma_t \varepsilon_t$ where $\varepsilon_t \sim N(0,1)$, under assumption of no drift we get $\varepsilon_t = \frac{r_t}{\sigma_t}$. In case of realized volatility, under assumptions mentioned above, we can say that error $\varepsilon_t = \frac{r_t}{\sqrt{RV_t}}$ should be approximately normally distributed white noise process.

In case of continuously observed prices and t getting close to zero, the realized variance approached the integrated variance on day t

$$IV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{2.6}$$

and it correspond closely to conditional variance for discrete sample returns.

As the prices are not observable in continuous time, in our case we need to compute realized variance in the discrete time.

However, financial time series exhibit larger movements usually associated with microeconomic and macroeconomic news or announcements, so called jumps, which largely affects estimates.

When we take jumps possibility into our price process, realized variation does not converge to the integrated variance even when it is really closely linked to it, but converges to quadratic variation, formally defined as

$$QV_t = IV_t + JV_t \tag{2.7}$$

where IV_t represent the variation from continuous part of returns and JV_t measures the variation coming from pure discontinuous part of returns - jumps

$$JV_t = \sum_{j=1}^{\gamma} J_{t,j}^2 \tag{2.8}$$

that occurred $j = 1, 2, \ldots, \gamma$ times over day t.

2.1.2. Realized Semivariance

However, in our work we would like to show not only the effect of whole sample variation, but also separately results from positive and negative variance. This decomposition is important due to fact that positive variance should have smaller effect on variation as the negative one. Kahneman and Tversky (1982) termed value function, when they studied relationship between values of various possible outcomes. They concluded that value function is steeper for losses then for gains. This can be explained by behavior of investors, which are more afraid to lose some money as to get gain. Their findings were supported by other studies - Kahneman and Tversky (1984), studying distress and joy of participants after experiencing losses or gains, and by Wells, Hobfoll and Lavin (1999) investigating gain and losses of resources where respondents were pregnant women. Their research showed that gains had no significant effect however the losses had much higher effect, which resulted in depression and anger.

Due to this different effect of positive and negative news, we use for measuring volatility so called semivariance. This estimator was proposed by Barndorff-

Nielsen, Kinnebrock and Shephard (2010) and it can capture the variation only of the negative or positive returns. These estimators are defines as sum of only positive (negative) intra-day returns for $RS^{+}(RS^{-})$.

$$RS^{+}_{t,M} = \sum_{i=0}^{M-1} r^2_{t,i} I_{[r_i>0]} \tag{2.9}$$

$$RS^{-}_{t,M} = \sum_{i=0}^{M-1} r^2_{t,i} I_{[r_i<0]} \tag{2.10}$$

These estimators provide decomposition of Realized variance $RV_{t,M} = RS^{+}_{t,M} + RS^{-}_{t,M}$. Authors' findings about negative semivariance seem in line with the conclusions of the Kahneman and Tversky(1982) that the negative semivariance is much more informative than the positive one.

Previous definition of realized semivariance counts only with integrated variance. Following Barndorff-Nielsen, Kinnebrock and Shephard (2010), the realized semivariance in continuous time converge into one-half of integrated variance plus jumps.

$$RS^{+}_{t,M} \xrightarrow{p} \frac{1}{2}\int_0^t \sigma^2_s ds + \sum_{0\leq s\leq t} r^2_{t,i} I_{[r_i>0]} \tag{2.11}$$

$$RS^{-}_{t,M} \xrightarrow{p} \frac{1}{2}\int_0^t \sigma^2_s ds + \sum_{0\leq s\leq t} r^2_{t,i} I_{[r_i<0]} \tag{2.12}$$

By analyzing 105 individual stocks and S&P 500 index, Patton and Sheppard (2013) confirmed that negative realized semivariance is: *"much more important for future volatility than positive semivariance, and disentangling the effects of these two components significantly improves forecasts of future volatility."*

The important finding to mention is that this kind of realized variance or realized semivariance measures only the variance in the intra-day returns. The inter-day changes between closing price and opening price are not included. These changes could be included, when we would calculate daily returns as difference between the last observation of previous day and pre-last observation of current day. However using this methodology, our results of realized variance could be biased due to the high variance of first observation in sample, including inter-day change in the price.

2.2. Value at Risk

Over the years, Value at Risk (VaR) as risk management tool became standard to measure market risk connected with portfolio. The advantage and reason for wide use of VaR is that it sums up all risk associated to portfolio into one number as a loss associated to given probability.

VaR is formally described as maximum expected loss over some time horizon with given probability.

$$Pr(r_t < -VaR_t \mid \Omega_t) = \alpha \tag{2.13}$$

where r_t is return of portfolio at time $t = 1, 2, \dots T$, information set at time t is denoted by Ω_t and α is probability level of risk, with confidence level $1-\alpha$. As we already mentioned, VaR is maximum loss of portfolio in the risk and it is defined usually on confidence interval 99%, 95% or 90% with probabilities α=1%, 5% or 10%, respectively. VaR at 90% is rarely used, as probability of loss is high and it is connected with high losses in portfolio value. On contrary, VaR at 99% confidence interval is highly used, for example Basel Committee on Banking Supervision in 2004 recommended Basel II, which uses VaR as a tool to estimate market risk exposure for setting capital requirements in financial system industry.

Engle and Manganelli (2004) describe VaR from statistical point of view: *"VaR estimation entails the estimation of a quantile of the distribution of returns."* However the return distributions are changing over time, which brings problems into the estimation. Due to this issue, it is better to directly estimate quantile of the distribution via semiparametric models. Except semiparametric models, VaR is estimated through parametric and nonparametric methods. Following Engle and Manganelli (2004) VaR methods split, we discuss main pros and cons of each method and the reasoning, why we use in our analysis the semiparametric one.

The most common parametric approached is Generalized Autoregressive Conditional Heteroskedasticity (GARCH), proposed by Bollerslev (1986), which allows the conditional variance to be time-varying, while constant unconditional variance and so improve the forecast results.

Another parametric model is RiskMetrics, developed by J.P. Morgan and Reuters (1996), which use the exponentially weighted moving average computation of variance. This moving average makes RiskMetrics model a special case for of GARCH. Parametric mean-VaR model is one of the simplest approached of the RiskMetrics model. Its simplicity comes from the usage of Gaussian normal distribution of returns.

$$VaR_t = E(r_t) - \sigma_t * qnorm(c) \tag{2.14}$$

where $E(r)$ is expected return, σ is standard deviation measured by realized volatility and $qnorm(c)$ is so called "cut-off" point, representing c-quantile of the standard normal distribution. The most frequent cut-off points are for the 90%, 95% and 99% confidence intervals with cut-off point 1.282, 1.645 and 2.326 respectively.

These methods usually underestimate the VaR approach, due to assumption of normally distributed residuals. Also, some other problems may arise while using these models, e.g. misspecification of the variance equation or that model errors may not be independently and identically distributed (i.i.d.), which is part of main model assumption.

For non-parametric models, most common method is Historical Simulation of VaR. The advantage of this approach is that it really simplifies VaR computations and also it does not make explicit assumptions about the portfolio returns. However if we look into procedure, the distribution of portfolio returns is constant, does not change and returns has the same weight. Boudoukh, Richardson and Whitelaw (1998) combined historical simulation with RiskMetrics to overcome problems. This hybrid approach does not use equal weights for all in-sample observation, but apply exponentially declining weights so take into account "age" of observations.

With semiparametric methods, for example with Extreme Value Theory, we can focus only on the tails of distributions and so we do not have to care about modelling the whole return distribution. The most common semiparametric model was introduced by Engle and Manganelli (2004) – Conditional Autoregressive Value at Risk (CAViaR), which does not model whole distribution of returns, but directly model the evolution of quantiles over time. With this model, we do not need to assume distribution of portfolio returns, but only assumption about correct specification of the quantile process. Other methods focusing on VaR are quasi-maximum likelihood GARCH models and methods based on quantile regression. The reason we work with semiparametric methods is that we do not need to use any assumptions about distribution.

Quantile VaR_α of a given portfolio can be characterized as

$$q_\alpha(r_{t+1}) = \sigma_t \Phi(\alpha) \tag{2.15}$$

where $q_\alpha(r_{t+1})$ is α-quantile of the return distribution, σ_t is volatility and $\phi(\alpha)$ denotes α-quantile of a standard normal cumulative distribution. In general, we do not want to make this assumption about distribution function of the underlying error term. That is the reason, why usage of quantile regression as nonparametric estimation is suitable in our case.

There are several other studies, focusing on regression quantile modelling in Value at Risk. Chernozhukov and Umantsev (2001) analyzed conditional market risk of stock price oil producer stock price and Dow Jones Industrial Average index, measuring conditional risk.

Taylor (2000) proposed approach to the estimation of the distribution of multiperiod returns by using historical returns of exchange rate data with three methods. Authors found that GARCH model with empirical distribution does not perform well. Overall, the GARCH models with Gaussian distribution and quantile regression approach have similar performance.

2.3. Linear Quantile regression model

In this section, we focus on the linear quantile regression originally proposed Koenker & Bassett (1978). We will follow Zikes and Barunik (2014) in use of this linear semiparametric model for α-quantile of future stock returns:

$$q_\alpha(r_{t+1} \mid \Omega_t) = \beta_0(\alpha) + \beta_\nu(\alpha)'\nu_{t,M} + \beta_z(\alpha)'z_t \tag{2.16}$$

where r_{t+1} is logarithmic return, Ω_t contains information known at time t, $v_{t,M}$ is set of quadratic variation, z_t is vector of weakly exogenous variables and $\beta_0(\alpha)$, $\beta_v(\alpha)$, $\beta_z(\alpha)$ are vectors to be estimated.

Previous equation is determined by parameters $\beta_0(\alpha)$, $\beta_v(\alpha)$, $\beta_z(\alpha)$ which solve minimizing problem of objective function:

$$\frac{1}{T}\sum_{t=1}^{T} \rho_\alpha(r_{t+1} - \beta_o(\alpha) - \beta_\nu(\alpha)'\nu_{t,M} - \beta_z(\alpha)'z_t) \tag{2.17}$$

where $\rho_\alpha(x) = (\alpha - 1\{x < 0\})x$.

Similar methodology was already used by Cenesizoglu and Timmermann (2008), who modelled the return distribution of US stock market. For quantile modelling they used similar specification as Zikes and Barunik (2014), but instead of quadratic variation they used last period's conditional quantile and absolute value of last period's return, firstly defined by Engle and Manganelli (2004). As we already described in previous section, the realized volatility has advantage of intraday changes, which can improve future modelling.

The main advantage of this approach is no distribution assumption. In the most of the models, the assumption about distribution has to be made. Moreover, the evidence from different quantiles can help to specify the economic source of return and due to that investor can include this information into its portfolio choice.

2.4. Forecast evaluation

As one of the main part of our work is dedicated to the forecast of variables, we need to monitor the forecast performance of our models. The evaluation provides essential feedback on the quality of forecasted data which can help us answer the question, if we used the correctly specified forecasting method and also if the forecasts are giving economic and statistical sense.

There is wide range of forecast evaluation methods used in the literature, which we can use to check the accuracy of the conditional quantile models underlying to this work. We take into account absolute and also relative performance of the models. For the absolute performance, which shows us the performance within model, we are going to use unconditional coverage test, independence test and combined conditional coverage test. As we also want to see the differences between different models, we are going to check the relative performance. The performance will be compared via unconditional coverage, the value of the tick loss function and the Diebold-Mariano test statistic for equal predictive accuracy. All of these performance measures are explained below.

2.4.1. Absolute performance

To evaluate the absolute performance of our models, we use three tests. The unconditional coverage test discussed by Kupiec (1995), Christoffersen (1998) independence test and test of conditional coverage.

Unconditional Coverage

Kupiec (1995)introduced performance tests based on proportion of failures or PoF test, which measures if the number of exceptions is consistent with the confidence level or not. Under the null hypothesis that the model is correctly specified, the number of failures follows the binominal distribution with probability α.

The probability (p) of observing the number of failures (x) in the sample with number of observations (n) and with the frequency of failures predicted by the model α, the null hypothesis for Proportion of Failures is:

$$Prob(x \mid \alpha, n) = \binom{n}{x} \alpha^x (1-\alpha)^{n-x} \tag{2.18}$$

Accurate estimates should have the property that the unconditional coverage, which is measured by $\alpha^* = x/n$ is equal to the desired coverage level α. Thus under the null hypothesis, $\alpha^* = \alpha$ which means that model predicts $\alpha * n$ violations. We can test the unconditional coverage hypothesis using the appropriate likelihood ratio test LR_{uc}

$$LR_{uc} = 2\left[log(\alpha^{*x}(1-\alpha^*)^{n-x}) - log(\alpha^x(1-\alpha^x)^{n-x})\right] \tag{2.19}$$

The LR_{uc} test statistics has an asymptotic χ_1^2 (chi-squared) distribution with one degree of freedom.

If the proportion α^* is below to the desired significance level α, then the unconditional coverage test reject the null hypothesis.

However, in case of dynamics are present in the higher-order moments, the unconditional coverage test is insufficient as it tests only the coverage of the interval. Christoffersen (1998) mentions that unconditional coverage *"does not have any power against the alternative that the zeros and ones come clustered together in a time-dependent fashion"*. Therefore we need to test independence assumption to have correct definition of the absolute performance of our forecasted data.

Test of independence

The null hypothesis of independence tells us the probability that the previous violations are not influencing the next violations. Christoffersen (1998) suggest to test the independence against first-order Markov alternative. At first, let's consider first-order Markov chain,I_t, with transition probability matrix

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \tag{2.20}$$

where $\pi_{ij} = Prob(I_t = j | I_{t-1} = i)$. For this process is likelihood

$$L(\Pi_1) = (1-\pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}} \tag{2.21}$$

Where n_{ij} is the number of observations with value i followed by j. To maximizing the log-likelihood function and solve for the parameters, we use following specified matrix:

$$\widehat{\Pi}_1 = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{10}}{n_{10}+n_{11}} \end{bmatrix} \tag{2.22}$$

Secondly, let's consider output sequenceI_t, from an interval model. It looks really similar to the Π_1 matrix. We test hypothesis that the sequence is independent by nothing that Π_2 corresponds to independence.

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ 1-\pi_2 & \pi_2 \end{bmatrix} \tag{2.23}$$

The likelihood of the second matrix under null hypothesis is therefore

$$L(\Pi_2) = (1-\pi_2)^{n_{00}} \pi_2^{n_{01}} (1-\pi_2)^{n_{10}} \pi_2^{n_{11}} \tag{2.24}$$

With the maximum likelihood estimate $\widehat{\Pi}_2 = \widehat{\pi}_2 = \frac{(n_{01}+n_{11})}{(n_{00}+n_{10}+n_{01}+n_{11})}$, the likelihood ratio test of independence is

$$LR_{ind} = -2log\left[\frac{L(\widehat{\Pi}_1)}{L(\widehat{\Pi}_2)}\right] \tag{2.25}$$

which is an asymptotically χ_1^2 (chi-squared) distributed with one degree of freedom.

With the test of independence we have tested the dynamics in the interval forecast, we can now proceed to the joint test of unconditional coverage and independence.

Conditional Coverage

Christoffersen (1998)also proposed, how to create this joint test, so called conditional coverage. It is basically just combination of the log-likelihood statistics:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{2.26}$$

with the null hypothesis about correctly specified model which are independent to previous violations. The LR_{cc} test has also an asymptotic χ_1^2 distribution with one degree of freedom.

This conditional coverage enables joint testing of randomness and correct coverage, while we can also study the effect of each subcomponent.

2.4.2. Relative performance

To compare the performance of used models, again we are going to use unconditional coverage test. Moreover, we are going to follow Giacomini and Komunjer (2005) in use of the loss function. As the last relative performance test we use Diebold and Mariano (1995) test for equal predictive ability.

Tick loss function

Giacomini and Komunjer (2005) implemented "tick" loss function τ_α. Let $\hat{f}_t$ be a forecast of the variable Y_{t+1}, conditional on the information set at time t. Tick loss function can be then defined as follows:

$$\tau_\alpha(e_{t+1}) = (\alpha - I(e_{t+1} < 0))e_{t+1} \tag{2.27}$$

where $I(.)$ is approximation to the indicator function, $e_{t+1} \equiv y_{t+1} - \hat{f}_t$, and $\hat{f}_t$ is the quantile forecast. Authors decided for this tick function specification, as the object of interest is the conditional α-quantile of the distribution of Y_{t+1}.

Authors in paper concluded that the use of tick loss function is better *"than the quadratic loss function in the definition of encompassing"* (Giacomini and Komunjer 2005). Therefore we will follow them in applying the tick loss function for the evaluation of our conditional modelling of returns.

Diebold-Mariano test statistics

Test developed by Diebold and Mariano (1995) is used to evaluate the equal predictive ability of different models. This test is based on the defined loss differential between the two forecasts, which is formally defined as:

$$d_t = L(e_1) - L(e_2) \tag{2.28}$$

where e_1 and e_2 are vectors of the forecast error from the first model and the second model respectively.

Under null hypothesis, two forecasts have equal accuracy if and only if loss differential has expected value zero for all t. Alternative hypothesis is that the two forecast have different accuracy level.

Considering the quantity T, the sample mean of the loss differential is

$$\bar{d} = \sum_{t=1}^{T} d_t \tag{2.29}$$

Diebold-Mariano test statistics is then:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}} \tag{2.30}$$

where $2\pi \hat{f}_d(0)$ is estimator of the asymptotic variance of $\sqrt{T}d$. Under null hypothesis, DM is normally distributed.

Diebold-Mariano test also use Newey-West estimator developed by Newey and West (1987) when using multi-step ahead forecast. It is important to use this estimator to account serial autocorrelation in the forecast errors, which occurs during forecasting.

2.5. Diversification

International diversification should allow us minimization of portfolio volatility in comparison to portfolio constructed by regional assets only. The reason is that on international market investors have bigger choice for investment. However, the option of diversification exists only in case of low cross-country or cross-market correlations. As the financial markets have become more integrated over the last years, this cross-market correlation should increase. We could evidence of this correlation during Global Financial Crisis, where we saw “Mexican wave” across financial markets. But the co-movements between markets depends does not systematically. We can see different correlations not only on cross-country level but also on cross-continent level or between countries with different maturities.

We believe that the effect of international diversification is slowly disappearing. Christoffersen, Errunza, Jacobs and Hugues (2012) support this hypothesis by using weekly return for developed (DMs) and emerging markets (EMs) over 36 years. They performed time-varying correlations and found that *"correlations have been significantly trending upward for both the DMs and EMs."* However, as emerging markets often experience financial crises, authors conclude that diversification benefits decreased in the DMs, EMs benefits stay the same.

The similar results were found by Bekaert, Hodrick and Zhang (2009), who examined international stick return co-movements using portfolio returns from 23 developed markets across globe (they consider North America, Europe and the Far East region) over period 1980-2005. They have not found evidence for an upward trend in return correlation for North America and for Far East regions, but for the European stock markets they found evidence. Evans and McMillan (2009) came to same conclusion after analyzing 33 international stock market indices. This information is important to know due to construction of our portfolio, consisting from European stock market indices.

In the previous researches, authors focused on the correlation of the whole distribution. However usually, several extreme observations in the tail of distribution can bias the overall correlations structure, as we are unable to determine what the extent of this biasness is. Therefore it would be best, as in case of forecasting specific quantile, use the correlation of the specific quantile. To do so, we are going to follow Campbell, Koedijk and Kofman (2000) in use of quantile correlation structure.

Let's assume normal return distribution. Then VaR quantile estimation r_c will not be exceeded with probability (1-c) %

$$r_c = \zeta_c \sigma \tag{2.31}$$

Where ζ_c is (1-c)% quantile of standardized normal distribution. Rewriting this formula as a portfolio quantile, we get

$$r_c^2 = \zeta_c^2 \left[w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \sigma \right] \tag{2.32}$$

where w_x and w_y are the weights in the portfolio.

By replacing the standard deviations with VaR quantile estimates, we get conditional correlation measure:

$$\rho_Q = \frac{r_Q^2 - w_x^2 r_{x,Q}^2 - w_{y,Q}^2 r_{y,Q}^2}{2 w_x w_y r_{x,Q} r_{y,Q}} \tag{2.33}$$

In case of normally distributed returns and equal weights in the portfolio, the correlation is constant across quantiles. In our work, we are going to use the equally weighted portfolio based on the Goetzmann *et al.* (2005), who showed that the equally weighted portfolio provide more diversification as capital-weighted portfolio on different markets.

As a benchmark model for our quantile regression analysis, we are going to use dynamic conditional correlation (DCC) from Garch model class, introduced by Engle and Sheppard (2001). The advantage of DCC model is the flexibility of univariate GARCH models with correlations, which were estimated by parametric models. Authors claim that DCC model perform well and provide sensible results in variety of situations.

They proposed the dynamic correlation structure (formula 2.34, 2.35 and 2.36). The time varying correlation matrix R_t will be be

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{2.34}$$

$$Q_t = (1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n)\bar{Q} + \sum_{m=1}^{M} \alpha_m (\epsilon_t -_m \epsilon_{t-m}^{'}) + \sum_{n=1}^{N} \beta_n Q_{t-n} \tag{2.35}$$

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{q_{22}} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \sqrt{q_{kk}} \end{bmatrix} \tag{2.36}$$

Where $\bar{Q}$ is a diagonal matrix composed of the square root of the diagonal elements of Q_t and Q_t^* is the unconditional covariance of the standardized residuals. The element of R_t should be in the form $\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii} q_{jj}}}$.

Chapter 3
Data

In our work, we will analyze the stock market data consisting from PX, BUX DAX and FTSE 100. These indices were chosen due to diversification of the data across Central and Western Europe. Also, we were limited during choosing process because of small number of high-frequency data over Central and Eastern Europe, as our source database does not possess needed prices for Warsaw Stock Exchange (WIG) and Vienna Stock Exchange (ATX) indices.

We characterize Central and Eastern Europe by PX and BUX indices. PX index represents Prague Stock Exchange index. The base of PX consists of 13 companies, where market capitalization share of top 4 companies is over 80%. PX index took over PX50 and PX-D indices in 2006 with base value of 1,000 in April 5, 1994.

Also BUX index, Budapest Stock Exchange index consists of 13 companies, with market capitalization of top 4 companies is almost 95%. BUX accounts approximately for 94% of Hungarian domestic market capitalization. BUX has base value of 1,000 as January 2, 1991.

Western Europe indices – DAX and FTSE 100 are characterized by much higher liquidity and market capitalization in comparison to Central market induces.

DAX index is German stock index, which consist of 30 most actively traded German companies traded on Frankfurt Stock Exchange. The DAX has base value of 1,000 as of July 1, 1988. The DAX represents around 80% of the market capital authorized in Germany.

FTSE 100 index (denoted as FTSE) is Financial Times Stock Exchange index, which includes 100 most highly capitalized companies listed in London Stock Exchange. Base value of index is 1,000 as of December 30, 1983.

3.1. High-frequency data

Over the last years, the significant increase of computer computational power enables us to gather data in milliseconds. However not all the time quantity can bring insights to data. Due to the bid-ask spread we experience in the high-frequency data micro-structure noise, which can largely bias our estimates. This noise however can be diminished by using appropriate filtering of the data. Bandi and Russell (2008) shows that *"there is a non-negligible probability of obtaining optimal sampling frequencies in excess of 5 minutes"*. This frequency is mainly because we add bid-ask spread only 78-105 times per trading day, as trading hours differ across markets. However the micro-structure noise is different for each stock. More liquid stocks have lower micro-structure noise (Aït-Sahalia & Yu 2009) as we use different indices on different markets, the optimal frequency could differ. Due to this fact, we follow Bandi and Russell (2008) in use of high-frequency 5-minute intraday data, which are commonly used between economists. All data were obtained on October 14, 2014, from Tick Data, Inc.

As we already mentioned, we cover period from January 1, 2008 until October 13, 2014. To summarize our data, we have 1,705 trading days with 136,887 closing prices for PX, 1,689 trading days with 157,585 closing prices for BUX, 1,727 trading days with 179,291 closing prices for DAX and 1,714 trading days with 174,103 closing prices for FTSE.

Trading hours and with it connected number of observations remained almost constant for DAX and FTSE. Only several exceptions occurred in the data. One of main the biggest outliers for DAX and for FTSE was last day of the year, when the trading hours were shorten. Moreover, DAX's number of observations was shortened several times in March 2009, when market was closed 30 minutes earlier. During this month, most of the markets hit the bottom and started to growth again. Otherwise DAX data were obtained since 9:05 am till 5:36 pm. Also FTSE index has been traded shorter during several days in March 2009, last trading day of the year, but also during Christmas Eve. FTSE experienced huge fall in terms of observations during February 25, 2011, when London Stock Exchange system was halted by computer problem for numerous hours. The change in the number of observations came with BUX. Till November 2010, BUX was trading since 9:00 am till 4:30 pm however due to change in December 1, 2010 Budapest Stock Market started to trade till 5:00 pm. This change in

Table 3.1.: Descriptive statistics of high-frequency daily logarithmic returns

	PX	BUX	DAX	FTSE
Mean	-0.0990	-0.0944	-0.0311	-0.0278
Median	-0.0620	-0.0838	0.0159	-0.0051
Min	-11.9665	-10.3103	-7.5587	-7.9355
Max	5.5317	6.7095	8.7943	7.6648
Std. Dev.	1.1125	1.4415	1.2899	1.0841
Skewness	-1.3322	-0.4413	-0.1131	0.0176
Kurtosis	13.4762	4.8864	4.7506	8.5443

Source: Author's computations

trading hours created additional 6 observations per day. PX changed trading hours several times since January 2008. Between January 1, 2008 and January 31, 2011 PX was trading since 9:30 am till 4:00 pm. Since February 1, 2011 till November 29, 2012 PX traded started 15 minutes earlier and since November 30, 2012 Prague Stock Index started to trade since 9:00 am. Now it starts to trade at the same time as DAX and FTSE indices.

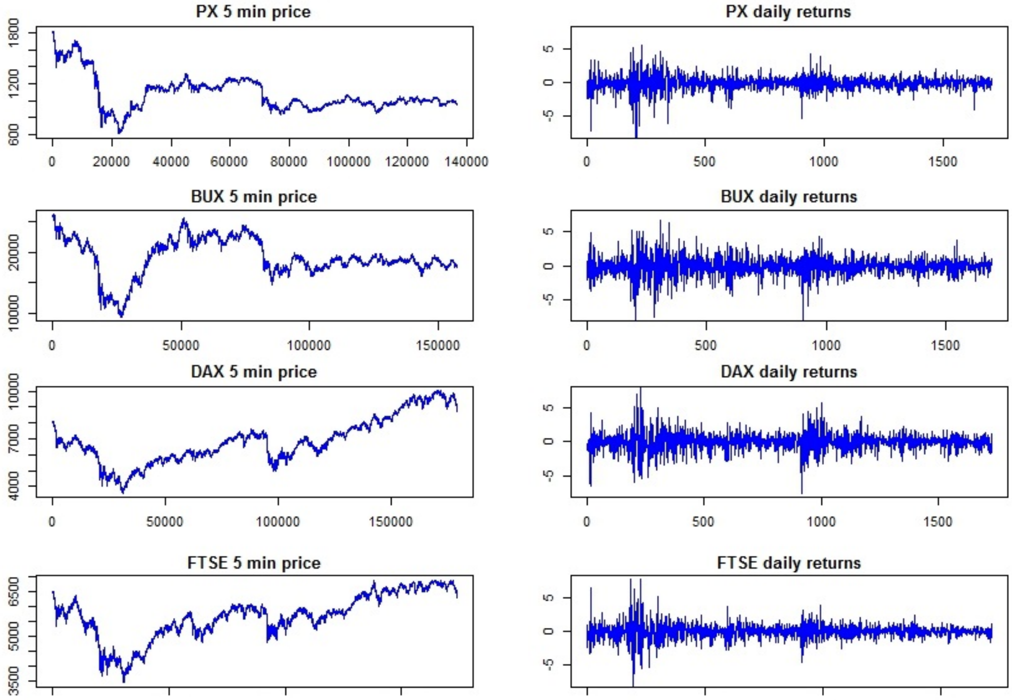
In general, we can observe that Western European markets are more traded as Central European markets. It's visible not only from number of trading days, but also from the length of trading per day.

The descriptive statistics of 5 minute closing prices and daily logarithmic returns are summarized in the Table 3.1.

From the descriptive statistics we can see that all indices except the FTSE index are negatively skewed. However FTSE's skewness is really close to the zero. Prague stock index has the highest negative skewness, which means that on this index is the highest probability of a bigger negative return.

Figure 3.1 shows 5 minute closing prices with corresponding daily returns since January 2008 till October 2014. We can see decreasing price over all stock indices till February 2009, when they hit the bottom. PX and BUX experienced highest falls, losing almost two thirds of its price since January 2008. The lowest decrease experienced FTSE index, which lost "only" 47 percent of its price. From the charts and from statistics in the Table X-X we can see that FTSE is also least volatile index with standard deviation of 1.08. This low volatility is visible also from daily returns in August 2011, when due to downgrade of United States and France ranking global market experienced fall of all main indices, FTSE in comparison to other markets experienced smaller variation in the returns.

Figure 3.1.: 5-minute closing prices and daily returns



Source: Author's computations

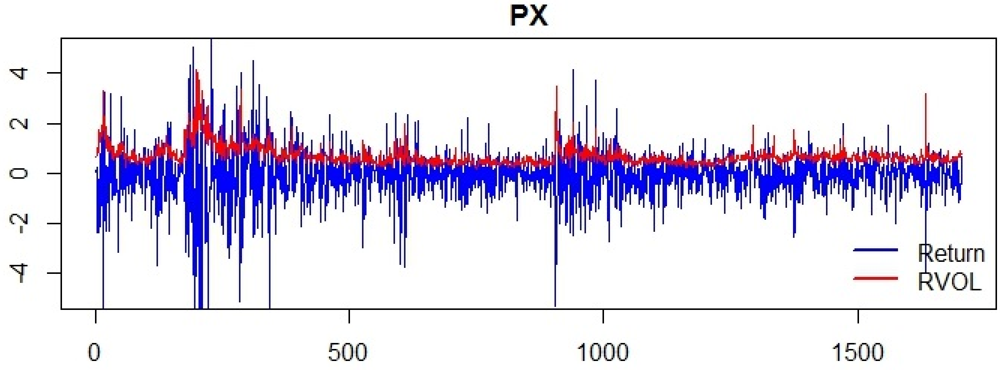
3.2. Realized variance and volatility

In this subsection, we will continue in constructing realized variation measures and components of RV. As discussed in the section 2.1.1, we calculate RV as sum of squared intra-day logarithmic returns. To compress the data, we will use realized volatility as a simple square root of realized variance. The comparison of daily returns and realized volatility is visible in Figure 3.2 below for the PX index, where we can see that realized volatility (red line) really nicely describes daily returns (blue line). For the rest of the indices, the graphical comparison can be found in Figure A.1 in Appendix.

Based on the figure comparison, we can see that realized volatility describes data well, except of periods with increased volatility during 2008 and 2011.

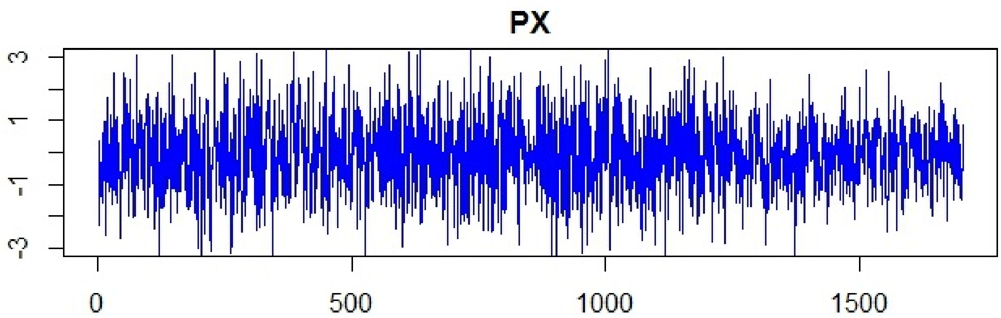
As we already mentioned earlier in section 2.1.1, realized volatility can be under assumption of no drift described as standard deviation. Thus error of

Figure 3.2.: Comparison of daily returns with realized volatility (PX index)



Source: Author's computations

Figure 3.3.: White noise assumption



Source: Author's computations

data calculated as daily return over realized volatility can be described as white noise process. Figure 3.3 shows that error of Prague Stock Exchange index can be described as white noise process (the results for the rest of indices are in Figure A.2 in Appendix).

Chapter 4
Empirical analysis

In the previous chapters we explained the underlying theory of VaR and forecasting, we proceed with our empirical analysis. We study the performance of stock index data between January 2008 and October 2014. During this time period, our data experienced several time intervals with significantly higher variance. For the modelling approach, it would be convenient to divide our sample into two parts and see, how the data are able to perform over these periods. For each of these periods we need to conclude in-sample and out-of-sample analysis. The in-sample period analysis is important to see the forecasting ability of used model and out-of-sample forecast will suit for "real" forecast.

However, due to lack of the data in our dataset (we have only around 1,700 trading days per index), we are going to perform analysis on the same dataset with changing the proportion of in-sample and out-of-sample period.

In our work we focus on lower quantiles 1%, 5% and 10% as 99%, 95% and 90% VaR are the most used risk metrics in the practice. Unfortunately, as only small number of observations are left for out-of-sample testing[1], we can expect not-well performing results for the 1% quantile. However as we want to see the performance of our models in the low quantiles, we are going to include this quantile into analysis.

In this chapter, we conclude 1-day VaR forecast using three models. At first linear quantile regression using realized variance, after that linear quantile regression using the realized semivariance and as a benchmark we are going to use VaR model with Gaussian return distribution, which we are going to call VaR_Gauss. After that we are going to change the horizon of forecasting and

[1] As 1% hit rate in approximately 500 observations is in fact 5 observations, our models can outperform in this low quantile.

going to perform 5-day and 10-day forecast. To compare the performance of the models, we are going to use absolute and relative performance metrics and conclude, which of the mentioned models has the best forecasting accuracy.

4.1. Linear quantile regression

At first we need to conduct linear quantile regression, which is based on Zikes and Barunik (2014) model. We are going to perform this regression on the whole sample size, to have description of the whole sample using the realized variance and after that we conclude analysis with usage of the realized semivariances. The exact realized variance linear quantile regression will be as follows:

$$q_\alpha(r_{t+1}) = \beta_0(\alpha) + \beta_\nu(RVOL_t) \tag{4.1}$$

The quantile regression can be estimated for the full distribution, however we are going to focus only on the main quantiles – left tailed 1%, 5%, 10%, 50% median regression, also known as least absolute deviation (LAD) and on right tailed 90%, 95% and 99%. We chose these quantiles due to fact that these quantiles shows tailed distribution and most of time these quantiles are the most sensitive for biased results.

The results of conditional linear quantile regression can be found in the Table 4.1 below and visually in the Figure A.3 in Appendix, where we can compare OLS estimate with the best fit to expected returns estimated by using quantile regression. The table report estimated coefficients for the variables with Student's t-statistics in the parentheses. We find that realized volatility is statistically significant across all lower and high quantiles, however we can see that on 50% quantiles, the significance level of estimated variables is low. This low significance on LAD supports our assumption of importance to use quantile regression. The estimated parameters of realized volatility have negative coefficients in the left-tail quantiles and positive coefficients in the right-tail quantiles. In general, we do not see any extreme values of regressions in individual quantiles. We can only conclude that PX has the heaviest left-tail realized volatility across analyzed indices. Coefficient of PX index on 1% quantile is almost 2.5 times larger as coefficient of DAX index. This can be the results of the market liquidity. As the PX is less tradable market, it is

Table 4.1.: Estimated results of conditional linear quantile regression with realized volatility

	quantile	0.01	0.05	0.10	0.50	0.90	0.95	0.99
PX	constant	-0.36	-0.30	-0.36	0.01	0.09	0.22	0.42
		(-0.82)	(-1.59)	(-2.84)	(0.19)	(0.91)	(1.41)	(1.33)
	$\sqrt{RV}$	-3.71	-2.01	-1.28	-0.12	1.38	1.59	2.63
		(-5.58)	(-4.53)	(-5.36)	(-1.16)	(7.71)	(6.12)	(4.78)
BUX	constant	-0.68	-0.32	-0.45	-0.12	-0.03	0.10	0.06
		(-1.01)	(-1.05)	(-2.88)	(-1.01)	(-0.17)	(0.41)	(0.12)
	$\sqrt{RV}$	-2.28	-1.66	-1.11	0.04	1.32	1.72	2.53
		(-2.52)	(-4.82)	(-7.17)	(0.35)	(7.21)	(6.7)	(4.24)
DAX	constant	-1.46	-0.50	-0.35	0.02	0.13	0.44	0.38
		(-5.97)	(-2.71)	(-2.36)	(0.37)	(1.25)	(2.52)	(1.27)
	$\sqrt{RV}$	-1.53	-1.42	-1.08	-0.01	1.14	1.25	2.23
		(-7.06)	(-6.97)	(-6.76)	(-0.12)	(9.38)	(6.85)	(6.36)
FTSE	constant	0.06	-0.27	-0.17	-0.02	0.21	0.28	0.22
		(0.47)	(-1.99)	(-1.49)	(-0.4)	(2.8)	(1.97)	(0.58)
	$\sqrt{RV}$	-2.99	-1.51	-1.13	0.03	0.98	1.19	2.40
		(-28.34)	(-6.91)	(-7.11)	(0.32)	(8.4)	(5.06)	(4.47)

Source: Author's computations

more sensitive to the information and though it is visible via higher changes in the extreme quantiles (0.01 and 0.99). As we look at the 50% quantile (LAD), PX has the lowest coefficient. This means that PX has more negative as positive returns. Surprisingly, BUX has the highest LAD coefficient, which could lead into assumption that there is higher probability of positive return from investing into this index.

For further analysis we are going to call model with realized volatility LQR_RV. Zikes and Barunik(2014) estimated linear quantile regression on S&P 500 index and WTI Crude Oil futures. Their estimated parameter had expected properties – sign and significance across all quantiles.

After we have model with the realized volatility, now we are going to use model with positive and negative volatility. As we discussed in the underlying theory, the differences can be found as we decompose realized volatility. Therefore we slightly change used realized volatility linear quantile regression model as follows:

$$q_\alpha(r_t) = \beta_0(\alpha) + \beta_{\nu 1}(RS_t^+) + \beta_{\nu 2}(RS_t^-) \tag{4.2}$$

Table 4.2.: Estimated results of linear quantile regression with squared root of realized semivariance

	quantile	0.01	0.05	0.10	0.50	0.90	0.95	0.99
PX	constant	-0.40	-0.31	-0.33	0.01	0.11	0.18	0.56
		(-2.36)	(-1.88)	(-2.56)	(0.14)	(1.19)	(1.2)	(2.42)
	$\sqrt{RS^+}$	-1.65	-1.70	-0.83	0.03	1.12	1.31	1.50
		(-3.49)	(-2.82)	(-2.17)	(0.19)	(4.17)	(3.25)	(3.57)
	$\sqrt{RS^-}$	-3.78	-1.20	-1.09	-0.20	0.81	1.09	2.15
		(-7.21)	(-1.96)	(-3.38)	(-1.64)	(3.28)	(2.89)	(2.77)
BUX	constant	-0.67	-0.32	-0.44	-0.11	-0.03	0.09	0.32
		(-0.96)	(-1.54)	(-2.44)	(-0.88)	(-0.16)	(0.39)	(0.65)
	$\sqrt{RS^+}$	-1.57	-1.32	-0.93	-0.12	1.19	1.07	1.35
		(-1.14)	(-8.28)	(-2.76)	(-0.56)	(3.72)	(2.8)	(1.32)
	$\sqrt{RS^-}$	-1.69	-1.04	-0.67	0.17	0.70	1.38	1.82
		(-1.31)	(-2.52)	(-2.06)	(0.76)	(2.39)	(3.08)	(2.25)
DAX	constant	-1.44	-0.59	-0.35	0.02	0.15	0.45	0.31
		(-5.27)	(-3.44)	(-2.42)	(0.33)	(1.39)	(2.64)	(1.27)
	$\sqrt{RS^+}$	-1.15	-0.37	-0.60	0.06	0.69	0.67	2.57
		(-1.91)	(-0.88)	(-1.85)	(0.32)	(2.52)	(1.62)	(3.53)
	$\sqrt{RS^-}$	-1.03	-1.52	-0.93	-0.07	0.90	1.08	0.80
		(-2.96)	(-4.78)	(-2.87)	(-0.45)	(3.71)	(3.04)	(2.16)
FTSE	constant	-0.06	-0.28	-0.19	-0.04	0.18	0.21	0.25
		(-0.34)	(-2.1)	(-1.7)	(-0.75)	(2.68)	(1.73)	(0.72)
	$\sqrt{RS^+}$	-1.32	-1.07	-0.49	-0.05	0.51	0.33	0.98
		(-2.41)	(-1.89)	(-1.19)	(-0.27)	(4.14)	(0.57)	(0.71)
	$\sqrt{RS^-}$	-2.69	-1.07	-1.10	0.13	0.92	1.47	2.38
		(-5.14)	(-1.88)	(-2.86)	(0.57)	(4.05)	(2.66)	(2.25)

Source: Author's computations

The results of this decomposition can be found in the Table 4.2 and Figure A.4and A.5in the Appendix. On 1% quantile, only for DAX is more important $\sqrt{RS^+}$ than $\sqrt{RS^-}$, while on the 5% confidence level, the negative volatility is already higher. BUX has similar coefficients at the 1% and on contrary, PX and FTSE indices has squared root of negative semivariance twice higher as positive one.

In the data, we can observe several significant changes in the coefficient of $\sqrt{RS^+}$ and $\sqrt{RS^-}$. For $\sqrt{RS^+}$, we can see significant change between 1% and 5% quantile in BUX and FTSE indices. On the other hand, for $\sqrt{RS^-}$, we can see significant changes on low quantiles for DAX and on upper quantiles (90% - 99%) for FTSE.

Our findings are consistent with the results of Zikes and Barunik(2014). They found that the realized downside volatility dominates positive volatility across all estimated quantiles. Therefore we can conclude that the $\sqrt{RS^{-}}$ on our CEE indices has the similar impact as Zikes and Barunik S&P 500 index. For this linear quantile regression model with semivariance we will further address as LQR_RS.

4.2. Forecast

We proceed with the one day forecast for our indices. For forecasting we are going to use rolling window forecast, when the size of window is equal to in-sample size. As we mentioned before, we are going to perform forecast for sample from January 2008 till October 2014. For this period we use 2 datasets, which will differ in the in-sample and out-of-sample proportion. This division we will use due to volatile returns in the 2011, which are visible in the Figure 3.2. For the first subsample, we use in-sample period size of 1,200 returns, while for the second subsample we use in-sample, with size of 800 returns. Due to this we can see the forecast performance for less and also for the highly volatile data. In this section we describe the results of subsamples in detail and after that we summarize the main findings.

For each sample we calculate in-sample and out-of-sample forecast using different models. At first we perform forecast using linear quantile regression using quadratic variation. The forecast will be estimated directly for the wanted quantile, derived from Koenker & Bassett (1978) and Zikes and Barunik (2014), to avoid assumption about return distribution.

$$q_{\alpha}(r_{t+1}) = \beta_{0,t}(\alpha) + \beta_{\nu}(RVOL_t) \tag{4.3}$$

As we mentioned in the theory, quadratic variation has a lot of information and should perform well, however we want to see the performance, we use the model with the semivariance.

$$q_{\alpha}(r_{t+1}) = \beta_{0,t}(\alpha) + \beta_{\nu 1}(RS_t^{+}) + \beta_{\nu 2}(RS_t^{-}) \tag{4.4}$$

In our case, as a benchmark model we are going to use VaR model with Gaussian return distribution, which is one the basic models used for forecasting

VaR.

$$VaR_{t+1}(\alpha) = E\left[r_t\right] - RV_t * qnorm(c) \tag{4.5}$$

For each of the sample we conduct forecast for 1%, 5% and 10% quantiles. The performance of the forecasts will be validated through absolute and relative performance tests, specified earlier in the Section 2.4.

Moreover we evaluate the relative performance of our models by using value of Giacomini and Komunjer tick-loss function (T-L) and Diebold-Mariano (DM) test statistic for equal predictive accuracy. For Diebold-Mariano test we consider two-sided alternative hypothesis, which says that the two compared models have different level of accuracy. As a benchmark model we are going to consider quantile regression model with the semivariance.

For the multi-step forecast, we consider two types of horizons, when we use 5 and 10-step-ahead forecasts. These forecasts are going to be estimated based on slightly changed methodology. At first we need to calculate cumulative returns for given horizon.

$$r_{h,t} = \sum_{i=0}^{h-1} r_{t+i} \tag{4.6}$$

Also we need to assume realized volatility for this horizon, which will be calculated based on given formula:

$$RVOL_{h,t} = \sqrt{\sum_{i=0}^{h-1} RVOL_{t+1}^2} \tag{4.7}$$

However, as we use cumulative returns, we have to consider the autoregression in our models. Therefore we evaluate multi-step ahead forecasts by using only relative performance metrics, as for absolute performance evaluation is used independence factor (in independence test and also conditional coverage test).

4.2.1. First sample forecast

At first we will evaluate the performance of our model on data with in-sample period of size 1,200 observations, covering both periods of higher volatility.

Table 4.3.: Absolute and relative performance of PX index on one-step-ahead forecast with in-sample period of 1,200 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0142	0.0142	0.1760	0.0041	0.0041	0.1072
L ind	1.3967	1.3967	0.9741	n/a	n/a	2.3013
L cc	3.2686	3.2686	848.5634	n/a	n/a	159.5064
Tick	0.0438	0.0433	0.1410	0.0286	0.0288	0.0572
DM	8.2050		18.5554	0.2466		27.4590
5%						
UC	0.0509	0.0500	0.1284	0.0289	0.0309	0.0598
L ind	1.1016	1.2466	0.7146	3.5938	7.0722	2.5438
L cc	1.1209	1.2466	111.1833	8.9381	11.3457	3.4679
Tick	0.1393	0.1391	0.1546	0.0876	0.0877	0.0837
DM	-3.2441		16.6369	-0.1092		19.7290
10%						
UC	0.0992	0.1009	0.0676	0.0495	0.0495	0.0351
L ind	0.1671	0.3468	0.4532	2.2303	2.2303	5.6338
L cc	0.1747	0.3580	16.1011	18.8120	18.8120	35.2103
Tick	0.2118	0.2118	0.2223	0.1374	0.1371	0.1621
DM	0.9779		-13.2277	-1.3660		-13.0008

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

One-step forecast

We start evaluation of forecast performance on the PX index. The results are summarized in the Table XX. On the left side of the table we can see the in-sample performance and on the right side of the table we can see out-of-sample performance.

On the 1% quantile we can observe that the VaR_Gauss model has the highest percentage of violations, which means it significantly underestimates the risk at a given confidence level. Therefore we reject hypothesis about correct specification in both in-sample and out-of-sample period. For LQR_RV and LQR_RS we cannot reject null hypothesis of unconditional coverage in in-sample and out-of-sample period, however for out-of-sample period we cannot perform independence test. This unavailability was caused by low number of violations and as no violations followed previous violations, the likelihood ratio became zero. The fact connected with this is that our out-of-sample period includes only 507 observations, therefore in case we would have wider out-of-sample period, some violation should arise. Diebold-Mariano accuracy test shows that in in-sample period our models do not have same forecasting accuracy, however in out-of-sample period LQR_RV and LQR_RS has same accuracy. DM hypothesis is rejected in case of VaR_Gauss.

For the in-sample period on the 5% quantile, we can see that models LQR_RV and LQR_RS have similar unconditional coverage, really close to 5% significance level, which means that these models are correctly specified. The benchmark VaR_Gauss model, on the other hand, has proportion of violations almost 13%. Therefore for VaR_Gauss model we can reject null hypothesis about correct specification. We can observe high likelihood ratio of conditional coverage test for the VaR_Gauss model, which supports the results of the unconditional coverage test. Nevertheless, this VaR_Gauss model is independent for previous violations. Between models comparison with Diebold-Mariano test for predictive accuracy tells us that in in-sample period we reject null hypothesis of same accuracy of the models. While between LQR_RV and LQR_RS is the difference really small, the comparison with VaR_Gauss shows significant difference supported also by tick-loss function. In the out-of sample period we see improvement of the VaR_Gauss model. The unconditional coverage felt down on approximately 6% violations and likelihood of conditional coverage for this model is the only one, which does not reject null hypothesis for the correct number of exceedances. The main factor behind the non-correct specification of the LQR_RV and LQR_RS models is the overestimation of the potential risk. Moreover, LQR_RS failed during the independence test.

On the 10% quantile, we can see similar results as for the 5% quantile. In the in-sample period, linear quantile regression models performs better as classical VaR_Gauss model. On contrary to 5% quantile, VaR_Gauss overestimate the potential risk. In the out-of-sample period, all our models are overestimating the potential risk, however in the LQR models, the overestimation is lower. Moreover, VaR_Gauss is also dependent to previous violations. Looking at the DM test, we just confirm the absolute performance tests. For the LQR_RS and LQR_RV we cannot reject null hypothesis of same accuracy.

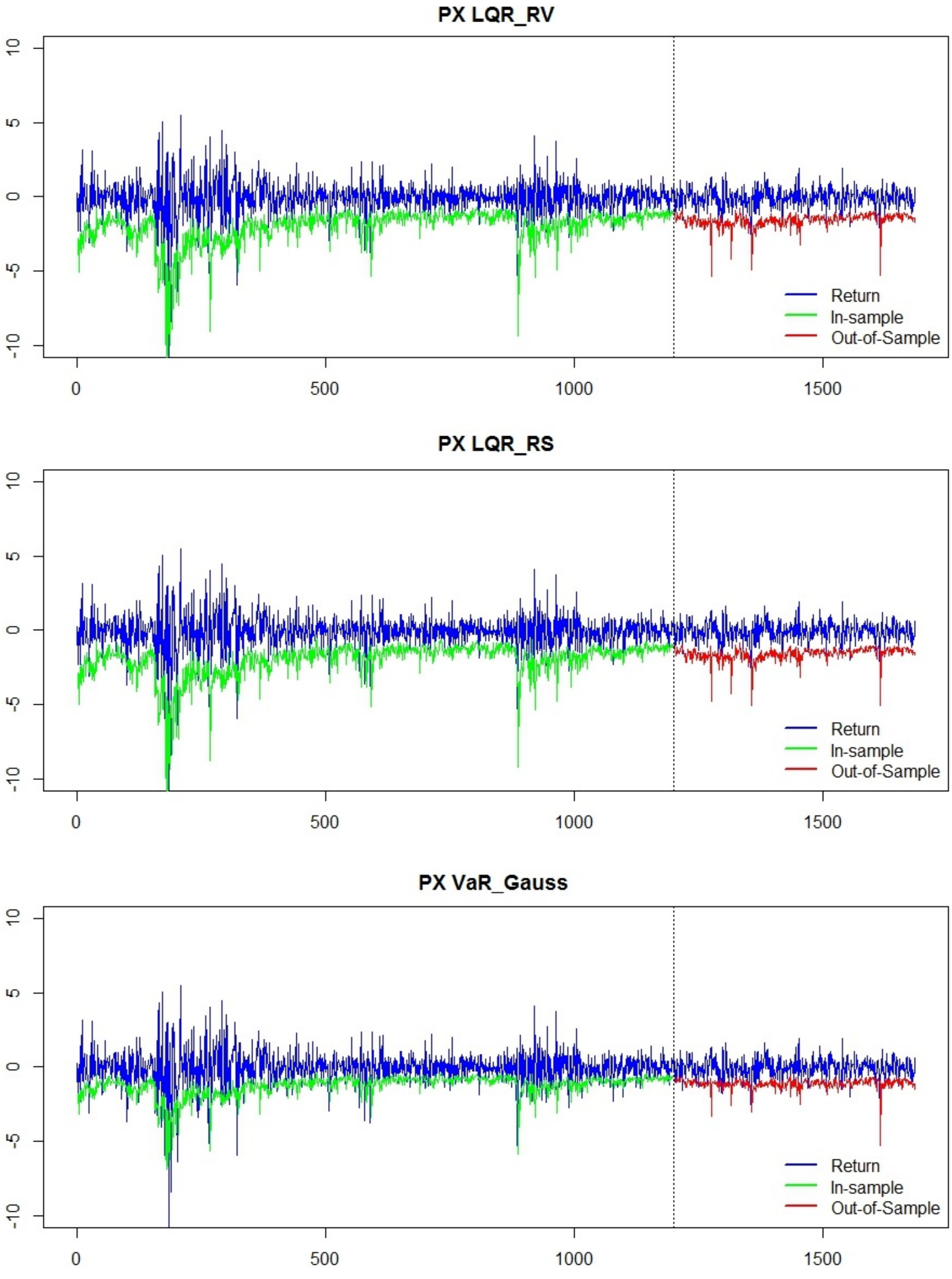
The performance of our models on PX index showed that linear quantile regression model with usage of realized semivariance perform best, followed by linear quantile regression model with usage of the realized variance. The VaR_Gauss model usually underestimates of overestimates the forecasts even in in-sample period, which can then cause significant violations in out-of-sample period. The graphical comparison of the PX forecast models is in Figure 4.1.

Results for BUX index we can see in Table B.1 in Appendix. At 1% quantile, linear quantile regression models perform really well, when LQR_RV predicts 1.7% violations and LQR_RS predicts 1.5% in in-sample period which is close to expected number of violations. However in the out-of-sample period, models predicted zero violations. This mean that LQR models overestimated risk. VaR_Gauss model on the other hand underestimated risk, with more than 10% violation in both samples. The independence test for quantile regression models cannot be concluded due to overestimation, however for the VaR_Gauss model the past violations does not influence future violations. From the relative performance perspective we can conclude that LQR_RS model has in out-of-sample period accuracy as LQR_RV, confirmed by Diebold-Mariano test value of 1.90. However this accuracy is influenced by observed number of violations.

On 5% quantile we have more consistent results of forecasts. Based on unconditional coverage, we accept null hypothesis about correct specification for LQR_RV and LQR_RS, while we reject this hypothesis for VaR_Gauss model, where the number of violations was 7.8%. However for the out-of-sample period, the number of violations is slightly overestimated for quantile regression models, while still a little underestimated for VaR_Gauss. The independence test shows that the violations are not influenced by previous violations. The Diebold-Mariano test confirms that the VaR_Gauss model is less accurate than quantile regression models, when in out-of-sample period the LQR models has same accuracy.

Based on the absolute performance on 10% quantile forecast of BUX index, in the in-sample period the VaR_Gauss model highly overestimates the risk, while LQR_RV and LQR_RS models were correctly specified. However in the out-of-sample period, all of our models overestimated the risk with 7.0%, 5.9% and 2.3% of violations for LQR_RV, LQR_RS and VaR_Gauss respectively. These violations haven't been influenced by the previous violations, confirmed by the independence test. Therefore the conditional coverage results mostly reject null hypothesis about correct specification and independence due to incorrect specification. By comparing our models by tick loss function and DM test, in the out-of-sample period quantile regression models had same accuracy while the VaR_Gauss model was significantly different.

Figure 4.1.: One-step-ahead PX forecast with in-sample size of 1,200 observations



Source: Author's computations

For all low quantiles of BUX we could see that the linear quantile regression models have better forecasting performance as classical VaR model with normally distributed returns. Moreover by using realized semivariance we can improve the forecasts even more.

During forecasting the DAX index data, we can observe similar performance as with PX and BUX data. The results are summarized in the Table B.2 in the Appendix of this document. For the 1% quantile, we see highly underestimated forecasts for the VaR_Gauss model with normally distributed returns. While LQR_RV and LQR_RS showed 0.8% violations for both models, VaR_Gauss model experienced more that 10% of violations (12.1% for the in-sample and 10.6% for the out-of-sample period). Due to the low number of violations, we cannot study their independence for linear quantile models, however for the VaR_Gauss model the independence hypothesis was not rejected. The relative performance moreover shows that even LQR_RV and LQR_RS models does not have same accuracy. The alternative hypothesis of more accurate model confirmed that LQR_RS model has better accuracy.

Moving to the 5% quantile, the unconditional coverage reject the null hypothesis of correct specification for the VaR_Gauss in in-sample period. The LQR models has 5.5% and 5.9% of violations for model with realized variance and semivariance respectively. These violations are independent in time for all models, therefore only for VaR_Gauss model we reject hypothesis of correct specification and independence. For the out-of-sample period, the conditional coverage confirms the correct specification and independence for all models. Looking at the DM test values we can reject hypothesis of the same accuracy between LQR_RV and LQR_RS, showing that LQR_RS has better performance. On contrary, LQR_RV has better accuracy in comparison to LQR_RS in out-of-sample period.

In the 10% quantile, we again reject only VaR_Gauss model, which shows overestimation of the risk. While LQR_RV and LQR_RS shows 10.3% and 10.6% violations respectively, VaR_Gauss model shows only 3.3% in the in-sample period. In the out-of-sample is the probability of violations similar as to the in-sample period. All models show independence between current and previous violations. DM test shows that only in the out-of-sample period has LQR_RV and LQR_RS same accuracy, while in the in-sample period is the LQR_RS model more accurate.

In the last forecast we analyzed FTSE index data. On 1% quantile, summarized in the Table B.3 in Appendix, the VaR_Gauss model underestimated the underlying risk, when allowed 11.1% and 10.7% violation in the in-sample and out-of-sample period respectively. Therefore we conclude that the model is not correctly specified. We have not been able to perform independence test for LQR_RS in the out-of-sample period, however other models show independence in all periods. Relative performance tells us that LQR_RV and LQR_RS have same accuracy in the in-sample period, however in the out-of-period, the LQR_RS is more accurate. VaR_Gauss model show the highest tick loss and DM test value, representing the least accurate model on 1% quantile.

On 5% quantile, linear quantile models again perform well, showing reasonable amount of violations in both in-sample and out-of-sample period. VaR_Gauss model underperform in the in-sample period, however in the out-of-sample period we cannot reject null hypothesis about correctly specified model. Forecasted violations are not influenced by previous violations in any of our models. From the DM test hypothesis we can conclude that LQR_RV and LQR_RS has same accuracy in the in-sample period. In the out-of-sample period, model with semivariance is the most accurate.

In-sample period on 10% quantile is correctly specified for linear quantile regression models, while VaR_Gauss model overestimated forecasts with unconditional coverage of 3.2% violations. In the out-of-sample period, all our model overestimate the risk. However while LQR_RV and LQR_RS forecasted 7.5% and 7.3% violations respectively, VaR_Gauss model estimated 2.4% violations leading to higher overestimation. The independence test have not rejected the independence of our estimates and conditional coverage test confirms that only two models are correctly specified and independent – LQR_RV and LQR_RS in the in-sample period. The relative performance tests show that linear quantile models have same accuracy with DM test values -1.05 and 0.01 in the in-sample and out-of-sample period respectively. The VaR_Gauss model on the other hand underperforms in comparison to LQR model with semivariance.

In the one-step ahead forecast we could observe that linear quantile models performed better in comparison to VaR model with Gaussian distribution. VaR_Gauss model mostly underperformed on 1% and 5% quantiles even in the in-sample period and on 10% quantile overestimated the risk. LQR models performed well on 1% quantile in the in-sample period, however in the out-of-sample period it underestimated risk, when on BUX index we have not observed any violations.

Multi-step forecast

In the multi-step forecast we are going to focus on the relative performance on the horizons 5 and 10 steps ahead and also will compare it to performance on one-step ahead forecast. The results for PX index are summarized in the Table 4.4 and results of BUX, DAX and FTSE are in the Table B.4, B.5 and B.6.

For PX index, the unconditional coverage reject correct specification except for LQR_RV out-of-sample forecast. VaR_Gauss model highly underestimated risk, with more than 28% violations in the in-sample period. LQR_RV model is the most accurate model in the in-sample period, however in the out-of-sample period it has same accuracy as LQR_RS model. On 10-step-ahead forecast, LQR models are correctly specified with 1.5% and 1.8% of violations, while VaR_Gauss model again underestimated risk with 12.3% violations. In the out-of-sample period, we reject correct specification for all models, when LQR_RV, LQR_RS and VaR_Gauss models estimated 3.2%, 13.2% and 12.2% violations. We can see that model with semivariance underestimated the risk the most from all models. Diebold-Mariano test with alternative hypothesis of more accurate model says that LQR_RV has better accuracy as LQR_RS on both samples. Moreover LQR_RS has in out-of-sample period same accuracy as VaR_Gauss model.

Moving to 5% quantile, the unconditional coverage test on horizon 5 days shows that in the in-sample period we reject correct specification for all models, while in the out-of-sample period we confirm correct specification for LQR_RV model. In the out-of-sample period LQR_RS model estimated 13.5% violations, while VaR_Gauss model only 7.9% of violations. Based on tick loss function and DM test we see that LQR_RV has better accuracy as model with semivariance in both in-sample and out-of-sample period, while VaR_Gauss shows same accuracy as LQR_RS. On horizon 10 we see LQR_RS model estimated 22% of violations instead of 5%, showing the high underestimation of the risk.

While on horizon one LQR models were correctly specified in the in-sample period, on horizon 5 we reject correct specification for all models. Following DM test results, LQR_RV and VaR_Gauss models has better accuracy in both in-sample and out-of-sample period compared to LQR_RS. On horizon 10, LQR_RV model has correct specification in the in-sample and out-of-sample period, while for LQR_RS and VaR_Gauss we reject null hypothesis of correct

Table 4.4.: Relative performance of PX index on horizon with in-sample size of 1,200 observations

VaR	*h=1*						*h=5*						*h=10*					
	in-sample			out-of-sample			in-sample			out-of-sample			in-sample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0142	0.0142	0.1760	0.0041	0.0041	0.1072	0.0184	0.0235	0.1424	0.0229	0.0500	0.1271	0.0151	0.0177	0.1228	0.0316	0.1326	0.1221
tick	0.0438	0.0433	0.1410	0.0286	0.0288	0.0572	0.0698	0.0732	0.2864	0.0740	0.1140	0.1660	0.0777	0.0663	0.3264	0.1036	0.2869	0.2694
DM	8.2050		18.5554	0.2466		27.4590	-5.1129		6.6048	-0.9022		3.3606	-6.2268		4.7536	-1.3124		2.2419
5%																		
UC	0.0509	0.0500	0.1284	0.0289	0.0309	0.0598	0.0645	0.0871	0.0938	0.0500	0.1354	0.0792	0.0547	0.0799	0.0791	0.0842	0.2211	0.0905
tick	0.1393	0.1391	0.1546	0.0876	0.0877	0.0837	0.2546	0.2480	0.3116	0.2220	0.2736	0.2196	0.3068	0.2603	0.3724	0.3405	0.5951	0.3402
DM	-3.2441		16.6369	-0.1092		19.7290	-4.9189		0.9658	-0.6565		1.0518	-4.9582		-1.7569	0.0199		0.9577
10%																		
UC	0.0992	0.1009	0.0676	0.0495	0.0495	0.0351	0.1181	0.1591	0.0486	0.0833	0.1958	0.0333	0.1060	0.1405	0.0336	0.1137	0.2800	0.0547
tick	0.2118	0.2118	0.2223	0.1374	0.1371	0.1621	0.4221	0.3994	0.4553	0.3468	0.4293	0.3910	0.5249	0.4431	0.6016	0.5383	0.8624	0.5581
DM	0.9779		-13.2277	-1.3660		-13.0008	-4.8687		-8.4238	-0.0981		-2.5483	-4.3730		-7.4558	0.4591		-1.7988

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

specification based on the unconditional coverage values, LQR_RS underestimated risk with 14% and 28% violations in the in-sample and out-of-sample period respectively, while VaR_Gauss overestimated the risk. DM test confirms results of the horizon 5, showing that LQR_RS has lower accuracy as LQR_RV and VaR_Gauss.

The result for the Budapest Stock Exchange index are similar to PX index. On 5-step-ahead forecast, only LQR_RV in the in-sample period is correctly specified on 1% quantile. The rest of the models underestimated risk – LQR_RS estimated 8.8% violations in the out-of-sample period and VaR_Gauss model in both samples estimated over 10% violations. DM test of model accuracy tells us that LQR_RV has better accuracy in comparison to LQR_RS while VaR_Gauss has lower accuracy. On horizon 10 steps ahead is LQR_RV the only model with correct specification in both samples. The difference is in the model accuracy, when LQR_RV is the most accurate and also VaR_Gauss model has better accuracy in the in-sample period in comparison to LQR_RS.

On 5% quantile and with h=5, only for LQR_RS model we reject null hypothesis of correct specification. In out-of-sample period, LQR_RS forecasted approximately 17% violations. Therefore based on tick-loss function, we say that LQR_RS is least accurate model in comparison to LQR_RV and VaR_Gauss models. We see similar results also for 10-step-ahead forecast, where we reject correct specification for the LQR_RS. VaR_Gauss is the only model, which on 5% quantile has correct specification in both – in-sample and out-of-sample periods, while LQR_RV underestimated risk with 7.6% violations. Based on Diebold-Mariano test value we reject null hypothesis of same accuracy in the in-sample period between our models. In the out-of-sample period, VaR_Gauss and also LQR_RV are more accurate than LQR_RS.

The unconditional coverage on 1-step-ahead forecast on 10% quantile is usually characterized by the correct specification in the in-sample period and overestimation in the out-of-sample period. On forecast with horizon 5 and 10 we see correct specification only for the LQR_RV model in the in-sample period, while LQR_RV in the out-of-sample period and LQR_RS model underestimate the risk. On contrary, VaR_Gauss model overestimated the risk with only 2% and 1.5% violations in in-sample and out-of-sample period on horizon 5 and 7.5% and 8.7% on horizon 10. DM test tells us that LQR_RV and VaR_Gauss models has better accuracy than LQR_RS.

We continue by analyzing DAX index. On the 1% quantile, only LQR_RV is correctly specified in the in-sample period with 5-step-ahead forecast. The rest of the models underestimated the risk. LQR_RV has also better accuracy than LQR_RS in both in-sample and out-of-sample period, while VaR_Gauss has lowest accuracy. On horizon 10 we have similar results, however VaR_Gauss model has better accuracy as LQR_RS in the out-of-sample period.

On the 5% quantile and horizon 5 days, VaR_Gauss model in the in-sample period is correctly specified, with 5% violations while LQR_RS model significantly underestimated the risk. Based on the tick loss function, VaR_Gauss and LQR_RV models has better accuracy in comparison to model with semi-variance. On horizon 10, our models has same accuracy in the out-of-sample period, while in the in-sample period, LQR_RV and VaR_Gauss are more accurate in comparison to LQR_RS.

VaR_Gauss model, as we already saw on PX and BUX indices, overestimates the risk on the 10% quantile. On horizon forecasts is this finding consistent, when in the in-sample period VaR_Gauss model estimated only 1% and 1.3% violations on horizon 5 and 10 days respectively. On the other hand, LQR_RS underestimated the risk with 23.5% and 32.2% violations in the out-of-sample period. LQR_RS is the least accurate model in both – in-sample and out-of-sample period, based on DM test and tick loss function.

The last index we performed horizon forecast is FTSE. On 1% quantile in the in-sample period is LQR_RV model correctly specified across all horizons, while for VaR_Gauss we reject null hypothesis of correct specification across all samples. Following tick loss function and DM test, on horizon 5 days has LQR_RV better accuracy than LQR_RS in both in-sample and out-of-sample period, while VaR_Gauss is the least accurate model. On horizon 10, LQR_RS and VaR_Gauss has same accuracy, while LQR_RV has the best one.

Continuing on 5% quantile, VaR_Gauss model is correctly specified in the in-sample period on 5 day and 10 day horizon, while LQR_RV model only on horizon 5. LQR_RS model forecasted over 24% violation in the out-of-sample period on horizon 10, therefore we reject correct specification. Following results of tick loss function and Diebold-Mariano relative performance, VaR_Gauss and LQR_RV are more accurate than LQR_RS on both horizons – 5 and 10.

Consistently with one-step-ahead forecasts, VaR_Gauss model overestimated risk on 10% quantile, while LQR_RS and LQR_RV underestimated risk in the out-of-sample period. On both horizons 5 and 10 days, we cannot reject null hypothesis of same accuracy for LQR_RV and LQR_RS model, while

VaR_Gauss model is more accurate in comparison to linear regression model with semivariance. While on horizon 5, LQR_RS model forecasted 23.5% violations in the out-of-sample period, on horizon 10 it forecasted 32.2% violations.

Forecast summary

For the one-step-ahead forecast, VaR_Gauss was the least accurate model across all indices, showing overestimation of risk on 10 % quantile, while underestimating it at lower quantiles. Linear quantile regression models performed well, when model with semivariance was the most accurate and model with realized variance had usually same accuracy proved by relative performance evaluation techniques. During 5 and 10 day horizon forecasts, the performance of LQR_RS model decreased, while the LQR_RV model performed best. The low accuracy of LQR_RS model could be caused by period with significant semivariance, which we included in the forecast. VaR_Gauss showed better accuracy as LQR_RS model, which could be caused by the assumption of the normal distribution, while LQR_RS model is based on more precise past stock performance.

4.2.2. Second sample forecast

After we evaluated performance of our models with in-sample period size of 1,200 observations, we decrease its size on 800 observations. Due to that we can observe the out-of-sample performance on more volatile returns.

One-step forecast

We start by evaluation forecasts, forecasted for Prague Stock Exchange index. The results are in the Table 4.5.

On 1% quantile, linear quantile models are specified well, when we failed to reject null hypothesis about correct model specification. On the other hand, with VaR_Gauss model we rejected the hypothesis about correct specification, when VaR_Gauss model forecasted in the in-sample period 16.5% of violations. This

Table 4.5.: Absolute and relative performance of PX index on one-step-ahead forecast with in-sample period of 800 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0138	0.0125	0.1652	0.0079	0.0079	0.1480
L ind	2.2131	2.5652	0.3516	n/a	n/a	2.8789
L cc	3.2381	3.0382	513.2945	n/a	n/a	482.4927
Tick	0.0484	0.0459	0.1549	0.0302	0.0302	0.0830
DM	-0.6586		16.2895	-0.1365		20.8914
5%						
UC	0.0501	0.0488	0.1214	0.0407	0.0407	0.0983
L ind	3.6454	1.9893	1.1544	1.3695	1.3695	1.5478
L cc	3.6455	2.0133	63.5478	3.0940	3.0940	35.8926
Tick	0.1554	0.1555	0.1713	0.0959	0.0968	0.1008
DM	5.2074		11.5101	2.3527		11.9470
10%						
UC	0.0964	0.0989	0.0638	0.0723	0.0746	0.0542
L ind	1.0890	0.7706	2.1730	0.0330	0.0012	0.7284
L cc	1.2072	0.7819	15.3263	8.2917	6.9090	25.0207
Tick	0.2347	0.2344	0.2458	0.1482	0.1485	0.1680
DM	-5.8114		-11.5681	1.8051		-16.0251

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

high number of violations can be explained by wider in-sample period, excluding second period of higher volatility. Moreover, in the out-of-sample period, VaR_Gauss model forecasted 14.8% of violations, which supports the incorrect specification in the in-sample period. Linear quantile regression models has same accuracy performance, with DM test value -0.65 and -0.13.

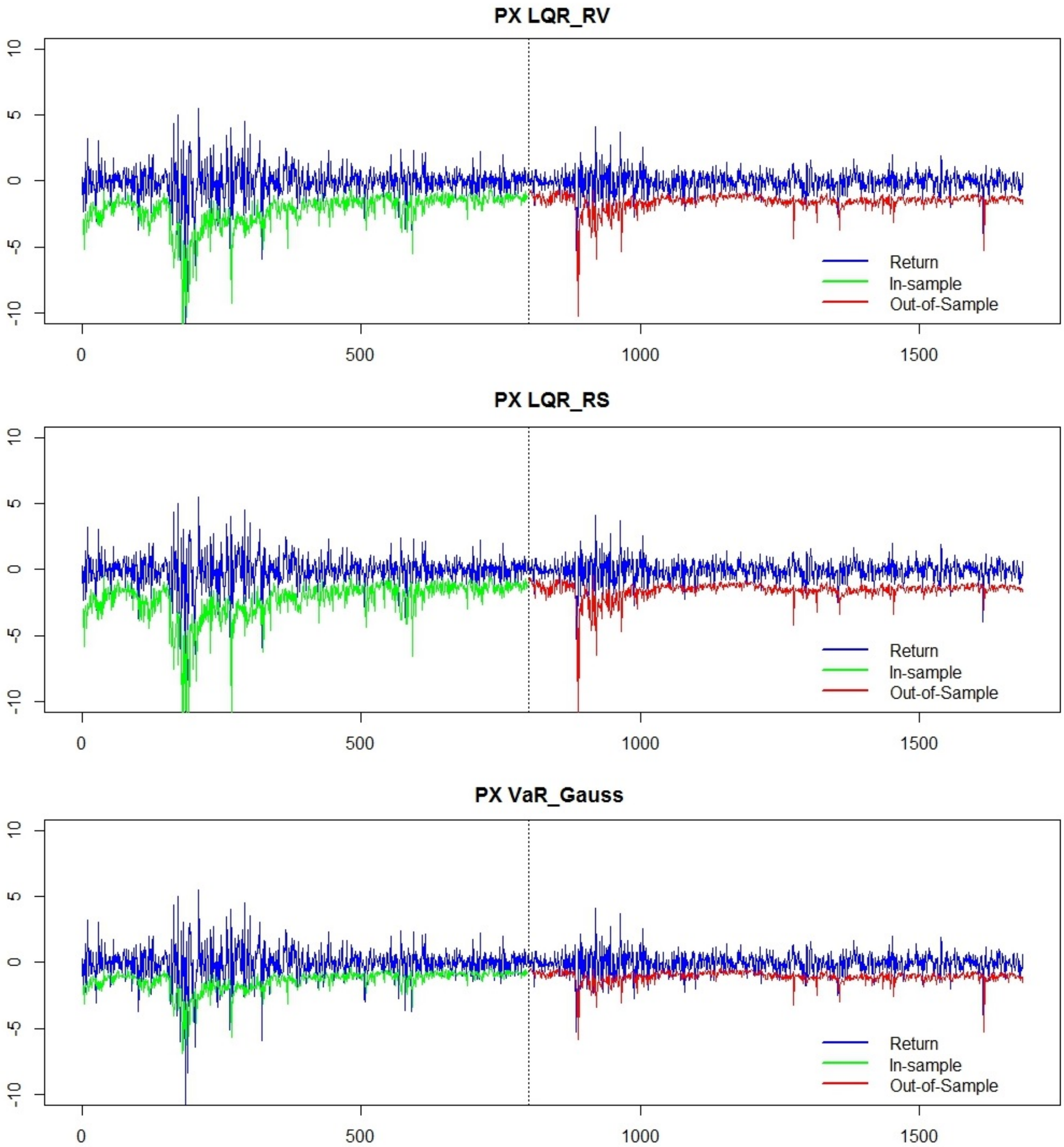
5% quantile follows the results of 1% quantile. LQR_RV and LQR_RS are correctly specified models in in-sample and also out-of-sample period, while we rejected null hypothesis for VaR_Gauss model, because instead of expected 5% violations, VaR_Gauss model estimated 12.1% and 9.8% violations in in-sample and out-of-sample period respectively. All our forecasted violations are independent in the time, therefore by conditional coverage we reject VaR_Gauss model. Moreover by relative performance we conclude that our models do not have same accuracy, when in in-sample and out-of-sample period LQR_RS model has the best accuracy from given models.

Forecasted Value at Risk values at 10% quantile are correctly specified for the linear quantile regression models in the in-sample period, while VaR_Gauss model underestimated the risk, when forecasted only 6.4% of violations. In the out-of-sample period, all our models underestimated given risk, however the underestimation was smaller in case of LQR models. Based on tick loss function we can conclude that VaR_Gauss model is least accurate and Diebold-Mariano test confirmed that LQR_RV model has better performance in both – in-sample and out-of-sample period.

Based on results from the PX index we can conclude linear quantile regression models has better forecasting potential than classical VaR model with normal distribution. We can make also graphical comparison from Figure 4.2. So we can conclude that VaR_Gauss model underestimates the risk in in-sample and also out-of-sample period.

We continue the forecast by analyzing BUX data on 1% quantile, which are summarized in Appendix in the Table B.7. VaR_Gauss model underestimated the potential risk, when forecasted 13.1% of violation in the in-sample period and 11.4% in the out-of-sample period, though LQR_RV and LQR_RS estimated approximately expected number of violations for this quantile. The VaR_Gauss estimated violations are independent to previous ones, while independence test for LQR models are not available due to low number of violations.

Figure 4.2.: One-step-ahead PX forecast with in-sample size of 800 observations



Source: Author's computations

By comparing DM test values we have to reject null hypothesis of same accuracy, VaR_Gauss model has the smallest accuracy and LQR_RS has greater accuracy as LQR_RV model.

While on 1% quantile, VaR_Gauss model underestimated risk in the in-sample and out-of-sample period, on 5% quantile it is correctly specified for out-of-sample period. The estimated number of violations are independent in time for all models, therefore based on conditional coverage test we reject only VaR_Gauss model in the in-sample period. Following the results for relative performance metrics, we conclude that out models do not have same accuracy. LQR_RV model has better accuracy in in-sample period, while LQR_RS has better accuracy in the out-of-sample period.

LQR_RV and LQR_RS have been correctly specified on the in-sample period, however in the out-of-sample period they overestimated the risk with 7.7% violations for both models. This violations are independent in time, therefore we reject conditional coverage hypothesis due to unconditional coverage likelihood part. VaR_Gauss model is overestimating the results as instead of expected 10% violation it forecasted 3.5% and 2.9% violation in the in-sample and out-of-sample period. Diebold-Mariano test confirms correct specification for the out-of-sample forecast between linear quantile regression models. In the in-sample period, the LQR_RV is more accurate.

The third analyzed index with in-sample size of 800 observations is DAX, which results are summarized in the Appendix in the Table B.8. Based on absolute performance metrics, 99% Value at Risk estimates on 1% quantile is correctly specified for the linear quantile regression models in both in-sample and out-of-sample period. On the other hand, VaR_Gauss model underestimated risk with more than 11% violations instead of 1%. Therefore for this model we reject null hypothesis of correct model specification supported by also conditional coverage metric. Following results of the tick loss value and DM test we can conclude that VaR_Gauss model has lowest accuracy, while LQR_RV and LQR_RS has same accuracy.

For 5% quantile we can observe similar results as for 1% quantile. While LQR models perform well, forecasting approximately required number of violations, model with normal distribution underestimated the risk with 8.8% and 7.2% in the in-sample and out-of-sample period respectively. Taking likelihood ratio of independence test, all violations for our models are independent to previous violations, therefore we cannot reject conditional coverage hypothesis for LQR models, while we reject it for VaR_Gauss model. In the in-sample period, LQR_RV and LQR_RS has same accuracy, while in the out-of-sample period, model with use of semivariance has better accuracy.

While for PX and BUX data linear quantile regression models overestimated risk on 10% quantile for out-of-sample period, in DAX case these model estimated correct probability of violations. However VaR_Gauss model still overestimated risk in both in-sample and out-of-sample period. Estimated violations are independent in time, therefore we reject conditional coverage null hypothesis only for VaR_Gauss model. Based on the Diebold-Mariano test, LQR_RV and LQR_RS has same accuracy in out-of-sample period, while in in-sample period LQR_RV is more accurate. VaR_Gauss has worst accuracy from the given models with DM test value of -11.8 and -14.8 in the in-sample and out-of-sample respectively.

The last index to analyze is FTSE index, which has similar results to DAX. On 1% quantile, VaR_Gauss underestimates the risk with 11.0% and 10.6% violations in the in-sample and out-of-sample respectively. On the other hand, linear quantile regression models are correctly specified in both samples. As we do not reject independence hypothesis, we reject conditional coverage hypothesis for VaR_Gauss model. Following the DM test value, in the in-sample period LQR_RV and LQR_RS models have same accuracy, while in the out-of-sample, the accuracy differs.

Linear quantile regression models on 5% quantile have been usually correctly specified, however LQR_RS overestimated the risk in the out-of-sample period. In the in-sample period, VaR_Gauss model underestimated risk with 6.6% of violations. Based on relative performance, we say that VaR_Gauss and LQR_RS models have same accuracy in the in-sample period. In the out-of-sample period, VaR_Gauss has lower accuracy as LQR_RS and so LQR_RV.

On 10% quantile, VaR is overestimated with VaR_Gauss model, while LQR models estimated correct number of violations. Exceptionally, we reject null hypothesis of independent violations for VaR_Gauss model in the out-of-sample period, with likelihood ratio of 2.81. Other models has independent violations in time. Based on tick loss value we say that VaR_Gauss model underperforms in comparison to LQR models, which has same accuracy based on Diebold-Mariano test.

Multi-step forecast

We continue with the horizon forecasts of PX index, which are summarized in the Table 4.6. On 1% quantile, we see consistent results for LQR_RV model, which is correctly specified across all quantiles in the in-sample period. For other models, only LQR_RS in the in-sample period on horizon 10 is correctly specified based on unconditional coverage. Following the Diebold-Mariano test, LQR_RV is more accurate than LQR_RS, while LQR_RS is more accurate than VaR_Gauss model. Therefore we can conclude that LQR_RV is the most accurate model on 1% quantile.

Moving to 5% quantile, for LQR_RV model we cannot reject null hypothesis of correct specification on horizon 5, while on horizon 10 we reject this hypothesis for out-of-sample period. LQR_RS is incorrectly specified on both horizons, when in the out-of-sample period it forecasted 15% and 20% violations on horizon 5 and 10 respectively. Based on the DM test, VaR_Gauss model has same accuracy as LQR_RS, while LQR_RV is more accurate in the in-sample period.

Similarly to 5% quantile, also on 10% quantile is the LQR_RV model correctly specified except for the out-of-sample period on horizon 10. LQR_RS model underestimated risk, when it allowed 26% violation on horizon 10 in the out-of-sample period and 20% in the in-sample period. On contrary, model with Gaussian distribution overestimated risk, allowing less than 6% of violations on both horizons. Based on tick loss function and alternative hypothesis of DM test we can say that LQR_RV and VaR_Gauss are more accurate in comparison to linear quantile regression model with semivariance.

We continue by analyzing BUX index, whose results are in the Table B.10 in Appendix. On 1% quantile, only LQR_RV is correctly specified in the in-sample period across all horizons. On horizon 5, LQR_RS and VaR_Gauss models underestimated risk with 7.6% and 11.5% violations in the out-of-sample period respectively, while on horizon 10 both underestimated risk with over 13% violations. Following the results of the DM test, LQR_RV is more accurate model than LQR_RS on horizon forecasts, while VaR_Gauss model is less accurate, except of the in-sample period on 5-step-ahead forecast.

On 5% quantile, VaR_Gauss model has correct specification in both in-sample and out-of-sample period during 5-step-ahead forecast, while LQR_RV only in the in-sample period and LQR_RS is not correctly specified. This is also

Table 4.6.: Relative performance of PX index on horizon with in-sample size of 800 observations

VaR	*h=1*						*h=5*						*h=10*					
	in-sample			out-of-sample			in-sample			out-of-sample			in-sample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0138	0.0125	0.1652	0.0079	0.0079	0.1480	0.0176	0.0277	0.1448	0.0284	0.0557	0.1375	0.0177	0.0177	0.1179	0.0423	0.1291	0.1314
tick	0.0484	0.0459	0.1549	0.0302	0.0302	0.0830	0.0750	0.0878	0.3348	0.1024	0.1412	0.2014	0.0857	0.0696	0.3632	0.1765	0.3079	0.3203
DM	-0.6586		16.2895	-0.1365		20.8914	-4.0666		5.8871	-1.2468		4.1452	-4.3716		3.6618	-2.0261		3.4270
5%																		
UC	0.0501	0.0488	0.1214	0.0407	0.0407	0.0983	0.0605	0.0869	0.0982	0.0648	0.1511	0.0875	0.0558	0.0862	0.0786	0.0869	0.2034	0.0971
tick	0.1554	0.1555	0.1713	0.0959	0.0968	0.1008	0.2774	0.2804	0.3544	0.2581	0.3076	0.2483	0.3322	0.2809	0.4118	0.3938	0.5970	0.3850
DM	5.2074		11.5101	2.3527		11.9470	-4.3691		1.0511	-1.4607		1.5216	-4.3077		-1.4631	-0.5726		1.2168
10%																		
UC	0.0964	0.0989	0.0638	0.0723	0.0746	0.0542	0.1108	0.1499	0.0504	0.1148	0.2080	0.0386	0.1039	0.1293	0.0342	0.1314	0.2594	0.0571
tick	0.2347	0.2344	0.2458	0.1482	0.1485	0.1680	0.4656	0.4459	0.5016	0.3864	0.4560	0.4076	0.5686	0.4761	0.6617	0.5856	0.8202	0.5945
DM	-5.8114		-11.5681	1.8051		-16.0251	-4.1746		-6.9676	-0.5090		-4.0655	-3.8092		-6.3587	0.0190		-3.2063

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

confirmed by alternative hypothesis of DM test, showing that LQR_RV and VaR_Gauss are both more accurate than LQR_RS. On horizon 10, all models are correctly specified in the in-sample period, while in out-of-sample period model underestimated risk. Similarly to horizon 5, VaR_Gauss and LQR_RV models are more accurate.

VaR_Gauss model overestimated risk on 10% quantile on horizon one and this trend is consistent also over higher horizons. On the other hand, LQR_RV is correctly specified in the in-sample period with 10.5% and 11.2% violation on horizon 5 and 10 respectively. Both VaR_Gauss and LQR_RV models are more accurate as the LQR_RS, which estimated in the out-of-sample period 20.9% violations on horizon 5 days and 25.8% on 10-step-ahead forecast, which is similar with PX index.

Following the results of DAX in the Table B.11 in Appendix, we can see consistent results with one-step-ahead forecast. On 1% quantile, VaR_Gauss model underestimated risk with more than 10% violation across horizons in both in-sample and out-of-sample period. LQR_RV is correctly specified in the in-sample period, while in the out-of-sample period it underestimated risk. Based on tick loss function, for LQR_RS and VaR_Gauss we cannot reject null hypothesis of the same accuracy, while LQR_RV model is the most accurate on both horizons.

On 5% quantile of horizon 5, only VaR_Gauss model in the in-sample period is correctly specified with 4.7% violations. For LQR models we reject hypothesis of correct specification on both horizons, when in the out-of-sample period of horizon 10, models underestimated risk with 11.8% and 24% violations by LQR_RV and LQR_RS respectively. DM test shows that LQR_RV and VaR_Gauss models are more accurate as model LQR model with semivariance.

10% quantile copy the results of the BUX and PX indices, when VaR_Gauss overestimated risk across sample for both horizons. For LQR_RV we fail to reject null hypothesis of unconditional coverage, therefore we say LQR_RV is correctly specified in the in-sample period. DM test with alternative hypothesis of better accuracy shows that both LQR_RV and VaR_Gauss models are more accurate than LQR_RS for 5 and 10-step-ahead forecasts.

The last index we analyze is FTSE, whose results are summarized in the Table B.12 in Appendix. On horizon 5 on 1% quantile, VaR_Gauss underestimated risk in both in-sample and out-of-sample periods, while LQR models were correctly specified in the in-sample period. LQR_RV is however more accurate as model with semivariance, while VaR_Gauss is less accurate model in comparison to LQR_RS. LQR_RS is incorrectly specified on horizon 10, which resulted into same accuracy with VaR_Gauss model.

As we move to higher quantiles, LQR_RS is decreasing accuracy in higher horizons. On 5% quantile, LQR_RS estimated 14.4% and 20.7% violation on horizon 5 and 10 respectively in the out-of-sample period. LQR_RV and VaR_Gauss are correctly specified in the in-sample periods. DM test value on 5-step-ahead forecast tells us that both LQR_RV and VaR_Gauss models are more accurate than LQR_RS and this is consistent also for the horizon 10 days, supported by high tick loss function values of LQR_RS.

10% quantile was characterized by overestimation of risk with VaR_Gauss model across indices. This is the same for the horizon forecasts in both in-sample and out-of-sample periods. LQR_RV is correctly specified in the in-sample period of 5 and 10-step-ahead forecasts, while LQR_RS model underestimated risk. On horizon 5, LQR_RV and VaR_Gauss models are more accurate than LQR_RS, while on the horizon 10 we cannot reject hypothesis of same accuracy for LQR models.

Forecast summary

The models performed similarly as on the dataset with larger in-sample period. On horizon one, VaR_Gauss was the least accurate model across all indices while on higher horizons it became more accurate. It was caused mainly by the usage of the cumulative returns. Linear quantile regression model with realized volatility performed well across all horizons, while model with semivariance was the most accurate on one-step-ahead forecast while underperformed on horizon 5 and 10.

4.3. Diversification

After we performed forecasts, we will focus on the diversification of the indices. As our indices are trading in different days, we need to filter data to be able to create the correlations. We choose only daily returns and realized variance for the days, when all of the indices were traded.

The plot of dynamic conditional correlation is in the Figure 4.3 (blue line). We can see that for the PX-DAX correlation is with mean 0.50 (+-0.2) and 0.44 (+-0.2) for the PX-FTSE.

For the quantile correlation, we modify Campbell *et al.* (2000) formula 2.33.

We use high-frequency data to define the realized variance. However from these data we cannot define the covariance of our portfolio, as the length of trading days differ and also countries of origin are in different time zones. Due to that we are going to use the covariance of the portfolio from the DCC model to define common quantile of the portfolio. However as we cannot mix the realized variance with covariance from the DCC model, we need to use also GARCH variance from dynamic conditional correlation model. The edited formula for quantile correlation is as follows

$$\rho_Q = \frac{r_Q^2 - w_x^2 qr_{x,Q}^2 - w_{y,Q}^2 qr_{y,Q}^2}{2 w_x w_y qr_{x,Q} qr_{y,Q}} \tag{4.8}$$

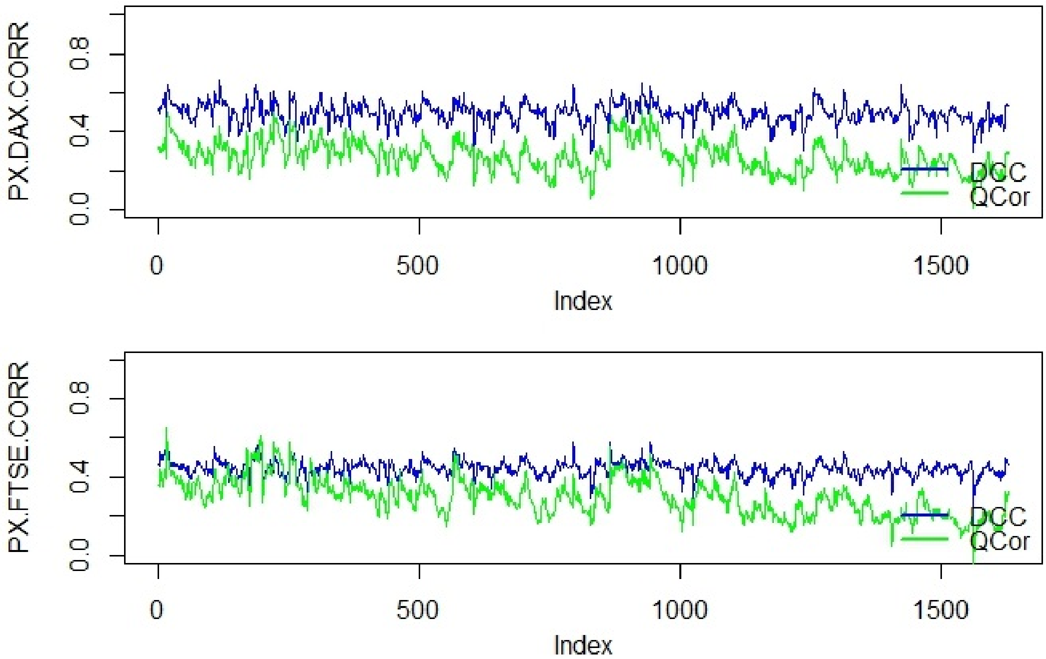
where qr_Q^2 are fitted values of quantile regression.

The results of quantile correlation is the green line in the Figure 4.3. We can see that in comparison to DCC model, quantile correlation shows smaller correlation between PX and Western European indices. Moreover, the correlation is decreasing over time.

Correlations charts for BUX index are in Appendix in the Figure A.6 The results are less volatile in comparison to PX correlations. DCC average correlation is 0.51 (+-0.12) with DAX index and 0.46 (+- 0.18) for FTSE index.

We found that realized measures improved correlation estimates, as in comparison to dynamic conditional correlation model does not assume normally distributed returns. Following the results of the quantile correlation, the diversification benefits are decreasing within European countries. Therefore we can confirm our hypothesis of globalization.

Figure 4.3.: Correlation of the PX with Western European indices



Source: Author's computations

Chapter 5
Conclusion

In this thesis we examined conditional Value at Risk modeling by employing flexible quantile regression framework on European indices during the period from January 2008 till October 2014. Our main contribution comes from Central and Eastern European indices, represented by PX and BUX index, together with Western European DAX and FTSE 100 with use of the high-frequency data. We follow Zikes and Barunik (2014) in application of linear quantile regression models for forecasting the Value at Risk and then we compare the quantile correlation between indices.

In the first part of the thesis, we present the theoretical background for realized measures and their obtaining from the high-frequency data. Further, we focus on Value at Risk concept, where we describe the approaches for its calculation and pros and cons of each methods. Followed by definition of linear quantile regression model with use of realized measures of realized variance and positive and negative semivariance, we introduce forecast evaluation methods of the absolute and relative performance. We conclude the theoretical part by defining quantile diversification framework.

The next part of the thesis starts with description of the data. Firstly, high-frequency data of studied indices is described in detail. Secondly, the realized variance and volatility of our dataset is presented.

After we explained theory and data, we continue with empirical application, where we analyzed the modelling performance of considered VaR models – linear quantile regression model with realized variance, semivariances and VaR model with assumption of normally distributed returns. We evaluated their performance in two subsamples, with difference in the size of in-sample period and described the results for all the indices for several horizons. The results of modelling are described in the detail for each of the sample with short summary at the end. Last part of the chapter focuses on diversification of indices over time.

The results of the modelling indicate that linear quantile regression models performed significantly better in both in-sample and out-of sample period compared to VaR model with normal distribution. The results are consistent across all indices, quantiles and subsamples during the one-day-ahead forecasts. However in the higher horizons, model with realized semivariances became less accurate, which can be caused by the large in-sample period, which includes periods with significant variance. The correlation findings from QC model show that diversification benefits are decreasing over time between European indices as we assumed in the beginning of the work.

In conclusion, we showed that modelling of conditional quantiles by quantile regression is reasonable VaR estimation, providing better results as different models. Therefore models with realized measures can be used as a more accurate tool for investors. The quantile correlation provided sufficient results, however as we used the variance and covariance from the dynamic conditional correlation model, it might be sensible to evaluate the diversification based on high-frequency data. We leave this improvement for future work.

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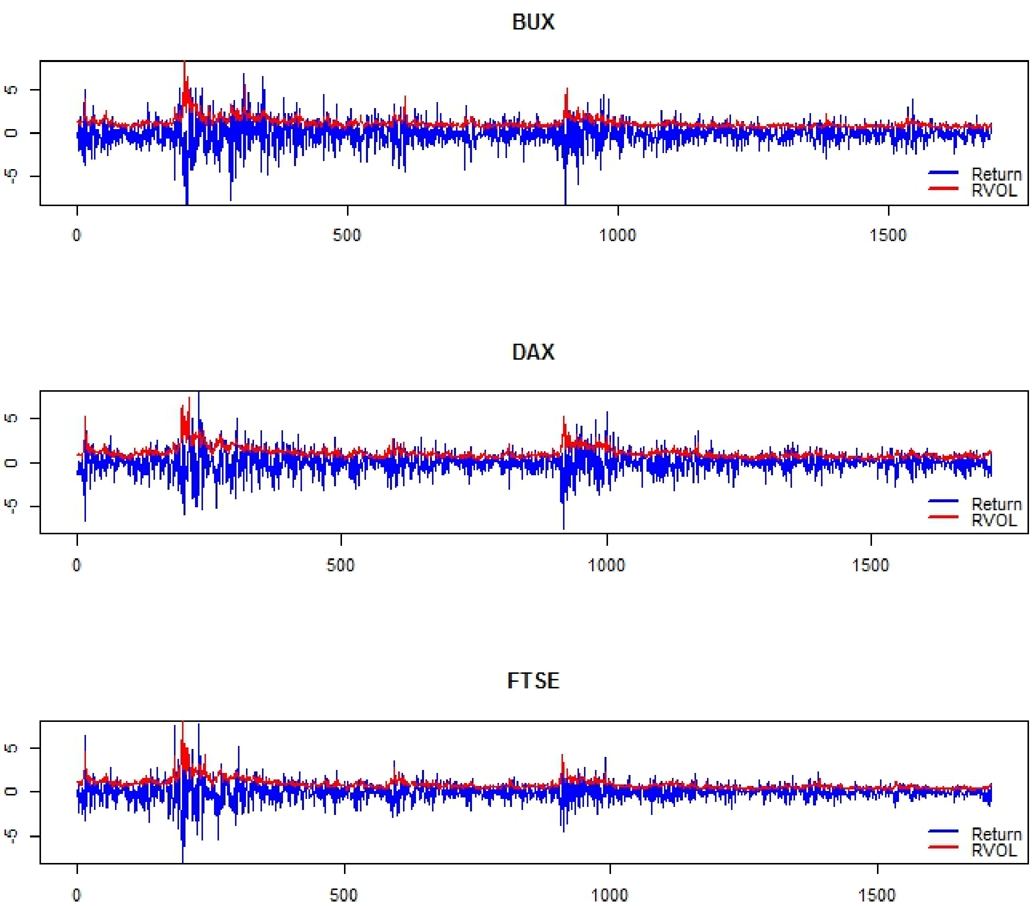
Appendix A

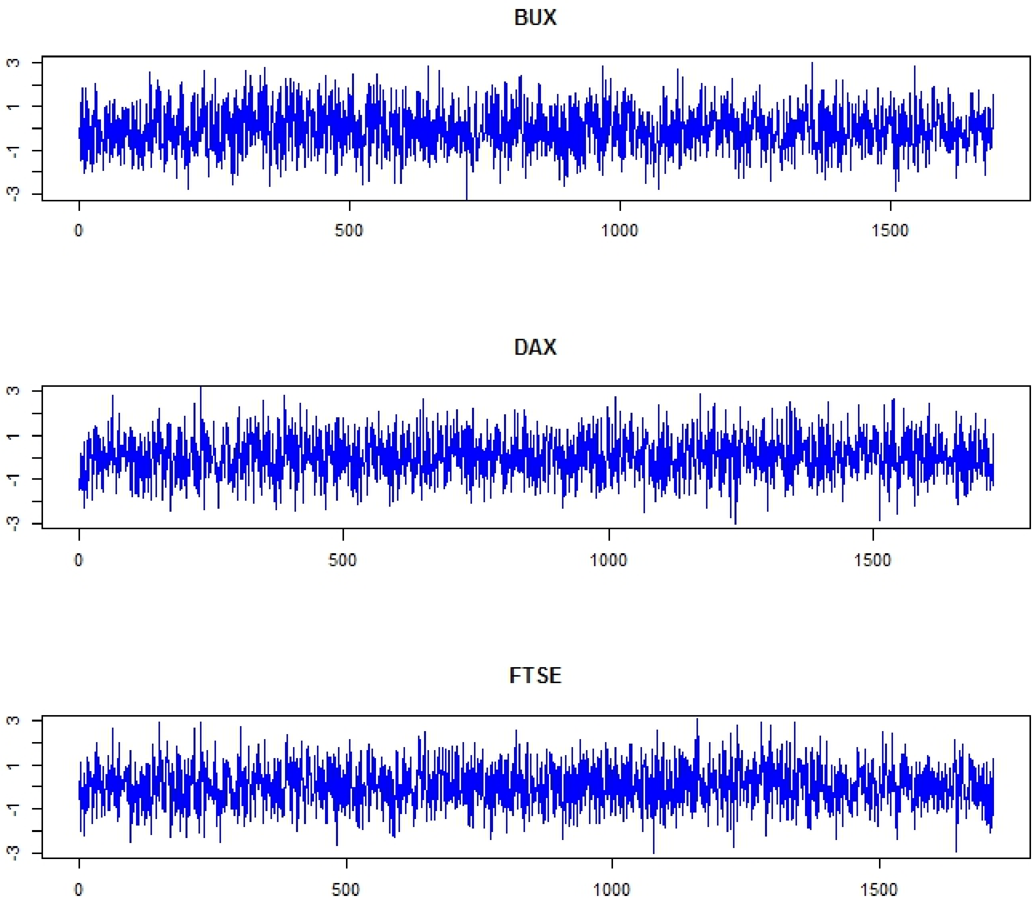
Figures

Figure A.1.: Comparison of daily returns with realized volatility (BUX, DAX amd FTSE indices)



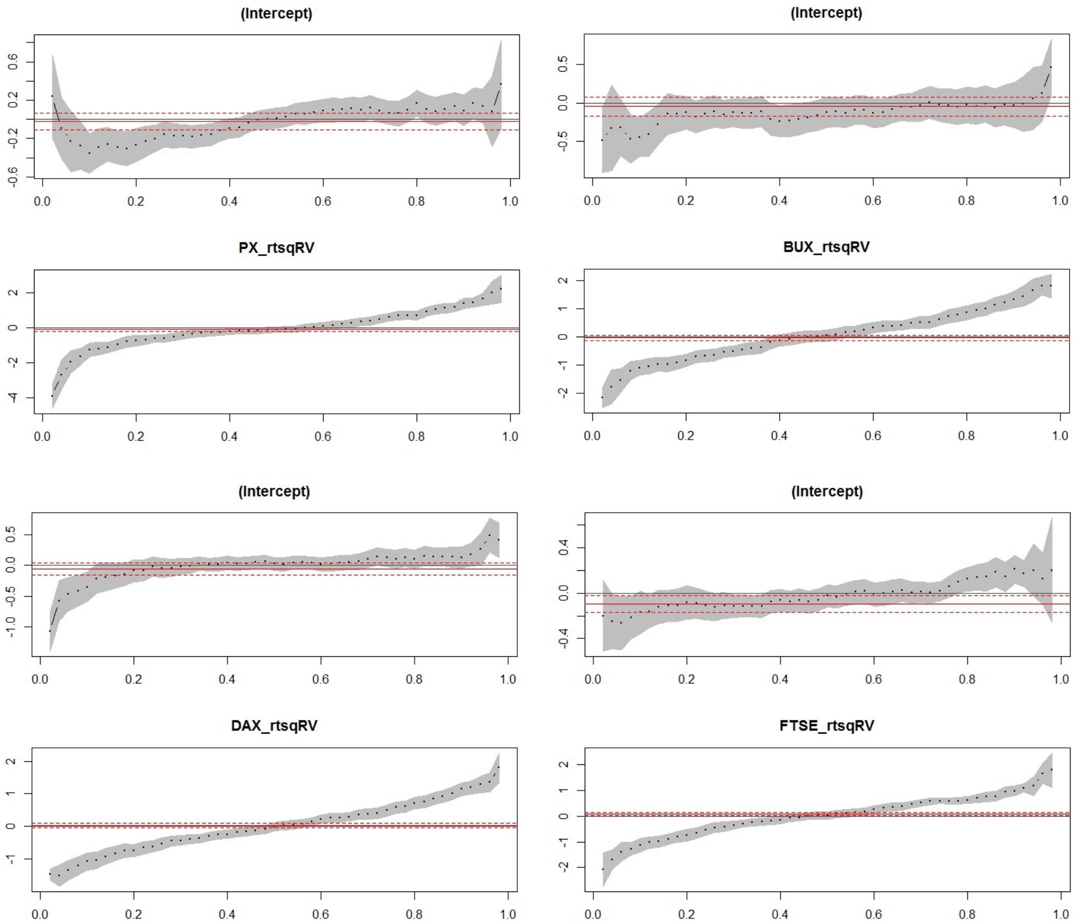
Source: Author's computations

Figure A.2.: White noise assumption for BUX, DAX and FTSE indices



Source: Author's computations

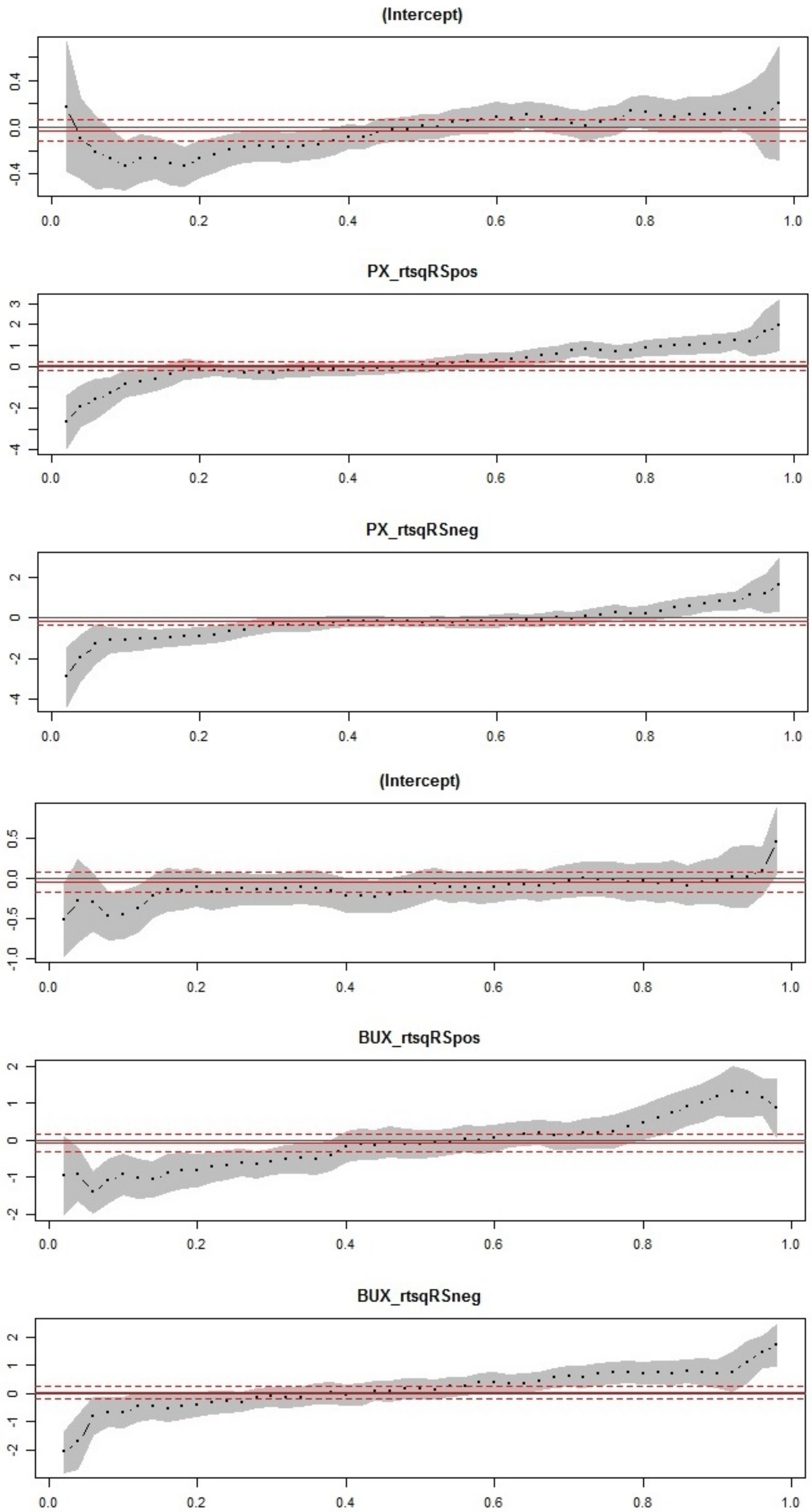
Figure A.3.: Results of conditional linear quantile regression with realized volatility



Note: PX__rtsqRV is realized volatility of PX index, DAX__rtsqRV is realized volatility of DAX index, BUX__rtsqRV is realized volatility of BUX index and FTSE__rtsqRV is realized volatility of FTSE index.

Source: Author's computations

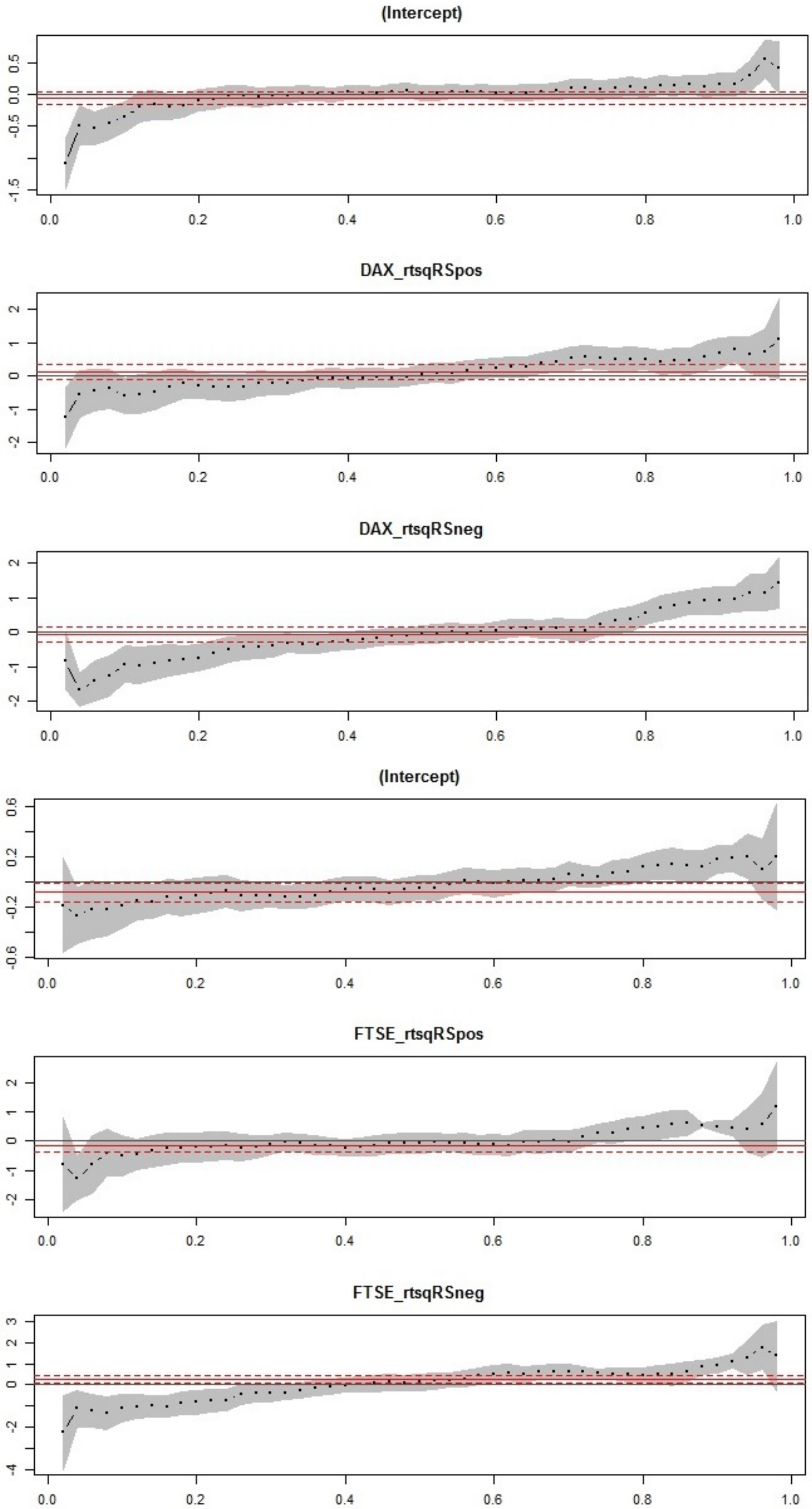
Figure A.4.: Results of linear quantile regression with squared root of realized semivariance



Note: PX__rtsqRSpos is realized positive semivolatility and PX__rtsqRSneg is realized negative semivolatility of PX index, BUX__rtsqRSpos is realized positive semivolatility and BUX__rtsqRSneg is realized negative semivolatility of BUX index.

Source: Author's computations

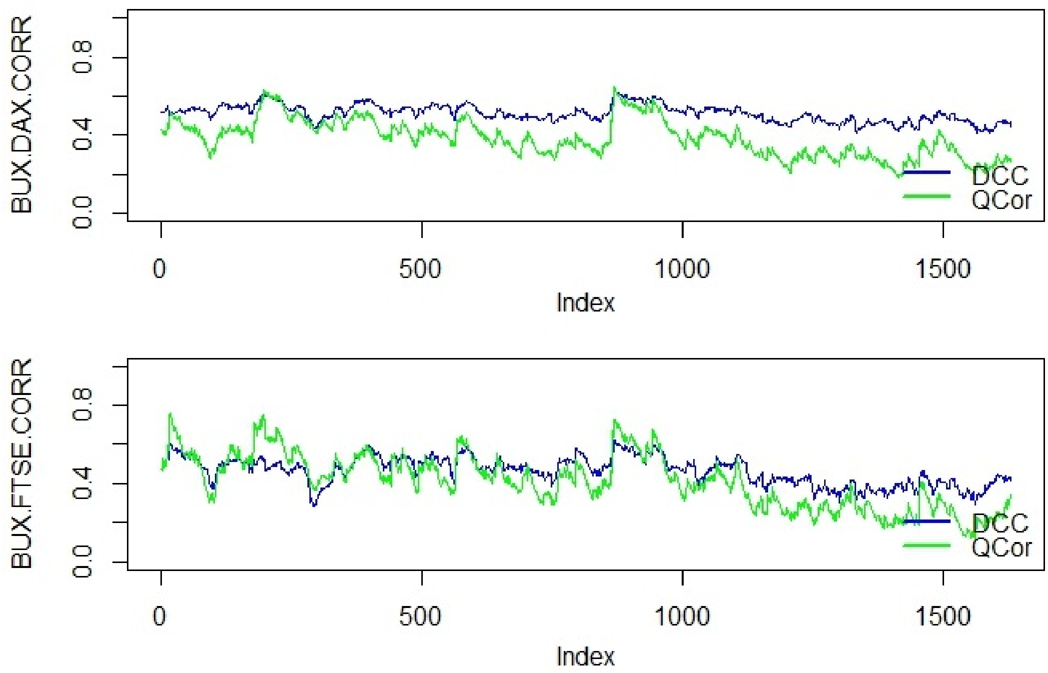
Figure A.5.: Results of linear quantile regression with squared root of realized semivariance



Note: DAX_rtsqRSpos is realized positive semivolatility and DAX_rtsqRSneg is realized negative semivolatility of DAX index, FTSE_rtsqRSpos is realized positive semivolatility and FTSE_rtsqRSneg is realized negative semivolatility of FTSE index.

Source: Author's computations

Figure A.6.: Correlation of the BUX with Western European indices



Source: Author's computations

Appendix B

Tables

Table B.1.: Absolute and relative performance of BUX index on one step ahead forecast with in-sample period of 1,200 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0167	0.0150	0.1293	0.0000	0.0000	0.1045
L ind	n/a	n/a	0.2780	n/a	n/a	0.3308
L cc	n/a	n/a	525.6238	n/a	n/a	146.0267
Tick	0.0525	0.0525	0.1342	0.0269	0.0270	0.0591
DM	6.4592		30.6375	1.9002		40.4325
5%						
UC	0.0500	0.0500	0.0784	0.0320	0.0320	0.0661
L ind	0.0000	0.0000	1.0067	0.4645	0.4645	0.4463
L cc	0.0001	0.0001	18.4943	4.1191	4.1191	2.7802
Tick	0.1776	0.1777	0.1805	0.0923	0.0922	0.0909
DM	-8.3852		28.2892	1.6246		29.5984
10%						
UC	0.0909	0.0909	0.0350	0.0704	0.0597	0.0235
L ind	0.0008	0.0008	0.1822	0.0555	0.3540	1.3285
L cc	1.1329	1.1329	73.3601	5.1086	10.1024	44.1961
Tick	0.2775	0.2778	0.3327	0.1558	0.1557	0.1953
DM	7.2559		-15.2318	1.0296		-23.5619

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.2.: Absolute and relative performance of DAX index on one step ahead forecast with in-sample period of 1,200 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0075	0.0083	0.1218	0.0099	0.0099	0.1065
L ind	n/a	n/a	0.0031	n/a	n/a	0.3162
L cc	n/a	n/a	477.5801	n/a	n/a	162.8786
Tick	0.0406	0.0401	0.1129	0.0276	0.0276	0.0735
DM	4.0591		39.7125	-8.4011		43.2381
5%						
UC	0.0550	0.0592	0.0851	0.0375	0.0394	0.0710
L ind	2.9166	1.5895	1.0918	1.7391	1.4526	0.8192
L cc	3.5398	3.6190	26.9819	3.5655	2.7300	5.0098
Tick	0.1528	0.1559	0.1606	0.1004	0.1001	0.1019
DM	-7.5125		4.4490	2.8415		30.3406
10%						
UC	0.1034	0.1059	0.0325	0.0809	0.0848	0.0355
L ind	0.3395	0.2049	0.0657	0.1554	0.0381	0.1875
L cc	0.4937	0.6641	80.1831	2.3471	1.4009	30.5969
Tick	0.2540	0.2546	0.3082	0.1574	0.1580	0.1890
DM	9.4499		-15.7617	-0.4087		-21.9370

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.3.: Absolute and relative performance of FTSE index on one step ahead forecast with in-sample period of 1,200 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0133	0.0150	0.1109	0.0121	0.0101	0.1073
L ind	1.5940	1.2176	1.4385	3.6882	n/a	1.0667
L cc	2.8201	3.8548	412.2663	3.9032	n/a	161.3626
Tick	0.0359	0.0359	0.0829	0.0193	0.0188	0.0435
DM	-1.6721		13.9054	5.1446		27.3715
5%						
UC	0.0484	0.0492	0.0676	0.0344	0.0344	0.0648
L ind	3.1133	2.8691	1.1931	0.2620	0.2620	1.6580
L cc	3.1808	2.8851	8.2366	3.0856	3.0856	3.7438
Tick	0.1242	0.1244	0.1265	0.0644	0.0642	0.0649
DM	-1.3387		2.9750	5.2698		34.9060
10%						
UC	0.1059	0.1068	0.0317	0.0749	0.0729	0.0243
L ind	1.7701	0.9561	0.4714	1.7419	2.0452	1.1340
L cc	2.2293	1.5523	83.0063	5.4961	6.4620	45.0344
Tick	0.2026	0.2050	0.2584	0.1032	0.1031	0.1278
DM	-1.0584		-12.4484	0.0055		-21.4441

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.4.: Relative performance of BUX index on horizon with in-sample size of 1,200 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0167	0.0150	0.1293	0.0000	0.0000	0.1045	0.0193	0.0268	0.1173	0.0151	0.0884	0.1013	0.0135	0.0252	0.1001	0.0240	0.1503	0.1111
tick	0.0525	0.0525	0.1342	0.0269	0.0270	0.0591	0.0928	0.0944	0.2434	0.0535	0.1371	0.1264	0.1074	0.1092	0.2853	0.0701	0.3040	0.1639
DM	6.4592		30.6375	1.9002		40.4325	-6.3153		5.3736	-4.5419		6.5939	-4.0132		1.9614	-2.9125		3.2219
5%																		
UC	0.0500	0.0500	0.0784	0.0320	0.0320	0.0661	0.0553	0.0871	0.0586	0.0690	0.1724	0.0603	0.0572	0.0715	0.0563	0.0763	0.2048	0.0545
tick	0.1776	0.1777	0.1805	0.0923	0.0922	0.0909	0.3434	0.3228	0.3455	0.2147	0.3405	0.2021	0.4315	0.3726	0.4444	0.2897	0.5762	0.2683
DM	-8.3852		28.2892	1.6246		29.5984	-6.2222		-5.2332	-0.8648		-1.7916	-4.0286		-3.5643	-1.2350		-1.9553
10%																		
UC	0.0909	0.0909	0.0350	0.0704	0.0597	0.0235	0.1080	0.1424	0.0201	0.1315	0.2091	0.0151	0.1051	0.1371	0.0076	0.1547	0.2527	0.0087
tick	0.2775	0.2778	0.3327	0.1558	0.1557	0.1953	0.5715	0.5179	0.7092	0.3475	0.4820	0.4391	0.7510	0.6209	0.9821	0.4904	0.7859	0.6083
DM	7.2559		-15.2318	1.0296		-23.5619	-5.1495		-10.1182	0.7165		-9.9136	-3.6692		-7.5503	-0.3535		-6.2033

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.5.: Relative performance of DAX index on horizon with in-sample size of 1,200 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0075	0.0083	0.1218	0.0099	0.0099	0.1065	0.0176	0.0318	0.0955	0.0737	0.1036	0.1454	0.0177	0.0269	0.0934	0.0865	0.1911	0.1127
tick	0.0406	0.0401	0.1129	0.0276	0.0276	0.0735	0.0816	0.0847	0.1878	0.0895	0.1567	0.2029	0.1174	0.0923	0.2986	0.1987	0.3910	0.2423
DM	4.0591		39.7125	-8.4011		43.2381	-4.9559		6.0047	-1.7275		5.8486	-4.1270		2.6428	-2.7305		1.7330
5%																		
UC	0.0550	0.0592	0.0851	0.0375	0.0394	0.0710	0.0611	0.0930	0.0503	0.1275	0.1753	0.1056	0.0664	0.0782	0.0463	0.1730	0.2797	0.0724
tick	0.1528	0.1559	0.1606	0.1004	0.1001	0.1019	0.3036	0.2841	0.3043	0.2699	0.3737	0.2475	0.4519	0.3342	0.4662	0.4266	0.6511	0.3408
DM	-7.5125		4.4490	2.8415		30.3406	-4.7899		-5.9913	-0.6030		-2.1628	-3.7262		-5.0147	0.8183		-1.3728
10%																		
UC	0.1034	0.1059	0.0325	0.0809	0.0848	0.0355	0.1064	0.1449	0.0101	0.1733	0.2351	0.0498	0.1068	0.1430	0.0135	0.1992	0.3219	0.0463
tick	0.2540	0.2546	0.3082	0.1574	0.1580	0.1890	0.5135	0.4742	0.6812	0.4175	0.5251	0.4121	0.7635	0.5614	0.9801	0.5710	0.8322	0.6093
DM	9.4499		-15.7617	-0.4087		-21.9370	-4.3697		-7.3313	0.8229		-8.9223	-3.6515		-5.8864	1.3858		-6.5522

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.6.: Relative performance of FTSE index on horizon with in-sample size of 1,200 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0133	0.0150	0.1109	0.0121	0.0101	0.1073	0.0151	0.0176	0.0888	0.0245	0.0634	0.1350	0.0160	0.0185	0.0833	0.0475	0.1632	0.1384
tick	0.0359	0.0359	0.0829	0.0193	0.0188	0.0435	0.0627	0.0613	0.1489	0.0585	0.0774	0.1175	0.0874	0.0704	0.2156	0.0999	0.2152	0.1704
DM	-1.6721		13.9054	5.1446		27.3715	-4.6634		3.0368	-4.8579		5.4808	-3.3539		1.6097	-3.2172		1.8949
5%																		
UC	0.0484	0.0492	0.0676	0.0344	0.0344	0.0648	0.0637	0.0779	0.0452	0.1207	0.1575	0.0736	0.0547	0.0723	0.0463	0.1033	0.2397	0.0764
tick	0.1242	0.1244	0.1265	0.0644	0.0642	0.0649	0.2525	0.2364	0.2532	0.1602	0.2060	0.1547	0.3539	0.2909	0.3583	0.2320	0.3966	0.2253
DM	-1.3387		2.9750	5.2698		34.9060	-1.6571		-1.4631	-0.9656		-3.2405	-1.8734		-1.2399	-0.5679		-2.1284
10%																		
UC	0.1059	0.1068	0.0317	0.0749	0.0729	0.0243	0.1080	0.1415	0.0168	0.1738	0.2290	0.0225	0.1077	0.1287	0.0126	0.1674	0.2955	0.0248
tick	0.2026	0.2050	0.2584	0.1032	0.1031	0.1278	0.4256	0.3912	0.5650	0.2539	0.3051	0.2975	0.5996	0.5041	0.8023	0.3648	0.5430	0.4274
DM	-1.0584		-12.4484	0.0055		-21.4441	-1.2850		-7.9772	-0.8437		-9.1094	-0.6798		-6.1457	0.7684		-6.4491

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.7.: Absolute and relative performance of BUX index on one step ahead forecast with in-sample period of 800 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0175	0.0150	0.1314	0.0035	0.0046	0.1139
L ind	n/a	n/a	0.5605	n/a	n/a	0.0558
L cc	n/a	n/a	359.8681	n/a	n/a	310.9898
Tick	0.0541	0.0546	0.1453	0.0370	0.0372	0.0835
DM	3.9290		26.4520	4.4226		28.6013
5%						
UC	0.0501	0.0526	0.0864	0.0380	0.0391	0.0621
L ind	0.0000	0.2858	0.7646	0.0597	0.0974	0.1318
L cc	0.0001	0.3947	19.2048	2.9346	2.4281	2.6436
Tick	0.1865	0.1874	0.1912	0.1233	0.1242	0.1221
DM	-7.7999		23.2581	3.9439		17.5952
10%						
UC	0.0901	0.0914	0.0350	0.0771	0.0771	0.0288
L ind	0.3970	0.3017	0.0003	0.3341	0.3341	0.1035
L cc	1.2916	0.9815	48.7392	5.7874	5.7874	66.3842
Tick	0.2958	0.2956	0.3491	0.1956	0.1957	0.2434
DM	5.1109		-12.0193	0.0136		-15.5623

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.8.: Absolute and relative performance of DAX index on one step ahead forecast with in-sample period of 800 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0050	0.0050	0.1189	0.0132	0.0132	0.1147
L ind	n/a	n/a	0.2014	n/a	n/a	0.1179
L cc	n/a	n/a	306.5043	n/a	n/a	328.0667
Tick	0.0382	0.0386	0.1139	0.0353	0.0351	0.0894
DM	-1.6217		37.0059	2.2411		31.8132
5%						
UC	0.0513	0.0526	0.0889	0.0474	0.0485	0.0728
L ind	0.7870	0.9096	2.5174	0.4396	0.3461	0.3223
L cc	0.8159	1.0185	23.3625	0.5699	0.3888	9.0535
Tick	0.1531	0.1532	0.1623	0.1220	0.1228	0.1259
DM	-0.8847		12.1723	3.8404		27.3254
10%						
UC	0.0989	0.1014	0.0350	0.1014	0.0992	0.0309
L ind	0.5588	0.8052	0.0003	0.8756	1.1913	0.0213
L cc	0.5701	0.8220	48.7392	0.8962	1.1973	64.2988
Tick	0.2549	0.2551	0.3091	0.1991	0.1994	0.2412
DM	-6.0521		-11.8191	-0.1439		-14.7896

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.9.: Absolute and relative performance of FTSE index on one step ahead forecast with in-sample period of 800 observations

VaR	in-sample			out-of-sample		
1%	LQR_RV	LQR_RS	VaR_Gauss	LQR_RV	LQR_RS	VaR_Gauss
UC	0.0163	0.0150	0.1101	0.0112	0.0089	0.1063
L ind	1.6280	1.9027	1.3091	2.7682	3.6421	2.6681
L cc	4.2955	3.6643	271.9193	2.8905	3.7456	288.2363
Tick	0.0399	0.0397	0.0907	0.0231	0.0231	0.0533
DM	-1.3777		11.5095	5.5826		15.4540
5%						
UC	0.0476	0.0501	0.0663	0.0369	0.0358	0.0649
L ind	2.2422	1.7540	0.6423	0.4572	0.5670	2.6154
L cc	2.3440	1.7540	4.7303	3.9889	4.7647	6.4392
Tick	0.1359	0.1360	0.1392	0.0800	0.0802	0.0809
DM	-4.5908		1.4312	4.8891		34.1237
10%						
UC	0.0976	0.1026	0.0338	0.0861	0.0872	0.0235
L ind	1.6597	0.3489	1.0548	1.8114	0.7793	2.8120
L cc	1.7103	0.4098	52.0675	3.8070	2.4598	84.4275
Tick	0.2212	0.2246	0.2834	0.1302	0.1315	0.1643
DM	0.3919		-10.3920	-0.5661		-13.5874

Absolute performance is evaluated by unconditional coverage (UC), likelihood of independence test (L ind) and likelihood of conditional coverage (L cc). Relative performance is evaluated by tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.10.: Relative performance of BUX index on horizon with in-sample size of 800 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0175	0.0150	0.1314	0.0035	0.0046	0.1139	0.0189	0.0290	0.1121	0.0220	0.0764	0.1146	0.0127	0.0253	0.0976	0.0303	0.1374	0.1315
tick	0.0541	0.0546	0.1453	0.0370	0.0372	0.0835	0.0986	0.1067	0.2674	0.0789	0.1347	0.1818	0.1046	0.1143	0.2925	0.1171	0.3323	0.2541
DM	3.9290		26.4520	4.4226		28.6013	-5.9084		6.1572	-5.9654		6.0441	-2.7270		1.7803	-3.2859		3.1104
5%																		
UC	0.0501	0.0526	0.0864	0.0380	0.0391	0.0621	0.0592	0.0856	0.0605	0.0718	0.1563	0.0625	0.0520	0.0646	0.0494	0.0838	0.2107	0.0698
tick	0.1865	0.1874	0.1912	0.1233	0.1242	0.1221	0.3665	0.3411	0.3716	0.2773	0.3718	0.2644	0.4330	0.3934	0.4588	0.3819	0.6584	0.3663
DM	-7.7999		23.2581	3.9439		17.5952	-4.8561		-4.1101	-3.0395		-2.5822	-3.0301		-2.7184	-2.3365		-1.9570
10%																		
UC	0.0901	0.0914	0.0350	0.0771	0.0771	0.0288	0.1045	0.1348	0.0202	0.1308	0.2095	0.0208	0.1128	0.1356	0.0051	0.1513	0.2584	0.0116
tick	0.2958	0.2956	0.3491	0.1956	0.1957	0.2434	0.6071	0.5445	0.7509	0.4376	0.5455	0.5384	0.7722	0.6588	1.0341	0.6359	0.9390	0.7561
DM	5.1109		-12.0193	0.0136		-15.5623	-4.1417		-8.3973	-1.4441		-8.2926	-2.9307		-6.1649	-1.1640		-6.1617

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.11.: Relative performance of DAX index on horizon with in-sample size of 800 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0050	0.0050	0.1189	0.0132	0.0132	0.1147	0.0176	0.0290	0.1058	0.0543	0.0965	0.1186	0.0190	0.0228	0.1052	0.0647	0.1538	0.1059
tick	0.0382	0.0386	0.1139	0.0353	0.0351	0.0894	0.0739	0.0766	0.1857	0.1234	0.2009	0.2208	0.1063	0.0840	0.3105	0.2425	0.4155	0.3300
DM	-1.6217		37.0059	2.2411		31.8132	-3.5398		3.0524	-3.5865		5.5127	-3.1309		1.4340	-3.0333		2.1547
5%																		
UC	0.0513	0.0526	0.0889	0.0474	0.0485	0.0728	0.0693	0.0869	0.0479	0.0909	0.1641	0.0787	0.0659	0.0773	0.0507	0.1182	0.2408	0.0658
tick	0.1531	0.1532	0.1623	0.1220	0.1228	0.1259	0.2948	0.2618	0.2961	0.3213	0.4405	0.3093	0.4421	0.3268	0.4634	0.5380	0.7525	0.4759
DM	-0.8847		12.1723	3.8404		27.3254	-3.6253		-4.6504	-3.0800		-2.7510	-2.9447		-3.9527	-2.1609		-1.9798
10%																		
UC	0.0989	0.1014	0.0350	0.1014	0.0992	0.0309	0.1071	0.1423	0.0076	0.1375	0.2339	0.0399	0.1077	0.1394	0.0127	0.1494	0.2809	0.0401
tick	0.2549	0.2551	0.3091	0.1991	0.1994	0.2412	0.5120	0.4544	0.6749	0.4924	0.6494	0.5676	0.7701	0.5498	0.9696	0.7504	0.9790	0.8582
DM	-6.0521		-11.8191	-0.1439		-14.7896	-3.4466		-5.4748	-1.2720		-6.7148	-3.0743		-4.5233	-1.4442		-4.3954

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations

Table B.12.: Relative performance of FTSE index on horizon with in-sample size of 800 observations

VaR	*h=1*						*h=5*						*h=10*					
	insample			out-of-sample			insample			out-of-sample			insample			out-of-sample		
1%	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss	LQR RV	LQR RS	VaR Gauss
UC	0.0163	0.0150	0.1101	0.0112	0.0089	0.1063	0.0139	0.0202	0.0970	0.0202	0.0596	0.1192	0.0165	0.0241	0.0925	0.0351	0.1222	0.1097
tick	0.0399	0.0397	0.0907	0.0231	0.0231	0.0533	0.0665	0.0659	0.1711	0.0661	0.0922	0.1195	0.0970	0.0796	0.2558	0.0988	0.2006	0.1725
DM	-1.3777		11.5095	5.5826		15.4540	-3.8621		2.8482	-5.5194		6.8547	-1.9025		1.3824	-3.8118		3.5304
5%																		
UC	0.0476	0.0501	0.0663	0.0369	0.0358	0.0649	0.0605	0.0869	0.0504	0.0832	0.1440	0.0664	0.0583	0.0837	0.0507	0.0724	0.2070	0.0690
tick	0.1359	0.1360	0.1392	0.0800	0.0802	0.0809	0.2760	0.2603	0.2776	0.1881	0.2425	0.1857	0.3960	0.3248	0.4032	0.2771	0.4224	0.2685
DM	-4.5908		1.4312	4.8891		34.1237	-1.0936		-0.7503	-2.9424		-4.7402	-1.0395		-0.6053	-3.0497		-3.8920
10%																		
UC	0.0976	0.1026	0.0338	0.0861	0.0872	0.0235	0.1058	0.1423	0.0139	0.1316	0.2182	0.0202	0.1115	0.1242	0.0127	0.1233	0.2455	0.0260
tick	0.2212	0.2246	0.2834	0.1302	0.1315	0.1643	0.4730	0.4320	0.6157	0.2979	0.3710	0.3845	0.6730	0.5651	0.8770	0.4274	0.6009	0.5460
DM	0.3919		-10.3920	-0.5661		-13.5874	-1.1144		-6.5397	-2.5679		-6.8482	-0.3168		-4.6589	-1.4912		-4.9911

Relative performance is evaluated by unconditional coverage (UC), tick loss function (Tick) and Diebold-Mariano test of accuracy (DM).

Source: Author's computations