Report on **Bachelor / Master Thesis**

Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague

Student:	**Bc. Barbora Máková**
Advisor:	**PhDr. Ladislav Krištoufek, Ph.D.**
Title of the thesis:	**Fractal Dimension and Efficient Market**

OVERALL ASSESSMENT

The master thesis by Barbora Máková represents an outstanding result of more than year work of author. Already at the first touch the extent of thesis indicates that the amount of endeavor required for its creation was very large. The work is devoted to highly actual topic – to the test of efficiency of capital markets. The topic is at the center of attention of not only dealers but also theoreticians because the results of somewhat older tools for this testing were not fully satisfactory. The new approach, presented in the thesis, is based on the idea of utilization of old, classical and by many generations of mathematicians employed properties of topological spaces and the idea of distance, frequently generated (sometimes indirectly) by the topology in question. Presumably it is not a coincidence that some returns to the roots of mathematics we witnessed in a couple of other branches of science. Let me recall the study of minimal distance estimators in robust statistics in seventies of last century or the research on the depth of data in the framework of not only statistics but of data processing as such.

The concept of dimension of (real) vector space was introduced in 1918 by Felix Hausdorff who left in mathematics a trace which can't be overlooked even by mathematicians who don't study topological or metric spaces. A sophisticated and imaginative generalization of this idea – fractal dimension and its application in the testing efficiency of capital markets is the main theme of the thesis. The reader is briefly introduced into the framework in which the study will be performed, the basic notions are shortly recalled and then the results of the main tool of the (applied) research – simulation studies – are offered to reader to make himself/herself the idea about ability of fractal dimension to reveal the (local) inefficiency of time series. The speed and capacities of memories of nowadays computational means gave researchers at hand a powerful tool for making the idea about the behavior of means for data processing in many branches of science. Of course, it does not mean that we should abandon the classical ways of study of these tools. Already in seventies Paul Halmos has warned on the danger of an exclusive reckoning on the results of Monte Carlo studies in his famous paper *Applied mathematics is a bad mathematics.* Nevertheless, the possibility to make ideas about the behavior of new estimators and tests on the finite sets, moreover intently damaged, i.e. intently deviating from the ideal assumptions, can't be ignored. The present thesis offers it in an ample extent.

The study took into account a wide scale of possibilities of behavior of time series under various circumstances. The simulations were carried out carefully and in a well-designed framework. The results are presented in an easy graspable form of graphs and tables. As the author concluded at the end of ``narrative'' part of thesis (but in fact at the middle of the volume) the hope that the fractal dimension can rather reliably and accurately reflect even local inefficiencies has been confirmed. One of the valuable contribution of thesis is a self-critical pegging of abilities of the method. All after, already in thirties of the last century Karl Popper wrote that the scientific method is such a method which clearly says when we cannot use it, i.e. not pretending that it is almighty.

The thesis could be a ground for writing doctoral thesis. On one hand some topics which were – for a sake of space – only briefly sketched – should be given in an exact form of definitions (in the present form the most of notions, starting with the Hausdorff dimension

Report on **Bachelor / Master Thesis**

Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague

Student:	**Bc. Barbora Máková**
Advisor:	**PhDr. Ladislav Krištoufek, Ph.D.**
Title of the thesis:	**Fractal Dimension and Efficient Market**

over the fractal dimension up to the ARFIMA process are really comprehensible only for the reader who knows them from some other source). There is always a hidden danger that something was overlooked if an exact, mathematical definitions of the notions are not given (remember the shock caused to Greek philosophy by Pythagoras' proof that square root of 2 can't be written as fraction of two integers). In other words, the doctoral thesis could complement the present results by theoretical background which Halmos' paper calls for. Moreover, doctoral thesis could discuss the problem from a bit larger distance, i.e. to start a discourse about the sense of some basic notions. E.g. the notion of efficiency which played (and still - from the pedagogical point of view, together with notions of sufficiency etc. - plays) important role was recognized to be impractical because even its founder, Ronald Aylmer Fisher knew that small deviation from the ideal conditions (of normality – even without leaving exponential family of densities) can destroy ``efficient'' behavior of classical statistics. Similarly the notion of correlation (which all after led to Box-Jenkins models of time series) is a notion which should be used with caution (let us recall that we can have two deterministically dependent random variables which are not correlated). Box-Jenkins models for time series are still accepted for time series without reliable verification that the series have really such character. Let us recall that already in nineties Graham Mizon (member of group of econometricians around David Hendry in Nuffield College) wrote a sophisticated paper with title *A simple message for autocorrelation correctors: Don't* (in paper he showed that although the simple tests – as D-W – indicated AR process, the data were generated in another way and it destroyed the results computed in a traditional way advised in all econometric textbooks).

At the end of my report I would like to stress once again that the thesis represents a nice piece of scientific work. Although it doesn't contain some surprising discoveries and sophisticated proofs of new mathematical theorems, it shows that the mapping of behavior of new tools for data processing by means of numerical simulations can bring inspiration for more theoretical treatment of the topic. Moreover, the scientific work is not based (at least not only based) on new inventions but it is also hard, diligent and frequently laborious work or even well-done craft. I personally believe that the work with the computer, i.e. collecting the information by means of well-designed and reliably generated random simulations (although it is still underestimated by theoreticians) is an important part of establishing the complex picture about newly proposed (data processing) methods. The theoretical and simulation part of research should be in some reasonable balance. Somebody is more attracted by theoretical side, somebody by ``experimental'' one.

It is out of any discussion that this diploma thesis is very good (if we reserve the word ``excellent'' for Nobel prize winners). I propose to appreciate it by the level A.

SUMMARY OF POINTS AWARDED *(for details, see below):*

CATEGORY		**POINTS**
Literature	*(max. 20 points)*	20
Methods	*(max. 30 points)*	30
Contribution	*(max. 30 points)*	30

Report on **Bachelor / Master Thesis**

Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague

Student:	**Bc. Barbora Máková**
Advisor:	**PhDr. Ladislav Krištoufek, Ph.D.**
Title of the thesis:	**Fractal Dimension and Efficient Market**

Manuscript Form	*(max. 20 points)*	18
TOTAL POINTS	*(max. 100 points)*	**98**
GRADE	**(1 – 2 – 3 – 4)**	**1**

NAME OF THE REFEREE: Jan Amos Visek

DATE OF EVALUATION: June 18, 2014

Referee Signature