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FACULTY OF SOCIAL SCIENCES

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Bachelor thesis

2014 **Tadeáš Kopecký**

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Inventory Control Problem with Random Demand

Bachelor thesis

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Author: **Tadeáš Kopecký**

Supervisor: **RNDr. Michal Červinka, Ph.D.**

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Abstrakt

Teorie skladu je důležitou součástí některých druhů podnikání. Slouží k efektivnímu snížení nákladů spojených s objednáváním a skladováním zboží. V této práci jsou popisovány algoritmy a modely, které jsou používány k určení optimálního pohybu zboží ve skladu. Také je prezentováno několik rozdílných metod určených k předpovědi poptávky. Tyto algoritmy a metody jsou aplikovány na reálná data. Cílem je ukázat způsob, jakým lze dosáhnout optimálního pohybu zboží ve skladu.

Klíčová slova

sklad, poptávka, předpověď, heuristiky, objednávka

Abstract

The inventory theory is important for certain kinds of business. Using the models from the inventory theory, we can effectively lower the costs associated with ordering and storing the goods. In this thesis, we describe the basic algorithms and models that are used to determine the optimal inventory policy. We also present several different methods used for the demand forecast. Finally, we use these algorithms and methods with the real data to show the possible way to propose the best inventory policy.

Keywords

inventory, demand, forecast, heuristics, order

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, July 21, 2014

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Signature

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I would like to express my deepest gratitude to my supervisor, RNDr. Michal Červinka, Ph.D. for helpfulness and guiding. I am also very grateful to my mother and her colleague who provided me the data for the case study. Last but not least I deeply thank to my family and friends. Without their support this thesis could have never been written.

Bachelor Thesis Proposal

In the thesis, the author will describe models suitable for modeling utilization of inventory capacity, material consumption (demand) and system of purchase orders. First, the author will summarize model with known demand, fixed or variable. Then, based on several time-series prediction techniques, the author will generalize the model to incorporate also stochastic demand. In the last part, the author will illustrate described solution techniques on an academic or a practical example. The theoretical part of the thesis will be based primarily on [1] and [2] and references therein.

Preliminary structure of the thesis:

1. Introduction
2. Inventory Control Problem with Constant Demand
3. Inventory Control Problem with Variable Demand
4. Methods of Demand Estimation and Prediction
    4.1. Moving Averages
    4.2. Exponential Smoothing
5. Numerical Example - Simulation
6. Conclusion

Literature:

[1] I. Morgenstern: Introduction Theory of Inventory Control, Faculty of Physics, University of Regensburg, 2007.

[2] F.S. Hillier, G.J. Lieberman: Introduction to Operations Research, 9th edition, Prentice Hall, 2010.

Contents

1 Introduction

The inventory theory is a very important part of almost every business that operates with the physical goods. For the wholesalers and the retailers, it is necessary to have enough goods ready for sale in order to avoid shortage and losses connected with it. The manufacturers need to have enough raw materials for production in order to avoid the penalties for the late deliveries. Many algorithms, formulas, and models have been developed to optimize these issues.

The year 1913, in which Wilson introduced the first formula for optimal quantity of the inventory, can be considered as a beginning of the inventory theory. However, the most important impulse for a new research came after the 2nd World War. In that time, the US Navy supported the research, as they wanted to optimize the number of spare parts carried on the ships. The producers and retailers starter to be interested in the inventory theory during the 1970's. Due to the increasing interest rates, the producers and retailers did not want to hold excessive inventory and they rather invested the spare capital. With the era of the modern computers, the process of solving the inventory control problems has become easier. Even small business can afford to have some software program that can help to optimize the inventory level.

The typical division of the models is into two large groups according to the nature of the demand. The deterministic models are based on a known (or a forecasted) demand. In the stochastic models, we know only the distribution of the demand. In the thesis we elaborate only on the deterministic models. We describe several different methods of the demand forecast, as it is important part of the inventory theory.

The goal of the thesis is to show that even with the simple methods, one can propose the effective inventory policy.

The thesis is structured as follows. First, we describe the basic deterministic model-the Wilson economic order quantity model for the static demand. In Section 3, we present the dynamic (the demand varies over the time) deterministic model developed by Wagner and Whitin. The forecasting methods are described in the Section 4. In the last section we apply the methods described in the thesis on the real data to show a possible way to optimize the inventory policy. The conclusion summarizes our findings.

2 Economic order quantity model

The family of deterministic models is very large, as over the 120 years many scientists have been trying to improve the algorithms and the models. All models are more or less based on the classical economic order quantity model developed by Harris (1915) and Wilson (1934) which will be described further in this chapter. The deterministic models operate with a known demand that can be either constant or it can vary over time periods. The downfall of deterministic models is their strong dependency on the quality of the demand forecast. Although there are many algorithms for effective demand forecasting, there will always be an error in the forecasts that can bias the results. In this chapter we will introduce classical economic order quantity model with constant demand. In the next chapter we will describe Wagner-Whitin model which allows the demand to vary. We will also present the heuristics that are used as an approximations for the Wagner-Whitin model.

2.1 Variables in our analysis

Before describing the models, we need to introduce all the variables we will use in the inventory optimization problem:

Customer demand (D) characterizes the number of units customers demand during certain time period.

Replenishment rate (P) characterizes the number of units that are filled into the storage room and are ready to be sold to the customers during certain time period.

Cycle time (T) characterizes the length of the time period between two deliveries.

Amount of units ordered (q_0) characterizes number of units that are delivered by one order. It is the control variable we want to optimize.

Order cost $(C(q_0))$ involves all costs that occur in the period between placing the initial order and the final delivery of the goods from the supplier. It consists of reorder cost $r(q_0)$ (administration work, transportation, quality check at the time of delivery) and price of the ordered goods that is proportional to the amount ordered. Therefore, we can therefore write the order cost such as:

$$C(q_0) = \begin{cases} r(q_0) + c(q_0)q_0 & q_0 > 0, \\ 0 & q_0 = 0, \end{cases}$$

where $c(q_0)$ is the cost of a single unit.

Storage cost (s) is the cost of storing one unit during certain time period. Storage costs are, e.g. depreciation of the goods, insurance payments, costs due to the damage, interest charges. It can be expressed as a proportion of the unit cost. Therefore we can write

$$s(q_0) = \alpha c(q_0),$$

where α is the inventory holding cost rate. The value of the constant α is usually lower than 1 (it can excess 1 in certain extreme cases, e.g., when the holding cost is larger that the unit cost).

Shortage costs (g) arise when the business is unable to meet the customer demand. Shortage costs are, e.g., general loss in sales, cost of additional fast delivery, or loss of goodwill.

Lead time (l) characterizes the time needed for order to be delivered from the supplier.

2.2 Economic order quantity model

The classical economic order quantity model is the simplest among all models in the inventory theory due to the many restrictions imposed on it. The classical EOQ model and all its variations optimize the quantity ordered and the length of the time cycle to minimize the total costs per unit of time. Ford W. Harris and R. H. Wilson independently developed the same model, thus the model is often called Wilson-Harris. We will introduce the classical EOQ model here. The simplifying assumptions of the classical EOQ model are following:

A.1 The customer demand is known, continuous and constant.

A.2 All costs are known and constant.

A.3 No shortage is allowed.

A.4 Goods are delivered instantly after placing the order (no lead time).

A.5 The replenishment of goods is instant (infinite replenishment rate).

A.6 We order single-unit only.

A.1 and **A.2** fix all variables at constant levels except for the quantity and the length of the time cycle. Further **A.1** guarantees that the rate at which the goods leave the inventory is constant and steady without any unexpected step-falls. **A.4** and **A.5** directly imply that the order is placed at the end of the cycle when the level of the inventory drops to zero, $q = 0$. At every moment t, $t \in [0, T]$, the inventory level q can be expressed as $q(t) = q_0 - Dt$ where q_0 is initial level of the inventory. At the end of the cycle we need to have $q(T) = 0$; therefore, we can express length of the cycle as $T = \frac{q_0}{D}$.

To minimize the total costs per unit of time, we first express the total cost per cycle as a sum of the order cost per cycle and the holding cost per cycle. The order cost per cycle with fixed reorder and unit costs is:

$$b(q_0) = r + cq_0. \tag{1}$$

The holding cost per cycle can be expressed as a sum of the holding costs at every moment t, $t \in [0, T]$. We can write:

$$h(q_0) = s\int_0^T [q_0 - Dt]\; dt = \frac{sq_0^2}{2D}. \tag{2}$$

Now we can express the total cost per cycle by adding **(1)** and **(2)**:

$$p(q_0) = r + cq_0 + \frac{sq_0^2}{2D}. \tag{3}$$

To obtain the total cost per unit of time, we divide the expression in **(3)** by $\frac{q_0}{D}$. We get:

$$f(q_0) = \frac{rD}{q_0} + cD + \frac{sq_0}{2}.$$

The function $f(q_0)$ is strictly convex since $\frac{\partial^2 f(q_0)}{\partial q_0^2} > 0$ for all $q_0 > 0$. Thus by taking derivative of $f(q_0)$ with respect to q_0 and setting it equal to zero, we obtain optimal ordered quantity that minimizes total costs per unit of time:

$$q_0^* = \sqrt{\frac{2rd}{s}}.$$

By plugging q_0^* into $T = \frac{q_0}{d}$ we obtain optimal length of the time cycle

$$T^* = \sqrt{\frac{2r}{sd}}.$$

We can now check the properties of the values q_0^* and T^*. With rising reorder cost, both optimal ordered quantity and optimal length of the cycle rise, as it is beneficial to make larger orders for longer time periods. With rising storage cost both optimal ordered quantity and optimal length of the cycle decrease, as it is desirable to make smaller orders for shorter time periods. With rising demand, the optimal ordered quantity rises and optimal length of the cycle decreases, as it is beneficial to make larger orders for shorter time periods.

We illustrate the use of the formulas for the optimal ordered quantity and optimal length of the cycle on the academic example.

Example 1

We have monthly demand for dental implants that is $D = 200$. The reorder cost associated with delivering the package of dental implants from the supplier by post service is approximatelly 350 CZK. The price of one dental implant is $c = 3500$ CZK. The shortage cost is 1.5% per year ($\alpha = 0.015$). Thus the monthly storage cost can be expressed as $s = \frac{0.015 \times 3500}{12}$.

The resulting optimal ordered quantity is $q_0^* = 179$ and the optimal length of the time cycle is $T^* = 0.89$. In the view of the EOQ model described above, to minimize total cost we need to place an order for 179 dental implants that covers demand approximately for 27 days.

2.3 Reorder level

The assumption of no lead time is very unrealistic, as always there is a necessity at least to transport the goods from the supplier. We assume that the lead time is constant. To meet the demand on time we have to make an order in advance to have it delivered at the time when $q = 0$. This ensures that the storage cost is minimized, as we do not hold more goods than it is necessary for current demand. The question is when to make an order. For this purpose, the most useful is to define reorder level of inventory. When the level of inventory drops to the reorder level we make an order. The lead time can be longer than one time cycle but every time it falls between two cycles. If the lead time falls between nth and $(n+1)$th cycle ($nT < L < (n+1)T$) we define the reorder level (R) as following:

$$R = dl - nq^* \tag{4}$$

Similarly to the EOQ model, we illustrate the use of the formula for the reorder level on the academic example.

Example 2

Let us now modify **Example 1** in such a way that we consider a lead time $l = 3$ days. The lead time is shorter than the optimal length of the time cycle. Thus $n = 0$ and reorder level is $R = 200\frac{3}{30} = 20$. Thus we need to place an order when inventory level falls to 20 units of dental implants.

The EOQ model has been expanded in many ways since it was firstly presented. We have models with allowed shortage, back-orders, or quantity discounts. One can find these models described, e.g., in Inventory Control and Management by Waters (2003).

3 Dynamic model

To this point we assumed that the demand is constant over time. In this section we describe the dynamic version of the deterministic inventory model with varying demand. It was firstly suggested by Wilson and Whitin (1934). The model has been improved in many ways since. We focus on the original model and heuristics that have been developed as an approximation for the rather complicated Wilson-Whitin (W-W) model.

3.1 Wagner-Whitin model

The W-W model considers demand to vary over the time. We assume that in each period t, $t = 1, ..., n$, the demand is denoted as D_t. We usually work with finite time horizon, as in the inventory control problems we rarely need to look more than one year ahead. For longer time horizons, we use statistical forecasting instead of inventory models. In the W-W model we want to find the optimal order quantity to minimize the total costs. If we order goods for the whole planning horizon through 1 to n, we could face high holding cost. On the contrary, if we make frequent orders, we could face high reorder costs. Thus we want to find optimal combinations of the time periods for which to make an order at once to have the reorder and the holding costs in balance. The assumptions of the W-W model are following:

A.1a The customer demand is known and strictly positive in every time period t, $t = 1, ..., n$.

A.2a The costs are known, strictly positive, and can vary in every time period t.

A.3a No shortage is allowed.

A.4a The goods are delivered instantly after placing the order (no lead time).

A.5a The replenishment of goods is instant (recall replenishment rate from Section 1.1).

A.6a The orders are for a single-unit only.

A.7a The time periods are discrete.

A.8a There is no capacity or investment constraint.

A.9a The beginning inventory equals to zero.

A.10a The storage costs are paid only for the inventory that is left at the end of the certain time period (if we order 200 units for two months ahead and sell 150 units during the first month, we will pay storage cost only for the 50 units).

Wagner and Whitin proposed an algorithm to solve the dynamic version of the classical EOQ model. The principle is to divide the problem into several sub-problems and solve it by the forward recursive method.

First, we solve the model only for $t = 1$. The beginning inventory equals to zero, so from **A.3a** and **A.9a** and from the fact that the demand D_1 is positive we always decide to place an order in $t = 1$. The question is whether to order for the first period only, or whether to order for more periods ahead. We add second time period and we solve two sub-problems-either we order in the first period for the first and the second period at once (we would not place an order in the second period); or we order in the first period for the first period only and consequently we have to place an order in the second period to cover D_2. Then we add the third period and we proceed in the same way up to the end of the planning horizon.This method was thought to be rather complicated, as we get 2^{n-1} possibilities to examine. It is the reason why many heuristics have been developed to ease the computation. However, with the modern computers and software packages the solving process becomes easier. Despite the fact that the W-W algorithm can be implemented easily using MS Excel, the heuristics are still widely used in practice. Axsater (2000) argues that it is because the heuristics are easier to understand and one can check the computation manually.

3.2 Heuristics

The W-W algorithm gives us the optimal solution to the dynamic inventory problem but it has been viewed by many researchers as too complex and difficult to solve for large n and many goods. Therefore, we can find several heuristics that are easier to solve but give us only an approximation of the right solution. The advantage of the heuristics is that they do not use the whole planning horizon which gives us less possibilities to examine. We consider initial order at $t = 1$ and then we examine whether it is beneficial to order for periods $t = 2, 3, .., n$, along with this order. If it is beneficial to order for k periods at once, we place an order and solve the problem again in the same way but taking $t = k+1$ as the initial time period. We repeat this algorithm till we get to the end of the planning horizon.

Lot-for-lot heuristic

The most straightforward and the simplest method is the lot-for-lot heuristic. The main idea is to order exactly the amount of goods that is needed in the next period. Thus $q_{0t} = D_t$ in every period t. Although this approach does not look very elaborate, it can be effectively used in the cases when the reorder costs are low. Using the lot-for-lot approach, we completely avoid holding costs which combined with possibly low reorder costs can result in the low total costs. On the other hand, if the reorder cost is high, this approach could lead to very ineffective results.

Part-period balancing heuristic

Part-period balancing heuristic was firstly proposed by Dematteis and Mendoza (1968). It balances the holding and reorder costs to find optimal number of time periods for which to place an order at once. We want to find the maximal number of periods with the holding cost is less or equal to the reorder cost for the period in which we place the order. We seek the highest τ, $\tau = t+1, ..., n$ such that

$$\sum_{m=t+1}^{\tau} \sum_{n=t}^{m-1} s_n D_m \leq r_t,$$

where t is the initial period in which we place an order. If we find such τ, we order for periods from t to τ. We set $\tau + 1$ as the current period and repeat the computation to the end of the planning horizon. If we do not find such τ, we order for one period only.

Silver-Meal heuristic

Silver and Meal (1973) suggested a heuristic that is close to the classical EOQ model, as it tries to minimize average cost per period. This heuristic is generally best known and by many researchers it is considered as the most accurate. Baker (1989) shows that in most situations the cost increase is only about 1-2%. The Silver-Meal heuristic tries to find when the average per period cost increases for the first time. We place an order at the time t and we want to find for how many periods it is optimal to place an order at once. We seek the highest τ, $\tau = t+1, ..., n$, such that

$$\frac{r_t + \sum_{m=t+1}^{\tau} \sum_{n=t}^{m-1} s_n D_m}{\tau - t + 1} < \frac{r_t + \sum_{m=t+1}^{\tau+1} \sum_{n=t}^{m-1} s_n D_m}{\tau - t + 2}. \quad (5)$$

Before we start to seek for τ, we need to check the following condition

$$\frac{r_t + s_t D_{t+1}}{2} < r_t \quad (6)$$

If condition (6) does not hold, it is optimal to order only for the first period t. If condition (6) holds and we find such τ for which (5) holds, we order for periods from t to τ. Then we set $\tau + 1$ as the current period and repeat the computation to the end of the planning horizon. If condition (6) holds and we do not find any τ for which (5) holds, we order for periods from t to $\tau = t + 1$ (for two periods at once). Then we set $\tau + 1$ as the current period and repeat the computation to the end of the planning horizon.

Least-unit-cost heuristic

The least-unit-cost heuristic is very similar to the Silver-Meal heuristic. It was firstly suggested by Groff (1979). In the least-unit-cost heuristic we want to minimize the average per unit costs. In other words we want to find when the average per unit cost increases for the first time. We place an order at the time t and we want to find for how many periods it is optimal to place an order at once. We seek the highest τ, $\tau = t+1, ..., n$ such that:

$$\frac{r_t + \sum_{m=t+1}^{\tau} \sum_{n=t}^{m-1} s_n D_m}{\sum_{m=t}^{\tau} D_m} < \frac{r_t + \sum_{m=t+1}^{\tau+1} \sum_{n=t}^{m-1} s_n D_m}{\sum_{m=t}^{\tau+1} D_m} \tag{7}$$

Before we start to seek for τ, we need to check the following condition

$$\frac{r_t + s_t D_{t+1}}{D_t + D_{t+1}} < \frac{r_t}{D_t} \tag{8}$$

If condition (8) does not hold, it is optimal to order only for the first period t. If condition (8) holds and we find such τ for which (7) holds, we order for periods from t to τ. Then we set $\tau + 1$ as the current period and repeat the computation to the end of the planning horizon. If condition (8) holds and we do not find any τ for which (7) holds, we order for periods from t to $\tau = t + 1$ (for two periods at once). Then we set $\tau + 1$ as the current period and repeat the computation to the end of the planning horizon.

In this section we described the basics of solving the dynamic inventory problems. The important variable in all methods is the demand. Thus in the next section we will focus on the different techniques of the demand forecasting.

4 Demand forecast

The demand forecast is very important part of the inventory control problem. There are two main reasons why we need an accurate demand forecast. Firstly, the assumption of no lead time that we used frequently in the previous models, will not often hold in the reality. Thus, it is desirable to have the information about the future demand, as we have to place the orders in advance. Secondly, if we have a demand that varies over time, it can be beneficial to order for more time periods at once. For this purpose, we again need the information about the future demand to be able to evaluate the costs associated with different combinations of ordering patterns.

The collection of the historical data is an important part of the forecasting. It is difficult to know the past demand precisely, so it is tempting to use the past sales instead. Past sales reflect the true demand only if there was no shortage in the past. Otherwise, the past sales underestimate the true demand and we should be careful about using them for the optimization.

The good demand forecast can help to lower the costs, as we can derive optimal ordering patterns and avoid the shortages. It is certainly one of the reasons why many techniques and approaches of the demand forecast have been developed. We can divide different approaches into the following four groups:

Judgemental approach-the judgemental approach is based on the subjective forecast of the future demand. This approach is usually implemented when there are no historical data. It can be a case while introducing a new product. The common procedure of the judgemental approach is the consensus method where a group of people who understand the market well, creates a general forecast through a discussion. This approach is not widely used in the inventory theory but can be used as a source of additional information for the forecasting by different and more precise techniques.

Experimental approach-the experimental approach is similar to the judgemental approach. It is used mainly when there is a new product and we need an estimate of its future demand. Common procedure of the experimental approach is a customer survey or a test marketing. In test marketing, the product is launched only in certain isolated area. Based on the results, the general forecast for the whole market is made. Again, in inventory theory, this approach is considered an addition to more precise forecasting techniques.

Causal approach-the causal approach tries to find a reason why certain product is purchased. If we find the factors that influence the sales (e.g., when we want to forecast demand for umbrellas, we need to collect data of rainfalls in certain area), we can develop the demand forecast. For this purpose we can use regression analysis. This approach can be very useful in understanding future demand but its use is very limited by the amount of the necessary data.

Time series approach-the time series approach uses mathematical methods to forecast the demand. The main advantage of this approach is that it can be conducted very easily with the computer. The downfall of the time series approach is an unexpected change in the demand pattern. The typical example can be a big advertisement campaign in next time period. The advertisement campaign can cause demand to be higher. Such a situation cannot be effectively forecasted by time series approach. Nevertheless, the time series forecast can be supported by the judgemental or the causal approach.

When conducting the demand forecast the important decision is how far in the future we want to look. The quality of forecast decreases with the longer horizon. Also different techniques are suitable only for a certain time horizon. The demand forecasts can be divided into three categories based on the length of their time horizon:

Short term forecast-looks from one week up to three months ahead. The technique mostly used is the time series approach.

Medium term forecast-looks from three months up to one year ahead. The technique mostly used is the causal and the time series approach.

Long term forecast-looks several years ahead. The technique mostly used is the judgemental approach.

In inventory theory we rarely look more than one year ahead therefore short and medium term forecasts are mostly used.

4.1 Time series forecasting

In this section we describe 6 basic forecasting methods. We start with the simple ones such as the naive forecast or the cumulative mean method. We also describe Brown's double exponential smoothing that can reflect a trend in the demand or the method of seasonal indices that can reflect a seasonality in the demand.

Naive forecast

The simplest method for the demand forecasting is the naive forecast. We can write the demand forecast in the period $t + 1$ as

$$D_{t+1}^{f} = D_t + \epsilon_t,$$

where ϵ_t is a random variable with zero mean and constant variance. Thus the demand forecast in period $t + 1$ is simply the actual demand in period t. The naive forecast works well when the demand follows a random walk process (no trend, no seasonality). In the real market, we usually can detect certain pattern in the demand for goods. Thus the naive forecast is not suitable for inventory control problems.

The cumulative mean

Another very simple method for the demand forecasting is the cumulative mean method. To effectively use it we need to have relatively stable demand over the time, preferably with no trend and no seasonality. The demand in period $t+1$ is defined by the independent random deviations from the historical average. We can write the demand forecast in period $t+1$ as

$$D_{t+1}^{f} = \frac{1}{n}\sum\nolimits_{t=1}^{n} D_t + \epsilon_t,$$

where ϵ_t is a random variable with zero mean and constant variance. This method works well for relatively stable demand and is not suitable for the data that contains trend or seasonality.

The moving average

The simple moving average method is similar to the cumulative mean with the difference that we do not use as many observations as possible for computing the average demand. We forecast the future demand by taking average of the demand for the last m periods.

$$D_{t+1}^{f} = \frac{1}{m}\sum\nolimits_{i=t-m+1}^{t} D_i + \epsilon_t.$$

The advantage of this method is that for every forecast we use only m latest demands. Thus the forecast can relatively quickly adapt to the changing patterns or trends. The problem is what m to choose. If we choose large m, the forecast will not respond quickly to the changing patterns but will cancel out the random variations in the demand. If we choose small m, the forecast will response quickly to the changing patterns but will not cancel out random variation in the demand. One possibility is choosing m with the smallest mean square error (for about the mean square error see Section 4.2). In general, we can say that a large m should be used in case of relatively stable demand and a small m when there is fluctuation or trend in the demand.

Moreover, it is reasonable to assume that the older the observation of demand is, the less influence it has to the future demand. The weighted moving average method works with the latest m observations in the same way as the moving average method but considers newer data as more significant for the forecast. We can write

$$D_{t+1}^f = \sum_{i=t-m+1}^{t} w_{t-i-1} D_i + \epsilon_t,$$

where $w_i \in \mathbb{N}$, $i = 1, .., m$ are the weights and $\sum_{i=1}^{m} w_i = 1$. Similarly to the simple moving average, we have to choose the optimal number of periods for the forecast. Additionally, we need to choose the right value of the parameters w_i. Again it can be done by the minimization of the mean square error.

Simple exponential smoothing

The method of moving averages demands relatively large amount of data to be stored to perform the forecast. To avoid this problem, Brown (1956) suggested the method of the simple exponential smoothing. It was further improved by Holt (1957). The method of exponential smoothing works only with one parameter α and to conduct the forecast we need less data than in the case of moving average. The forecast consists of weighted linear combination of the actual demand in the last period and of the forecast in the last period. Therefore, we can drop all the older data because they are already contained in the last period forecast. We can write

$$D^f_{t+1} = \alpha D_t + (1 - \alpha) D^f_t + \epsilon_t,$$

where $\alpha \in (0, 1)$ is a parameter we have to choose. In practice, we usually choose the value of α between 0.1 and 0.5. However, it is prefered to test different values of α on historical data and choose the one which provides the lowest mean square error. The value of α influences the nature of the forecast. With a large value of α, the forecast will be sensitive to the changing pattern of the demand but also will be very sensitive to random variations in demand. With a low value of α, the forecast will slowly adapt to the changing patterns but will not respond to the random variations.

The exponential smoothing is not suitable for long term forecast. We can write the forecasted demand for the period $t+h$ such as $D^f_{t+h} = D^f_{t+1}$ for $h > 1$. The forecast will be the same for all the periods in the future. That could deliver poor results in case that there is a seasonality or a trend in the data.

Brown's double exponential smoothing

Brown (1963) suggested the method that is similar to the simple exponential smoothing but can work better with the trended data. The method is still simple as we need only one parameter α. We can write the demand forecast in period $t+1$ as

$$D^f_{t+1} = 2a_t - b_t + \frac{\alpha}{1-\alpha}(a_t - b_t) + \epsilon_t,$$

where $a_t = \alpha D_t + (1-\alpha)a_{t-1}$, $b_t = \alpha a_t + (1-\alpha)b_{t-1}$ and $a_1 = b_1 = D_1$. Similarly to the simple exponential smoothing, we only have to decide what value of α to choose. If we want a forecast for more than one period ahead (for period $t+h$), the formula is as follows:

$$D^f_{t+h} = a_t + hb_t,$$

for $h > 1$.

Method of seasonal indices

If we have trended data, we can effectively use the Brown's double exponential smoothing. For the data with seasonality we can use the method of seasonal indices. For each month (it can be also done for other time period such as quarter of the year) we construct the seasonal index which will be used for the forecast. The seasonal index for the month i in the year 1 looks as follows:

$$s_{1i} = \frac{D_i}{\frac{\sum_{i=1}^{12} D_i}{12}}, i = 1, ..., 12.$$

It is desired to have the data for more than one year back. Then we can get different values of the seasonal index for the ith month $s_{1i}, s_{2i}, ..., s_{ni}$. With the average of the seasonal indices for the ith month, we can write the forecast as:

$$D^f_{t+i} = \bar{s}_i \bar{D} + \epsilon_t$$

4.2 Safety stock

The forecasted data will never be 100% same as the reality. It is the reason why the businesses usually have the extra stock referred as the safety stock (or the buffer stock). With the safety stock we can lower the risk of the shortage. The formula for the level of the safety stock is following:

$$Z = f\sigma,$$

where f is a service factor and σ is the standard deviation of the demand. To obtain the service factor, we need to determine the service level. It expresses the probability that a certain level of safety stock will not lead to stock-out. We need to convert the service level to the service factor. To do so we use the inverse normal distribution. E.g., the service level of 0.95 means that we want to avoid the shortage in 95% of the time. In the statistical tables we can find the z-value for the service level which turns to be the service factor (1.645 in our case).

4.3 Accuracy measures

The quality of the forecast can be quantified by the so-called accuracy measures. The accuracy measures can be used as a criterion for choosing the best forecasting method or for choosing the right coefficients of the particular methods. The accuracy measures can also be used to quantify uncertainty of the forecast. The accuracy measures are based on the forecasting error that is defined as $e_t = D_t - \widehat{D_t^f}$. Makridakis and Hibon (1995) mention 14 different accuracy measures. They recommend the mean square error (MSE) and symmetric mean absolute percentage error (sMAPE) as the most useful ones. These two will be described here and used later in the last part of the thesis devoted to a case study.

The MSE is defined as follows:

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2.$$

The MSE gives more weight to the large errors due to the squaring. Thus it is widely used in inventory theory where large error is less desirable than two or more smaller errors. The formula for the MSE also provides information about the uncertainty of the forecast, as it is similar to the statistical measure of the variance.

The sMAPE is defined as follows:

$$\text{sMAPE} = \frac{100}{n} \sum_{t=1}^{n} \frac{|e_t|}{\frac{(d_t + d_t^f)}{2}}.$$

The sMAPE measures average relative size of the absolute forecasting error as a percentage of the corresponding actual data. The sMAPE is a relative measure. It means that can use it for comparison across forecasting horizon and different series.

The rule of thumb for sMAPE is that we have a very good forecast if sMAPE $< 10\%$, we have a reasonable forecast with sMAPE $< 30\%$, and we have an inaccurate forecast with sMAPE $> 50\%$.

5 Application of the models to the real life problem

In the last section of the thesis, we show how the previously described methods can be used in practice. We collected the monthly data for the number of dental implants introduced in a dental center in the Czech Republic for the past three years (2011-2013). During those three years, the average waiting time to get the implants was at maximum one month and no customer wanted to switch the dental center due to the waiting time. Thus it is possible to consider the number of dental implants introduced in a month as a monthly demand for dental implants. We also collected the values of the variables, such as the unit cost, the holding cost, and the reorder cost, which are necessary for the inventory optimization problem.

We show the possible procedure for determining the optimal forecasting method and the way of choosing the best ordering policy according to the forecasted data. First, we examine the accuracy of the forecasting methods on the historical data. We will choose the best methods according to the sMAPE. With these methods we will make forecasts for 12 month ahead. Then we will use the heuristics and W-W algorithm to decide the best ordering policy. Also, we will compare the accuracy of these methods. We will go through the same procedure with the actual data that corresponds to the 12 months for which we make the forecast. We will calculate the total costs of the inventory policies and compare them. Our goal is to show that even with the basic methods described in the thesis, the forecasted data will not significantly differ from the reality.

Demand forecasting

The monthly demand for dental implants is depicted in the Figure 1. We can detect a possible seasonality from the pattern of the demand. Every year during the July and August, there is a significant drop in the demand. On the other hand, we can rule out any trend, as all values fluctuate around the mean 205 dental implants per month. From this first insight we can assume that the best forecasting method could be the forecast with seasonal indices to cover the seasonality.

We divide our data into three groups. The data from 2011 represents the base data, as some forecasting methods require a larger amount of the past data (mostly simple or weighted average). Based on the data from 2011 and 2012 we will conduct a forecast for each month in 2012. This way we can determine the accuracy of the different methods the optimal coefficients where needed (e.g. exponential smoothing, weighted moving average).

To determine the optimal number of periods used in the moving average method, we compare the MSE of the forecast for 2012 using different number of periods. The minimum number of pe-

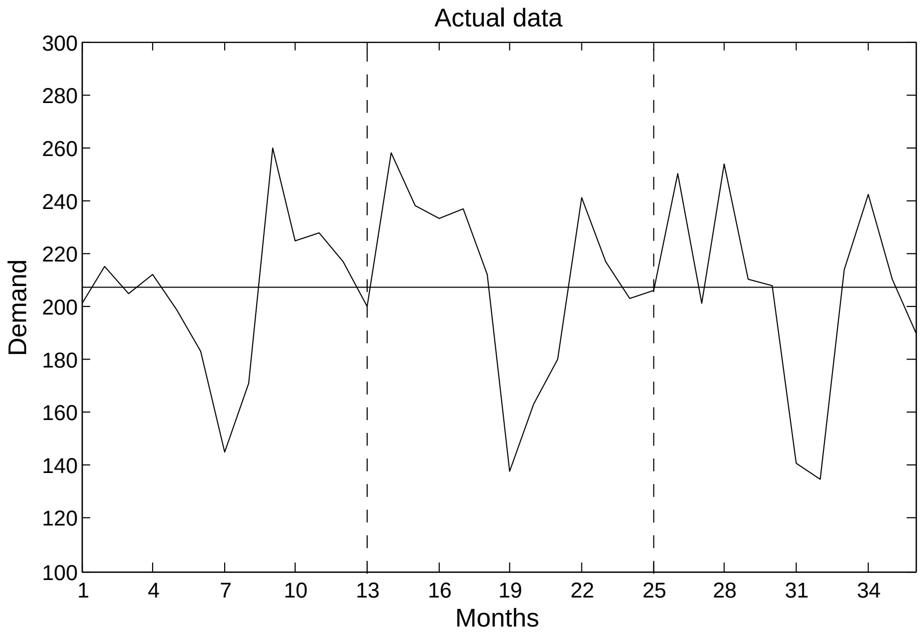

Figure 1: Actual data

riods is two. If we used only one period, we would have gotten the naive forecast. The maximum number of periods is twelve, as we are limited by the amount of historical data. The resulting MSEs that correspond with the different numbers of time periods are depicted in the Appendix A in the Table 4. The lowest MSE is for 12 periods. It provides us with rather smoothed forecast that does not react to the changing pattern much. It is understandable as our data does not contain any trend. Thus there is no need for a quick adaptation to the new values of the demand.

Similarly to the moving average method, we need to determine the optimal number of periods used for weighted moving average. We also need to determine the optimal weights. We can do this by using, e.g., the Solver in the MS Excel. All weights had to satisfy $a \geq b \geq c \geq ... \geq l$, where a is the weight for the most recent period, b is the weight for the second most recent period, etc. We again compared the MSEs of the forecasts using different number of time periods combined with optimal weights. The resulting MSEs that correspond to the different number of time periods and different weights are depicted in the Appendix A in the Table 5. The lowest MSE is for 12 periods and for the weights $\frac{1}{26}(15, 1, ..., 1)$. Again, it gives us rather smoothed forecast which support our findings about fluctuations around the mean and no trend.

The last coefficients we need to determine are the values of α in the exponential smoothing and Brown's double exponential smoothing method. The forecast from these methods depends on the last observation and on the last forecast. Therefore it is important to decide what value to take as the last forecast when conducting the first forecast. There is no strict rule for choosing the starting point. We tried to use averages from 12, 9, 6 and 3 last periods and the demand from the last period (in this case the last observation and the last forecast is the same). We used the Solver to find the optimal values of α and we compared the MSEs. The resulting MSEs that correspond with different starting periods are depicted in the Appendix A in the Table 6 and 7. For exponential smoothing in the view of the lowest MSE, it is the best to use the average of last 12 months as the starting point and $\alpha = 0.01$. For Brown's double exponential smoothing, it is the best to use the average of last 9 months as the starting point and $\alpha = 0.01$. These values of α cause that the forecast does not respond to recent changes in the demand almost at all and we get the smoothed forecast again.

In the theoretical part, we explained that the value of alpha can be chosen also based on the past experience. Thus we tried also different values of alpha that do not smooth the forecast so much (we tried the value of alpha 0.1, 0.3, 0.5 and 0.7). But as we need to make the forecast for the whole year 2013 ahead, we got very similar results for each value of alpha (the difference was at maximum 2 implants per month in the year 2013). Doing this for the Brown's double exponential smoothing would yield the similar results. Moreover the Brown's double exponential smoothing is not suitable for our data, as we have no trend.

The Figures 2-8 in Appendix A show the difference between the actual and the forecasted data for each method for the year 2012. Also, the graphs show the forecast for the year 2013 for each method (the dashed line represents the actual data, the full line represents the forecasted data).

The sMAPE for each method is depicted in the Table 1. For each method it is between 11 and 15 percent. That means that all our forecasts are reasonable. The Table 1 shows the lowest sMAPE in case of the weighted moving average method and the method of seasonal indices. These two forecasts are not smoothed

Table 1: sMAPE

Forecasting method	**sMAPE (%)**
Naive forecast	14.29
Cumulative mean	13.76
Moving average	14.46
Weighted moving average	11.15
Exponential smoothing	13.98
Brown's smoothing	13.72
Seasonality	11.45

as much as the other forecasts. This fact can be a cause of the better sMAPE. The smoothed forecasts (exponential smoothing, Brown's double exponential smoothing, cumulative mean and moving average) feature very similar sMAPE around 14%. In the next part we will use the forecasted data from the weighted moving average method and from the method of seasonal indices as they feature the lowest sMAPE.

Optimal ordering pattern and optimal costs

In this section we determine the optimal ordering pattern based on the previously forecasted data. We work with the assumption that we have to place an order at the beginning of the year 2013 and we cannot change the order during the year. It implies that the forecast cannot be updated with new data obtained during the 2013. The forecast is based only on the data from 2011 and 2012. We will apply the W-W algorithm and the heuristics to the forecasted data presented in the previous section. We will find the optimal ordering policy for this forecasted data to determine the total variable costs. We will also determine the optimal ordering policy for the actual data from 2013 to be able to compare how costly it is to use the forecasted data.

To use the W-W algorithm and the heuristics properly we need to know the reorder and storage cost of the dental implants. The costs were estimated by consulting with several dentist. They are based mainly on their experience. The reorder cost is estimated to be 650CZK. It includes the shipment fees, the packaging, the insurance, and the administration work. The storage cost is estimated to be 1.5CZK per month (18CZK per year). The cost of the single dental implant is aproximatelly 3000CZK thus the storage cost is only 0.6% of the unit cost. Such low storage cost is not very common in the inventory theory. The storage cost in our case prevents from ordering many implants at once. Moreover, the dental implants do not depreciate over time. They are very small, so we do not need any special storage room which also lower the storage cost. The storage cost will be calculated from the inventory remaining at the end of each month. As mentioned above, we assume that the order for 2013 is placed at the beginning of the year. The delivery is instant and is made at the beginning of the month (therefore, we have maximum of 12 deliveries).

To determine the total variable cost we sum up all the reorder costs and all the storage costs that occur during the 12 months. The forecasted data over/underestimates the actual demand in certain months. In some month we can have more implants stored than it is necessary or on the other hand we can face a shortage. To correct a possible overestimation, the ending inventory in each month will be calculated as a difference between the total inventory at the beginning of the month t and actual demand in the month t. With this we obtain the actual storage costs rather than the estimated storage costs. To avoid shortage we need to establish a safety stock. We want as little shortage as possible, thus the service level will be set to 99.9%. Corresponding service factor is 3.09. Together with standard deviation of 32, the safety stock is 99 dental implants. At the beginning of the 2013 we need to order additional 99 dental implants to cover for possible shortage. Each month, we will pay the storage cost of what is left from the safety stock. The variable costs for the actual data will be calculated without the safety stock, as it is not needed.

The complete results that capture the total variable costs of certain ordering pattern for each forecasted series and for each heuristics can be found in Appendix A in the Tables 8-10. The ordering pattern is described by series of 0s and 1s where 1 stands for ordering in a certain period and 0 stands for not ordering in the period. In the Table 2 one can find the lowest variable costs for each forecasted series together with the ordering pattern.

With the actual data we got different ordering pattern with

Table 2: Optimal ordering pattern and variable costs I

Forecasting method	Ordering policy	Var. cost (CZK)
Actual demand	(1,1,0,1,0,1,0,0,1,0,1,0)	5781
Weighted moving average	(1,0,1,0,1,0,1,0,1,0,1,0)	6379.5
Seasonality	(1,0,1,0,1,0,1,0,1,0,1,0)	6622.5

each heuristic. The lot-for-lot heuristic proved to be worst fit (7800CZK). It would be more suitable if the storage costs were high and reorder cost was low. The other three heuristics-Silver-Meal, part-period-balancing and least-unit-cost-are much closer to the optimum derived by W-W algorithm. The variable cost derived by the least-unit-cost heuristic, which is the worst one from the three heuristic mentioned earlier, is only by 6% more expensive than policy derived by W-W algorithm. We can see that the usage of heuristics does not dramatically higher the expenditures.

For the forecasted data we got the same ordering pattern in all cases, except for the Silver-Meal heuristic applied on the data forecasted by the seasonal indices. The reason of the same ordering pattern could lie in the nature of the forecasted data. Although the data forecasted by seasonal indices and by weighted moving average is not smoothed as much as the data from other methods, still they do not vary as much as the actual data. In each month, the forecasted demand is very similar to the other months. We, therefore, get the same repeating ordering pattern where we order for two months every time.

If we knew the demand in advance precisely, the total variable cost would be 5781CZK. The variable costs of the forecasted data are only 10-15% higher. The higher costs are caused mainly by the safety stock, which is not needed in the case of known demand. Surprisingly the costs are lower for the data computed by the weighted moving average method. It is only the result of the safety stock. Using the data from the weighted moving average method we almost deplete the safety stock after 5 month. Thus the storage cost for the safety stock is low. For the data from the seasonal indices method we never deplete the safety stock only to the 60 dental implants at maximum. Thus we have to pay high storage cost for the safety stock. It would be more than enough to have the safety stock of 50 units (approximately one half of the original safety stock). The total variable costs are then only 6181.5CZK. This reasoning is possible for us to do only because we have the actual data for 2013. Without them we would probably keep the larger safety stock in order to avoid the shortage. Consequently we could lower the safety stock in the next years based on the past experience.

So far we assumed that we have to place the order for a whole year ahead. Now we can look what will happen if we are abble to place updated orders throughout the year. More specifically, before each delivery we will be able to re-forecast the demand with the additional data from the 2013 and place the new order.

We will do this for the two forecasting methods with the lowest sMAPE and only for W-W algorithm. Additionally we will try to calculate the variable costs also with the data forecasted by the simple exponential smoothing using different values of alpha (0.01, 0.1, 0.3, 0.5, and 0.7). The forecast for the year 2013 using the simple exponential smoothing would not be a constant anymore as we can use the new data obtained during the year 2013. We also want to examine whether it is better to use the value of alpha found above by comparing the MSE or it is better to use alpha based, e.g., on the past experience. The results can be found in Table 3.

Table 3: Optimal ordering pattern and variable costs II

Forecasting method	**Ordering policy**	**Var. cost (CZK)**
Actual demand	(1,1,0,1,0,1,0,0,1,0,1,0)	5781
Weighted moving average	(1,0,1,0,1,0,1,0,1,0,1,0)	6907.5
Seasonality	(1,0,1,0,1,0,1,0,1,0,1,0)	6622.5
Exp. smoothing ($\alpha = 0.01$)	(1,0,1,0,1,0,1,0,1,0,1,0)	6516
Exp. smoothing ($\alpha = 0.1$)	(1,0,1,0,1,0,1,0,1,0,1,0)	6591
Exp. smoothing ($\alpha = 0.3$)	(1,0,1,0,1,0,1,0,1,0,1,0)	6717
Exp. smoothing ($\alpha = 0.5$)	(1,0,1,0,1,0,1,0,1,0,1,0)	6999
Exp. smoothing ($\alpha = 0.7$)	(1,0,1,0,1,0,1,0,1,0,1,0)	7164

The safety stock is again implemented (99 units of dental implants delivered in the first period). The cost for seasonal indices method is not changed, as we would need new data for a whole year at once to update it. Again, the cost is relatively high because of the safety stock. For the exponential smoothing the lowest cost is for the value of alpha 0.01 which was found by comparing the MSE. For the value of alpha 0.5 and 0.7 there is no need for safety stock of 99 units. Similarly to the method of seasonal indices the safety stock of 50 units would be sufficient enough. However the costs would be still larger than the cost for the value of alpha 0.01 (6558CZK for the value of alpha 0.5 and 6723CZK for the value of alpha 0.7). The cost for the weighted moving average method is significantly higher than the previous cost calculated above. But again it would be sufficient enough to use the safety stock of 50 units. The cost is then 6465.5CZK. It is still higher than the previous one but only for less than 100CZK.

It is surprising that the possibility of updating the forecast do not lower the costs for the weighted moving average. One possible explanation would be that the forecasted data are lagged behind the actual data. E.g., the low demand in July and August is projected also to September when the actual demand is back on the value somewhere around the average.

In the last part of the case study we examine how the change in the reorder and storage cost influence the ordering pattern. From the analysis above we can conclude that the forecast that is closest to the reality is produced by the method of seasonal indices. Thus in this part we will compare the ordering pattern of the actual data to the ordering pattern forecasted by the method of the seasonal indices. First, we changed the reorder cost keeping the storage cost fixed. The results can be found in Table 11. We changed also the storage cost keeping the reorder cost fixed. The results can be found in Table 12.

The results are very similar both for the actual and the forecasted data. If the reorder cost is four times lower (162.5CZK) than the original one, it is optimal to place the order every month both for actual and forecasted data. The same ordering pattern is for the storage cost that is six times higher (6CZK) than the original one. The other extreme case, ordering only in the first period for the whole year, occurs if we have the reorder cost 17 times higher (11050CZK) than the original one for the actual data and 18 times higher (11700CZK) for the forecasted data. The same ordering pattern is for storage cost that is 17 times lower (0.088CZK) for the actual data and 18 times lower (0.083CZK) for the forecasted data.

Briefly, we describe the way of managing the inventory on the dental clinic from which we have the data. They use the software that alerts them whenever the number of the dental implants fall under certain level. Then they make the order. The orders are made at least once a month. Moreover, they do not keep any safety stock. Thus, few times a year, they face the shortage. To avoid the possible delays caused by the shortage, the implants are quickly delivered. The cost of the express delivery is approximately the same as for the standard delivery. Only a few implants is ordered with the express delivery to avoid the shortage before the standard order is delivered (the standart order has longer delivery time). Thus this procedure doubles the reorder cost. In the best scenario the variable cost is 7800 CZK (no shortage during the whole year and twelve orders per year). The cost is still higher than the cost computed above. By using the W-W algorithm or the heuristics they would be able to lower the costs by more than 1000CZK. Another possible solution to lower potential high variable cost could be at least to implement the safety stock. They would not need to worry about the shortage and express deliveries.

In this part of the thesis we showed that even with the simple tools such as MS Excel and with the basic methods, one can propose the inventory policy that is close to the reality and could be applied in small businesses without any need to buy expensive inventory control software.

6 Summary

The inventory theory should be without any doubt an important part of the business. We can lower the costs associated with holding and ordering the goods using the methods described in the thesis. The business should pay attention to the right inventory policy. The spare capital can be used, e.g., to invest to the advertisement campaign.

In the thesis, we present the algorithms to determine the best ordering policy. The heuristics work well and their results are similar to the results derived by the W-W algorithm. Nevertheless it seems better to use the W-W algorithm. It is easily implemented while using the Excel and it provides the most accurate result.

We also described several different methods of demand forecast. Each method is suitable for different nature of the demand data. Therefore, it is desirable to try the methods on the historical data to find the best fit. The past experience with the forecasting can be important, as we can support the time series forecast with the additional information.

In the case study we suggest the ordering policy that could lower the cost for the dental clinic we have the data from. At least we recommend to implement the safety stock to avoid the unnecessary express deliveries.

In general, we show the simple method of solving the inventory problem that can be easily implemented by the small businesses.

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Appendix A

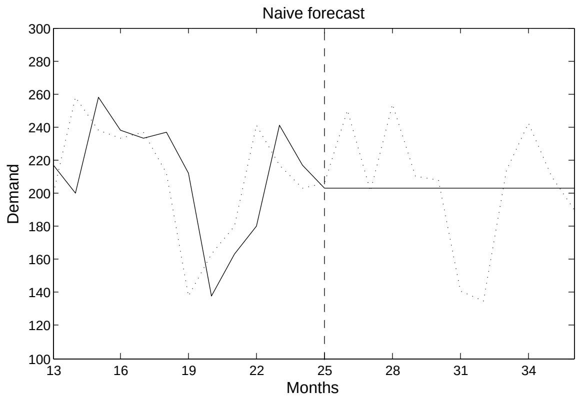

Figure 2: Naive forecast

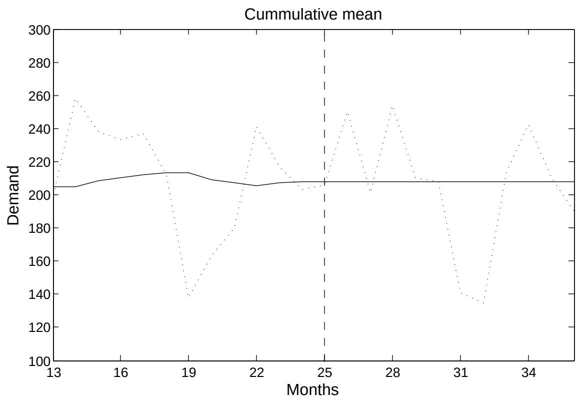

Figure 3: Cummulative mean

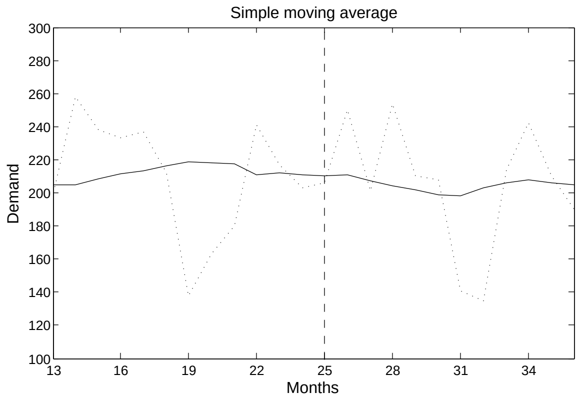

Figure 4: Simple moving average

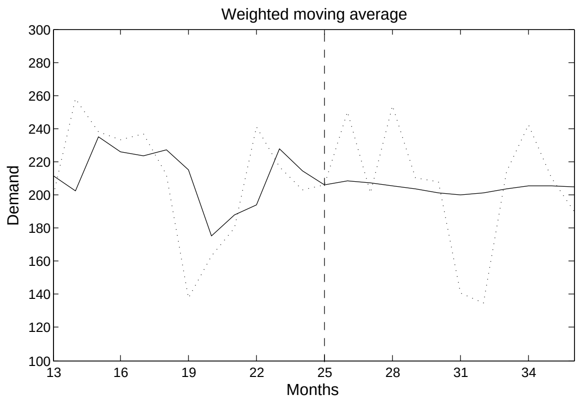

Figure 5: Weighted moving average

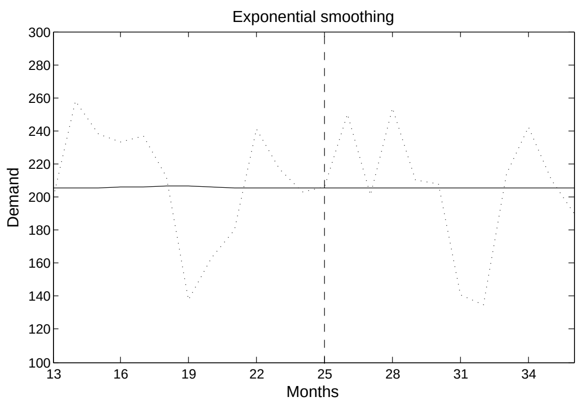

Figure 6: Simple exponential smoothing

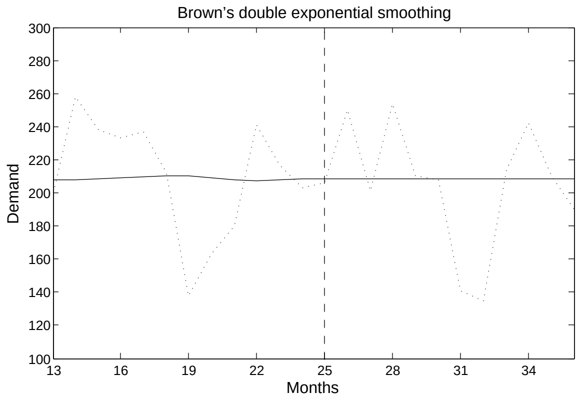

Figure 7: Brown's double exponential smoothing

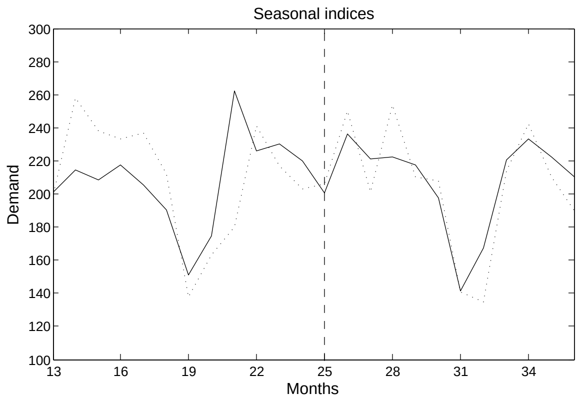

Figure 8: Forecast with seasonal indices

Table 4: MSE for different number of periods

Number of periods	**MSE**
2	1487
3	1617
4	1665
5	1538
6	1432
7	1432
8	1426
9	1492
10	1568
11	1516
12	1392

Table 5: MSE for different weights and different number of periods

Number of periods	**Optimal weights**	**MSE**
2	$\frac{1}{9}(8,1)$	1285
3	$\frac{1}{12}(10,1,1)$	1260
4	$\frac{1}{13}(10,1,1,1)$	1215
5	$\frac{1}{13}(9,1,1,1,1)$	1139
6	$\frac{1}{14}(9,1,...,1)$	1097
7	$\frac{1}{16}(10,1,...,1)$	1097
8	$\frac{1}{18}(11,1,...,1)$	1082
9	$\frac{1}{22}(14,1,...,1)$	1103
10	$\frac{1}{26}(17,1,...,1)$	1114
11	$\frac{1}{26}(16,1,...,1)$	1074
12	$\frac{1}{26}(15,1,...,1)$	1024

Table 6: MSE for different starting point - exp. smoothing

First value	Optimal alpha	MSE
Actual last period value	0.01	1205
3-month average	0.77	1272
6-month average	0.79	1269
9-month average	0.79	1269
12-month average	0.01	1182

Table 7: MSE for different starting point - Brown's double exp. sm.

First value	Optimal alpha	MSE
Actual last period value	0.05	1284
3-month average	0.01	1177
6-month average	0.01	1196
9-month average	0.01	1291
12-month average	0.01	1211

Table 8: Optimal cost and ordering pattern for actual data

Type of heuristic	Ordering pattern	Variable cost (CZK)
W-W algorithm	(1,1,0,1,0,1,0,0,1,0,1,0)	5781
Silver-Meal	(1,0,1,0,1,0,0,1,0,1,0,1)	6027
Least unit cost	(1,0,1,0,1,0,1,0,0,1,0,1)	6127.5
Part period balancing	(1,0,1,0,1,0,1,0,1,0,1,0)	5818.5
Lot for lot	(1,1,1,1,1,1,1,1,1,1,1,1)	7800

Table 9: Optimal cost and ordering pattern for weighted moving average

Type of heuristic	Ordering pattern	Variable cost (CZK)
W-W algorithm	(1,0,1,0,1,0,1,0,1,0,1,0)	6379.5
Silver-Meal	(1,0,1,0,1,0,1,0,1,0,1,0)	6379.5
Least unit cost	(1,0,1,0,1,0,1,0,1,0,1,0)	6379.5
Part period balancing	(1,0,1,0,1,0,1,0,1,0,1,0)	6379.5
Lot for lot	(1,1,1,1,1,1,1,1,1,1,1,1)	9007.5

Table 10: Optimal cost and ordering pattern for method of seasonal indices

Type of heuristic	Ordering pattern	Variable cost (CZK)
W-W algorithm	(1,0,1,0,1,0,1,0,1,0,1,0)	6622.5
Silver-Meal	(1,0,1,0,1,0,0,1,0,1,0,1)	6768
Least unit cost	(1,0,1,0,1,0,1,0,1,0,1,0)	6622.5
Part period balancing	(1,0,1,0,1,0,1,0,1,0,1,0)	6622.5
Lot for lot	(1,1,1,1,1,1,1,1,1,1,1,1)	9429

Table 11: Optimal ordering pattern for different reorder cost

Reorder cost	Actual data	Seasonal indices
162.5	(1,1,1,1,1,1,1,1,1,1,1,1)	(1,1,1,1,1,1,1,1,1,1,1,1)
217	(1,1,1,1,1,1,1,0,1,1,1,1)	(1,1,1,1,1,1,1,0,1,1,1,1)
325	(1,1,0,1,1,0,1,0,1,1,1,0)	(1,1,1,1,1,1,0,1,1,1,1,0)
1300	(1,0,0,1,0,0,1,0,0,1,0,0)	(1,0,0,1,0,0,1,0,0,1,0,0)
2600	(1,0,0,1,0,0,0,0,1,0,0,0)	(1,0,0,1,0,0,0,0,1,0,0,0)
4550	(1,0,0,0,0,0,0,1,0,0,0,0)	(1,0,0,0,0,0,0,1,0,0,0,0)
11050	(1,0,0,0,0,0,0,0,0,0,0,0)	(1,0,0,0,0,0,0,1,0,0,0,0)

Table 12: Optimal ordering pattern for different storage cost

Storage cost	**Actual data**	**Seasonal indices**
$\frac{1.5}{17}$	(1,0,0,0,0,0,0,0,0,0,0,0)	(1,0,0,0,0,0,0,1,0,0,0,0)
$\frac{1.5}{17}$	(1,0,0,0,0,0,0,1,0,0,0,0)	(1,0,0,0,0,0,0,1,0,0,0,0)
0.375	(1,0,0,1,0,0,0,0,1,0,0,0)	(1,0,0,1,0,0,0,0,1,0,0,0)
0.75	(1,0,0,1,0,0,1,0,0,1,0,0)	(1,0,0,1,0,0,1,0,0,1,0,0)
3	(1,1,0,1,1,0,1,0,1,1,1,0)	(1,1,1,1,1,1,0,1,1,1,1,0)
4.5	(1,1,1,1,1,1,1,0,1,1,1,1)	(1,1,1,1,1,1,1,0,1,1,1,1)
6	(1,1,1,1,1,1,1,1,1,1,1,1)	(1,1,1,1,1,1,1,0,1,1,1,1)