Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Time-scale analysis of sovereign bonds market co-movement in the EU

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, July 27, 2014

Signature

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Abstract

The thesis analyses co-movement of daily 10Y sovereign bond yields of 11 EU members (Greece, Spain, Portugal, Italy, France, Germany, Netherlands, Great Britain, Belgium, Sweden and Denmark) divided into the three groups (the Core of the Eurozone, the Periphery of the Eurozone, the states outside the Eurozone). In the center of attention are changes of co-movement in the crisis period, especially near the two significant dates - the fall of Lehman Brothers (15.9.2008) and the day, when increase of Greek public deficit was announced (20.10.2009). Main contribution of the thesis is usage of alternative methodology - wavelet transformation. It allows to research how co-movement changes across scales (frequencies) and through time. Wavelet coherence is used as well as wavelet bivariate and multiple correlation. The thesis brings three main findings: (1) co-movement significantly decreased in the crisis period, but the results differ in the groups, (2) co-movement significantly differs across scales, but its heterogeneity decreased in the crisis period, (3) near to the examined dates sharp and significant decrease of wavelet correlation was observable across lower scales in some states.

JEL Classification	C32, C49, C58, H63
Keywords	Co-movement, Wavelet Transformation, Sovereign Debt Crisis, Sovereign Bond Yields, Eurozone

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Abstrakt

Práce analyzuje vzájemnou závislost mezi denními výnosy desetiletých vládních dluhopisů jedenácti zemí EU (Řecka, Španělska, Portugalska, Itálie, Francie, Německa, Nizozemska, Velké Británie, Belgie, Švédska a Dánska) rozdělených do tří skupin (Jádro Eurozóny, Periférie Eurozóny, země mimo Eurozónu). V centru pozornosti jsou změny vzájemné závislosti v období krize, zejména blízko dvou významných událostí - pádu Lehman Brothers a veřejného ohlášení zvýšení řeckého deficitu. Hlavní přínost práce tkví ve využití alternativní techniky - waveletové transformace. Tato metoda dovoluje zkoumat, jak se liší vzájemná závislost výnosů na jednotlivých škálách (frekvencích). Ke zkoumání je použita waveletová koherence a korelace. Práce přináší tři hlavní zjištění: (1) vzájemná závislost se signifikantně snížila v období krize, avšak jsou zde vidět rozdíly mezi Jádrem a Periférií, (2) vzájemnou závislost se signifikantně liší napříč škálami, ale heterogenita výsledků je v období krize menší,(3) u obou zmíněných událostí bylo detekováno signifikatní snížení waveletové korelace napříč nižšími škálami.

Klasifikace JEL	C32, C49, C58, H63
Klíčová slova	Vzájemná Závislost, Waveletová Transformace, Dluhová Krize, Výnosy z Vládních Dluhopisů, Eurozóna

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Acronyms

ADF Augmented Dickey-Fuller (test)

ARIMA Autoregressive Integrated Moving Average

CWT Continuous Wavelet Transformation

DCC Dynamic Conditional Correlation

DWT Discrete Wavelet Transformation

EMU Economic and Monetary Union (or European Monetary Union)

ESM European Stabilisation Mechanism

EU European Union

GARCH General Autoregressive Conditional Heteroscedasticity

GDP Gross Domestic Product

GIIPS Greece, Italy, Ireland, Portugal, Spain

MBS Mortgage-Backed Securities

MODWT Maximal Overlap Discrete Wavelet Transformation

MRA Multi Resolution Analysis

OLS Ordinary Least Squares

SGP Stability and Growth Pact

STFT Short Time Fourier Transformation

WPS Wavelet Power Spectrum

QMF Quadrature Mirror Filter

XWT Cross Wavelet Transformation

Master Thesis Proposal

Author	Bc. Filip Šmolík
Supervisor	Mgr. Lukáš Vácha, PhD.
Proposed topic	Time-scale analysis of sovereign bonds market co-movement in the EU

Topic characteristics Sovereign debt, its service and development is actual topic in overindebted Europe. This thesis will be aimed on relationship between sovereign bond yields (which are in fact cost of the debt) - how yield paid by one state influences yield of another one. In the centre of my attention is especially change of these relationships after crisis, in other words change of comovement between sovereign yields of the EU states. Literature in this field is very rich as well as amount of using techniques and followng results. There are studies, which observe significant change of comovement after the crisis. Gilmore et al. (2010) as well as Dias (2012) used minimal spanning tree analysis to show that there is strong increase in intensity of comovement in Eurozone members contrary to decrease in the states outside. Bicu and Candelon (2012) used serial correlation common feature method and showed that change of comovement because of crisis is not significant. Caporin et al, (2013) differ even more: using quantile regression they showed that comovement on sovereign market yields significantly declined after crisis. Bhanot et al. (2011) realized that correlation between GIIPS states decreased. Results of Cipollini et al. (2013) are the same. From previous text is can be seen heterogeneity of the methods. My thesis will enrich the discussion using different methodology, which combine time and frequency domain approach – wavelet analysis. Using wavelets I will be able to compare how the results change through different frequencies, thus I will be able to discriminate between long and short run. Moreover, we will use more recent dataset and try to compare bonds with different maturities.

Hypotheses

1. After the crisis sovereign yield in EU states became more volatile.
2. Comovement between yields of particular states significantly changed after the crisis.
3. Comovement is stronger if both states are Eurozone members.
4. Comovement is stronger in case of sovereign bonds with shorter maturity.
5. Results of hypotheses 1-4 are different across the frequencies.
6. There is significant contagion, whose origin is in Greece.

Methodology For the thesis I want to use wavelet methodology. This approach is unique, because it combines time-domain information (which is typical for majority of time series analysis techniques) and frequency-domain information known from spectral analysis. Wavelet coherence allows us to discover how relationship between two time series is evolving through time end frequencies. Moreover, direction of causality between time series can be identified. Another great advantage is that it can be applied on non-stationary time series. Robustness of the results can be checked by employing various types of wavelets (Mexican hat, Morlet,...) and techniques (the discrete wavelet transformation (DWT), the continuous wavelet transformation (CWT), the maximal overlap DWT). Presence of contagion can be detected by rolling wavelet correlation, used for example in Rua and Nunes (2009). I will use either MATLAB or R-Project software for analysis and computation, because both of them have large amount of toolkits from which I can choose. Summary of previous text is following: 1) Hypothesis 1 will be analysed by wavelet power spectrum. 2) Hypotheses 2 – 5 will be analysed by wavelet coherence map and cross-correlation. 3) Hypothesis 6 I will analyse with help of rolling wavelet correlation.

Outline

1. Introduction
2. Sovereign debt and sovereign yields
3. Methodology description – wavelets
4. Empirical analysis
5. Conclusion
6. Appendix

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Author Supervisor

Chapter 1

Introduction

Since its beginning the Eurozone was created to strengthen the financial and macroeconomic integration between their members. Part of the integration was harmonization of sovereign debt markets (Pagano & von Thadden 2004). A result was a high degree of co-movement[1] between yields (rate of return obtained by investor holding the bond until maturity) of sovereign bonds (Missio 2013). When Lehman Brothers – one of the biggest investment bank all around the world – collapsed and went bankrupt, it meant a new chapter in the book of financial markets. Subsequent financial and banking crisis spread through the whole world and theories of risk management were shaken. Another shock came later. Greece – fiscally undisciplined Eurozone member state – admitted that budget deficit will be 12.5 % of GDP - much higher than it was expected.[2] Later it was revealed that the government accounting was falsified and the true volume of public debt was systematically lowered in order to achieve Euro.[3] Progressively more countries became dangerously close to bankruptcy – Spain, Portugal, Italy, Ireland or more recently Cyprus. Financial crisis shattered trust of lenders toward corporations as well as banks and Greek revelation acted in a similar way toward states of European Union (EU). Suddenly borrowing via government bonds became more expensive, because textbook assumption that sovereign bonds of developed states are riskless seemed to be obsolete. Important question arose: how previously mentioned events affected unification of co-movement of yields on government bond's mar-

[1] LSE financial dictionary defines it in the following way: *The tendency of two variables, e.g. the returns from two investments, to move in parallel.*

[2] http://www.businessweek.com/news/2012-09-05/greek-crisis-timeline-from-maastricht-treaty- to-ecb-bond-buying

[3] http://www.bloomberg.com/news/2011-05-26/greece-cheated-to-join-euro-sanctions-since-were-too-soft-issing-says.html

ket inside the EU?

There are many ways how to measure co-movement – from simple ones (e.g. sample correlation) to more advanced techniques, which are able to capture changes of co-movement through time (e.g. DCC GARCH or dynamic copula to name a few). All of them have one thing in common – they focus only on time-domain aspect of data and usually ignore frequency-domain aspect, hence the results could not be complete. Macroeconomic theory often distinguishes between long and short run. Hence it is desirable to make something similar even in an econometric study related to the topic.

The main goal of the thesis is to enrich the discussion by employing alternative methodology – wavelet analysis, which is suitable for previously mentioned task. This unique method allows us to decompose time series into different frequencies, which can be explored separately and then the results will be compared. Hence using wavelets we are able to discriminate between short run and long run co-movement. Moreover, we will employ wavelets for detection of contagion[4] in the EU. We will apply previously mentioned method on 10Y[5] sovereign bond yields of 11 member states of the EU - Greece, Spain, Portugal, Italy, Germany, France, Netherlands, Great Britain, Belgium, Denmark and Sweden[6] Those time series cover period from 1.1.2001 to 31.12.2013 with daily sampling.

The thesis is structured as follows. Chapter 2 describes debt crisis in the EU, sovereign bonds market, determinants of sovereign yields and provides literature review. Chapter 3 describes wavelet methodology – theory behind it and its application in economics. Chapter 4 provides basic analysis of data and motivates choice of wavelet methodology. Chapter 5 is dedicated to coherence analysis. Chapter 6 employs wavelet multiple correlation. Chapter 7 uses wavelet correlation for detection of a contagion. In Chapter 8 we examine how heterogeneity of wavelet correlation across scales changes in time. Finally, Chapter 9 concludes and describes the opportunities for another research in this area. The thesis contains Appendix, where mathematical prerequisities and additional results can be found.

[4]Contagion will be described in Chapter 2.
[5]It denotes a bond with maturity equal to 10 years.
[6]In-depth motivation of this choice will be described in Chapter 4.

Chapter 2

Background and Literature Review

The aim of this chapter is to describe topics related to the research questions stated in Chapter 1. The chapter itself is organized as follows. Firstly we shortly describe global financial crisis, subsequent sovereign debt crisis in the EU, the determinants of sovereing yields and their spreads[1] as well as the sovereign bonds market in the EU. Then we provide literature review related to co-movement of sovereign yields before and after the crisis has begun and clearly formulate our hypotheses.

2.1 Sovereign Debt Crisis in the EU

In the previous decade sovereign debt crisis hit the EU. This type of crisis as well as subsequent defaults of European countries were common in history. According to Reinhart & Rogoff (2008) from year 1300 to 1800 no state in Europe did successfully manage its debt and all of them defaulted at least once (France – 8 defaults, Spain – 6 defaults). Similar situation was in years 1800 - 1980 (Reinhart & Rogoff 2010). But there is a big difference between past and present debt crises. The majority of borrowed money was used for financing of war (Dincecco 2009), hence past debt crises occurred during and after the wars. Now the situation in Europe is quite different. There was no conflict since World War II in the states of the EU,[2] but government debt is the highest in their history (Reinhart & Rogoff 2008), which is one of the factors that lead to

[1]A spread is a difference between a yields of a particular bond and a yield of a benchmark bond.

[2]Except Croatia, which became member of the EU recently.

current debt crisis.

Its predecessor was global financial and banking crisis.[3] Many essays were written about the topic, hence in the thesis only the most basic facts needed for further empirical analysis will be stated. Mishkin (2010) recognizes two main phases. First one was the crisis on the market with sub-prime mortgages in the USA. The problems began to erupt in February 2007, when Federal Home Loan Mortgage Corporation ended to buy the most risky mortages and mortgage-backed securities (MBS).[4] In June 2007 Bear Stearns, influential brokerage firm and investment bank, announced that it suspends redemption[5] in its hedge funds. In August 2007 BNP Paribas made it too. In March 2008 the already mentioned Bear Stearns collapsed. According to Mishkin (2010) investors on financial markets were nervous, but they did not panic, because in their opinion MBS market represented only a small share of the total financial market. And then the second phase came. On 15.9.2008 Lehman Brothers – the fourth biggest investment bank all over the world – went bankrupt. The main reason was that it suffered high losses, which stemmed from holding of toxic assets – including already mentioned MBS. US government let Lehman Brothers fall and instead of helping it provided bail-out to another investment bank in danger – Merill Lynch. The fall of Lehman Brothers was a trigger of a global financial crisis, which spread across the world via financial linkages.

Sovereign debt crisis can be considered as another of its stages as well as global financial crisis was its trigger (Lane 2012). The author distinguishes between three phases of the debt crisis in the EU. First one is pre-2007 phase of accumulating debt and loosening fiscal discipline despite attempts such as the Stability and Growth Pact (SGP). In this phase governments in the developed EU states created huge budget deficits. Modern economic theory was always interested in the topic. There are many opinions why budget deficits occur in developed states. One possible reason was proposed by Buchanan & Wagner (1977) and it is little bit pessimistic. According to them debt is a result of fiscal illusion created by political parties in order to maximize probability of victory in the next election. Politicians count that people do not fully understand laws of fiscal policy and tend to support public spending. Persson & Svensson (1989) as

[3]In their study Reinhart & Rogoff (2010) demonstrated that it is a common pattern in history.

[4]http://timeline.stlouisfed.org/index.cfm?p=timeline

[5]It means that an institution prevents investors to withdraw their money from a fund.

well as Alesina & Tabellini (1990) proposed models with multiple parties. The parties differ in sovereign debt preference – its size (Persson & Svensson 1989) or timing (Alesina & Tabellini 1990). Debt is used as an instrument,which purpose is to influence fiscal policy of a successor. The main results of the works are twofold:

1. higher antagonism of preferences of parties implies higher deficit
2. result of stronger disagreement of citizens with expenditure policy leads to higher deficit too.

Academical works proposed by e.g. Alesina & Drazen (1989) or Roubini & Sachs (1989) examined the impact of coalition rule on the size of s budget deficit. They agree that coalitions increase it. Roubini & Sachs (1989) showed that weaker governments tend to have higher deficits and coalitions are weaker. Balassone & Giordano (2001) showed the importance of similar ideologies within the coalition for prevention of excessive deficit. The alternative explanation has Velasco (1999), who showed that if there is a dispersion of power over fiscal policy in the sense that policy dealing with revenues is centralized and the policy dealing with expenses is decentralized, then there is a higher tendency to borrow and spend.

In the second phase (2008 – 2009) the already mentioned global financial crisis began to rage in the USA and spread to the whole world. Uncertainty increased and borrowing became more expensive. The third phase began in the end of 2009. The majority of the EU governments lowered expected tax revenues, hence deficits became larger (Lane 2012). Moreover, on 20th October 2009 Greek minister of finance Giorgos Papakonstantinou publicly stated that the expected budget deficit will be 12.5 % of GDP, more than two times higher than it was previously announced. The first downgrade of rating followed - on 22th October Fitch lowered Greek rating from A to A- and it was only the first of many downgrades. Situation became critical when it was revelead that the government debt is higher than it was stated. The situation of Greek public finances went worse. In April 2010 Greece was forced to apply for international financial aid from International Monetary Fund (IMF), European Central Bank (ECB) and the EU itself.[6] The help was conditioned by strict austerity measures and structural reforms. Another bail-out came in February 2012 and the

[6]http://www.businessweek.com/news/2012-09-05/greek-crisis-timeline-from-maastricht-treaty- to-ecb-bond-buying

restructuring of Greek debt was negotiated.

Moreover, it was revealed that more EU states have problems with sovereign debt service. Even before the crisis the economy of Portugal limped beyond other members of the Eurozone (Reis 2013). When the financial crisis stroke, all structural weaknesses were revealed, especially large public sector, low investment to education and low productivity of labor. Moreover, two of the most important banks - Banco Privado Portuguęs (BPP) and Banco Portuguęs de Negócios (BPN) – started to write-off their toxic assets. Their situation became unsustainable and thus Portuguese government provided bail-out to both of them. After Papakonstantinou's speech the distrust of investors toward government bonds increased and Portugal was another victim. Since the beginning of 2010 all rating agencies began to downgrade sovereign bonds of Portugal and it became impossible to borrow on international financial markets. Hence in 2011 Portugal was forced to receive financial aid from the IMF and the EU.[7]

Spain was a different case. Its economy grew before the crisis and the state had budget surpluses, although there were severe structural weaknesses too (Neal & Garcia-Iglesias 2012). An especially dangerous weakness represented real estate bubble, which crashed in the end of 2008.[8] The crash had hard impact on Spanish banking sector and several banks (e.g. Bankia, NCG Banco) had to be saved in 2012.[9] Moreover, Spain fell into recession – GDP growth ended and unemployment increased, especially unemployment of youth. Hence it is not surprising that public debt of Spain sharply jumped – from 40.2 % of GDP [10] in 2008 to 93.9 % in 2013. Moreover, similarly to Greece and Portugal the rating of Spanish bonds was gradually downgraded.[11] All events caused that Spain needed financial aid from the outside in 2012.

Italy was one of the founders of European Community (EC) but its economy was not considered to be as strong as economy of France or Germany (Denk 2013). The state had problems with public debt, which was extremely high (e.g. 114.9 % in 1998). Similarly to previously mentioned states financial

[7] http://www.businessweek.com/news/2012-09-05/greek-crisis-timeline-from-maastricht-treaty-to-ecb-bond-buying

[8] http://www.economist.com/node/12725415

[9] http://www.bbc.com/news/business-20523753

[10] All statistics used in following text related to debt are taken from *http://countryeconomy.com.*

[11] http://en.citizendium.org/wiki/Eurozone_crisis/Timelines

crisis showed necessity of reforms. The crisis lowered state's revenues and government debt increased to 127 % of GDP in 2012. Rating agencies considered its bonds to be risky and downgraded them – firstly on 13th January 2012 by S&P, tightly followed by other rating agencies.[12]

The previously mentioned states are often labeled as GIIPS or PIIGS[13] and they are given as an example of fiscal irresponsibility. On the other hand states outside GIIPS had problems too (Chang 2011). In September 2008 states of Benelux nationalized important Belgian bank Fortis. In the same month a bail-out for another Belgian bank – Dexia – was announced. Problems in Belgian banking sector did not vanish and in October 2008 another help was provided for bank KBC. Those actions burdened public finance and debt-to-GDP ratio increased from 84 % in 2007 to 99.6 % in 2012. Other states began to suffer too. Even such important state as France was heavily criticized in 2011 and 2012 for lack of austerity measures and its bonds were downgraded.[14]

To secure financial stability in the EU new fiscal tool was established on 8th October 2012 - European Stabilization Mechanism (ESM). It was created instead of two other ones – European Financial Stability Mechanism and European Financial Stability Facility (EFSF).[15] Its main goal was to secure bail-outs for members in distress.

2.2 Sovereign Bonds and Their Market in the EU

In previous text we have written that there are the states in the EU, which have problems with repayment of debt. Now we focus on an important tool of debt service - sovereign bonds. In the center of our attention are especially sovereign yields, which represent cost of borrowing that has to be paid by the states. They are affected by long run and short run determinants.[16] According

[12]http://www.standardandpoors.com/ratings/articles/en/us/?articleType=HTML&assetID=1245327302187

[13]Ireland was hit by the crisis too, but it is not part of our dataset, thus we will not describe it in the thesis.

[14]http://www.reuters.com/article/2011/08/12/us-crisis-france-idUSTRE77B2YJ20110812

[15]http://www.esm.europa.eu/press/releases/20121008_esm-is-inaugurated.htm

[16]The following text has no ambition to substitute deep literature review. We only want to paint a picture of the determinants affecting the yield.

to Poghosyan (2014) there are two main determinants of long run sovereign yields. The first one is the potential output growth. Higher potential output growth increases sovereign yields of a state.[17] The second one is the government debt. The debt increases sovereign bond yields in two ways. Firstly crowding-out of private investment takes place, thus marginal product of capital increases and thus the real interest rate rises too. The second way of yield increasing is through default risk – investors are aware that a state will not be able to meet its obligation and thus they demand higher risk premia (this was theoratically proven by Eaton & Gersovitz 1981). As short run determinants Poghosyan (2014) uses changes in volatility index (VIX) representing global uncertainty, debt ratio, inflation and short-term real rate. Sovereing debt has got a higher impact in the short run, which is in accordance with Eichler and Maltritz (2013). According to Poghosyan (2014) those determinants are considered to be crucial. But additional ones are added in other studies. Kilponen *et al.* (2012) or Büchel (2013) demonstrated that even comments made by fiscal and monetary authorities in the EU have an impact on yields. Another rich branch of yield research (e.g. Afonso *et al.* 2012) showed that rating agencies have influence on yields too. Afonso *et al.* (2011) observed negative impact of current account on yields of developed states.[18] Favero (2013) discovered that exchange rate expectations affect the spreads in EMU[19] too. But in the crisis period the sensitivity of yields on the determinants changed. Financial and subsequent sovereign debt crisis in the EU caused that the importance of determinants – especially related to fiscal policy - increased according to von Hagen *et al.* (2011), Bernoth & Erdogan (2012) or Beirne & Fratzscher (2013). Moreover, Germany was again considered to be safe haven for the investors (Bernath & Erdogan 2012). All those studies were listed because determinants of yields affect their co-movement too (Piljak 2013).

There are two types of markets related to the sovereign bonds (Dunne *et al.* 2007). The first one is the primary market. On the primary market newly issued bonds are bought. The secondary market is a place, where the already issued bonds are traded between investors. There are studies (e.g. Broner *et*

[17] Poghosyan (2014) derives it using Ramsey-type growth model and its Euler equation, where for a closed economy change in consumption can be substituted by potential output growth. Similar result holds even for an open economy.

[18] *...it could reflect rapid accumulation of fixed investment, which should lead to higher growth and improved sustainability over the medium term."[p5]* Afonso *et al.* (2011)

[19] Economic and Monetary Union

al. 2006) that demonstrated a positive role of the secondary market related to access of borrowers to debt financing.

Government institutions have three main tasks related to the sovereign bonds market:[20]

1. **Government debt management** - an institution, which decides if, when and how many sovereign bonds will be issued. Usually it is in the juristiction of the ministry of finance.

2. **Issuance of government bonds** – an institution is responsible for realization of issuance itself. It is common to use network of banks and other private subjects. Usually the task is executed by a specialized agency subordinated to the ministry of finance or the central bank in some cases.

3. **Market supervision** is usually provided by a central bank.

Since 1999 the sovereign bonds of the Eurozone membes had to be denominated in Euro (Pagano & von Thadden 2004). According to the authors one of the results was that investors began to count the whole Eurozone as their "home", hence the home bias - an empirically observable tendency to prefer bonds denominated in their country - disappeared for investors within the Eurozone; exchange rate risk vanished.

2.3 Co-movement and Contagion in the EU

Previous section about sovereign bonds market lead us near to the main topic - and the research question - how described crises affected co-movement between sovereign yields of the EU members on various scales (frequencies). Firstly we will establish a research background from older works focused on the topic of co-movement and contagion. Before we start with the literature review itself, we have to properly define key term of the thesis - contagion.

2.3.1 Definition of Contagion

Present-days financial markets are highly connected. Hence financial crisis which erupted in one country spreads through various channels to other coun-

[20] EFC Sub-Committee on EU Sovereign Debt Markets[Online]: *http://europa.eu/efc/sub_committee/index_en.htm*, the list of national authorities related to sovereign debt market can be found there.

tries. This phenomenon is called contagion. According to Kaminsky *et al.* (2003) contagion accompanied Mexican crisis in 1994, Russian crisis in 1998 and Asian crisis in 1999. Aloui *et al.* (2011) studied the contagion during global financial crisis in 2008 on stock markets. The topic itself is not fully developed and its theory is not fully established. Even the definition of contagion itself is not properly unified. For example Pericoli & Sbracia (2003) give five different definitions of a contagion. For the purpose of the thesis we will use the following definition from influential paper written by Forbes & Rigobon (2002):

Definition 1 (Forbes & Rigobon 2002) *Contagion occurs if there is significant increase in cross-market linkages after a shock to an individual country (or group of countries).*

This type of contagion is sometimes called shift-contagion because contagion arises from the shift of inter-market linkages. Moreover, literature distinguishes between “fundamental-based” and “pure” contagion. The first type refers to a spread of contagion through real linkages, e.g. according to Kumar & Persaud (2002) through trade links or common external shocks.[21] Pure contagion means that the crisis in one country is spread without the change of the fundamentals of financial markets. For example according to the previously mentioned study very important channel for a pure contagion is risk-appetite. The contagion of this type is spread by a phenomena observed in behavioral finance – herding behavior, loss of confidence, self-fullfilling prophecies, multiple equilibria etc. The main problem is how to distinguish between contagion and a normal co-movement. The question how to detect contagion has been presented in economics since seminal works written by Calvo & Reinhart (1996) and Eichengreen *et al.*(1996). Gallegati (2012) provides a list of various techniques, which are used for its identification (multivariate GARCH, copula or probit to name a few). Moreoever, he states that standard time-domain techniques have problems with distinguishing between a co-movement and a contagion. On the other hand the wavelet method has got a good ability to detect contagion because of its properties, which allow us to decompose time series into different scales. The reason will be explained in Chapter 3.

[21] In literature this type is often labeled as ”spillovers”.

2.3.2 Co-movement of sovereign bond yields – literature review

In the previous section we defined terms, which are crucial for further analysis. Now we will provide review of literature, which analyses co-movement of sovereign bond yields or their spreads. We will focus more on recent papers, especially on their methodology. Moreover, we try to focus on the articles not employing additional variables (because we will not employ them too).

In the beginning of the year 1993 Maastricht treaty became effective and it has built the road to monetary union. As we mentioned earlier, governments of the Eurozone were obliged to denominate their debts in Euro. The primary goals were to increase credibility and eliminate exchange rate risk and thus to make sovereign yields fall. Hence it is not surprising that the works focused on the period – e.g. Laopodis (2008), Gilmore *et al.* (2010) or Missio (2013) - using different methods showed that there was high co-movement between yields after the creation of the Eurozone.

Then the financial crisis came and the sovereign debt crisis spread. Hence the research question arose: was there a change in degree of co-movement and in the integration of the market? To answer the question of integration Dias (2012) employed statistical method called minimum spanning tree analysis.[22] He used 10Y daily data from 2007 to 2010 from 19 countries and compared the results with a previous study made by previously mentioned Gilmore *et al.* (2010), who used monthly data from 1993 to 2008. It was seen that sovereign bonds market is shattered into smaller groups (Eurozone Core vs Eurozone Periphery) and co-movement decreased. It is the important change, because the older pre-crisis studies showed that before the crisis the market was more integrated. Dias (2012) observed that state, which is the most connected with others, is Netherlands. Moreover, Netherlands remains strongly connected with France and Germany even in the crisis period. Different situation is with Greece, Portugal, Ireland and Spain. Those four states became isolated. Broader class of spanning trees was used by Dias (2013) on crisis era data from 2009 to 2012. The results are the same: overall co-movement decreased and disintegration of sovereign bonds market increased - two groups were established. The first one

[22] It is statistical application of graph theory, which is able to compute degree of connection between observations and clusterize them into groups.

consists of Germany, Austria, Finland, Sweden, France and Netherlands, the second group involves other members of the EU. Antonakakis (2012) analysed co-movement of sovereign yields spreads (2007 - 2012) in the EU using multivariate GARCH, more precisely its DCC variant. He showed that dynamic conditional correlation of spreads often follows an inverted U-shaped curve - sharp rise and subsequent decrease. Dajcman (2013a) used a rolling window exceedance correlation approach to demonstrate that correlation of sovereign yields is not symmetric – different correlation is observed in case of a positive change and a different value is seen in case of a negative change. Moreover, he observed that that in the crisis period co-movement decreased. This finding is in accordance with findings obtained by Inoue *et al.* (2013). They showed that conditional correlation between sovereign yields of the states most severely hit by the crisis and Germany significantly decreased. Moreover, co-movement among GIIPS states decreased through the crisis, but co-movement between Spain, Portugal and Belgium remained strong even in the crisis period. Similar findings related to the three states were obtained by Claeys & Vašíček (2014) using Factor Augmented VAR. Moreover, they observed higher heterogeneity among non-Eurozone members in comparison with its member states. Christiansen (2014) measured integration of the EU members by explanatory power of a portfolio (using R^2).[23] She discovered that in the crisis integration decreased in the Eurozone. In the new members (included Greece, Spain and Portugal) the effect is stronger than in the old ones. Finally, we mention papers using alternative methods. Terceńo *et al.* (2013) employed a technique called self-organizing map (type of neural-network model). They observed a decrease of integration on the sovereign bonds market in the EU. Moreover, in the crisis period higher heterogeneity of the results is seen among non-Eurozone states too. On the frontier of econometric research there is a paper written by Bariviera *et al.* (2013) using complexity-entropy causality plane approach. Again, the study confirmed disintegration of the market.

Now we provide a short review of literature dealing with contagion on sovereign bonds markets in the EU. The results are dependent on version of contagion definition. Hence we will focus only on works using definition of Forbes & Rigobon (2002). Bhanot *et al.* (2012) analysed 5Y daily data four years before and after June 2007. They used vector autoregression (VAR) model augmented with time-varying volatility. Other variables such as the implied option volatil-

[23] Coefficient of determination

ity as well as CDS spread were employed. Then the impulse response analysis was made and results are interpreted. The main finding of the paper is following. According to them unconditional correlation between spreads increased, but employed impulse response functions showed that impact of a shock in one one country leads to a significantly smaller impact on yields in another country, which speaks against contagion hypothesis. Dajcman (2013b) argues that standard contagion detectors based on correlation have important flaw: they do not distinguish between co-movement of normal and extreme values. Hence he – using 10Y daily data - proposed a measure of co-exceedance (large positive changes) based on extreme value theory. He observed high co-exceedance in following cases: Ireland-Portugal, Italy-Spain, France-Italy, and Germany-France. In the opposite direction the lowest co-exceedance was observed in cases of France-Portugal and Germany-Portugal. Using 10Y daily data from 1999 to 2012 Gómez-Puig & Sosvilla-Rivero (2014) employed Granger causality and observed new causality patterns in the crisis period plus intensification of the older ones, which the authors interpret as a proof of contagion. They used endogenous structural breaking points for each pair of tested time series. The interesting result for the purpose of the thesis is that 2/3 of the breaking points were identified after the day of budget deficit revelation. Missio (2013) - using 10Y yields and DCC GARCH - observed sharp increase (very short-time) of conditional correlation after July 2010 between Greece-Spain, Greece-Portugal and Greece-Ireland. On the other hand according to the author significant drop between Greece-Germany, Greece-Italy and Greece-Netherlands correlation occurred in the same time. Decrease of correlation took place after the fall of Lehman Brothers To make the literature review complete several studies using large multi-factor models are listed. Arghyrou & Kantonikas (2012) used panel data method and detected contagion in crisis period, especially in Portugal, Spain and Italy. Mink & de Haan (2013) showed that news about Greece and its bailout affects sovereign spreads of the Eurozone states. Beirne & Fratzscher (2013) found evidence of both type of contagion. Already mentioned Claeys & Vašíček discovered that billateral spillovers in the EU are more larger in the crisis era, but it does not automatically imply contagion. Except the last one all those studies use monthly frequency of data.

Many techniques were used and they brought different results. The authors are not unanimous in all aspects, but even there exist the similarities:

1. There is a tendency to segmentation of sovereign bonds markets into groups based on geographical and economic criteria.

2. Overall level of co-movement decreased in the EU (while using simple sovereign yields).

3. Contagion (at least some of its form) occurred on sovereign bonds market in the EU during the crisis.

Based on the literature review we precisely formulate our hypotheses, which will be tested in the empirical part of the thesis:

I Overall co-movement between sovereign yields decreased in the crisis period.

II Sovereign bonds market became more decentralized after the crisis – there is a segmentation toward groups (defined geographically as well as economically).

III **The results of the previous two hypotheses are different across frequencies (scales).**

IV **After the beginning of the crisis there was a contagion, at least in some EU sovereign bonds markets.**

V **The results of hypothesis III differ through time. In other words - heterogeneity of co-movement across scales (frequencies) changed in the crisis period.**[24]

Boldly highlighted hypotheses represent our specific contribution to the recent literature. Hypothesis III uses alternative methodology to decompose time series into different scales (frequencies), analyses them separately and compares them. Hypothesis IV applies methodology proposed by Gallegati (2012) on sovereign yields topic. In our knowledge the methodology was not applied on the topic before. Hypothesis V compares heterogeneity of the results. The way how to test the hypotheses will be explained in detail in Chapter 3.

[24]See Chapter 4 for demonstration how heterogeneity changes through time

Chapter 3

Methodology

This chapter is dedicated to the description of the tool of the thesis – wavelet transformation. It is organized as follows. At first we introduce frequency-domain analysis using Fourier transformation. Then philosophy of wavelet analysis will be explained in general and we will show how it differs. Then we will define and describe the continuous wavelet transformation (CWT), the discrete wavelet transformation (DWT) and the maximal overlap discrete wavelet transformation (MODWT). The next part will be dedicated to wavelet coherence, correlation and multiple correlation. In the end a short review of their application will be provided, with special focus on the application in the fields of macroeconomics and finance.

3.1 Time and Frequencies

Let $x(t)$ denotes time series.[1] If we use time-domain information, we observe changes of values in time. Using time-domain analysis we can observe how the process depends on previous observations, where the structural breaking points are and other useful information. Spectral analysis uses completely different approach. In 1807 French mathematician Joseph Fourier in his work *Treatise on the propagation of heat in solid bodies*[2] presented a new view on the periodic signal.[3] It can be decomposed into a sum of weighted sines and cosines, thus the approach enables to express signal in terms of different frequencies. It exploits

[1]In this chapter $x(t)$ denotes continuous time series and x_t stands for discrete time series.
[2]The work was published in later - in 1822.
[3]Signal is another expression for continuous time series and it is often used in technical disciplines

the following fact (which is implied by Taylor polynomial)

$$e^{-i\omega t} = cos(\omega t) + i \cdot sin(\omega t), \tag{3.1}$$

where $i = \sqrt{-1}$ and ω denotes angular frequency.[4] Relationship between time and frequency representation of signal $x(t)$ can be written in the following form[5]:

$$X(f) = \langle x(t), e^{-i2\pi ft} \rangle = \int_{-\infty}^{+\infty} x(t)e^{-i2\pi ft}dt, \tag{3.2}$$

or inversely

$$x(t) = \langle X(f), e^{i2\pi ft} \rangle = \int_{-\infty}^{+\infty} X(f)e^{i2\pi ft}df. \tag{3.3}$$

An expression $\langle \cdot \rangle$ is called inner product. For our analysis it can be interpreted as a degree of "similarity" between two series.[6] Hence $X(f)$ measures how successfully a particular frequency fits the signal.

Fourier transformation has got its discrete variant too:

$$DFT(f_n) = \frac{1}{N}\sum_{t=0}^{N-1} x_t e^{-i2\pi f_n t\Delta T} \tag{3.4}$$

$$x_t = \frac{1}{\Delta T}\sum_{f_n=0}^{\frac{N-1}{T}} DFT(f_n)e^{i2\pi f_n t\Delta T}, \tag{3.5}$$

where T is the length of the signal, ΔT is the length of the interval used for sampling, $N = \frac{T}{\Delta T}$ denotes the number of samples and $f_n = \frac{n}{T}, n = 0, 1, 2, \ldots N-1$ stand for discrete frequency components.

The transformation was found useful by many econometricians (e.g. nonlinear ARIMA estimates, see Ludlow & Enders 2000) and especially in business cycle analysis (e.g. Harvey & Jaeger (1993), Canova (1998) or Raihan *et al.* 2005). It allows the researchers to decompose total variance of time series as a sum of variances of different frequencies. The proof of this proposition can be found in Hamilton (1994).

In the context of practical usage both analyses have their disadvantages. Time-

[4] $\omega = 2\pi f$, where f is frequency. See Appendix A.1

[5] Notation in this section is adopted from Gao & Yan (2011).

[6] More precisely, it denotes similarity of directions of the series, see Appendix A.1.

domain analysis is able to describe the value of time series in particular time, but tells us nothing about frequencies. The opposite problem is with frequency analysis – we can evaluate how particular frequency is important for given time series, but we are not able to provide time localization, because sines and cosines are defined for all real numbers. Hence we are for example able to say that given frequency plays important role in seasonal component of US GDP, but we will hardly identify an important change after some significant date. Moreover, we cannot analyse non-stationary time series, hence we have to extract trend and thus we will lose long-run information. The second disadvantage is a complete loss of all information related to time-domain. In other words – we are able to identify the most significant frequency in business cycle analysis, but we cannot be sure if this frequency remains significant through time.

To overcome these problems Gabor (1946) used Short time Fourier transformation (STFT). The idea of this improved Fourier transformation was to divide series into smaller subsamples (using time-localized window function $g(t-\tau)$ with positive real parameter of localization τ) and analyse them separately in the following way:

$$STFT(\tau, f) = \int_{-\infty}^{+\infty} x(t)g(t-\tau)e^{-i2\pi ft}dt. \tag{3.6}$$

But even STFT was a target of critique. For example Raihan *et al.* (2005) criticized STFT because of the resolution, which is the same in different frequencies. Hence - "*we analyse either high frequency components using narrow windows (wideband frequency analysis), or low frequency components using wide windows (narrowband frequency analysis), but not both"[p3].*(Polikar 1999). Nevertheless, it was first the step toward methodology, which will be employed in the thesis – wavelets.

3.2 What Are Wavelets?

Let us begin with two borrowed descriptions of wavelets. Percival and Walden (2000) state that wavelet is a *"small wave, which grows and decays essentially in a limited time period"[p2].* Gallegati *et al.* (2011) define wavelets as "… *mathematical functions that transform the data into a mathematically equiva-*

lent representation and cut up data into different frequency components, with a resolution matched to its scale"[p491]. To make wavelet transformation more understandable the following analogy is sometimes mentioned (e.g. Crowley 2007). The result of wavelet analysis can be compared to musical notation. If we look into musical notes, it can be seen that three main information are obtained from them:

1. The notation shows which tone (frequency) is played in particular time.

2. Intensity (amplitude) of music in particular time.

3. Length of a tone.

It is clear that musical notes combine time-domain and frequency-domain approach. We will demonstrate that wavelets provide the same type of information.

First wavelet is attributed to German mathematician Alfred Haar (1910). The purpose of its wavelet function was to give an example of orthonormal process in $L^2(\mathbb{R})$ space. During the first half of the 20th century few authors contributed to the topic, for example Littlewood & Paley (1931) or Ricker (1953). But those papers were mathematically oriented without the focus on practical applications. It was partially changed in 1964, when Alberto Calderon used wavelets in his work, which was dedicated to harmonic analysis. The core of the work was later used by Goupillaud, Grossman & Morlet (1984) and important type of wavelets was established. Their work inspired other scientists, e.g. Daubechies (1988), Mallat (1989) and many others. In the second half of the 80ths wavelets became an important tool for analysis of data in various disciplines – geophysics, medicine, biology, criminology and later in economics and finance, which are in the center of our attention. Now there are two main branches of wavelet transformations – continuous and discrete.

3.3 The Continuous Wavelet Transformation

The section introduces an important tool of the thesis – continuous wavelet transformation (CWT). We start with the basic properties and then popular Morlet wavelet will be described. In the following text we use Gencay *et al.* (2002) and Addison (2002) as the main sources of references.

3.3.1 General properties of the CWT

In the previous text we mentioned that wavelet transformation uses a function of the same name for transformation of a signal. Now we mathematically describe it for the continuous case. At first we have to describe wavelet function $\psi(t)$, which is sometimes called mother wavelet. We want to compare $\psi(t)$ with the signal $x(t)$. A function is a wavelet if it has got several properties. The first and the most important one of them is called the admissibility condition:

$$0 < C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(f)|^2}{f} df < \infty, \tag{3.7}$$

where $\hat{\psi}(f)$ is a Fourier transformation of the function. If $f \to 0$, then $C_\psi \to 0$ (Addison 2002). The second necessary condition is a result of the admissibility condition. It says that the integral of its wavelet function has to be equal to zero, in mathematical notation

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0. \tag{3.8}$$

The third condition is related to square integration. Let $L^2(\mathbb{R})$ be a set of square integrable functions. According to the third assumption

$$\int_{-\infty}^{+\infty} \psi^2(t)dt = 1 < \infty. \tag{3.9}$$

In words – wavelet function is squared integrable. It implies that the function has got finite energy.[7] In the case of the wavelets we can say more precisely that it has got unit energy. Another property is that wavelets should have compact support[8]. We say that function $\psi : \mathbb{R} \to \mathbb{R}$ has got compact support if it is zero everywhere except closed and bounded interval. The importance of the property will be explained later. We further assume that the signal is squared-integrable too, thus

$$\int_{-\infty}^{+\infty} x^2(t)dt < \infty. \tag{3.10}$$

[7]Energy of a function is defined as integral of the squared function.

[8]It is not exactly true, because for example Shannon wavelet hasn't got compact support. But all other wavelet function mentioned in the thesis will have it.

To preserve time-domain information and to obtain frequency-domain information we have to add time localization parameters τ together with scaling parameter s. The first one helps to preserve time information, the second parameter allows to discover how the function behaves in different scales (and thus frequencies). For both of them holds that $\tau, s \in \mathbb{R}^+$. Location parameter affects where wavelet lies (in other words where wavelet function will be compared with $x(t)$), scaling parameters define length of wavelet (in other words how "long" wavelet will be used for previously mentioned comparison). Now we transform the signal using the CWT (Addison 2002):

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right) dt, \tag{3.11}$$

where $\psi^* \left(\frac{t-\tau}{s} \right)$ denotes modified wavelet function with complex conjugate and the meaning of τ and s was explained earlier. $W_x(\tau, s)$ uses as a projection continuous wavelet function $\frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right)$ having several desirable properties, which $e^{-i\omega t}$ lacks. Thanks to compact support together with location parameter we are able to preserve time-domain information. The scaling parameter helps us to obtain time-frequency trade-off. On the other hand similarly to Fourier transformation the CWT has got simple interpretation too, because it holds that[9]

$$W_x(\tau, s) = \langle x(t), \psi^*_{\{\tau,s\}} \rangle. \tag{3.12}$$

Hence the CWT is an inner product of time series and modified mother wavelet function. Again, from definition of inner product we can deduce that the CWT can be interpreted as degree of similarity between the series and mother wavelet. Wavelet transformation has got the following pattern:

1. We choose a small scale parameter s and a location $t - \tau$ and compute how similar is the given wavelet to the measured signal. Higher value represents higher degree of similarity between wavelet function and original time series.

2. Choose different location parameter to cover the whole domain of the signal.

3. Repeat operations one and two with different scale until all possible scales are employed.

[9] $\psi_{\{\tau,s\}} = \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right)$ denotes localized and scaled version of wavelet function

Admissibility condition guarantees - according to Calderon-Grossman-Morlet's theorem - that time series can be reconstructed again (Mallat 1999), hence inverse function exists:

$$x(t) = \frac{1}{C_\psi} \int_0^{+\infty} \left(\int_{-\infty}^{+\infty} W_x(\tau, s) \psi^*_{\{\tau,s\}} d\tau \right) \frac{ds}{s^2}. \tag{3.13}$$

After the transformation we will obtain a time-scale map. Until now we have used the term "scale" when we worked with wavelet transformation and the term "frequency" when we described Fourier transformation. But between those two terms there is a close linkage (Gencay *et. al* 2002). Choosing the smaller scale s implies that we space out more wavelets into the signal, hence we examine its behavior on higher frequencies. And equivalently – using larger scales means that lower frequencies are examined. To properly interpret the result we will define wavelet power spectrum:

$$WPS_x = |W_x(\tau, s)|^2. \tag{3.14}$$

WPS shows how energy (variance) is distributed across time and scales. Examples of wavelet power spectrum is given in Figure 3.1. Artificial nonlinear process AR(1) with the following parameters was generated:

$$y_t = \begin{cases} 0.7y_{t-1} + \varepsilon_t & \text{if } t < 500 \\ -0.4y_{t-1} + \varepsilon_t & \text{otherwise,} \end{cases} \tag{3.15}$$

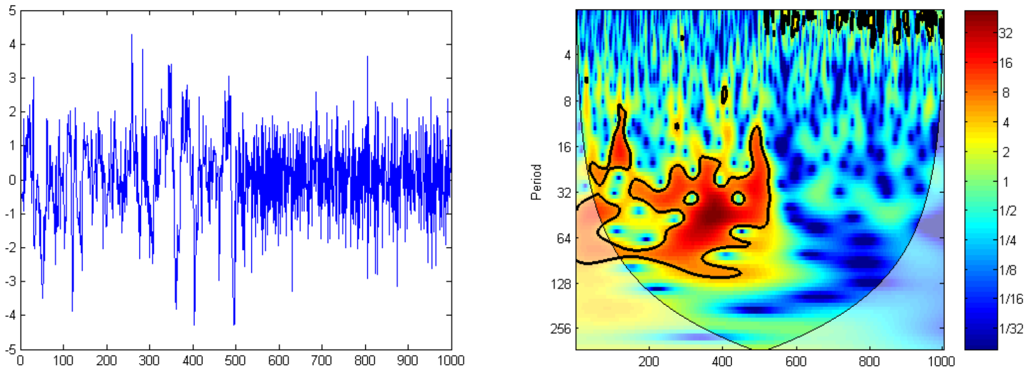
where ε_t denotes gaussian white noise. The process is plotted in Figure 3.1 as well as its wavelet power spectrum.

Advantage of the CWT over Fourier transformation is demonstrated in Figure 3.1. We can identify a structural break - first half of the time series tells us that the variance is significant on lower frequencies (high scales). In contrast, after the structural break significant variance can be found on higher frequencies (lower scales).[10] If we apply it on real time series, we are able to obtain information about its volatility in shorter and longer run and we are able to identify a point, where volatility changed.

Looking at WPS of AR(1) process defined by the equation 3.15 we see that the results outside the cone are hazy. The cone is called "cone of influence".

[10] Although in this particular case it has not to be seen on first time.

Figure 3.1: Example: Nonlinear AR(1) process



(a) AR(1) process (b) WPS of AR(1) process

Source: Own computation via Matlab.

Outside the cone the so-called edge effect is presented, because of the discrete nature of data contrary to continuous nature of a function (Torrence and Compo 1998). The effect pollutes the results, thus the results will not be interpreted outside the cone of influence.

3.3.2 Morlet wavelet

Morlet wavelet was proposed by Goupillaud *et al.* (1984) for the purpose of seismic signal analysis. The wavelet function has got the following form

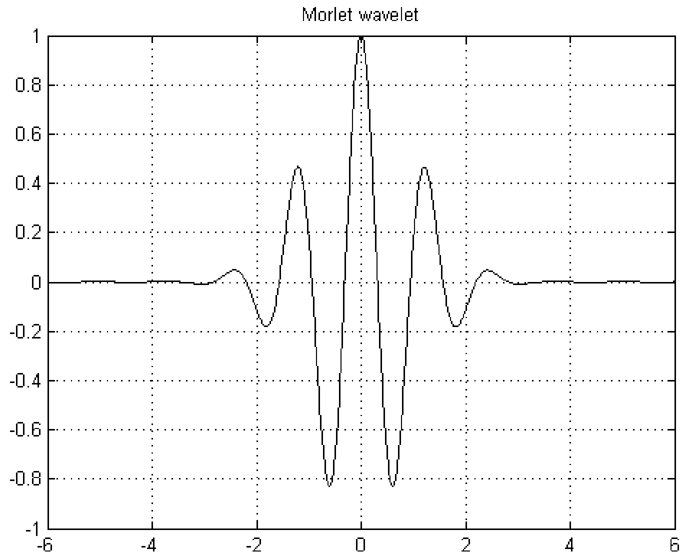
$$\psi(t) = \pi^{-\frac{1}{4}}(e^{2\pi f_0 i\ t} - e^{-(2\pi f_0)^2/2})e^{-\frac{t^2}{2}}. \tag{3.16}$$

In the equation above f_0 denotes central frequency of mother wavelet. According to Addison (2002) if $f_0 > 0$, then the equation can be rewritten into simpler form:

$$\psi(t) = \pi^{-\frac{1}{4}} e^{2\pi f_0 i\ t} e^{-\frac{t^2}{2}}. \tag{3.17}$$

This equation is easier to interpret. It has got three elements. The last one represents Gaussian "bell" shape, the second term is a complex sinusoidal wave. Finally, the first term is so-called normalization factor, which purpose is to guarantee the unit energy of the wavelet.

Figure 3.2: Morlet wavelet



Source: Own computation via Matlab.

Translated and scaled version of the process is obtained via simple replacement of $\frac{t-\tau}{s}$ instead of simple t, hence

$$\psi\left(\frac{t-\tau}{s}\right)=\pi^{-\frac{1}{4}}e^{2\pi f_0 i\ \left(\frac{t-\tau}{s}\right)}e^{-\frac{\left(\frac{t-\tau}{s}\right)^2}{2}}. \tag{3.18}$$

The wavelet is very popular among the researchers (Addison 2002). To understand why and which desirable property this wavelet transformation has, we have to slightly digress to define Heisenberg box. Let $|\psi_t|^2$ and $|\hat{\psi_f}|^2$ are energy density in time-domain, respectively frequency domain. Moreover, let σ_t and σ_f denote their spreads. Then rectangle with perimeter equal to $2(\sigma_t+\sigma_f)$ is called Heisenberg box. It represents the already mentioned trade-off between frequency and time-domain spread. The idea comes from physicist Heisenberg (1928). His work showed that on a time-frequency map it is impossible to represent signal as a single point. Representation is possible only in a rectangle. The desirable wavelet transformation minimizes Heisenberg box and hence it has got optimal joint time–frequency concentration (Aguiar-Conraria *et al.* 2012). And now the answer as to why Morlet wavelet transformation has got such popularity – it minimizes the Heisenberg box (Mallat 1999).

3.4 The Discrete Wavelet Transformation

In this section we closely examine the Discrete wavelet transform (DWT). We start with a general definition and then we proceed with description of frequently used wavelets. The section uses notation from Percival & Walden (2000) and partially from Gencay *et al.* (2002). The concept of the DWT is similar to the CWT, but there are significant differences and additional assumptions. Similarly to the CWT the DWT works like filters, which concept is described in Appendix A.4, but sampling is different as can be seen from the following equation (Crowley 2007):

$$\psi_{j,\tau} = 2^{-\frac{j}{2}} \psi^* \left(\frac{t - 2^j \tau}{2^j} \right). \tag{3.19}$$

Now scaling parameter s is replaced by 2^j, where $j \in \mathbb{N}$. But - while working with the DWT - literature usually uses filter notation.

3.4.1 The DWT – general properties

The DWT is similar to the CWT in imposed conditions. Let $\{h_l\}_{l=0}^{L-1}$ be a wavelet filter, where L is the number of nonzero elements (length). Similarly as in case of the CWT, the following conditions of the DWT wavelet filter have to be satisfied:

1. $\sum_{l=0}^{L-1} h_l = 0$
2. $\sum_{l=0}^{L-1} h_l^2 = 1$
3. $\sum_{l=0}^{L-1} h_l h_{l+2n} = 0.$

where n is a positive integer. The first two conditions are discrete equivalents to their continuous counterparts. The third one is new and together with the second one it says that the transformation has to be orthonormal.[11] Beside those three conditions an additional one has to be imposed on time series itself – it has to have length $N = 2^J$, where $J \in \mathbb{N}$. Hence only time series with length equal to the power of two can be analysed using the DWT. Let $X = (X_0, \ldots, X_{N-1})^T$ be a $N \times 1$ vector, which represents a dyadic time series. Then DWT transformation can be written in the following form:

$$\mathbf{W} = \mathcal{W} X, \tag{3.20}$$

[11] Orthonormality is defined in Appendix A.2.

where $\mathcal{W}$ is $N \times N$ wavelet transformation matrix, which elements are the filters and zeros. $\mathbf{W}$ denotes $N \times 1$ vector of wavelet coefficients. Before we start with deeper analysis, we have to mention two properties, both implied by orthonormality of transformation matrix $\mathcal{W}$. First of them is formula

$$X = \mathcal{W}^T \mathbf{W}, \tag{3.21}$$

which means that time series can be reconstructed using wavelet coefficients. The second result of transformation's orthonormality is:

$$||X||^2 = X^T X = \mathbf{W}^T \mathbf{W} = ||\mathbf{W}||^2, \tag{3.22}$$

where $||X||$ and $||\mathbf{W}||$ stand for *norm* of X, respectively $\mathbf{W}$.[12] It means that the total energy of time series is equal to the total energy of wavelet coefficients. This result is important for further work with wavelet spectrum of the process. Let us assume that the length of X is equal to 2^J. Then we decompose $\mathbf{W}$ and $\mathcal{W}$ in the following way:

$$\mathbf{W} = \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_J \\ V_J \end{pmatrix} = \begin{pmatrix} \mathcal{W}_1 \\ \mathcal{W}_2 \\ \vdots \\ \mathcal{W}_J \\ \mathcal{V}_J \end{pmatrix} X. \tag{3.23}$$

First J elements of $\mathbf{W}$ are $\frac{N}{2^j} \times 1$ wavelet coefficients vectors (mother wavelet), the last one - V_J - is a scaling coefficient. The previously defined wavelet filters h_l belong to the first J elements of the matrix $\mathcal{W}$. Moreover, $\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_J$ are $\frac{N}{2^j} \times N$ matrices. In contradiction the typical nonzero element of $1 \times N$ vector $\mathcal{V}_\mathbf{J}$ is scaling filter g_l - so-called "quadrature mirror filter" (QMF) - related to wavelet filter in the following way:

$$g_l = (-1)^{l+1} h_{L-1-l}. \tag{3.24}$$

Let us return to the second result implied by orthonormality. After rewriting energy preserving condition using decomposition on wavelet and scaling

[12] See Appendix A.2 for explanation of the relationship and definition of a *norm*.

coefficients, we will obtain:

$$||X||^2 = ||\mathbf{W}||^2 = \sum_{j=1}^{J} ||W_j||^2 + ||\mathcal{V}_{\mathbf{J}}||^2. \quad (3.25)$$

This energy preserving condition can be used to variance decomposition between the scales using relationship $N\bar{X}^2 = ||\mathcal{V}_{\mathbf{J}}||^2$:

$$\hat{\sigma}^2 = \frac{\sum_{i=0}^{N-1}(X_i - \bar{X})^2}{N} = \frac{||X||^2}{N} - \bar{X}^2 \quad (3.26)$$

$$\hat{\sigma}^2 = \frac{||X||^2}{N} - \frac{||\mathcal{V}_J||^2}{N} \quad (3.27)$$

$$\hat{\sigma}^2 = \frac{1}{N}\left(||X||^2 - ||\mathcal{V}_J||^2\right) \quad (3.28)$$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{j=1}^{J} ||W_j||^2, \quad (3.29)$$

where $\bar{X}$ stands for a sample mean. The last equation denotes the empirical wavelet power spectrum, in other words contribution of particular scales to the total variance.

3.4.2 Pyramid algorithm

In practice the pyramid algorithm proposed by Mallat (1989) is used for computation of the DWT. There are J − 1 stages. In the first stage there is a decomposition of the original time series X into two vectors. The first one is $\frac{N}{2} \times 1$ vector of wavelet coefficients W_1, which satisfies $W_1 = \mathcal{W}_1 X$. The matrix $\mathcal{W}_1$ is $\frac{N}{2} \times N$ matrix of wavelet transformation, which consists of wavelet filters h_l and zero elements. In each next row the filters are circularly shifted. The second obtained vector is $\frac{N}{2} \times 1$ vector of scaling coefficients V_1 satisfying $V_1 = \mathcal{V}_1 X$.

The next stage of the algorithm is similar to the first one. We make another decomposition, but not with original time series, but with V_1. Again, we will obtain two new series - a $\frac{N}{4} \times 1$ vector of wavelet coefficients W_2 and a $\frac{N}{4} \times 1$ vector of scaling coefficients V_2. It is used instead of the original time series because it is faster.

We will follow this approach in all stages of the algorithm and then we will obtain $N \times 1$ vector of wavelet coefficients $\mathbf{W}$ from the equation 3.20. Obtained wavelet coefficients can be used for an additive decomposition of time series - Multiresolution Analysis (MRA). Pyramid algorithm and orthonornality imply that

$$X = \mathcal{W}_1^T W_1 + \mathcal{W}_2^T W_2 + \ldots + \mathcal{W}_J^T W_J + \mathcal{V}_J^T V_J = \mathcal{D}_1 + \mathcal{D}_2 + \ldots + \mathcal{D}_J + \mathcal{S}_J, \quad (3.30)$$

where $\mathcal{D}$ denotes so-called wavelet detail and $\mathcal{S}$ stands for smooth detail. The equation 3.30 is the core equation of the MRA.

Empirical literature using wavelets usually prefers non-matrix notation. Then (following Gencay *et al.* 2002) we can write first stage of the DWT as[13]

$$w_{1,t} = \sum_{l=0}^{L-1} h_l X_{(2t+1-l) mod N} \quad \text{for} \quad t = 0, 1, 2, \ldots, \frac{N}{2} - 1 \quad (3.31)$$

$$v_{1,t} = \sum_{l=0}^{L-1} g_l X_{(2t+1-l) mod N} \quad \text{for} \quad t = 0, 1, 2, \ldots, \frac{N}{2} - 1 \quad (3.32)$$

where *mod* denotes *modulo* operator properly defined in Appendix A.4. The equations above are **circular convolution** of wavelet filter h (respectively g) and time series X.[14] While applying the same logic as before we write the second step of the transformation as:

$$w_{2,t} = \sum_{l=0}^{L-1} h_l v_{1,(2t+1-l) mod \frac{N}{2}} \quad \text{for} \quad t = 0, 1, 2 \ldots, \frac{N}{4} - 1 \quad (3.33)$$

$$v_{2,t} = \sum_{l=0}^{L-1} g_l v_{1,(2t+1-l) mod \frac{N}{2}} \quad \text{for} \quad t = 0, 1, 2 \ldots, \frac{N}{4} - 1 \quad (3.34)$$

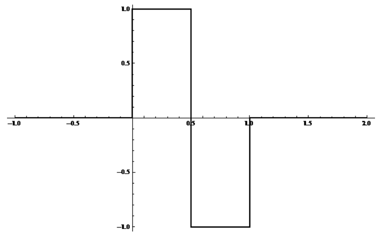
All other steps are executed in the similar way. Previous equations imply that the DWT is a **cascade filter**[15]. Now we describe two important discrete wavelets – Haar and Daubechies.

[13]The results of the following transformations are elements of the previously defined vectors $W_1, W_2 \ldots$, respectively $V_1, V_2, \ldots$. For example $W_1 = (w_{1,0}, w_{1,1}, \ldots, w_{1,\frac{N}{2}-1})^T$.

[14]Again, the concept of convolution is described in Appendix A.1

[15]See Appendix A.4

Figure 3.3: Haar wavelet



Source: Own computation via Mathematica 9

Haar Wavelet

Haar wavelet is the oldest known wavelet (Gencay *et al.* 2002). Its wavelet function has got the following form:

$$\psi_t = \begin{cases} 1 & \text{if } t \in [0, \frac{1}{2}) \\ -1 & \text{if } t \in [\frac{1}{2}, 1) \\ 0 & \text{otherwise.} \end{cases} \tag{3.35}$$

Its scaling (father wavelet) function is represented by this equation:

$$\phi_t = \begin{cases} 1 & \text{if } t \in [0, 1) \\ 0 & \text{otherwise.} \end{cases} \tag{3.36}$$

Haar wavelet is obtained from general representation if we set $L = 2$, $h_1 = -\frac{1}{\sqrt{2}}$ and $h_0 = \frac{1}{\sqrt{2}}$.

Let us assume that length of X is 8, thus $J = 3$. Then Haar wavelet transformation matrix has got the following form:

$$\mathcal{W} = \begin{pmatrix} \mathcal{W}_1 \\ \mathcal{W}_2 \\ \mathcal{W}_3 \\ \mathcal{V}_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & -\frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\ \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \end{pmatrix} \tag{3.37}$$

Using the equation 3.20 wavelet coefficients are computed in the following way:

$$\mathbf{W} = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(X_1 - X_0) \\ \frac{1}{\sqrt{2}}(X_3 - X_2) \\ \frac{1}{\sqrt{2}}(X_5 - X_4) \\ \frac{1}{\sqrt{2}}(X_7 - X_6) \\ \frac{1}{2}(X_3 + X_2 - X_1 - X_0) \\ \frac{1}{2}(X_7 + X_6 - X_5 - X_4) \\ \frac{1}{\sqrt{8}}(X_7 + X_6 + X_5 + X_4 - X_3 - X_2 - X_1 - X_0) \\ \frac{1}{\sqrt{8}}(X_7 + X_6 + X_5 + X_4 + X_3 + X_2 + X_1 + X_0) \end{pmatrix} \tag{3.38}$$

The matrix shows that wavelet coefficients have simple interpretation.[16]. First four elements are the first differences divided by $\sqrt{2}$. The fifth and the sixth element can be interpreted as "joint first difference" - the first difference of two commutated elements divided by 2. An alternative interpretation is possible - the difference of two nonadjacent elements. In a similar way we look at the seventh coefficient. The last one - scaling coefficient - is a simple sum of all elements of X divided by $\sqrt{8}$.

Daubechies wavelet

This type of discrete wavelet transformation was proposed by Daubechies (1988). It was derived from compact support criterion with maximum vanishing moments[17]. According to Gencay *et al.* (2002) the most obvious formal definition of Daubechies wavelet is through its square gain function[18]:

$$\mathcal{H}(f) = 2sin^L(\pi f) \sum_{l=0}^{\frac{L}{2}-1} \begin{pmatrix} L/2\text{–}1 + l \\ l \end{pmatrix} cos^{2l}(\pi f). \tag{3.39}$$

There are two main types of Daubechies wavelets – extremal phase denoted by $D(\cdot)$ and the least asymmetric. The least asymmetric version $LA(\cdot)$ of

[16]especially in this particular case.

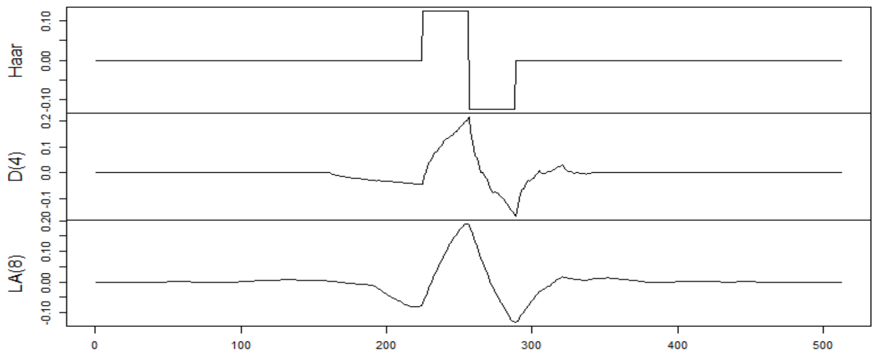
[17]Function $\psi(t)$ has P vanishing moments if

$$\int t^p \psi(t) dt = 0$$

for $p = 1, 2, \ldots, P$–1. Higher amount of vanishing moments implies that wavelet filter can be more effectively applied on non-stationary time series.

[18]Again defined in Appendix A.4

Figure 3.4: Daubechies wavelets - comparison



Source: Gencay *et al* (2002).

Daubechies wavelet corrects the problem of asymmetry, which causes that some observations are considered to be more important that others and it corrupts the results (Ramsey & Thong 2012). All of the previously mentioned types have got 10 versions: D(2),D(4),D(6),...,D(20) and LA(2),LA(4),LA(6),...,LA(20). Higher number in bracket implies smoother wavelet function. The previously described Haar wavelet is special case - D(2) - of Daubechies wavelet. Or reversely we can say that Daubechies wavelet is a generalization of Haar wavelet.

3.5 The Maximal Overlap Discrete Wavelet Transformation

The maximal overlap discrete wavelet transformation (MODWT) is a technique, which is a modified version of a standard discrete wavelet transformation. In many textbooks and academical works (e.g. Percival & Walden (2000), Gencay *et al.* (2002), Gallegati *et al* (2011) and many more) coefficients, filters and matrices related to the MODWT are denoted by "$\tilde{til}de$". Hence the central MODWT equation is rewritten in the following form

$$\tilde{\mathbf{W}} = \tilde{\mathcal{W}} X, \tag{3.40}$$

where $\tilde{\mathbf{W}}$ and $\tilde{\mathcal{W}}$ stands for $(J+1)N \times 1$ vector, respectively $(J+1)N \times N$ matrix. Similarly to the DWT :

$$\tilde{\mathcal{W}} = \begin{pmatrix} \tilde{\mathcal{W}}_1 \\ \tilde{\mathcal{W}}_2 \\ \vdots \\ \tilde{\mathcal{W}}_J \\ \tilde{\mathcal{V}}_J \end{pmatrix}. \tag{3.41}$$

Again, first J elements are matrices of wavelet filters, the last one is a matrix of QMF. All of them are $N \times N$ matrices (Gencay *et al.* 2002). The MODWT filters are a rescaled variant of the DWT filters mentioned in the previous text (Percival & Walden 2000):

$$\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}} \tag{3.42}$$

$$\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}. \tag{3.43}$$

where $h_{j,l}$ and $g_{j,l}$ are filters belonging to particular scale. Percival and Mofjeld (1997) state four main advantages of the MODWT. Firstly – and this is the most important change – analysed time series does not have to be dyadic. It means that we are not limited by the length of time series. The second advantage is related to shift invariance. It can be proven (Percival & Walden 2000) that the original DWT is not shift invariant, hence the results are affected by a shift of the series. The third advantage is that the estimator of variance made by the MODWT is asymptotically more efficient (Percival 1995). The last one is that the MODWT is a zero phase filter, which implies that "… *feature in the original time series may be properly aligned with features in the multiresolution analysis. "[p135]*(Gencay *et al.* 2002).

Similarly to the DWT pyramid algorithm will be used for computation of coefficients. Firstly let us define a variable

$$L_j = (2^j - 1)(L - 1) + 1 \tag{3.44}$$

denoting length of a filter on particular scale. The first step is (Percival & Walden 2000):

$$\tilde{w}_{1,t} = \sum_{l=0}^{L_1-1} \tilde{h}_{1,l} X_{(t-l) mod N} \quad t = 0, 1, 2, \ldots, N-1 \tag{3.45}$$

$$\tilde{v}_{1,t} = \sum_{l=0}^{L_1-1} \tilde{g}_{1,l} X_{(t-l) mod N} \quad t = 0, 1, 2, \ldots, N-1, \tag{3.46}$$

Then the second step can be written as

$$\tilde{w}_{2,t} = \sum_{l=0}^{L_2-1} \tilde{h}_{2,l} \tilde{v}_{1,(t-l) mod N} \quad t = 0, 1, 2, \ldots, N-1 \tag{3.47}$$

$$\tilde{v}_{2,t} = \sum_{l=0}^{L_2-1} \tilde{g}_{2,l} \tilde{v}_{1,(t-l) mod N} \quad t = 0, 1, 2, \ldots, N-1. \tag{3.48}$$

Similarly all other steps are executed.

3.6 Wavelet Transformation and Analysis of Co-movement

In the previous text we have described how wavelets can be used for analysis of time series, especially for variance analysis. Now we describe techniques using wavelet transformation, which purpose is to analyse co-movement between several time series: wavelet coherence, correlation and multiple correlation.

3.6.1 Coherence

Let x_t and y_t be two time series of a length N. If we use simple Pearson's correlation analysis coefficient

$$\operatorname{corr}(X, Y) = \frac{\sum_{i=1}^{N} (x_t - \overline{x})(y_t - \overline{y})}{\left[\sum_{i=1}^{N} (x_t - \overline{x})^2 \sum_{i=1}^{N} (y_t - \overline{y})^2 \right]^{1/2}}, \tag{3.49}$$

where $\bar{x}$ and $\bar{y}$ are sample means, we will obtain a number between -1 and 1, which tells us how those time series co-move. But we are not able to see how this relationship holds on different frequencies. Moreover, we are not able to detect how the coefficient of correlation changes with time nor how one series is delayed after another. Wavelet coherence analysis allows us to do it and it is the reason why it is suitable for testing the hypotheses of the thesis. We will use standard notation like e.g. Torrence & Compo (1998) or Grinsted *et al.* (2004). Let us assume that both time series $x(t)$ and $y(t)$ are locally stationary (Grinsted *et al.* 2004). At first we have to define cross-wavelet transformation (XWT) as a product of two wavelet tranformation

$$W_{xy} = W_x(\tau, s)W_y^*(\tau, s), \tag{3.50}$$

where $W_x(\tau, s)$ and $W_y(\tau, s)$ are continuous wavelet transformations of both time series and $*$ denotes complex conjugate. Then the term

$$CWS_{xy} = |W_{x,y}(\tau, s)| \tag{3.51}$$

is called cross wavelet power spectrum. It is a wavelet equivalent of covariance. Now we will define wavelet coherence itself:

$$R^2(\tau, s) = \frac{|(S(s^{-1}W_{xy}(\tau, s))|^2}{S(s^{-1}|W_x(\tau, s)|^2)S(s^{-1}|W_y(\tau, s)|^2)}, \tag{3.52}$$

where S denotes smoothing operator. According to Grinsted *et al.* (2004) the operator can be defined by the following equation:

$$S(W) = S_{scale}(S_{time}(W(s))). \tag{3.53}$$

In the previous equation S_{scale} and S_{time} denote smoothing along scale axis, respectively smoothing in time.[19] In the thesis we defined squared coherence, but equivalently it can be written in extracted-root version (for example Aguiar-Conraria *et al.* 2012). In both version – squared and not-squared - the interpretation is the same. It holds that $0 \leq R^2 \leq 1$ as well as $0 \leq R \leq 1$. High level of coherence implies that there is a strong co-movement between the time series. Low values mean that the series are linearly independent on a particular scale.[20] Horizontal axis shows time, on vertical axis lies the scales (or frequency). Hence we are able to see how the relationship between the time

[19] According to Gristed *et al.* (2004) without smoothing coherence would be 1 across all scales.

[20] The word linearly is crucial – we cannot say that there is no relationship between the series, only there is no linear relationship.

series evolves through time and scales.

The important information is whether our results are statistically significant. Although there were some formal significance tests developed (see Ge (2007) and Ge 2008), according to Aguiar-Conraria *et al.* (2012) their null hypotheses are too restrictive and hence they are not suitable for wavelet analysis in economics. In the thesis Monte Carlo simulations will be used.[21] The significant parts of the wavelet coherence estimates are inside black lines. Last but not least we are interested in solving the question – which of the two series precedes the second one. This is examined by the phase difference, which can be computed in the following way:

$$\Xi(\tau, s) = \tan^{-1}\left(\frac{\mathfrak{I}(S(s^{-1}|W_{xy}(\tau,s)|))}{\mathfrak{R}(S(s^{-1}|W_{xy}(\tau,s)|))}\right), \tag{3.54}$$

where $\mathfrak{R}$ and $\mathfrak{I}$ denote real part, respectively imaginary part of smoothed wavelet cross-wavelet transformation. On the picture phases are the black arrows pointing in different direction. Figure 3.5 gives insight how to interpret behavior of the time series based on the direction of the arrows.

3.6.2 Wavelet correlation: a detector of contagion

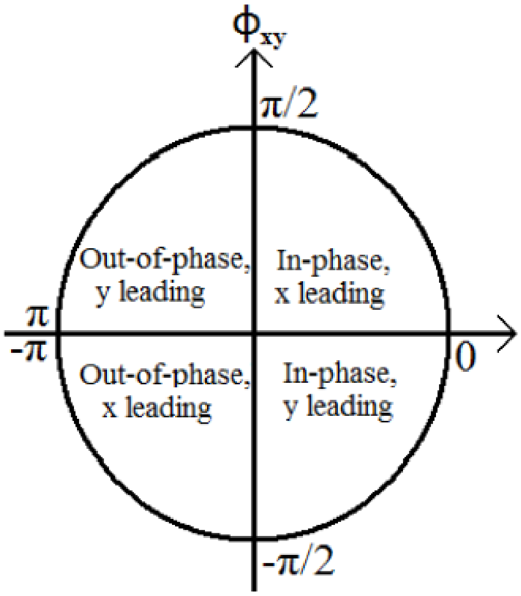
In the previous text we showed how the DWT and then the MODWT can be used for decomposition of variance into different scales. Now we use this knowledge to analyse the relationship between the two variables. Let X_t and Y_t be two stationary stochastic time series. Then – following Gencay *et al.* (2002) and Gallegati (2012) - we define

$$\sigma_X^2(\lambda_j) = \frac{1}{\tilde{N}_j}\sum_{t=L_j-1}^{N-1}\tilde{w}_{X,j,t}^2 \tag{3.55}$$

$$\sigma_Y^2(\lambda_j) = \frac{1}{\tilde{N}_j}\sum_{t=L_j-1}^{N-1}\tilde{w}_{Y,j,t}^2 \tag{3.56}$$

[21] "*We generate a large ensemble of surrogate data set pairs with the same AR1 coefficients as the input datasets. For each pair we calculate the wavelet coherence. We then estimate the significance level for each scale using only values outside the cone of influence."[p565]* Grinsted *et al.* (2004)

Figure 3.5: Phases of wavelet coherence



Source: Aguiar-Conraria & Soares (2014)

as variance of coefficients from a scale λ_j obtained via the MODWT procedure.[22] Firstly let us recall that L_j denotes length of wavelet filter of scale λ_j. With previous knowledge we can define

$$\tilde{N}_j = N – L_j + 1, \tag{3.57}$$

which is the amount of wavelet coefficients not affected by boundary[23]. Further we define scale-by-scale covariance of the two series:

$$\gamma_{XY}(\lambda_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{w}_{X,j,t} \tilde{w}_{Y,j,t}. \tag{3.58}$$

Then we can compute scale-by-scale correlation between the two series:

$$\rho_{XY}(\lambda_j) = \frac{\gamma_{XY}(\lambda_j)}{\sqrt{\sigma_X^2(\lambda_j)\sigma_Y^2(\lambda_j)}}. \tag{3.59}$$

According to Whitcher *et al.* (1999) the estimator has got many important attributes - it is unbiased, consistent and asymptotically normal. Now the esti-

[22]Gencay *et al.* (2002) as well as large amount of papers use this λ_j notation. Hence we will employ it too.

[23]For the discussion about boundary see Gencay *et al.* (2002), Chapter 4.6.3.

mator will be used for a test of contagion presence in the way it was described by Gallegati (2012). At first let us denote the point in time, when the contagion began. We define $\rho^{(I)}_{XY}(\lambda_j)$ as a correlation before the point and $\rho^{(II)}_{XY}(\lambda_j)$ as correlation after the point. We will compare pre-crisis and crisis values. Now we test hypothesis that the both values are the same[24]:

$$H_0 : \rho^{(I)}_{XY}(\lambda_j) = \rho^{(II)}_{XY}(\lambda_j) \tag{3.60}$$

$$A : \rho^{(I)}_{XY}(\lambda_j) \neq \rho^{(II)}_{XY}(\lambda_j). \tag{3.61}$$

To test the hypothesis we compute confidence interval of pre-crisis and crisis estimators. The interval is computed via the following formula (following Whitcher *et al.* (1999) and Gencay *et al.* 2002):

$$\left[\tanh\left(\tanh^{-1}\left(\rho_{XY}(\lambda_j)\right) \mp \frac{\Phi^{-1}(1\text{–}p)}{\sqrt{N_j - 3}}\right)\right], \tag{3.62}$$

where $\Phi^{-1}(1\text{–}p)$ is a quantile of the standard normal distribution[25]. Big advantage of the method is its robustness to non-Gaussian features of data (Gallegati 2012). On the other hand according to Gencay *et al.* (2002) the formula is valid only if there is no trend or other non-stationary feature. It is clear that hypothesis will be rejected (and thus contagion detected) if confidence intervals of pre-crisis and after-crisis correlation do not overlap.

Gallegati's approach follows philosophy of the methods proposed by Bodart & Candelon (2009) and Orlov (2009). Both mentioned works are specific - they decompose analysed time series into different frequencies, because they are key for distinguishing between contagion and interdependency. Changes of comovement on higher frequencies (lower scales in wavelet framework) represents temporary changes - a contagion, lower frequencies (higher scales) describes only change of interdependence.

3.6.3 Wavelet multiple correlation

Final part of the methodology chapter will be dedicated to wavelet multiple correlation. We will follow Fernandez-Macho (2012a), who proposed the method. Let Z represents $N \times K$ matrix of multivariate time series. Then

[24] A denotes an alternative of a null hypothesis.

[25] $tanh(x) = \frac{1-e^{-2x}}{1+e^{-2x}}$ is hyberbolic tangent.

$W_{jt} = (w_{1jt}, w_{2jt}, \dots, w_{Kjt})$ will be the matrix of wavelet coefficients on scale λ_j obtained by the already known MODWT procedure. Then estimator of the wavelet multiple correlation is:

$$\varphi_Z(\lambda_j) = \sqrt{1 - \frac{1}{\max \operatorname{diag} P_j^{-1}}}. \tag{3.63}$$

Matrix P_j denotes correlation matrix between elements of W_{jt}. Max diag P$_j$ is the highest diagonal element of inverted matrix P_j. The idea behind the formula is the following. On all scales K auxiliary regressions[26] are performend and their $R^{2\prime}s$ are obtained using relationship[27]:

$$R_i^2 = 1 - \frac{1}{\rho^{ii}} \tag{3.64}$$

where ρ^{ii} represents i-th diagonal element of P. Then the multiple wavelet correlation can be estimated in following way:

$$\varphi_Z(\lambda_j) = \frac{Cov(\tilde{w}_{ijt}, \hat{\tilde{w}}_{ijt})}{\sqrt{Var(\tilde{w}_{ijt})Var(\hat{\tilde{w}}_{ijt})}}, \tag{3.65}$$

where $\tilde{w}$ denotes the MODWT coefficients and $\hat{\tilde{w}}$ represents fitted values from the auxiliary regressions mentioned above. Covariance and both variances are computed via the same procedure as the bivariate wavelet correlation. According to Fernandez-Macho (2012a) the estimator is consistent and asymptotically normal. Confidence intervals are computed in a similar way as in case of bivariate multiple correlation:

$$\left[\tanh\left(\tanh^{-1}\left(\varphi_Z(\lambda_j)\right) \mp \frac{\Phi^{-1}(1{-}p)}{\sqrt{\frac{N}{2^j} - 3}}\right)\right] \tag{3.66}$$

3.7 Wavelets in Macroeconomics and Finance

In the previous text the principle of wavelet analysis was described. Now we provide a short review of literature using the technique for the purpose of economics. Let us begin with the applications of the CWT. The pioneering work

[26]For example i-th regression has got following form:w_{ijt} is dependent variable and w_{hjt} - where $h \neq i$ - are its regressors.

[27]Using fact that coefficient of multiple correlation is square root of coefficient of determination is

was a paper written by Aguiar-Conraria *et al.* (2008), where wavelet coherence analysis was firstly used for the purpose of demonstrating that relationship between monetary variables changes through time and scales. Rua (2010) used wavelets for analysis of comovement between business cycles of European countries. Aguiar-Conraria *et al.* (2012) employed coherence to analyse the relationship between main macroeconomic variables (GDP, interest rate, M2) and components of yield curve (slope, level, curvature) on US data from 1961 to 2011. Main result is that there were periods, where co-movement between these variables diminished. Rua & Lopes (2012) used wavelets to estimate the degree of cohesion (business cycles co-movement) between the EU and USA. They showed heterogeneity across the frequencies. Wavelet coherence was used even for analysis of markets with commodities (e.g. Vácha & Baruník (2012), Vácha *et al.* 2013). Hence it can be seen that the CWT became popular technique in economics and finance.

But there are many econometric articles employing its discrete counterparts too. Ramsey & Lampart (1998) decomposed crucial economic relationship between expenditures and income into different scales. It was one of the first attempts to use the DWT for the purpose of macroeconomic analysis. Gallegati & Gallegati (2007) analysed volatility of GDP in G7 countries using the MODWT methodology. They found out that the variance of wavelet coefficients differs through countries and scales. Durai & Bhaduri *et al.* (2009) tested a hypothesis of Fama (1981). The hypothesis says that there is a negative correlation between stock prices and inflation. The authors employed wavelet correlation and discovered that the hypothesis holds only for some of the scales. Wavelet correlation was also used by Gallegati (2012) to detect contagion on stock markets during the financial crisis in 2007. According to his findings all tested markets were hit by the crisis, but only in some of them (Japan and Brazil) a significant change of co-movement was detected on all scales. Ramsey & Thong (2012) performed discrete wavelet transformation analysis on international trade structure relationship – between term of trade and trade balance. They were able to empirically confirm that the so-called S-Curve proposed by Backus, Kehoe & Kydland (1994) exists. Gallegati *et al.* (2011) tested the Phillips wage curve in the USA. They an used multiresolution analysis for OLS regression in scale-by-scale sense. They found that the estimates differ across frequencies, which can be interpreted as a sign of nonlinearity. A similar procedure was used in Gallegati *et al.* (2013) for an analysis of a relationship between output and real interest rate. But the wavelet coefficients have broader application. The DWT was used for a construction of a long memory estimator (Jensen 2000). Gencay & Signori (2012) employed the MODWT and constructed a new multiscale test for serial correlation, which in Monte Carlo simulations outperformed Box-Ljung test and Box-Pierce test.

There is similar pattern in literature using continuous and discrete wavelets tools for time series analysis. Firstly the authors find several time series, which are long enough (because the problem with macroeconomic data is that they often have quarterly or yearly frequency, financial data are more suitable). Then they choose between the CWT and the DWT techniques. Coherence as well as correlation only allows us to analyse two time series at the same time, but wavelet multiple correlation can be used as well as it is possible to apply additional methods (e.g. OLS). Usually the main finding of the work is that the results are different across scales, which means that some hypothesis can hold only in a short/long run. Using wavelets it is quite obvious to test previously developed hypotheses.

Chapter 4

Data: Preliminary Analysis and Motivation

In our analysis we use daily data of bid yields of 10Y sovereign bonds on a secondary market. A bid yield is a minimal yield demanded by an investor on the secondary market, which makes her to buy a bond and thus it mirrors investor's attitude toward issuer. In the following empirical study we use only the word ”yields” for clarity's sake. All data were downloaded from Reuters Wealth Manager database.[1] The dataset starts on 1st January 2001 and ends on 31th December 2013. Firstly we motivate the choice of data. Then we will compute descriptive statistics and visualize the time series. At last we will show why it is good idea to employ wavelet transformation.

4.1 Selection of Data – Motivation

4.1.1 Selection of countries

We chose sovereign yields of 11 states, preliminarily divided into the three groups:

1. **the Core of the Eurozone** – Germany(GE), France(FR), Italy(IT) and Belgium(BE). These countries were the founders of European Communities (EC), their economies are considered to be highly developed.

2. **the Periphery of the Eurozone** – Greece(GR), Spain(SP), Portugal(PT) and Italy(IT). The group is often called by acronym PIIGS or

[1] Account for IES students was used.

GIIPS[2]. They are the states, which problems with government debts were described in Chapter 2.

3. **the states outside the Eurozone** –Great Britain(GB), Denmark(DM) and Sweden(SW). These three countries are highly developed economies, which stand out of the Eurozone. We chose them to test how the relationship between them differs from others.

The division of the states into the Core and the Periphery was executed based on the reasons described above as well as in the literature review - the states of the Core constitute one of the two obtained groups[3] according to Dias (2012). The already mentioned four states were considered to be the Periphery in Bhanot *et al.* (2012) and many more studies. Of course there are more countries outside the Eurozone, but we favored older and more developed members of the EU.

Data are fully synchronized. To replace missing values approximation via cubic splines from ***R*** package *ZOO* was used (following Ramsey & Thong 2012).

4.1.2 Which measure should we use?

As a first step we have to decide whether to use differentials or not. In general wavelet transformation does not require stationarity, the process should be only locally stationary. To obtain sensible confidence intervals of wavelet correlation we need data without non-stationary features[4] and it is important when we want to test data for the presence of a contagion based on overlapping intervals. Hence we will test the time series for stationarity and then decide if use differentials or not.

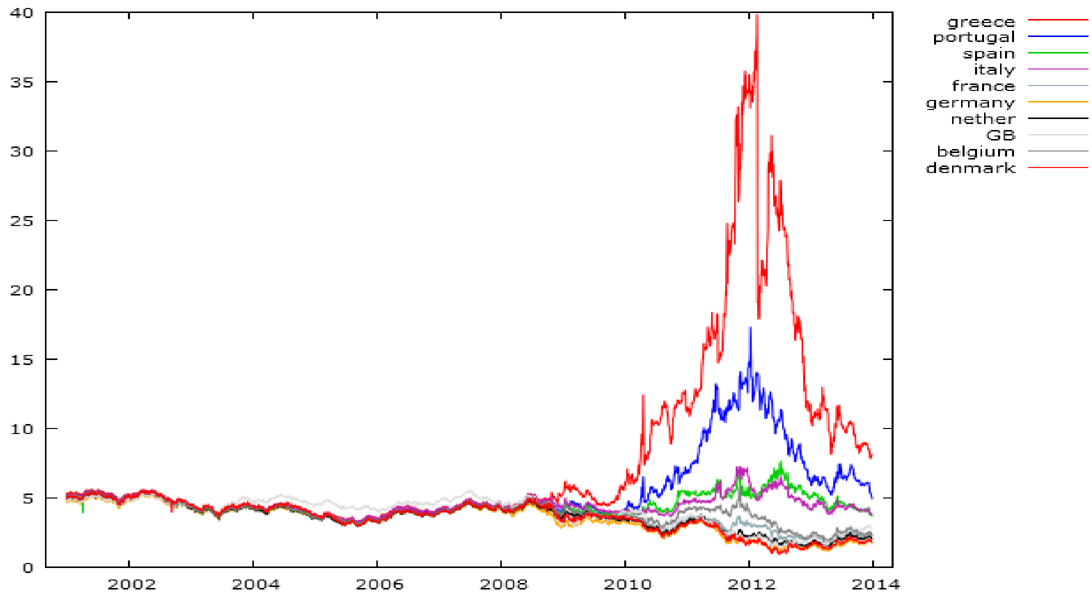
The second question is whether we will analyse the interaction between yields or their spreads? Spreads are used more likely in larger panel data studies with multiple dependent variables. But all works in the literature review define yield spread as a difference between a yield of a country and a yield of German sovereign bonds and we would like to analyse Germany too, because it is

[2]The last state is Ireland. Original acronym was PIGS and Ireland was not its part. It obtained this status - in our opinion - undeservely because of bubble, not structural weaknesses, Hence we prefered to analyse only old "members " of PIGS.

[3]except Belgium.

[4]See Gencay *et al.* (2002).

Figure 4.1: Development of yields



Source: Reuters Wealth Manager, visualized in *Gretl*

probably the most important economy in the EU. Moreover, we would like to model "real" yields, which are demanded by investors. Yield spread represents risk premium of a bond, thus practically it is a different variable. Large panel data studies focusing on the contagion use spreads. Literature dealing with bivariate co-movement of yields (without additional explanatory variables) is not numerous. Large panel data studies focused on contagion use spreads. While comparing works of Inoue *et al.* (2013) and Antonakakis (2012) we see that same method (DCC GARCH) brings different results.

Hence in our study we will compare the results primarily with the similar studies (Inoue *et al.* (2013), Dias (2012 & 2013), Gómez-Puig & Sosvilla-Rivero 2014, Dajcman 2013a & 2013b). Comparison with the studies using spreads will be made only if the results are similar (especially if we are able to observe specific behavior of a particular state, e.g. Belgium).

4.2 Descriptive Statistics

In the beginning of the preliminary analysis we plot all time series together. From the beginning of the dataset in 2001 until 2008 it can be seen that there

Table 4.1: Descriptive statistic of yields

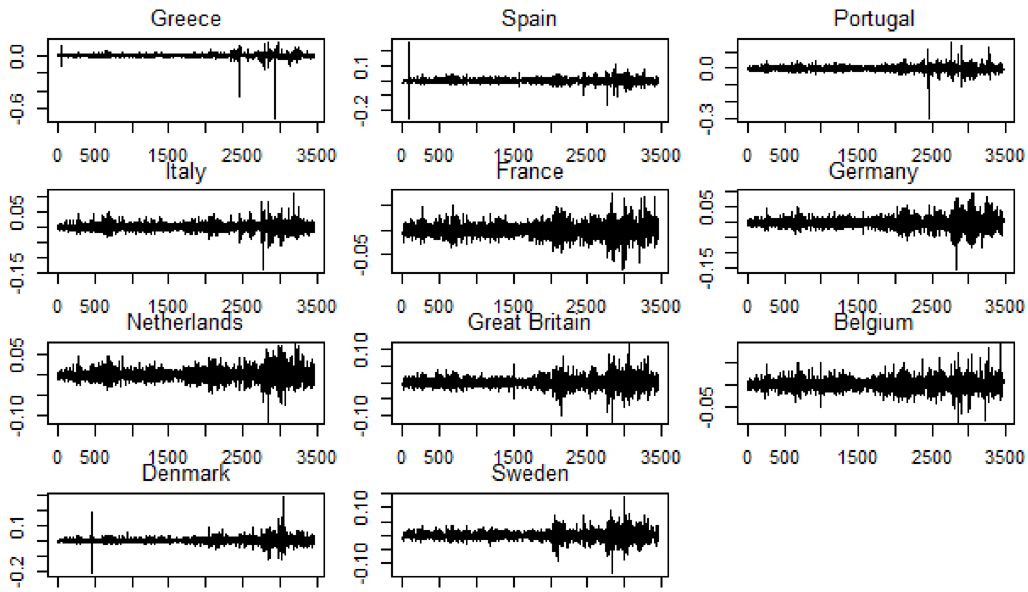
State	Mean	S.D.	Median	Minimum	Maximum
GR	7.95331	6.7614	4.95000	3.22400	39.8500
PT	5.51522	2.4183	4.57100	3.10400	17.3450
SP	4.49075	0.7413	4.31100	3.00200	7.58000
IT	4.53437	0.6553	4.42100	3.21000	7.31100
FR	3.74103	0.8319	3.78600	1.66703	5.34700
GE	3.46683	1.0588	3.70100	1.16200	5.28646
NL	3.65083	0.9689	3.82200	1.49200	5.40000
GB	3.99046	1.0502	4.39300	1.43800	5.57600
BE	3.96421	0.7793	4.05900	1.92758	5.86500
DM	3.62768	1.1273	3.82000	0.96700	5.51215
SW	3.62420	1.1350	3.7300	1.1250	5.7700

Source: Author's computation using Gretl

is high degree of co-movement of all 11 time series (see Figure 4.1). After the beginning of the crisis we observe that yields became more heterogeneous. Since 2009 yields of several states (the states we considered to be the Periphery) sharply increased, especially the yields of Greece. This sharp increase corresponds to the time, when Greek government stated that the sovereign debt of his country is much higher than it was expected.

Table 4.1 shows descriptive statistics of data. We observe that the yields of Germany have the lowest mean. It is not surprising as German bonds are considered to be the benchmark bonds. The other three states with means below 3.75 % are France, Netherlands, Sweden and Denmark. France and Netherlands are classified as the Core. Greece stays on the opposite side with mean of its yields higher than 7 %. Again, it should not surprise us, because Greece is the in a serious debt crisis. The second highest mean has got Portugal, another country in the Periphery. Interestingly, GB has also relatively high mean. Looking at Figure 4.1, we can see that before the crisis the yields of Great Britain were higher than yields of the Eurozone members. This situation changed after 2008. Standard deviation represents volatility of time series. According to Table 4.1 the most volatile time series are yields of Greece and Portugal. Their standard deviations are several times higher than in the other cases. On the opposite side are Italy, Spain and Belgium.

Figure 4.2: Development of transformed yields



Source: Reuters Wealth Manager, visualized in R

To test stationarity we used Augmented Dickey-Fuller (ADF) test. According to the results all variables are non-stationary, because null hypothesis of the test (unit root is presented) cannot be rejected.[5] In order to guarantee stationarity we transformed data in the following way:

$$y_t = \log\left(\frac{x_t}{x_{t-1}}\right) = \log\left(x_t\right) - \log\left(x_{t-1}\right).$$

It can be interpeted as a log-difference of yields and shows how yields change[6]. This transformation should guarantee stationarity[7].

Figure 4.2 depicts the graphs representing log-differentiated time series. We observe several jumps in the following four states: Greece, Portugal, Spain, Denmark. We checked the data and realized that no jump occurred as a result of the approximation via cubic splines. On the first look we see that the series are more clustered in crisis era. Table 4.2 shows descriptive statistic of transformed time series. It can be seen that all means are close to zero. Only two of

[5]The results are in Appendix B.1.

[6]$y_t = \log\left(\frac{x_t}{x_{t-1}}\right) = \log\left(1 + \frac{x_t - x_{t-1}}{x_{t-1}}\right) = \log\left(1 + R_t\right)$, where R_t stands for percentual change.

[7]To be sure we will the tests of stationarity.

them are higher than zero - Greece and Portugal. Standard deviation is - again - the highest in Greece. ADF test is performed and now the results are different - we reject null hypothesis in all cases and thus all time series are stationary. To be sure we used Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test, which did not reject null hypothesis [8] Moreover, Jarque-Bera test was performed on all 11 time series and null hypothesis was rejected in all cases.[9]. The results are clear - no time series is normally distributed, which is in accordance with Christiansen (2014). It is not surprising, because high excessive kurtosis is observable in all cases, especially in case of Greece, Spain and Portugal. High excessive kurtosis tells us that the distribution of time series has fatter tails than the standard normal distribution and exteme values occur more likely. The implication of this result will be discussed in the next chapter.

Table 4.2: Descriptive statistic of transformed yields

State	Mean	S.D.	Min	Max	Skew.	Exc. Kurt.	ADF H_0	JB H_0
GR	0.00013399	0.022344	-0.71549	0.15010	-12.214	371.70	rejected	rejected
PT	4.2361e-005	0.015953	-0.30185	0.15902	-1.2580	48.933	rejected	rejected
SP	-6.4191e-005	0.014848	-0.25407	0.25784	-0.28813	59.548	rejected	rejected
IT	-7.7926e-005	0.012406	-0.14112	0.10809	0.00087905	10.031	rejected	rejected
FR	-0.00021270	0.013125	-0.081616	0.074986	0.021069	3.8502	rejected	rejected
GE	-0.00026995	0.017232	-0.15587	0.095825	-0.0057566	5.7125	rejected	rejected
NL	-0.00023724	0.014302	-0.11472	0.074758	0.14049	4.1635	rejected	rejected
GB	-0.00013988	0.015500	-0.00046062	0.11522	0.14633	5.8020	rejected	rejected
BE	-0.00021027	0.012604	-0.081736	0.081007	0.21694	4.5969	rejected	rejected
DM	-0.00028048	0.018666	-0.21218	0.29127	1.1421	32.246	rejected	rejected
SW	-0.00019247	0.015776	-0.13383	0.14073	0.20371	7.8546	rejected	rejected

Source: Author's computation using Gretl

4.3 Pearson's Correlation - Why Wavelets Should Be Useful

In the beginning we have to introduce an important convention - since now we label time series obtained via previous log-transformation as "yields". Now we motivate the usage of wavelet methodology. Table 4.3 shows Pearson's correlation[10] between yields of the states in our dataset.

[8] Null hypothesis of KPSS test: time series is stationary, see Appendix B.1.

[9] Test statistics are in Appendix B.1.

[10] This type of correlation is used in the empirical studies analysing time series.

Table 4.3: Pearson's correlation - whole sample

	GR	PT	SP	IT	FR	GE	NL	GB	BE	DM	SW
GR	1.00	0.41	0.27	0.22	0.08	-0.01	0.08	0.02	0.16	0.04	0.03
PT	0.41	1.00	0.45	0.39	0.19	0.07	0.21	0.09	0.27	0.09	0.07
SP	0.27	0.45	1.00	0.58	0.30	0.07	0.29	0.11	0.39	0.11	0.12
IT	0.22	0.39	0.58	1.00	0.49	0.16	0.19	0.15	0.60	0.03	0.05
FR	0.08	0.19	0.30	0.49	1.00	0.71	0.67	0.59	0.78	0.52	0.49
GE	-0.01	0.07	0.07	0.16	0.71	1.00	0.78	0.79	0.53	0.76	0.72
NL	0.08	0.21	0.29	0.19	0.67	0.78	1.00	0.65	0.51	0.77	0.70
GB	0.02	0.09	0.11	0.15	0.59	0.79	0.65	1.00	0.45	0.65	0.63
BE	0.16	0.27	0.39	0.60	0.78	0.53	0.51	0.45	1.00	0.35	0.33
DM	0.04	0.09	0.11	0.03	0.52	0.76	0.77	0.65	0.35	1.00	0.67
SW	0.03	0.07	0.12	0.05	0.49	0.72	0.70	0.63	0.33	0.67	1.00

Source: Author's computation using R

Even a simple analysis reveals the interesting results. Firstly, yields of the states of the Core seem to be more correlated between themselves than with the Peripheral states. Relatively high values are observed, e.g. correlation between France and Germany is *0.71*, France-Netherlands correlation is *0.67* and the value of Germany-Netherlands correlation is *0.78*. Yields of Belgium are highly correlated with yields of France, but co-movement with Germany and Netherlands is lower. Intra-group correlation of the Periphery is lower than in the previous cases (Portugal-Greece – *0.41*, Spain-Portugal - *0.45*), but much higher than correlation between the different groups, which is close to zero. All states outside the Eurozone have highly correlated yields between themselves as well as with the Core, but the correlations with countries from the Periphery are very low. In the end we note that Belgium differs from the other Core states – lower intra-group correlations, lower correlations with the non-Eurozone countries and higher correlations with the Periphery.

The results obtained from the previous analysis give us only limited information, because the observations from the pre-crisis period were analysed together with the crisis-era period. Now we split our sample into the two subsamples. The first one is from the pre-crisis period and consists of 2009 obsevations. The second subsample with 1370 observations represents data from crisis era (counted from the fall of Lehman Brothers). This simple action brings an important change of the previous results as can be seen in Tables 4.4 and 4.5. Before the

Table 4.4: Pearson's correlation - before the fall of Lehman Brothers

	GR	PT	SP	IT	FR	GE	NL	GB	BE	DM	SW
GR	1.00	0.63	0.48	0.29	0.31	0.30	0.62	0.28	0.29	0.52	0.60
PT	0.63	1.00	0.65	0.49	0.50	0.50	0.89	0.45	0.45	0.72	0.69
SP	0.48	0.65	1.00	0.35	0.36	0.35	0.66	0.32	0.31	0.55	0.52
IT	0.29	0.49	0.35	1.00	0.93	0.91	0.53	0.73	0.87	0.39	0.47
FR	0.31	0.50	0.36	0.93	1.00	0.93	0.53	0.75	0.89	0.40	0.47
GE	0.30	0.50	0.35	0.91	0.93	1.00	0.53	0.74	0.86	0.39	0.48
NL	0.62	0.89	0.66	0.53	0.53	0.53	1.00	0.44	0.48	0.72	0.67
GB	0.28	0.45	0.32	0.73	0.75	0.74	0.44	1.00	0.72	0.37	0.44
BE	0.29	0.45	0.31	0.87	0.89	0.86	0.48	0.72	1.00	0.36	0.44
DM	0.52	0.72	0.55	0.39	0.40	0.39	0.72	0.37	0.36	1.00	0.59
SW	0.60	0.69	0.52	0.47	0.47	0.48	0.67	0.44	0.44	0.59	1.00

Source: Author's computation using R

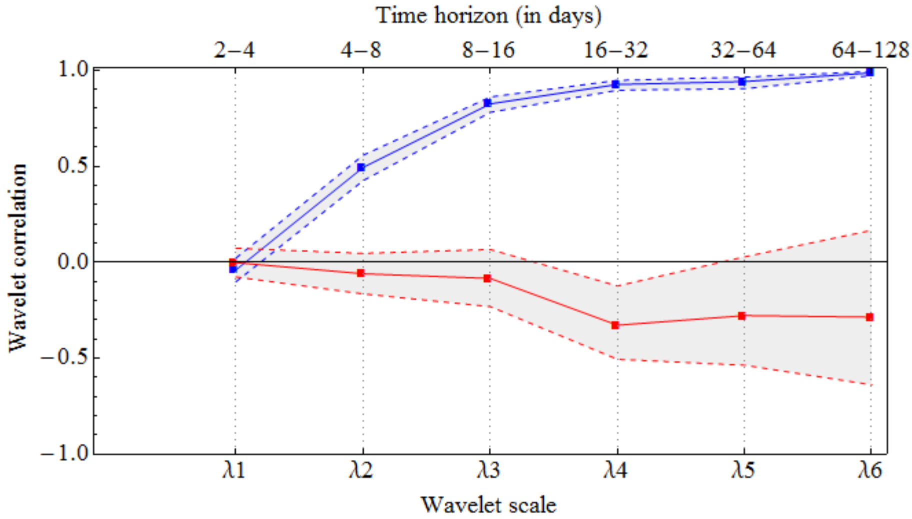
Table 4.5: Pearson's correlation - after the fall of Lehman Brothers

	GR	PT	SP	IT	FR	GE	NL	GB	BE	DM	SW
GR	1.00	0.37	0.24	0.22	0.03	-0.06	-0.03	-0.03	0.14	-0.05	-0.07
PT	0.37	1.00	0.38	0.35	0.08	-0.04	-0.01	0.00	0.21	-0.08	-0.09
SP	0.24	0.38	1.00	0.72	0.27	-0.05	0.09	0.02	0.44	-0.08	-0.05
IT	0.22	0.35	0.72	1.00	0.28	-0.09	0.05	-0.05	0.46	-0.10	-0.09
FR	0.03	0.08	0.27	0.28	1.00	0.65	0.73	0.54	0.73	0.56	0.50
GE	-0.06	-0.04	-0.05	-0.09	0.65	1.00	0.86	0.80	0.42	0.85	0.78
NL	-0.03	-0.01	0.09	0.05	0.73	0.86	1.00	0.73	0.53	0.79	0.72
GB	-0.03	0.00	0.02	-0.05	0.54	0.80	0.73	1.00	0.35	0.73	0.68
BE	0.14	0.21	0.44	0.46	0.73	0.42	0.53	0.35	1.00	0.35	0.29
DM	-0.05	-0.08	-0.08	-0.10	0.56	0.85	0.79	0.73	0.35	1.00	0.69
SW	-0.07	-0.09	-0.05	-0.09	0.50	0.78	0.72	0.68	0.29	0.69	1.00

Source: Author's computation using R

fall of Lehman Brothers the correlation between yields is higher in majority of elements of the correlation matrix. However, the differences between the Core and the Periphery are observable again, but they are lower. Completely different results can be seen in case of Netherlands – high co-movement with Portugal and lower co-movement with the Core states. From the post-Lehman Brothers sample we can conclude that the results are closer to the original correlation matrix. Correlation between the states from different groups decreased. Contrary to the previously observed results intra-group correlation increased in some cases, e.g. the value of Spain-Italy correlation was *0.35* before the crisis and *0.72* in the second subsample. A similar example from the Core group is the correlation between yields of Netherlands and Germany, where a jump

Figure 4.3: Wavelet correlation - example



Source: author's computation in R(package waveslim) and *Mathematica*. Blue lines denote estimate plus its confidence interval before the bankruptcy of Lehman Brothres, red ones stand for the same after the event.

from *0.53* to *0.86* is observable.

From the previous text we easily conclude that the results are diferent in the both subsamples. Now we motivate our usage of wavelets on an example of correlation between yields of Greece and Germany. Figure 4.3 shows wavelet (MODWT) correlation and confidence intervals on the first six scales[11] obtained from the subsamples.

The weakness of the previously employed method is revealed. Let us look at the pre-Lehman Brothers period.[12] The correlation on the first scale is only -*0.04*, on the scale λ_6 it is *0.979* - almost perfect correlation. It is easily observable that the post-Lehman Brothers estimate is different. On almost all scales (except λ_4) the correlation is negative, but with confidence intervals including zero line, which means that we are not able to decide whether the correlation is significantly different from zero. Generally we observe that heterogeneity among the particular scales is much lower than in the pre-Lehman Brothers period.

[11]First six scales $(\lambda_1, \ldots, \lambda_6)$ represent 2-128 day band. Here I have to thank mgr. Lukáš Vácha, PhD for provision of *Mathematica* code.

[12]All results are in Appendix B.1

The example in Figure 4.3 clearly indicates that the results differ across scales. Examining the pre-Lehman Brothers subsample we can see that the correlation on the scale λ_1 is close to zero and it is statistically insignificant. But on the other scales the correlation increases and on the sixth scale it can be seen almost linear relationship. The curve representing estimates from the second subsample shows a completely different story. The estimated correlation is negative, but the estimated confidence intervals contain zero (except the scale λ_4), which means that we cannot guarantee that correlation is different from zero. On the scale λ_4 correlation is *-0.32*. Hence use of wavelet analysis is justified. Without using wavelets we would not have discovered that in the longer run co-movement is almost linear. Ordinary Pearson's method shows that correlation is close to zero, but we see that significant negative correlation is presented.

Chapter 5

Wavelet Coherence Analysis

In this chapter data will be examined using wavelet coherence and the hypotheses I to III formulated in Chapter 2 will be tested. The methodology enables only to make a pairwise comparison, thus firstly we will discuss coherence within the groups established in the previous section (intra-group co-movement) and then analyse their inter-group relationships. In the inter-group analysis we examine the relationships between the Periphery and the other states. We will not focus on co-movement between the Core and the non-Eurozone states.

Let us start with the several technical notes related to the chapter. Firstly, the Matlab toolbox made by Grinsted, Moore & Jevrejeva[1] is used. Morlet wavelet, which advantage was described and explained in Chapter 3, will be employed. Secondly - we will not interpret (and for the sake of clarity not show) the phases. The reason is that we are primarily interested in the intensity of co-movement. The third note is related to the scope of the analysis in terms of periods (time horizon). We use daily data, hence the numbers on the scale/frequency vertical axis represent periods (cycles[2]) in days. We will not focus on scales representing periods higher than 256 days.[3] The reason is that the results above the level is difficult to interpret because of the cone of influence described in Chapter 3. Fourth note is about labelling of scales. In the following text we will comment how co-movement changed on the different scales through time. In order to make the comparison clearer, we will use the following notation:

[1]http://noc.ac.uk/using-science/crosswavelet-wavelet-coherence

[2]We will interpret the results in this chapter using terminology similar to Aguiar-Conraria *et al.* (2012).

[3]We follow Vácha & Baruník (2012).

1. scales, which represent time horizon between 2 and 8 days (thus periods approximately to one week) are labeled as the low scales.

2. scales representing the cycles between 8 and 32 days (approximately one week to one month) are labeled as the medium scales.

3. scales, which represent the periods between 32 and 128 days (approximately one month to 1/3 of a year) are labeled as the high scales.

4. scales representing other the periods up to 256 days (2/3 of a year) are considered to be the extremely high scales.

Another note points towards the two dates significant for the European debt crisis. Both of them were already mentioned in Chapter 2. First one is 15.9.2008 - the day when Lehman Brothers went bankrupt and which is considered to be the start of the "hot" phase of the global financial crisis (Mishkin 2010). Moreover, according to Lane (2012) it was the trigger of the sovereign debt crisis too. The second date is 20.10.2009. It was the day when the minister of finance Papakonstantinou announced that the budget deficit will be much higher than it was expected. Both dates will be labeled with the vertical lines.[4]

Last note is related to the figures in the chapter. We know that it is difficult to read the text inside, hence all pictures were uploaded to SIS in higher resolution.

5.1 The Core of the Eurozone

In the earlier text we classified Germany, Netherlands, Belgium and France as the Core. Firstly we will summarize common features of all their time-scale maps (Figure 5.1). The states were highly integrated since 2001. It means that coherence is significant on all examined scales.[5] It lasted through the whole non-crisis period. In the crisis era – after 20.10.2009 – coherence temporarily disappeared on the lower and medium scales, but remained (at least partially) on the higher scales.

[4] Colours of the lines change based on visibility and contrast.

[5] On the other hand black spaces indicate that on the lowest scales coherence was not significant all the time.

Now we mention specifics of the particular maps. For now we exclude Belgium from the analysis. For the other five states coherence is significant on all scales in the non-crisis era. Even when Lehman Brothers fell we are not able to see sharp decrease of co-movement (exception is Germany-Netherlands on the cycles of 16-32 days). After 20.10.2009 no massive decline of co-movement is observable too. In the case of Germany-France as well as France-Netherlands we observe significant drop of co-movement across scales, but later – after 2011. Only a low decrease is seen on the timescale map of Germany-Netherlands.

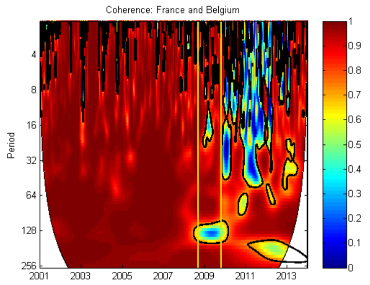
The time-scale maps with sovereign yields of Belgium look differently. After 20.10.2009 on the low and medium scales significant coherence almost disappeared. The weakest effect is observed between France and Belgium. But in the other two cases – Germany and Netherlands – the drop was more massive (in wider band) and the effect lasted longer. On the other hand it can be seen that in the end of the dataset co-movement increased and coherence is significant again.

To interpret the intra-group decrease of interdependence in the Core we have to recall several facts stated in Chapter 2:

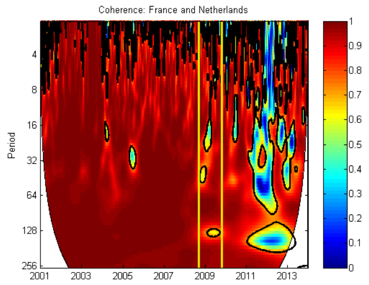
- France was in 2011 and 2012 heavily criticized by the rating agencies and owned a large share of Greek debt.
- Belgian banking sector suffered from toxic assets and had to be bail-outed. Moreover, Belgium had high pre-crisis debt-to-GDP ratio.

The first information should explain sudden drop of significant coherence of French yields with yields of Netherlands and Germany in 2011. The described Belgian situation can be the reason why its yields behave differently than the rest of the Core. Interesting finding is high co-movement between Germany and Netherlands despite warning messages related to Netherlands (Denk 2011).

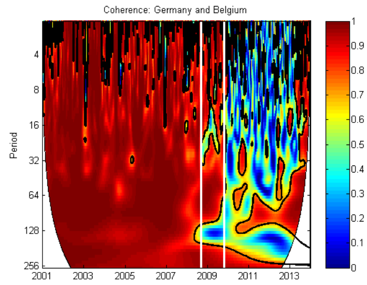
Figure 5.1: Intra-group coherence: The Core



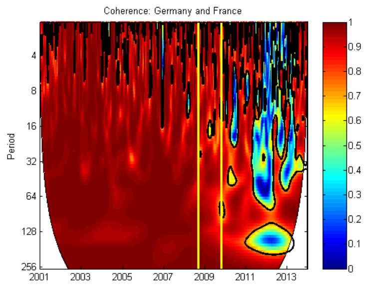
(a) France-Belgium



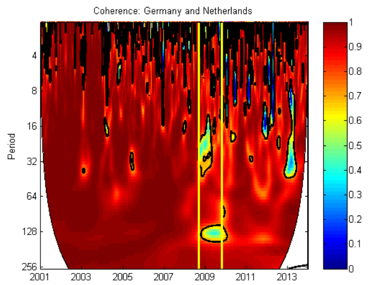
(b) France-Netherlands



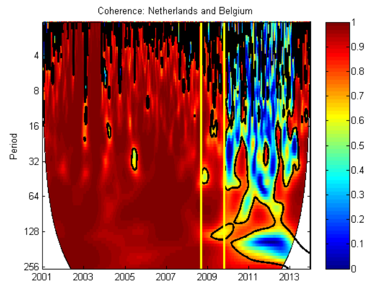
(c) Germany-Belgium



(d) Germany-France

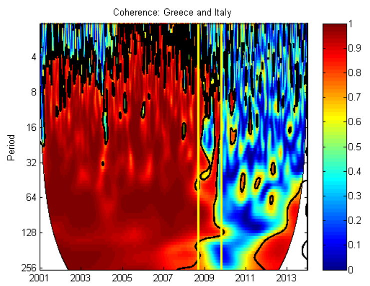


(e) Germany-Netherlands

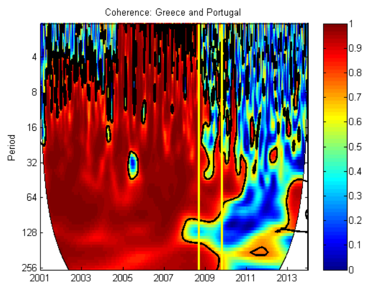


(f) Netherlands-Belgium

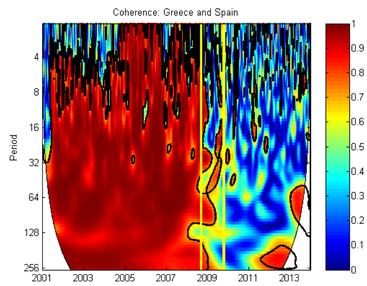
Figure 5.2: Intra-group coherence: The Periphery



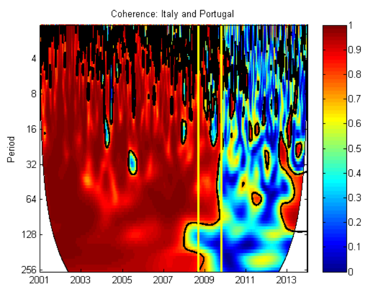
(a) Greece-Italy



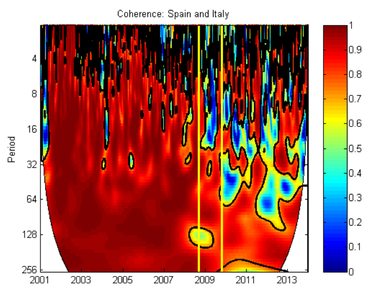
(b) Greece-Portugal



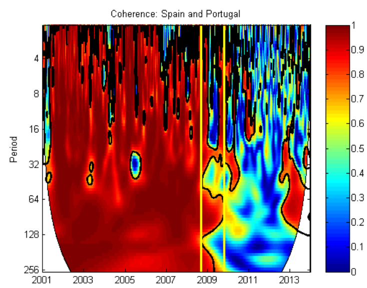
(c) Greece-Spain



(d) Italy-Portugal



(e) Spain-Italy



(f) Spain-Portugal

*Source:*Author's computation via Matlab

5.2 The Periphery of the Eurozone

In Chapter 4 we selected Greece, Spain, Portugal and Italy to be the Periphery and performed coherence analysis (Figure 5.2). As in the previous case we firstly point out the common signs of the pictures. Contrary to the Core we are not able to observe significant coherence since 2001 on all scales. When the final stage of the Eurozone was established, co-movement increased on majority of the scales, but we see a larger heterogeneity across scales than in the previous case, especially on the low and medium scales. Co-movement between Greece and the other three states increased after 2005 in 2-4 day band.

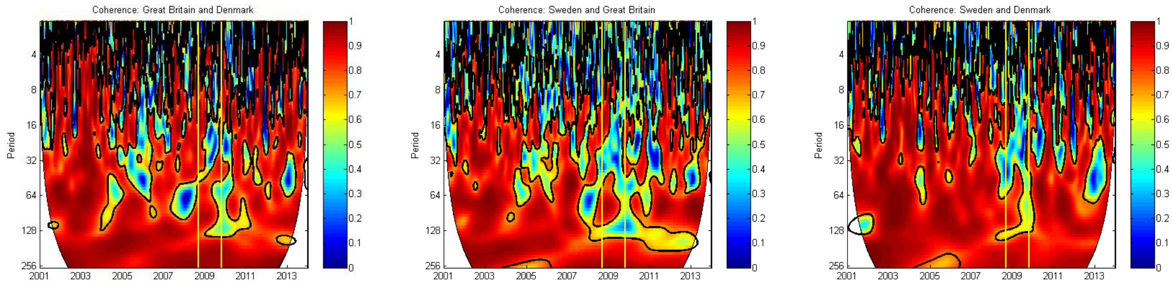
After the fall of Lehman Brothers coherence decreased, especially on the low scales and especially in case of Greece-Spain timescale map. After 20.10.2009 a sudden decrease occurred on Italy-Portugal and Greece-Italy map. Different case is the Spain-Italy co-movement, which did not decrease so much and coherence remained significant on the low scales and 64-256 day band. Moreover, in 2012/2013 spaces with high coherence occurred on the medium, high, and extremely high scales, but the results differ across the timescale maps (e.g. Greece-Italy: 64-256 day band, Italy-Portugal and Spain-Portugal: 16-128 band). Except these cases the co-movement between the Peripheral states was low in the crisis period and the pre-crisis integration was not established again (contrary to the Core states).

From the previous analysis we conclude that investors demanding rate of return carefully distinguished between development of situation in the Periphery. Only exception (in accordance with Dajcman 2013b) is the relationship between Spain and Italy, which exhibits high degree of co-movement even in the crisis.

5.3 The States Outside the Eurozone

The third group represents developed states, which are not the members of the Eurozone yet – Denmark, Sweden and Great Britain (see Figure 5.3). The significance of coherence on the lower scales varies, especially between Sweden and Great Britain. Even on the medium scales (8-32 days) the co-movement does not seem to be strong all the time. On the scales representing the cycles

Figure 5.3: Intra-group coherence: states outside the Eurozone



(a) Great Britain-Denmark (b) Great Britain-Sweden (c) Sweden-Denmark

Source: Author's computation via Matlab

of 128-256 days the degree of co-movement is stable and high even after the beginning of the crisis. The pattern of all three timescale maps is clearly different than the patters of the previous two groups.

It is hard to judge if the crisis affected the co-movement. After the bankruptcy of Lehman Brothers a decrease of coherence can be seen, especially on the medium and high scales (16-128 day band), but the effect is not as strong and persistent as in case of the Periphery. The most interesting is the Sweden-Denmark timescale map. There is a clearly observable decrease of coherence after the fall of Lehman Brothers as well as an **increase** after 20.10.2009.

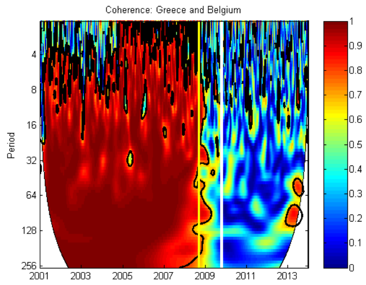
From Figure 5.3 we see that the non-Eurozone states were less integrated than the Core and the Periphery before the crisis came, but in the crisis period their mutual co-movement seems to be stronger than on the Periphery. The finding is in accordance with Clayes and Vašíček (2014). The possible reason, why co-movement on high frequencies is relatively low and volatile, can be presence of the exchange rate risk (in contrast to the Eurozone, where Euro wiped it out for investors from the inside).

5.4 Inter-group Analysis

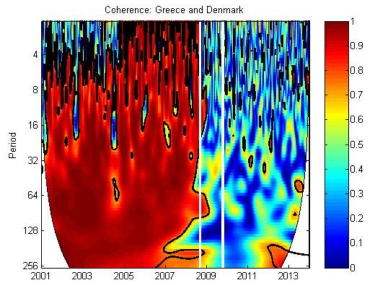
5.4.1 Greece

The next part of the chapter is dedicated to the inter-group coherence analysis. In the following section we focus on co-movement between the Peripheral states

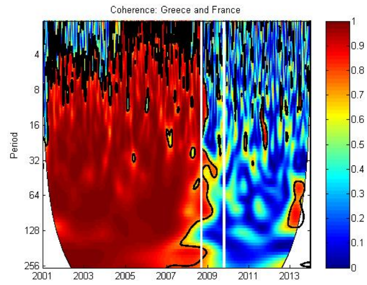
Figure 5.4: Inter-group coherence: Greece and Spain



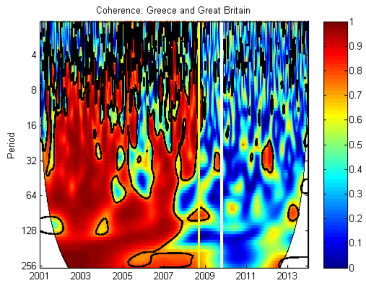
(a) Greece-Belgium



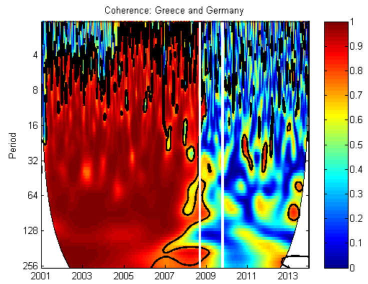
(b) Greece-Denmark



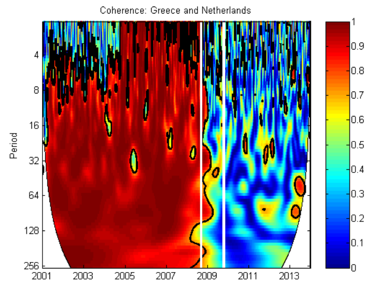
(c) Greece-France



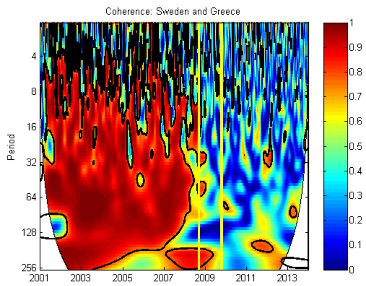
(d) Greece-Great Britain



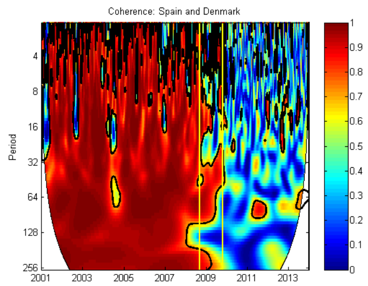
(e) Greece-Germany



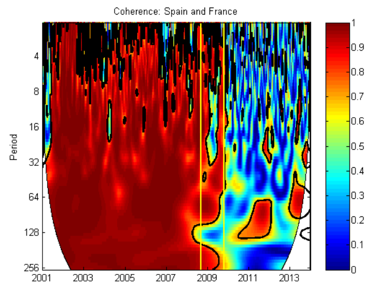
(f) Greece-Netherlands



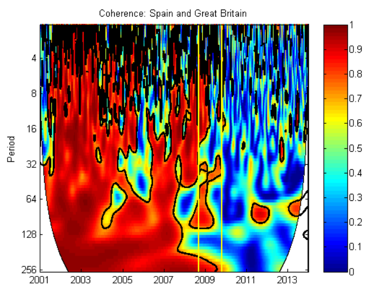
(g) Greece-Sweden



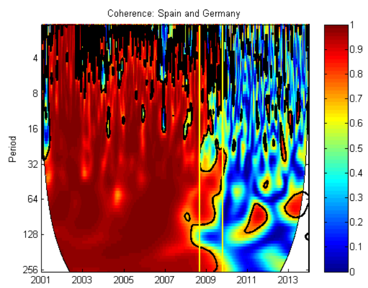
(h) Spain-Denmark



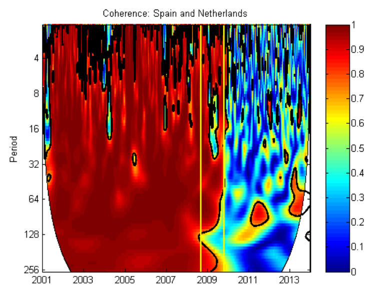
(i) Spain-France



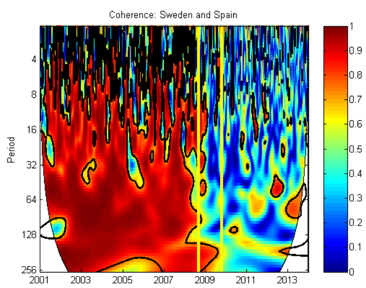
(j) Spain-Great Britain



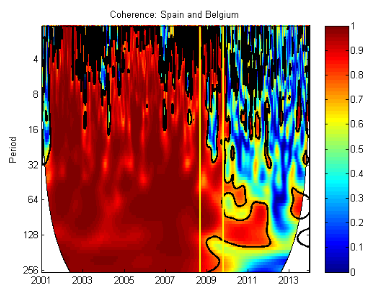
(k) Spain-Germany



(l) Spain-Netherladns



(m) Spain-Sweden



(n) Spain-Belgium

Source:Author'scomputation via Matlab

and the other countries from our dataset. Seven pictures in Figure 5.4 show interdependence between sovereign yields of Greece and Great Britain, Denmark, Germany, Belgium, Netherlands, France and Sweden. In all seven pictures a similar pattern is observable. From 2000 until 2005 almost no significant coherence can be measured on the lowest scales (2–4 day band). In the 8-256 day band coherence is high. After 2005 (according to Kouretas & Vlamis (2010) economic boom took place in Greece) coherence on the low scales increased while coherence on the others remained high, thus Greece became more integrated with the other states with the only two exceptions - Great Britain and Sweden. After the fall of Lehman Brothers coherence sharply decreased on all scales representing the cycles up to 256 days. On the other hand after 20.10.2009 no additional massive decrease can be observed on the timescale map. No important island of high coherence is seen in the crisis period. However, there are smaller ones, especially in the 16-128 day band. Since the autumn of 2012 (where the ESM was launched) slightly larger significant spaces occurred on the high scales (Greece-France, Greece-Belgium, Greece-Netherlands). But the majority of the timescale maps shows low coherence. Hence we are allowed to say that the crisis shattered integration of Greek sovereign yields with other yields.

Let us focus on the relationship between Greece and Great Britain, which is an illustrative example why wavelet coherence is useful. In the beginning of the sample period coherence was not significant on scales representing the cycles lower than 16 days and on the lowest scales (2-4 day band) it became insignificant until 2005. In contrast, we can see a significant co-movement in 8-256 day band, which lasted until 2005. In this year the changes across all scales are observable and coherence on the medium and high scales disappeared, but there is a higher amount of islands of significance on the lowest scales. After 2007 coherence is significant on the scales, which represent the cycles of 4-32 days, but on higher scales it disappeared. We can conclude that yields of Greece and Great Britain were not fully integrated - there was always some scale, where co-movement is low. Since the fall of Lehman Brothers the timescale map looks like the previously analysed one - majority of coherence disappeared before 20.10.2009.

5.4.2 Spain

Let us analyse the timescale maps related to Spain (Figure 5.4). Again, on all pictures we observe initial increase of co-movement on the scales representing the cycles of periods up to 16 days and a sudden decrease on the majority of scales following in the crisis era, in which coherence remained low until the end of our dataset. Similarly to Greece there is a clear difference between Spain-Great Britain and Spain-Sweden timescale maps and the rest of the pictures.

Co-movement with yields of Germany, France, Denmark and Netherlands has very similar pattern. Shortly before 15.9.2008 coherence vanished around the cycle of 128 days, on medium scales disappeared after the date. But we see that a major drop occurred after 20.10.2009 on all scales. The drop is visible especially on Spain-France timescale map.

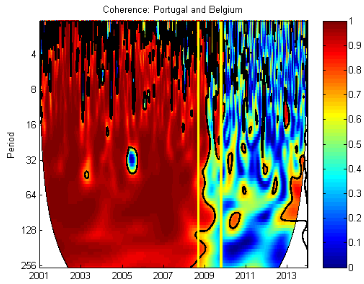
Yields of Belgium behaved differently. After the fall of Lehman Brothers a decrease is observable on the medium scales, but the overall drop of co-movement is much lower than with the other states, especially on the high scales. A large amount of coherence disappeared after 20.10.2009, but – and this is the main difference – it remained significant on 64-128 day band long after the date. Then gradually disappeared in 2012, when massive downgrades of EU member states (Spain and Belgium included) took place.[6]

5.4.3 Portugal

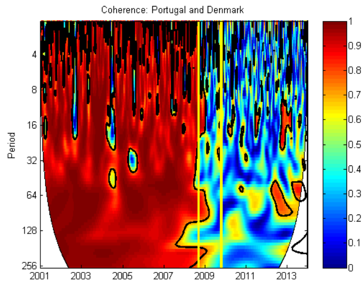
The next examined state is Portugal. The results of wavelet coherence analysis are presented in Figure 5.5. In 2001 Portugal was more integrated on the European sovereign bonds market than Greece and Spain. Except low scales in the beginning of the dataset significant coherence is observable on all cycles of period up to 256 days. On the lowest scales an era of significant coherence was usually followed by a period of low coherence. Only the time-scale maps of Portugal-Sweden and Portugal-Britain differ. It can be seen large amount of insignificant coherence, especially on the scales representing the cycles of 4-64 days. After 15.9.2008 coherence began to decrease on all scales and generally in

[6] We do not want to claim that decrease was because of downgrade itself, only that both events coincide.

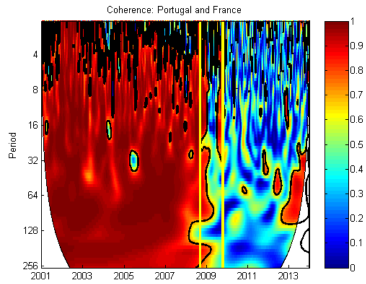
Figure 5.5: Inter-group coherence: Portugal and Italy



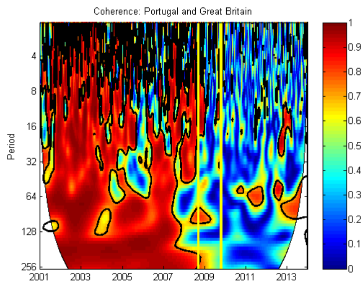
(a) Portugal-Belgium



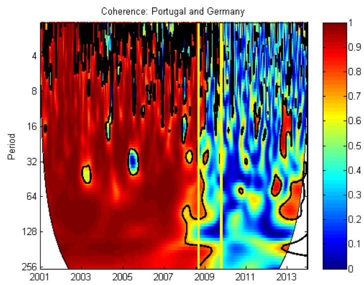
(b) Portugal-Denmark



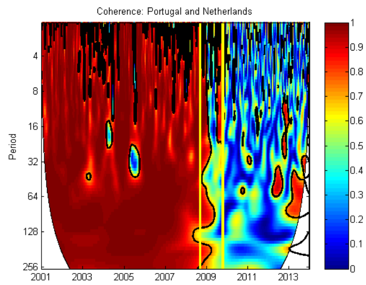
(c) Portugal-France



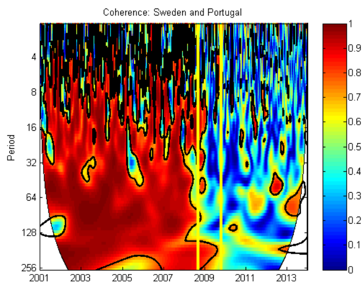
(d) Portugal-Great Britain



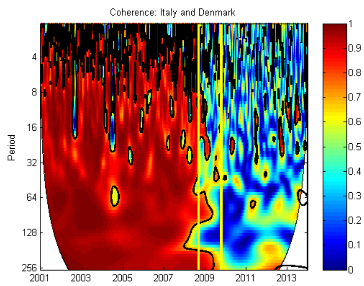
(e) Portugal-Germany



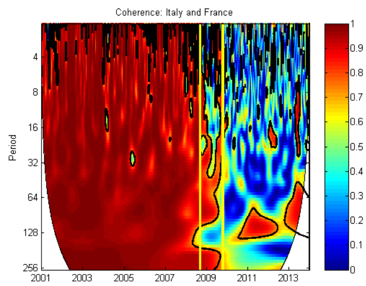
(f) Portugal-Netherlands



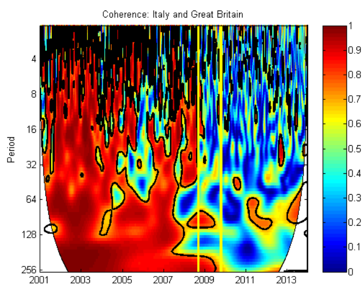
(g) Portugal-Sweden



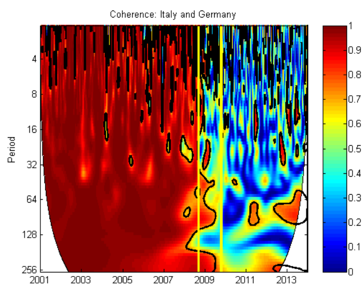
(h) Italy-Denmark



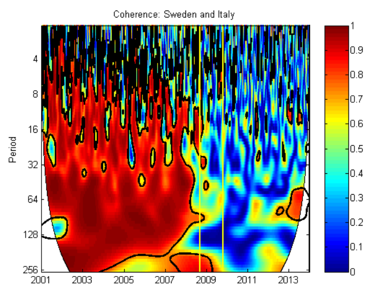
(i) Italy-France



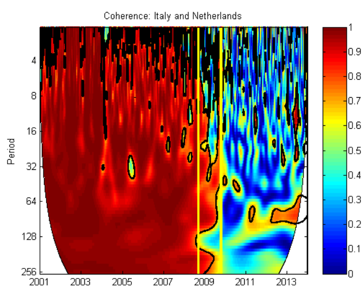
(j) Italy-Great Britain



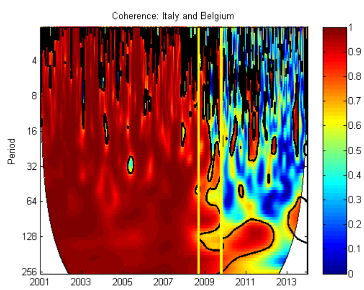
(k) Italy-Germany



(l) Italy-Sweden



(m) Italy-Netherlands



(n) Italy-Belgium

Source: Author's computation via Matlab

16-256 day band disappeared before 20.10.2009. Shortly after the second date coherence vanished even on the low and medium scales. Similarly to Spain only smaller islands of significant coherence occurred in the end of the dataset (2012-2013).

5.4.4 Italy

The last country, which yields and their co-movement will be analysed in Figure 5.5, is Italy. Its pre-crisis results are similar to the results of Spain and Portugal. After the bankruptcy of Lehman Brothers we see an immediate drop on Italy-Denmark, Italy-Germany and Italy-Great Britain timescale maps. Immediately after the speech of Papakonstantinou coherence jumped down in case of France, Netherlands and Belgium (2-64 day band). In the crisis era almost all significant coherence disappeared. There are only two more important exceptions. The first one is Italy-Belgium timescale map, where it remained even in the crisis era (similar pattern as in Spain-Belgium relationship), the second one is an island on Italy-France map in the 64-128 day band.

All seven pictures demonstrate a drop of coherence between Italian yields and yields of the other states - the same pattern was observable on the Periphery overall. Italy did not suffer a banking crisis like Spain or Portugal or did not falsify its accounting like Greece. But we clearly see that investors feel that Italy belongs to the Periphery.

5.5 Summary of Chapter 5

In the chapter we have used wavelet coherence to analyse co-movement of yields between 11 states of the EU. We have divided them into the three groups – the Core, the Periphery and the states outside the Eurozone. Then we executed the intra- as well as the inter-group analysis. Especially we studied how co-movement behaved near two significant dates – 15.9.2008 and 20.10.2009.

The results indicate that the big differences were in the Eurozone since it was established. The Core states were fully integrated – high coherence was observed on all measured scales. On the other hand states from the Periphery did not emanate similarly high integration, especially on the lowest scales. But even on the lowest scales coherence gradually increased on the Periphery. Integration inside the Eurozone can be clearly seen if we look at co-movement between yields of the states outside the Eurozone. Hence one can conclude that the Eurozone unified co-movement of sovereign yields of the member states.

Things changed when the crisis came and Lehman Brothers fell. Again the differences between the Core and the Periphery were manifested. Except Belgium we did not observe any massive decline of co-movement in the Core. Coherence decreased later - in 2011, thus long time after the disclosure of public deficit - but before 2014 it seems that the degree of co-movement is high again. The timescale maps of the Periphery states tell us different stories. There was a massive drop of co-movement, almost all significant coherence disappeared. An interesting finding is that in case of Greece it disappeared after the fall of Lehman Brothers, in some timescale maps of the other states after the speech of Papakonstantinou. But in the majority of cases (except the Spanish-Italian relationship) high co-movement was not established again. Coherence between the non-Eurozone members was not affected too much and in the crisis period their co-movements seem to be stronger. Since October 2012 on many timescale maps (especially in intra-group analysis) we can observe bigger red islands on the scales, which represent the cycles of periods of 32–128 days. It corresponds to the time when the ESM was established. If significant coherence was really result of a creation of the ESM, then the action had effect only in longer run, not in short run. Coherence between the groups decreased too in the crisis era. Possible reason why it vanished is that the economies of the Peripheral states were more severely hit by the crisis. Moreover, according to the studies reviewed in Chapter 2 sensitivity on national macroeconomic determinants increased in the crisis – investors more carefully distinguieshed between the Eurozone states. De Grauwe & Ji (2013) proposed an alternative explanation. According to them the Peripheral states are trapped in the self-fulfilling prophecies – they are the victims of negative market sentiments. Effect of the global uncertainty can be presented too.

The results are in accordance with the articles reviewed in Chapter 2. Bhanot *et al.* (2012) point out on a significantly strong relationship between yields of Spain and France. Antonakakis (2012) as well as Dias (2012) and Clayes & Vašíček (2014) pointed out that Belgium differs from the other Core states. We explained it by a remediation of banking sector and a high debt-to-GDP ratio. They observed heterogeneity of the results among the non-Eurozone states.

Moreover, Denmark is considered to be closer to the Eurozone than the other states. According to Claeys & Vašíček (2014) main reason is that Denmark participates in ERM II.[7]

Now we will closely compare the results with the paper written by Inoue *et al.* (2013).[8] They observed massive decrease of conditional correlation between Germany and the Peripheral states. After 20.10.2009 massive drop of co-movement is observable in the following cases: Spain, Portugal and Italy. On the other hand co-movement with two states of the Core – France and Netherlands - decreased only temporarily. Moreover, the correlation with Belgium is more similar to the development on the Periphery. Even the previously mentioned studies observed that Belgium differs from the other states of the Core. That is exactly what we have observed. Using wavelet coherence we only add that in case of France the drop was observed only on the low and medium scales. In the end of 2013 (which is not included in dataset of Inoue *et al.* 2013) coherence is again significant on the majority of the scales. The findings differ when Inoue *et al.* (2013) compared conditional correlation between Italy-Belgium and Italy-Spain. Their results exhibit a high degree of similarity even in the crisis period – co-movement decreased, but not as much as in case of correlation between Germany and Periphery states. Using wavelet coherence **our results are completely different**. Coherence in the crisis part of Italy-Belgium timescale map is almost completely insignificant. Contrary to it the timescale map of Italy and Spain has large amount of significant spaces across all scales. The results can be explained by the differences of the both methods - coherence is an unconditional measure of co-movement.

In the end of Chapter 2 we stated three hypotheses related to sovereign yields of bonds of the EU member states. Let us recall them and decide if they were proven to be correct or not.

Hypothesis 1 *In the crisis period co-movement between sovereign yields decreased.*

The thesis proved that the hypothesis is correct. A significant decrease (at least on some scales) is observable in all cases.

[7]Exchange Rate Mechanism 2 is a system (part of EMU), whose aim is to preserve stabilty of exchange rate toward Euro.

[8]The article is the most similar to ours: no spreads, daily 10Y data, no additional explanatory variables. Only difference is that DCC produces conditional correlation.

Hypothesis 2 *The results of the hypothesis 2 differ based on the group to which a particular country belongs.*

Although there is a heterogeneity in the groups we clearly observe that the Core is less disintegrated than the Periphery after the fall of Lehman Brothers. A specific pattern is observable in a group of the states outside the Eurozone. Before the crisis co-movement on the Periphery used to be (at least on the scales representing the cycles of 2-8 days) weaker. Situation changed during the crisis, when almost all significant coherence between the Peripheral states vanished. These findings support the hypothesis. On the other hand we are not able to decide whether the intra-group relationships in the Periphery are stronger or weaker than the inter-group relationships.

Hypothesis 3 *The results of previous two hypotheses differ across scales.*

This is the key hypothesis of the thesis and wavelet coherence analysis is a suitable tool for its testing. From the previous summary it can be seen that there is some heterogeneity between the scales. It holds especially for the Core. The relationship between yields of Belgium and the other states differs across scales (e.g. Belgium-Spain relationship and peninsula of significant coherence in 128-256 day band). The other timescale maps obtained from the intra-group analysis of the Core show that co-movement decreased on low and medium scales, but not on large scales. Another example is a significant co-movement between Italy and Greece on the high scales, which began in the crisis era. Similarly we can see the bigger island of significant coherence starting after after the ESM was established on 32-128 days band. Both examples are in contrast with insignificant coherence estimated on the lower scales. In the crisis era we observed increase of significant coherence on high an extremely high scales, which corresponds to the time when the ESM was launched. Hence our analysis showed that there are differences between the scales and thus the hypothesis is confirmed.

In the end let us state the shortcomings of the analysis. When we described the results, we in fact distinguished (as all previously mentioned articles using the method) only between two states - coherence is either significant or insignificant. We are hardly able to comment how co-movement differs in fully significant or insignificant area. This disadvantage will be compensated using wavelet multiple correlation in the next chapter. The second shortcoming is related to confidence intervals obtained via Monte Carlo simulation. The authors of the toolbox warn that if coherence is tested against AR(1) with Gaussian white noise, then the distribution of the examined time series should be normal too, otherwise some significant coherence on low scales can be labeled as insignificant. As we know from Chapter 4, Jarque-Bera test rejected normality in all cases. Hence the results from the low scales should be interpreted with caution. Moreover, we have to repeat the disclaimer stated earlier. We only point out that the significant changes in coherence occur near to the previously mentioned events. Wavelet coherence is not a precise tool for the rigorous identification of structural breaking points. But there are other tools (e.g. wavelet bivariate or multiple correlation), which are more accurate in terms of confidence intervals. The question of confidence intervals will be discussed in the next three chapters.

Chapter 6

Wavelet Multiple Correlation Analysis of Co-movement

The previous chapter was about the pairwise comparison of the states from our dataset. New methodology developed by Fernandez-Macho (2012a) allows us to extend our previous analysis and examine the development of co-movement of the whole groups (the Core, the Periphery, the states outside the Eurozone) plus overall co-movement of all states in our dataset together. Fernandez-Macho (2012a) applied the method on the analysis of 11 stock markets in the EMU. According to the results (daily data from 2000 to 2009) multiple correlation significantly differs across scales. Tiwari *et al.* (2013) applied it on co-movement between Asian stock markets with the similar results. Moreover, these works state that multiple correlation is high and increasing with scales. We want to discover, if interdependence within the groups significantly changed in the crisis period. While writing an empirical analysis we have to put stress on the word *significantly.* Using wavelet multiple correlation we compute confidence intervals on 95 % level of confidence. If there is at least one case when the confidence intervals of the estimated correlation on a particular scale do not overlap, then we are allowed to claim that the results differ across scales. We will begin with splitting our sample into the two subsamples - before and after the fall of Lehman Brothers. In some studies (e.g. Gallegati 2012) a crisis sample starts in 2007. But the fall of Lehman Brothers is considered to be the trigger of the second - and more intensive - crisis phase[1] (Mishkin 2010). Moreover, the results from Chapter 5 justify our choice - no massive decrease of coherence is obsevable before 15.9.2008. Let us recall that the period before

[1]Hence for simplicity we will denote subsample from period before Lehman Brothers as ”pre-crisis”.

contains 2009 observations, the period after consists of 1370 observations.

Now let us mention how multiple correlation supplements previously used coherence method. In Chapter 5 we employed wavelet coherence for pairwise comparison. Contrary to it with wavelet multiple correlation we are able to analyse the whole group together. Another advantage is that we will obtain exact values and thus we able to decide whether correlation differs or not through scales and groups. Similarly to Chapter 5 let us point to the several technical decisions related to the analysis. Firstly we have to choose the wavelet filter. Based on Fernandez-Macho (2012a) and Tiwari *et al.* (2013) the Least Asymmetric LA(8) wavelet filter proposed by Daubechies (1988) will be used. Its advantages were described in Chapter 3. Secondly we have to decide how deeply (in terms of scales) we will go in our analysis. We will use the MODWT, more precisely the first six wavelet scales $(\lambda_1, \lambda_2, \ldots, \lambda_6)$, because the amount of the observations in the second subsample does not allow to use more.[2] These scales - while using daily data - represent the cycles of 2-4, 4-8, 8-16, 16-32, 32-64 and 64-128 days. For computation **R** package *wavemulcor* written by Fernandez-Macho (2012b) is employed. The numerical results are in Appendix *B*.2

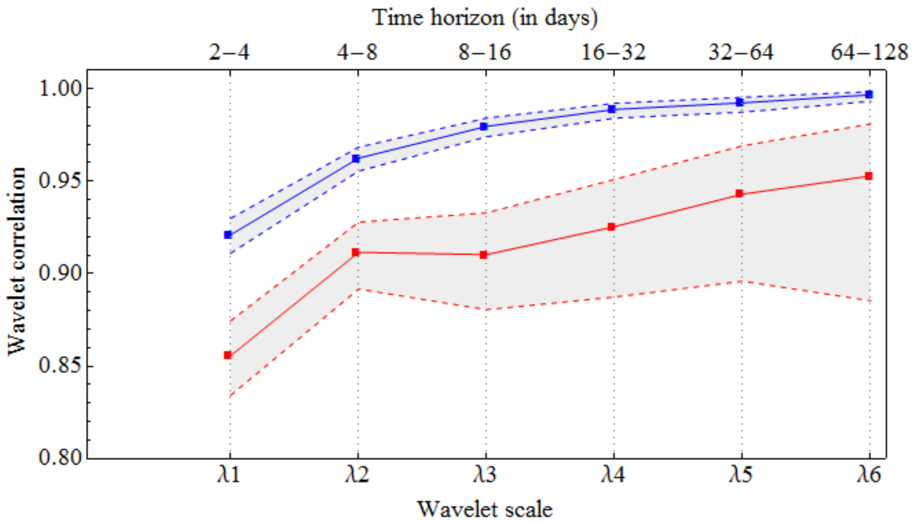
6.1 The Core of the Eurozone

Firstly we will analyse multiple correlation in the Core states. The results are presented in Table B.4 and Figure 6.1. Generally, we observe that pre-crisis co-movement is high. Moreover, the results show us that before the fall of Lehman Brothers multiple correlation differed across scales, because their confidence intervals do not overlap. The lowest value is seen on the first scale, but even the value is high – *0.9210*. On the scale λ_6 the value is close to perfect correlation. The estimates are increasing with scales.

It is evident that in the post-Lehman Brothers subsample the results changed. Estimated multiple correlation decreased on all scales. The decrease is statistically significant, because the confidence intervals of the pre- and the crisis subsample do not overlap on any scale. It is in accordance with the results

[2]The confidence intervals are too wide.

Figure 6.1: Wavelet multiple correlation: the Core



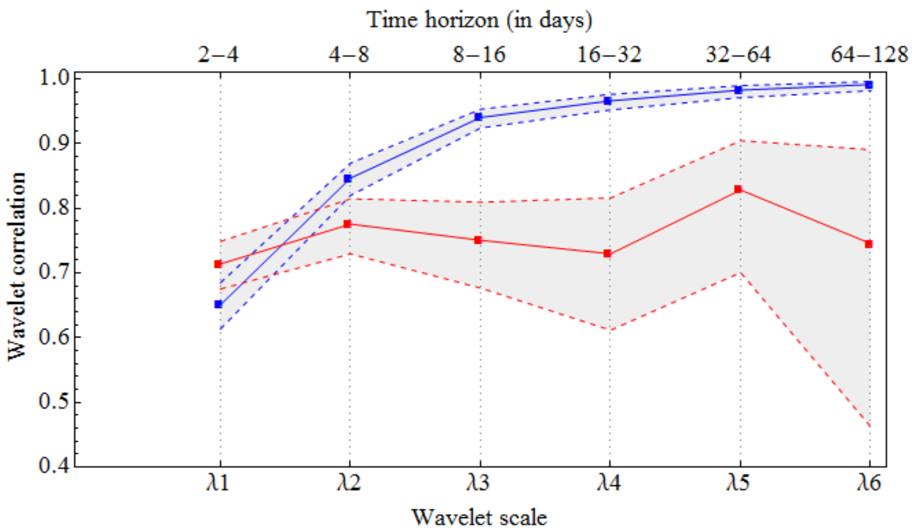
*Source:*Author's computation via R (wavemulcor). Blue lines denote multiple wavelet correlation estimate plus its confidence intervals computed from the subsample before the fall of Lehman Brothers, red lines stand for the same after the fall.

obtained in the previous chapter. Another important change is that now the confidence intervals – except the first scale – overlap. It means that in the crisis period we are not allowed to say that multiple correlation differs across scales between λ_2 and λ_6. Contrary to this fact multiple correlation on the scale λ_1 is significantly lower. Hence – again – the results differ across scales, but the heterogeneity is limited now.

6.2 The Periphery of the Eurozone

The chapter follows to provide similar analysis for the states on the Periphery (see Figure 6.2 and Table B.5). Again, let us firstly describe the pre-crisis results. It can be seen that the difference between the lowest and the highest estimated values is higher than in case of the Core. On the scale λ_1 correlation is only *0.6505*, on the scale λ_6 it is - again - almost 1. It is in accordance with the results from coherence analysis, where we measured low co-movement on the low scales (represented by the scales λ_1 and λ_2 in this chapter) and high co-movement on the medium and partially on the higher scales (which represent

Figure 6.2: Wavelet multiple correlation: the Periphery



*Source:*Author's computation via R (wavemulcor). Blue lines denote multiple wavelet correlation estimate plus its confidence intervals computed from the subsample before the fall of Lehman Brothers, red lines stand for the same after the fall.

the periods of 16-64 days). From Figure 6.2 we can clearly read that multiple correlation on the first scale is different than the others. Statistical properties of the confidence intervals confirm it – from the lower and the upper borders we read that the first three scales have statistically different multiple correlation.

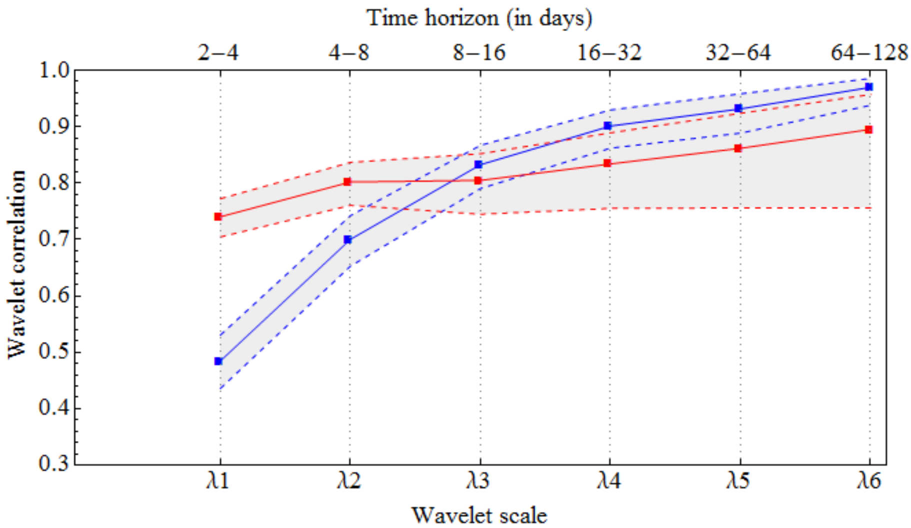
After the fall of Lehman Brothers we observe a decrease of multiple correlations on all scales, except the first one, where the estimate increased (insignificantly). Contrary to the first scale, the decrease of multiple correlations on all other scales is statistically significant.[3] Moreover, if we look at the confidence intervals on all scales in the crisis subsample, it can be seen that all of them are heavily overlapping. Hence the hypothesis that multiple correlation is heterogenous across scales cannot be confirmed after the fall of Lehman Brothers.

6.3 The states outside the Eurozone

Now we compute and compare the multiple correlations of the non-Eurozone states for the both subsamples. From Table 6.3 several important information

[3]The drop on the scale λ_2 is significant, but the gap between the confidence intervals is very small.

Figure 6.3: Wavelet multiple correlation: the states outside the Eurozone

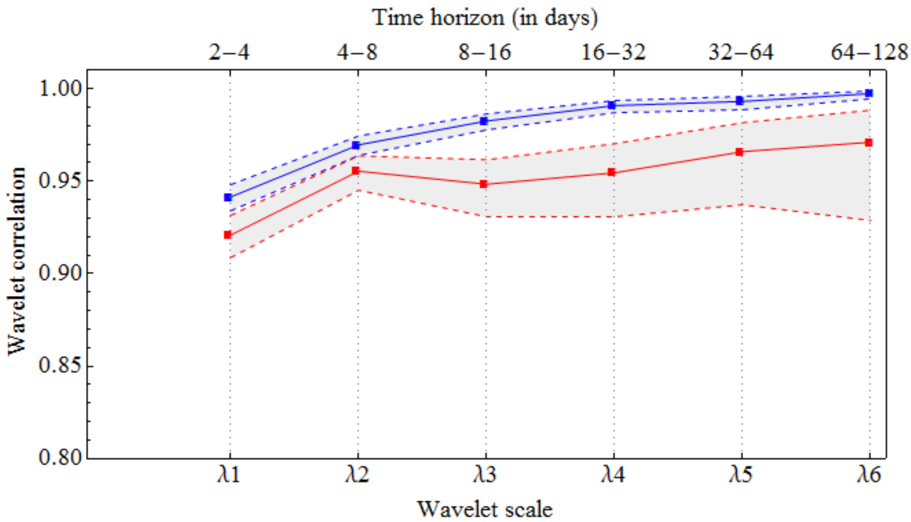


*Source:*Author's computation via R (wavemulcor). Blue lines denote multiple wavelet correlation estimate plus its confidence intervals computed from subsample before the fall of Lehman Brothers, red lines stand for the same after the fall.

can be read. Firstly – and it is not too surprising with respect to the results of Chapter 5 – in the pre-crisis period correlation is lower than in case of the other two groups. It should carefully lead us to the conclusion that the integration of the market after the creation of the Eurozone had no effect on the older EU members with its own currency.[4] However, the pattern of scale-by-scale correlation is similar to the other two groups. It is increasing with the scales and again we are allowed to say that the results differ across scales.

From the post-Lehman Brothers subsample we have obtained a completely different picture. The most important change can be seen on the first scale – short-run correlation between the yields rapidly and significantly increased. Similar commentary can be made in case of λ_2, only with the difference that the change is not so striking. On the other scales a decrease is observable, but we are not able to confirm it statistically – the intervals are overlapping. Moreover, in the crisis period heterogeneity of co-movement across scales decreased.

Figure 6.4: Wavelet multiple correlation: all states together



Source: Author's computation via R (wavemulcor). Blue lines denote multiple wavelet correlation estimate plus its confidence intervals computed from subsample before the fall of Lehman Brothers, red lines stand for the same after the fall.

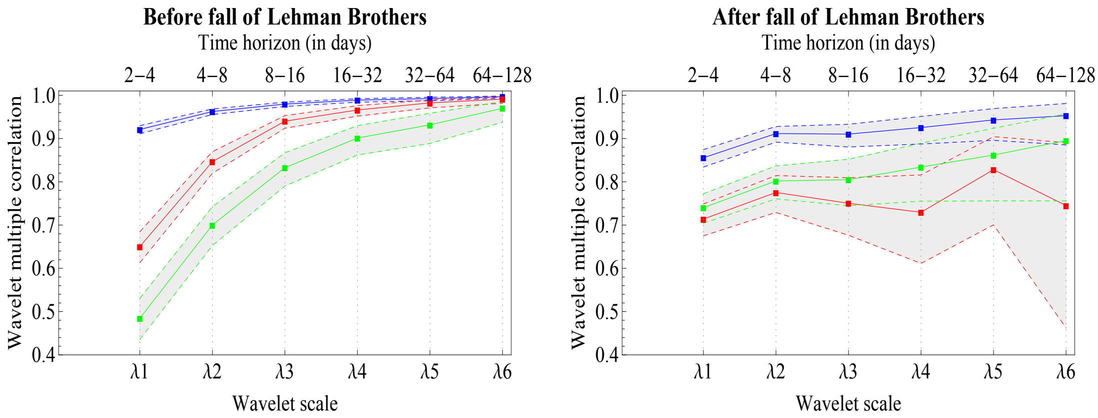
6.4 All states together

In the end we will analyse multiple correlation of all states in our dataset (Figure 6.4 and Table B.7). Before the crisis overall multiple correlation was significantly different across scales. Similarly to the Core correlation is increasing with scales. The values are high – the lowest one is *0.9415*. On the sixth scale co-movement is almost linear.

It is not surprising that in the second subsample the estimates changed. Again, we see that values of multiple correlation are higher than *0.9*, but all of them fell in comparison with the pre-crisis era. Comparison of the confidence intervals from the both subsamples shows that the drop of multiple correlation is significant on all scales (but a gap on the scale λ_2 is almost negligible).

[4]We are not allowed to generalize for all non-Eurozone states because of data selection, as we have written in Chapter 4.

Figure 6.5: Wavelet multiple correlation: comparison of all groups



*Source:*Author's computation via R (wavemulcor). Green lines stand for correlation and confidence intervals of the groups of the states outside the Eurozone.

6.5 Summary of Chapter 6

Wavelet multiple correlation analysis was employed to bring the answers on the three questions:

1. Has co-movement between sovereign yields decreased in the crisis era?
2. Are the results different for the different groups?
3. Are the results different across scales?

The answer on the first question is "YES" for the Core and the Periphery. Except the first scale decrease is observable on the Periphery. Different situation is with the non-Eurozone members, where correlation on the two lowest scales significantly increased and the changes of the rest of the results are statistically insignificant. The findings are consistent with coherence analysis made in Chapter 5 and support the previous findings.

The answer on the second question compares all three groups (see Figure 6.5). If we compare their multiple correlations, we will realize that the estimates of the Core are higher than the ones from the other two groups on all scales before the fall of Lehman Brothers. Multiple correlation of the Periphery group is higher that correlation of the states outside the Eurozone. After the fall the confidence intervals are overlapping, but we are allowed to say that multiple correlation is the highest - again - in the Core on the first three scales.

The third question is crucial for the thesis. During this chapter we have compared the confidence intervals of the estimates from the different scales and explored whether they overlap. In the pre-Lehman Brothers era the significant differences are observable in the Core and some scales significantly differ on the Periphery as well as in the states outside the Eurozone. The pre-crisis graphs of the both groups are very similar to the results obtained by Fernandez-Macho (2012a) and Tiwari *et al.* (2013) for stock indices - generally the results increase with scales and are significantly heterogenous. In the second subsample situation changed, especially on the Periphery and the states outside the Eurozone. Now we are not able to statistically confirm that multiple correlation is heterogenous. Hence heterogeneity of the results decreased in the crisis era.

Chapter 7

Contagion on the Sovereign Bonds Market in the EU: Wavelet-based Approach

In this chapter we will test the hypothesis whether the contagion from Greece was presented during the crisis. We will analyse the changes of correlations betwen yields of Greece and the other states from our dataset (10 relationships in total). The procedure follows the already mentioned Gallegati (2012) and was described in Chapter 3. Before we start, we have to make several important decisions related to the procedure. Firstly we have to choose the date when the contagion occurred. We will use the same two dates as in Chapter 5 - 15.9.2008 and 20.10.2009. The second question is how many scales we should use for our test? We will follow Gallegati (2012). He argues that a contagion occurs in the short run. Hence he recommends usage of the following scales: λ_1 (2-4 days), λ_2 (4-8 days) and λ_3 (8-16 days). Contrary to the previous chapter we will use a window with the length equal to one trading year[1] (250 observations) before as well as after 15.9.2008 and 20.10.2009. Main argument for justification of this approach is following. In Chapter 2 we wrote that the first phase of the global financial crisis occurred since 2007 (Mishkin 2010). Using a window with the length equal to 250 observations our pre-Lehman Brothers sample is not the really pre-crisis one.[2] Hence we compare wavelet correlation in the three crisis periods:

- Detection of the contagion during the bankruptcy of Lehman Brothers is

[1]It is length of a window recommended by Ranta (2010).
[2]But we will refer to them in this way for simplicity.

made by comparison between the latent-crisis period (the window before the fall of Lehman Brothers) and the period of the "hot" phase of the financial crisis.

- When detecting the contagion near to 20.10.2009 we compare the window partially covering the after-Lehman Brothers subsample from the previous test with another window representing the sovereign debt crisis in the EU.[3]

Hence the approach allows us to decompose a potential jump or a drop of correlation between two events. Last question is which wavelet will be used. Similarly to multiple wavelet correlation LA(8) wavelet filter will be employed as in Gallegati (2012). However, in the cases of the positive results their robustness will be checked by employing other two wavelet filters (following Gallegati *et al.* 2013) – Daubechies D(4) and Haar.

The analysis itself is made using **R** package *waveslim* created by Whitcher (2013). We perform the test on 95 % level of confidence. In the chapter we show only the graphical results.[4]

7.1 Contagion After the Fall of Lehman Brothers

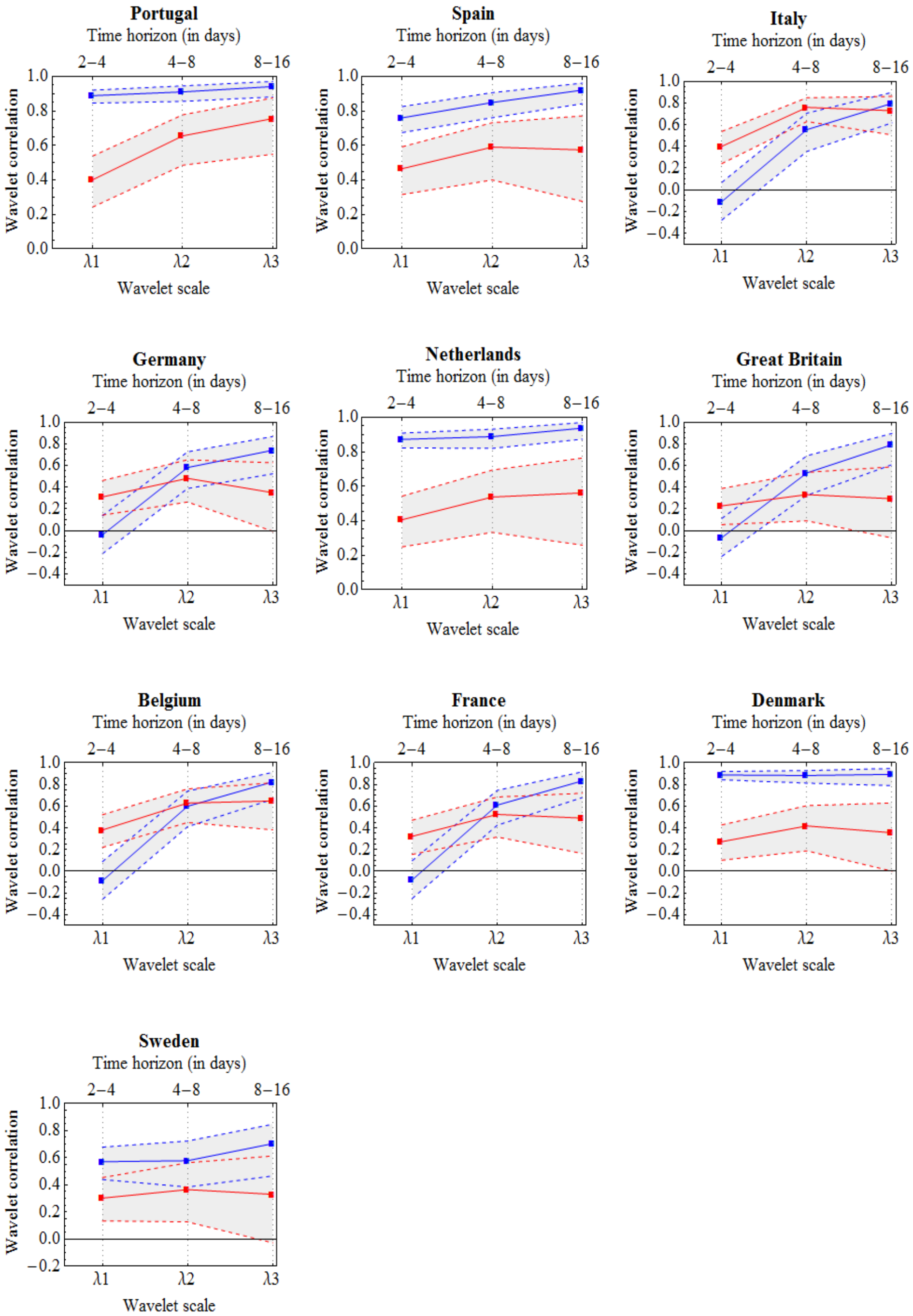
Figure 7.1 consists of 10 pictures and shows estimated wavelet correlation between yields of Greece and the other states on the different scales together with their confidence intervals. Blue ones are computed from the window before the contagion point, red ones are estimated based on the observations after the point. Notation is adopted from Whitcher (2013). Again, we have to mention that a contagion occurs only if the both confidence intervals are not overlapping on **all** scales. Firstly our contagion point will be 15.9.2008 - the bankruptcy of Lehman Brothers.

Now we will analyse the results. In the period before the fall of Lehman

[3]At least its beginning

[4]All related tables were uploaded to SIS.

Figure 7.1: Contagion between Greece and other states after fall of Lehman Brothres on 10Y sovereign bonds market



Source: Author's computation via R (package waveslim). Blue lines denote wavelet correlation estimate plus its confidence intervals computed from the window before the contagion point, red lines stand for the same after the contagion point.

Brothers the correlation between Netherlands and Greece used to be high and relatively homogeneous, similarly to Spain, Portugal and Denmark (see Figure 7.1). In the post-Lehman window the situation changed and the correlation significantly decreased in all previously mentioned cases. The confidence intervals are not overlapping, thus we detected contagion in all four states. These results are interesting, because the countries mentioned earlier belong to the different groups – the Core, the Periphery as well as the states outside the Eurozone. The other six states, where the contagion was not detected, have the similar pattern of the results (except Sweden). In the pre-crisis window estimated correlation was lower in general and increasing with the scales. The results in the crisis window differ on the first scale, where all correlations increased. Moreover, the rise of correlations with Italy, Belgium and France is significant. The results on the scale λ_2 seem to be same for the both windows, whereas correlations on the third scale are lower, but not significantly. On the scale λ_3 all estimated correlations decreased.

To check the robustnes of results to wavelet filters we will use Haar and D(4). The graphical outputs are in Appendix B.3.

Table 7.1: Contagion - fall of Lehman Brothers: all results

State	LA8	D4	Haar
GB	NO		
Belgium	NO		
Denmark	YES	YES	YES
France	NO		
Germany	NO		
Italy	NO		
Netherlands	YES	YES	YES
Portugal	YES	YES	*YES*
Spain	YES	YES	**NO**
Sweden	NO		

Source: Author's computation using R(waveslim)

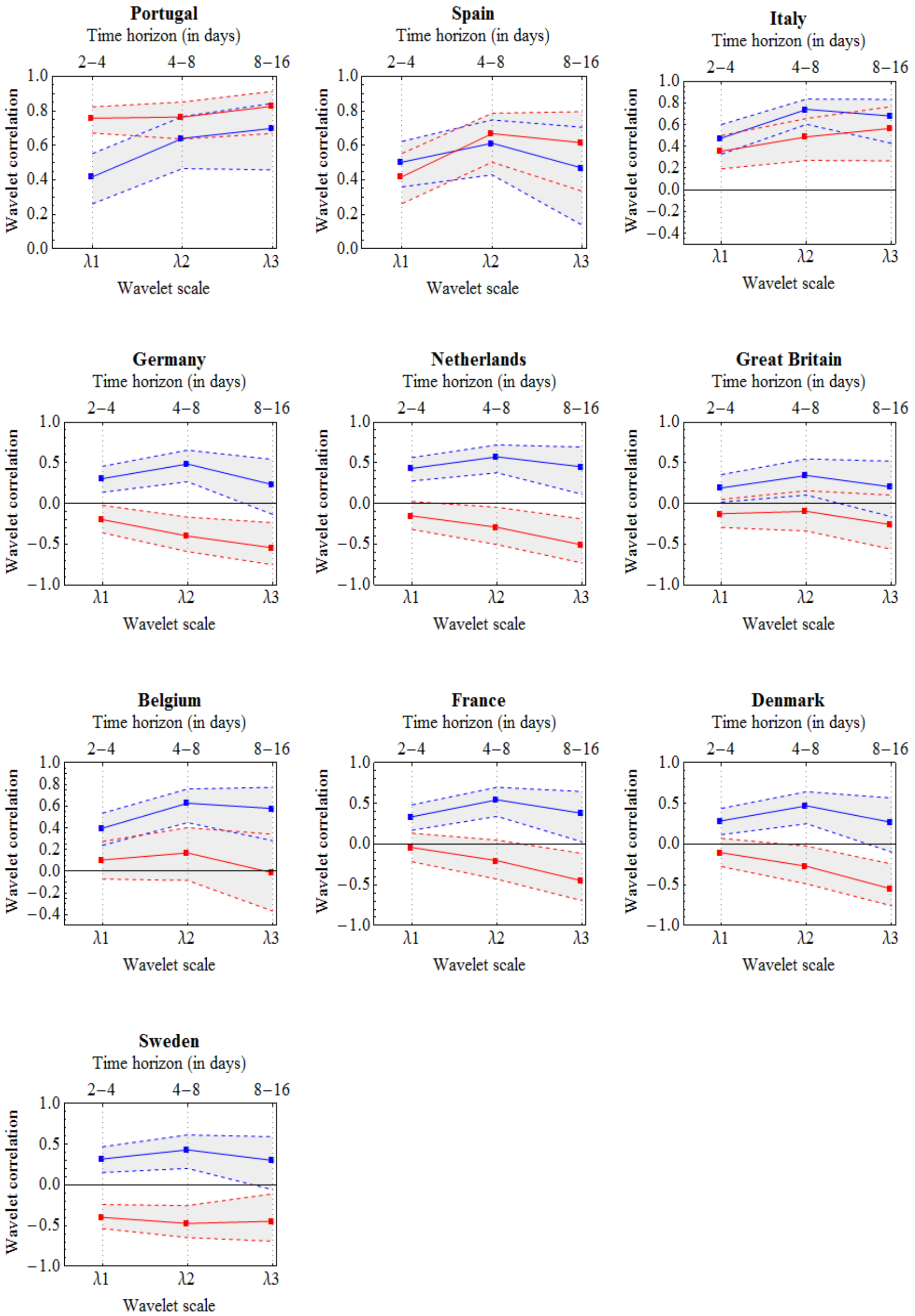
The results obtained via all filters are in Table 7.1 and we see that already mentioned tightness of the results are problematic in case of Spain, where overlap was detected using Haar. Correlation with yields of Portugal does not have this problem, but using alternative wavelets distance between the lower bound of pre-crisis estimator and the upper bound of crisis-era estimator is almost negligible. On the other hand the results of the other two tested states – Denmark and Netherlands – exhibit the robustness, because a significant shift occurred and the gaps between the confidence intervals are big.

7.2 Contagion After the Speech of Papakonstantinou (20.10.2009)

We perform similar contagion analysis again, but we change the contagion point. Now it will be 20.10.2009 - the day, when Greek minister of finance declared new budget deficit. All graphical results are presented in Figure 7.2. We observe that there are five pictures where the confidence intervals do not overlap – Germany, Netherlands, France, Sweden and Denmark. Moreover, in case of Belgium the results are very tight – there is a only minimal overlapping between the confidence intervals. Three of the countries are listed in the Core. Denmark and Sweden are the highly developed states outside the Eurozone. Hence – contrary to the bankruptcy of Lehman Brothers – no contagion on the Periphery was detected. However, it is still interesting to make a comparison within the Peripheral states, especially between Spain and Portugal. In the pre-speech window the results seem to be similar. In the second window their results are completely different. On the scale λ_1 correlation between yields of Greece and Portugal significantly increased. Moreover, in no other case is a jump up statistically significant. On the other two scales (λ_2 and λ_3) correlation increased too, but the confidence intervals are overlapping.

The analysis shows that after the speech of Papakonstantinou the significant change of co-movement was detected only in the highly developed states. A possible reason can be that long before this event investors saw that there are states in the EU, which are fiscally more stable than others. Hence when shocking news came, all relationships between Greece as an example of fiscally irresponsible state on one side and those five states on the opposite side have already been broken. Moreover, correlation became negative. Some authors (e.g. Kilponen *et al.* 2012) call this phenomenon "FLIGH-TO-QUALITY". An

Figure 7.2: Contagion between Greece and other states on 20.10.2009 (disclosure of debt by Papakonstantinou) on 10Y sovereign bonds market



Source: Author's computation via R (package waveslim). Blue lines denote wavelet correlation estimate plus its confidence intervals computed from the window before the contagion point, red lines stand for the same after the contagion point.

increase of co-movement on the lowest scale between yields of Greece and Portugal can imply that the second mentioned state stopped being trustworthy.

Again Haar and D4 will be used for checking of the robustness. Now we will test (except all states where the contagion was detected) an additional state – Belgium, where the overlap was tight. According to the new results[5] summarized in Table 7.2 contagion was confirmed in the following states: Germany, Denmark, Sweden and Netherlands. Both employed filters do not rejected null hypothesis, hence contagion was not presented in Belgium around 20.10.2009. The most important result is related to France – now confidence intervals overlap on the scale λ_1. It means that the original results are not too robust and have to be interpreted with caution.

Table 7.2: Contagion 20.10.2009: all results

State	LA8	D4	Haar
GB	NO		
Belgium	NO	NO	NO
Denmark	YES	YES	YES
France	YES	**NO**	YES
Germany	YES	YES	YES
Italy	NO		
Netherlands	YES	YES	YES
Portugal	NO		
Spain	NO		
Sweden	YES	YES	YES

Source: Author's computation using **R** (waveslim)

7.3 Summary and Discussion of Chapter 7

The chapter was dedicated to the study of the contagion between Greece and the other states. We used the same methodology like Gallegati (2012). The results are very interesting. Wavelet correlation shows that in the two countries - Netherlands and Denmark - contagion occurred during both events. Netherlands is a state, which has got - according to Dias (2012) - a central position in the minimum spaning tree graph. After 20.9.2009 the contagion was detected only in the Non-peripheral countries. On the other hand after the fall

[5]Graphs are in Appendix B.3.

of Lehman Brothers the phenomenon occurred on the Periphery too - in Spain and Portugal. The findings related to the contagion near to the speech of Papakonstantinou can be explained using Dias (2012). The author compared minimum spanning tree from two subsamples - before 2010 and after 2010. He realized that since 2010 the exclusion of GIIPS was finished. Hence it can be the reason why no contagion occurred - the Periphery was already established according to the investors. It is important to note that the event can be called a contagion only if we use the definition from Gallegati (2012) - *"significant change of co-movement"* and do not strictly follow Forbes & Rigobon (2002), whose definition is *"significant increase in cross-market linkages"*. Moreover, the results in the section document how careful should economists be while analysing co-movement only using time-domain techniques. Using only the scale λ_1 we could conclude that after the fall of Lehman Brothers co-movement increased in many cases.

The results are not in accordance with Gómez-Puig & Sosvilla-Rivero (2014). As was written in Chapter 2 they identified many structural breaks in a period after the announcement, but only few of them were close to the date and Greece was not involved.[6] A similar situation is with contagion around the bankruptcy of Lehman Brothers. The results of Inoue *et al.* (2013) are different too. Their results indicate that the fall of Lehman hit fiscally strong states and speech of Papakonstantinou hit the Peripheral countries.

An important question arises: can we trust the results? The answer is a cautious "YES". Firstly we have to recall that Daubechies LA(8) wavelet should be the most suitable for the purpose - neither it is as simple as Haar, nor contaminates data with large asymmetry as D(4). On the other hand robustness means that the obtained estimates should be relatively similar. If a gap between confidence intervals is big, then there is no problem and the contagion was detected again. But in case of Spain or France the differences are negligible, hence we should be careful with the quick judgments. Moreover, we *a priori* set the important parameter – the length of the window. Nevertheless, we can observe the massive changes of co-movement during the days close to 15.9.2008 and 20.10.2009, thus in this sense the hypothesis IV holds.

[6] Only with Austria, which is not in the dataset.

Chapter 8

Heterogeneity of the Results Across Scales: the Pre-crisis and the Crisis Period Comparison

Eight chapter brings a supplementary analysis to the contents of Chapter 5 and Chapter 6. We have pointed to the shortcoming when wavelet coherence is used to test the hypothesis that co-movement differs across scales. Hence in Chapter 6 we employed wavelet multiple correlation and revealed that heterogeneity of the results across scales decreased in the crisis era. But the topic deserves to be developed in the stand-alone chapter.

8.1 Methodology

Overall sample is divided into 13 subsamples representing the trading years (2001 - 2013) - similarly to Baruník *et al.* (2013). We will be interested if there is significant heterogeneity of the results. The null hypothesis and the alternative of the test are following

$$\begin{aligned} H_0 &: \rho_{XY}(\lambda_1) = \rho_{XY}(\lambda_2) = \ldots = \rho_{XY}(\lambda_J) \\ A &: H_0 \text{ does not hold,} \end{aligned}$$

where $\rho_{XY}(\lambda_j)$ stands for wavelet correlation of time series X and Y on a particular scale λ_j. Similarly to Chapter 6 we will examine if the confidence intervals overlap across scales or not. But now we will use normal, not multiple correlation. Moreover, the length of our windows will be smaller, hence only the lower scales ($\lambda_1, \ldots, \lambda_4$) will be used.

The procedure has the following pattern: in each subsample wavelet bivariate correlation between all states will be estimated[1] on the first four scales. Then we make pairwise comparison and detect if the confidence intervals of the estimates from different scales overlap or not and count the cases when **they are not overlapping**.[2] Hence – using this simple index - we are able to compare degree of heterogeneity – *6* denotes maximal heterogeneity, zero value implies that **the hypothesis of homogeneity of the results across scales is not rejected**. Based on the numbers obtained in Chapter 6 our initial guess is that the index will decrease in the crisis era. While applying the approach on the example from Chapter 4 (see Figure 4.3), we will realize that the pre-crisis index of heterogeneity is 6, the index of post-Lehman Brothers period is zero.

To make the results more lucid we will use *MatrixPlot* fuction built in *Mathematica 9*. The horizontal axis represents the states in our dataset, on the vertical axis are the years (2001 – 2013), colors denote intensity of the heterogeneity. Following color scheme is used: 0 – WHITE, 1 – LIGHT BLUE, 2 – BLUE, 3 – YELLOW, 4 – ORANGE, 5 – RED, HIGHER THAN 5 was not seen in any case. Hence the white spaces imply homogeneous estimates across scales.[3] In the analysis we will focus mainly on the two states– Germany and Greece and thus in this time the hypothesis of homogeneity is not rejected.

8.2 Results

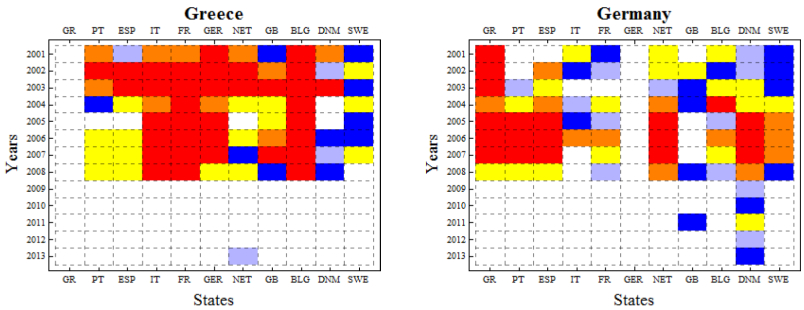
Firstly we focus on the two states - Greece a Germany, then we will summarize the results of the other 9 states. Let us begin with Greece, which results are in Figure 8.1. Generally we observe that in the years 2001 – 2008 (non-crisis period) co-movement between Greek yields and yields of other states was het-

[1]Again MODWT and LA(8) will be used.

[2]It means that we analyse if there is an overlap between the confidence intervals of λ_1 and λ_2 estimates, λ_1 and λ_3 estimates, etc. It is 6 relationships in total.

[3]When we compute wavelet correlation between two exactly same time series, then they are perfectly homogeneous.

Figure 8.1: Heterogeneity of the results across scales: Greece, Germany and their wavelet correlation with the other states.



Source: Author's computation via R(waveslim) and *Mathematica 9*. Colors denote: 0 – WHITE, 1 – LIGHT BLUE, 2 – BLUE, 3 – YELLOW, 4 – ORANGE, 5 – RED, where a number stands from the number of significantly different estimates. Non-white color implies that null hypothesis of homogeneity is rejected.

erogeneous across scales. Only exception is the year 2005, where the hypothesis of homogeneity was not rejected four times. Relatively low degree of heterogeneity emanates co-movement with Denmark, but even there the estimates differs across scales. On the opposite side are the states of the Core plus Italy – high values of the index of heterogeneity (5 and 4) are observed whole non-crisis period. It is in accordance with Chapter 5, where we detected lack of significant coherence on the low scales between Greece and the Core together with significant coherence on the medium and higher scales. Situation rapidly changed since 2009. Heterogeneity among the scales almost completely disappeared. From Chapter 5 we know that significant coherence disappeared too.

Even in case of co-movement between Germany and the other states we see how the heterogeneity fell since 2009 (see Figure 8.1). The highest differences in the obtained results are between Germany and Greece. This finding is supported by the motivation example from Chapter 4, where large spread between estimated values on the different scales was observed. The most homogeneous is the relationship with yields of Great Britain. In the crisis period heterogeneity remained only between Germany and Denmark.

The results for the other 9 states are presented in Figure 8.3. In the non-crisis era heterogeneity of the results is observed in all states. Generally we can say that the heterogeneity gradually increased. Its peak is between 2005 and 2007, where the colors are the most intensive. Let us highlight several information

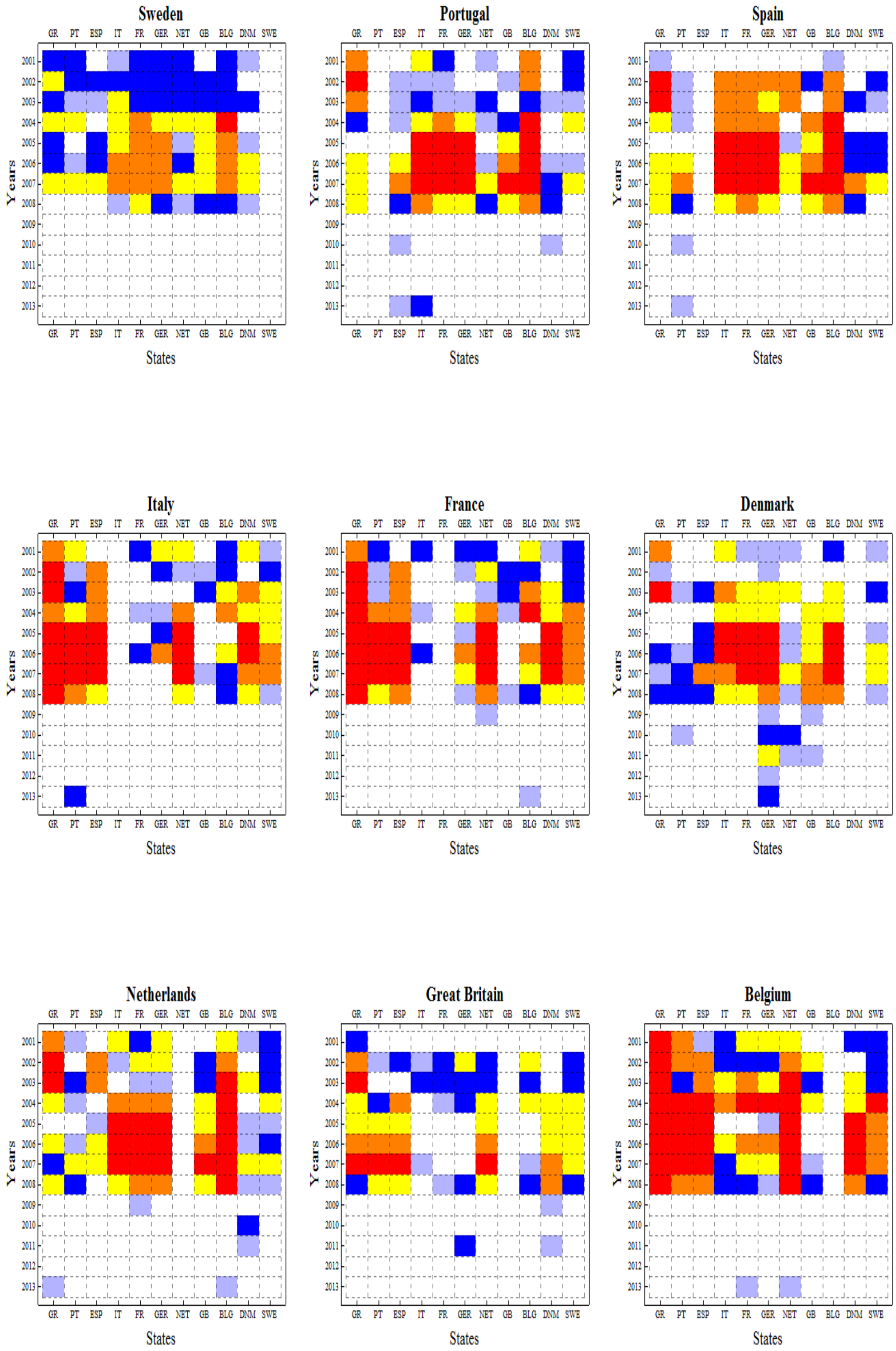
1. In the case of Great Britain the index of heterogeneity is lower than the indices of the other states.

2. Spain, Portugal, Denmark and Sweden had high valuea of the index in case of correlation between them and the Core (except Netherlands) plus Italy.

3. In addition, co-movement of Netherlands with the other Core states is highly heterogeneous. But co-movement inside the other three states of the Core is less heterogeneous.

Similarly to the previous results we see sharp decrease of the index since 2009, hence after the fall of Lehman Brothers. It implies that we proved the hypothesis V. Only exception is already mentioned co-movement between Germany and Denmark, which remained significantly heterogeneous until the end of our dataset. The findings are in accordance with the findings from Chapter 6, where the estimates from the crisis era are statistically less heterogeneus than the pre-crisis estimates. Barunik *et al.* (2013) interpret it as increase of a panic on the market in the crisis period – market sentiments in the shorter and longer horizon converge. Moreover, the results imply that the usage of wavelet bivariate correlation on the crisis-time data brings almost no additional statistically significant results.

Figure 8.2: Heterogeneity of the results across scales: other states



Source: Author's computation via R(waveslim) and Mathematica 9

Chapter 9

Conclusion

The thesis was dedicated to the analysis of co-movement between sovereign bond yields of the EU members. Our dataset contains daily observations from 1.1.2001 to 31.12.2013 of the 11 members of the EU (Greece, Portugal, Spain, Italy, France, Germany, Netherlands, Great Britain, Belgium, Denmark, Sweden), which were divided into the three groups (the Core, the Periphery, the states outside the Eurozone). In the centre of our attention were changes of co-movement during the financial crisis and the sovereign debt crisis. We employed wavelet transformation, which is able to decompose time series into different scales and then co-movement can be estimated scale-by-scale. Main advantage of the approach is that we are able to distinguish between shorter and longer time horizon - we obtain information about development of co-movement on frequencies while not losing information about development in time, thus we can compare the results through time and across scales.

We brought the three main findings as a contribution to the current stream of literature dedicated to the topic. Firstly – the alternative methodology was used and it confirmed that in the crisis period co-movement significantly decreased, which supports the hypothesis I. Coherence was significant on the majority of scales in the non-crisis period, contrary to the crisis era, when significant coherence almost completely disappeared, especially on the Periphery and between the groups. The next chapter was dedicated to multiple wavelet correlation method proposed by Fernandez-Macho (2012a). Using the technique we estimated the correlations of the whole groups together. We observed that overall degree of co-movement decreased after the fall of Lehman Brothers. But the testing of the hypothesis inside the groups brought the different results. In the Core multiple correlation significantly decreased on all scales, on the Periphery the significant drop of correlation occurred only on the medium and higher scales. Completely different findings are obtained in the group of the states outside the Eurozone. Significant increase is observable on the scales representing the cycles of 2-8 days, whereas the other scales exhibited a decrease (statistically insignificant). This is in accordance with the results obtained from the pairwise intra-group analysis provided in Chapter 5. Moreover, inter-group coherence decreased in the crisis era. This implies that the hypothesis II holds – co-movement differs across the groups. It implies that the integration of sovereign bonds in the Eurozone suffered hard blow. Yields of the Core exhibit high degree of co-movement in the end of 2013, integration of the yields on the Periphery was shattered and not restored.

The second important finding fully exploits the ability of wavelet transformation. It shows that co-movement significantly differed across scales (frequencies). We have described the shortfalls of wavelet coherence as a detector of heterogeneity of the results in case of large significant (or insignificant) coherence regions. Nevertheless, heterogeneity is clearly observable in the several cases. Firstly – coherence is lower on the lowest scales (2-8 days) in comparison with other scales (on the Periphery and in the states outside the Eurozone) in the pre-crisis era. Secondly – in the crisis there are smaller islands of coherence near to the significant events (e.g. launching of the ESM) on high and extremely high scales. Thirdly – in the Core decrease of coherence is more apparent on the lower scales. Wavelet multiple correlation shows us that before the fall of Lehman Brothers the results are similar to the ones obtained by Fernandez-Macho (2012a) and Tiwari *et al.* (2013) – multiple correlation is increasing and concave function of the scales. These findings support the hypothesis III stated in Chapter 3. In the crisis subsample situation changed and the heterogeneity across scales decreased. Whole Chapter 8 was dedicated to the study of the hypothesis. We created simple index of heterogeneity and provided pairwise wavelet correlation of all time series. The findings speak clearly – since 2009 statistically significant heterogeneity almost completely disappeared. Hence the hypothesis V was proven to be correct.

Third contribution was an application of the Gallegati's test on sovereign bond yields data. Using two different contagion points we have obtained interesting results. After the fall of Lehman Brothers contagion spread to the states belonging to all three groups - Spain, Portugal, Denmark and Netherlands. Different situation is with the day of Papakonstantinou's speech. Now all states with presence of the contagion are from the Core or they are non-Eurozone states - Netherlands, Denmark, Sweden, Germany and France. Moreover, two countries – Netherlands and Denmark – suffered contagion in both cases. Moreover, after 20.10.2009 correlation of all five states became negative ("FLIGHT-TO-QUALITY" phenomenon). On the other hand we pointed to the problems of robustness related to some results. Moreover, we discussed abuse of the term contagion in our thesis, because contagion primarily means increase of inter-market linkages (Forbes & Rigobon 2002). Nevertheless, significant changes of co-movement occurred during the days close to the examined dates.

In the thesis we demonstrated that despite the shortcomings described earlier both wavelet transformations - the CWT and the MODWT - are useful tools for the analysis of interdependence between sovereign bond yields. Main message is that in similar type of analysis frequency domain has to be taken into account. Aim of the thesis was to provide large study with many states, hence we did not fully exploit all possibilites of wavelet analysis. We did not focus on phases and causality, hence scale-by-scale analysis using Granger causality similarly to Gallegati *et al.* (2011) can be performed. Moreover, using wavelet coherence we focused more on the states from the Periphery and omitted relationship between the Core and the non-Eurozone countries. The results obtained by Missio (2013) and Gómez-Puig & Sosvilla-Rivero (2014) indicate that the contagion occurred later - in 2010, hence it would be interesting to test another date for it. Different approach is to model risk premia using spreads. We would expect, based on provided literature review, that the results would be different and it would be interesting to compare the findings.

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Appendices

Appendix A

Mathematical prerequisites

A.1 Spaces and Products

Definition 2 *An inner product of a real vector space V is an assignment $V \times V \rightarrow C$, where C is complex field that for any two vectors $u, v \in V$, there is a map $\langle u, v\rangle$ that has got following properties:*

1. $\langle au + bw, v\rangle = \langle au, v\rangle + \langle bw, v\rangle$
2. $\langle u, v\rangle = \overline{\langle v, w\rangle}$
3. $\langle u, u\rangle \geq 0$ *and* $\langle u, u\rangle = 0$ *if and only if* $u = 0$.

Example 1 *Let $x(t)$ and $y(t)$ be continuous functions. Then their inner product is*

$$\langle x(t), y(t)\rangle = \int_a^b x(t)y(t)dt \tag{A.1}$$

where $x(t), y(t) \in C[a, b]$.

Example 2 *Let $\{x_t\}_{t=1}^T$ and $\{y_t\}_{t=1}^T$ be discrete variables. Then their inner product is*

$$\langle x_t, y_t\rangle = \sum_{t=1}^{T} x_t y_t \tag{A.2}$$

where $x_t, y_t \in C[a, b]$.

Definition 3 *Norm $||u|| = \sqrt{\langle u, u\rangle}$ denotes a length of vector u.*

Consequence 1 *From the relationship $||x_t - y_t||^2 = ||x_t||^2 - 2\langle x_t, y_t\rangle + ||y_t||^2$ we see that the squared distance between x_t and y_t is minimized iff their inner product is maximized.*

Definition 4 *Subset* $S = \{u, v\}$ *is called orthogonal if*

$$\langle u, v \rangle = 0 \tag{A.3}$$

Definition 5 *Moreover, if* $||u|| = ||v|| = 1$*, then it is also called orthonormal.*

A.2 Orthonormal matrix and its properties

In chapter 3 we defined wavelet transformation even in the matrix form. Those matrices have very important property – orthonormality. The property itself was defined earlier. Now we will define it even in matrix form according to Percival and Walden (2000).

Definition 6 *Let* $\mathcal{O}$ *is* $N \times N$ *matrix of real values. Then the matrix is orthonormal if following condition is satisfied:*

$$\mathcal{O}^T \mathcal{O} = \mathcal{O}\mathcal{O}^T = I_N \tag{A.4}$$

where I_N *is a unit matrix.*

Orthonormality has got several important consequences.

Consequence 2

$$||\mathcal{O}||^2 = \langle \mathcal{O}, \mathcal{O} \rangle = \mathcal{O}^T \mathcal{O} = I_N \tag{A.5}$$

If $\mathcal{O}$ *is* $N \times 1$ *vector, then its energy is 1. The second property is related to energy too and it is implied by the previous consequece.*

Consequence 3 *Let us define real-valued matrix* O *in the following way*

$$O = \mathcal{O} X \tag{A.6}$$

where X *is* N *matrix. Then*

$$||O||^2 = O^T O = (\mathcal{O}X)^T (\mathcal{O}X) = X^T \mathcal{O}^T \mathcal{O} X = ||X||^2 \tag{A.7}$$

It means that energy of the transformed matrix O *is equal to energy of the original matrix* X*, hence the orthonormal transformation is energy-preserving.*

A.3 Trygonometry

We have to define basic terms, which are essential for further analysis. Let x_t *be a periodic function in the following form:*

$$x_t = \sin(a + 2\pi f) \tag{A.8}$$

The function is affected by those parameters:

1. ***frequency***: *f = 1/p where p denotes time, which is the process needs to complete one oscillation. Then frequency f is a number of oscillations per unit of time. Sometimes a different definition of frequency is used:*

   $$\omega = 2\pi f \tag{A.9}$$

   In this case ω *is called* ***angular frequency***. *The highest frequency that can be measured -* $f = \frac{1}{2}, \omega = \pi$ *is called* ***Nyquist frequency***.

2. ***Phase*** *a denotes a shift from the beginning (which is zero). If there is no phase, then a is equal to zero.*

A.4 Convolution and Filters

The following notation comes from Percival & Walden (2000).

Definition 7 (Discrete Convolution) *Let {a} and { b} denote two infinite sequences, which satisfy*

- $\sum_{-\infty}^{+\infty} |a_t|^2 < \infty$
- $\sum_{-\infty}^{+\infty} |b_t|^2 < \infty$

Then convolution of those two sequences is defined as:

$$a * b_t = \sum_{u=-\infty}^{+\infty} a_u b_{t-u} \tag{A.10}$$

Continuous version can be defined too, but for rest of the Appendix we will use only a discrete variant. Using wavelets we have to define circular convolution too.

Definition 8 (Circular convolution) *Let $\{a\}$ and $\{ b\}$ be thw sequences from the previous definition. Then circular convolution is defined as*

$$a * b_t = \sum_{u=-\infty}^{+\infty} a_u b_{(t-u) mod N} \tag{A.11}$$

In the previous definition mod represents modulo operator. It is defined in the following way - if positive integer j lies between 0 and $N-1$, then $j mod = j$, otherwise $j mod = k + n \cdot N$. The term $n \cdot N$ has to satisfy $0 \leq j + n \cdot N \leq N - 1$.

A.4.1 Discrete filters

Let us begin with filters and filtration. In its broadest sense the process of filtration can be classified in the following way (Gencay et al.(2002))

$$x_t \rightarrow FILTER \rightarrow y_t \tag{A.12}$$

We put time series x_t into a black box and obtain different (filtered) series. In the thesis there are two ways how to denote the filtering process – filter as some sequence and filter as gain frequency function. In the first mentioned case there are sequences $\{b\}$ and $\{a\}$, first of them is the series that will have to be filtered, the second one is the filter and sometimes called impulse response sequence. Then the process of filtration has got the following form

$$\{b_t\} \rightarrow \{a_t\} \rightarrow \{a * b_t\} \tag{A.13}$$

where $\{a * b_t\}$ is filtered series and a result of the previously defined convolution. A filter which consists of two and more sub-filters is called a ***cascade filter****. Similarly the process of filtration can be defined as*

$$\{b_t\} \rightarrow A(f) \rightarrow \{a * b_t\} \tag{A.14}$$

$A(f)$ denotes the result of Fourier discrete transformation. In the filtration process it is called a transfer function. It can be decomposed into two terms:

$$A(f) = |A(f)| e^{i\theta(f)} \tag{A.15}$$

The first term is called a gain function, second one is a phase function. Then $|A(f)|^2$ denotes a ***square gain function****.*

Appendix B

Additional results

B.1 Chapter 4

Jarque-Bera test

Null hypothesis: *time series is normally distributed (it has got skewness equal to 0 and kurtosis equal to 3.)*
Alternative: *time series is not normally distributed.*

Table B.1: Jarque-Bera test

	Original time series		Transformed time series	
State	**Statistic**	**Hypothesis**	**Statistic**	**Hypothesis**
Greece	6906.26	rejected	19536402	rejected
Portugal	3706.908	rejected	338009.8	rejected
Spain	383.5581	rejected	499285.2	rejected
Italy	882.591	rejected	14167.77	rejected
France	97.29463	rejected	2087.358	rejected
Germany	242.1919	rejected	4594.407	rejected
Netherlands	180.9415	rejected	2451.643	rejected
Great Britain	457.7317	rejected	4751.622	rejected
Belgium	107.9906	rejected	3001.68	rejected
Denmark	236.5438	rejected	147129.7	rejected
Sweden	131.7153	rejected	8714.205	rejected

*Source:*Author's own computation via R

ADF test

Null hypothesis: *unit root is presented in time series (it implies non-stationarity).*
Alternative: *presence of unit root is rejected (stationarity is not rejected).*

Table B.2: ADF test

	Original time series		Transformed time series	
State	**Statistic**	**Hypothesis**	**Statistic**	**Hypothesis**
Greece	-2.018674	not rejected	-15.14227	rejected
Portugal	-1.582441	not rejected	-15.96001	rejected
Spain	-2.535941	not rejected	-14.64791	rejected
Italy	-2.852052	not rejected	-13.97596	rejected
France	-2.74329	not rejected	-14.45931	rejected
Germany	-2.525721	not rejected	-15.73451	rejected
Netherlands	-2.403587	not rejected	-15.04801	rejected
Great Britain	-2.359631	not rejected	-15.38081	rejected
Belgium	-2.371403	not rejected	-14.98479	rejected
Denmark	-2.43375	not rejected	-15.31836	rejected
Sweden	-2.82095	not rejected	-14.23549	rejected

*Source:*Author's own computation via R

KPSS test

Null hypothesis: *time series is stationary.*
Alternative: *time series is not stationary.*
Test statistics (level of confidence): *10% - 0.347, 5% - 0.463, 1% - 0.739.*

Table B.3: KPSS test

	Original time series		Transformed time series	
State	**Statistic**	**Hypothesis**	**Statistic**	**Hypothesis**
Greece			0.1544719	not rejected
Portugal			0.127781	not rejected
Spain			0.06473551	not rejected
Italy			0.05818086	not rejected
France			0.03749797	not rejected
Germany			0.05059178	not rejected
Netherlands			0.04838691	not rejected
Great Britain			0.07452943	not rejected
Belgium			0.05255431	not rejected
Denmark			0.04977732	not rejected
Sweden			0.05567951	not rejected

*Source:*Author's own computation via R

Example: wavelet correlation of Greece-Germany

Scale	$wavecor_0$	$lower_0$	$upper_0$	$wavecor_1$	$lower_1$	$upper_1$
λ_1	-0.0401	-0.1017	0.0218	-0.0024	-0.0773	0.0725
λ_2	0.4937	0.4246	0.5572	-0.0605	-0.1655	0.0458
λ_3	0.8244	0.7802	0.8604	-0.0848	-0.2319	0.0661
λ_4	0.9254	0.8953	0.9471	-0.3286	-0.5062	-0.1241
λ_5	0.9409	0.9034	0.9641	-0.2786	-0.5371	0.0276
λ_6	0.9863	0.9714	0.9934	-0.2864	-0.6391	0.1657

B.2 Chapter 6

The following notation is used: wavemulcor$_0$ *and wavemulcor*$_1$ *denote estimates of wavelet multiple correlation before, respectively after the default of Lehman Brothers,* lower *and* upper *denote lower and upper bound of a confidence interval.*

Table B.4: Wavelet multiple correlation: The Core

scale	$wavemulcor_0$	$lower_0$	$upper_0$	$wavemulcor_1$	$lower_1$	$upper_1$
λ_1	0.9210	0.9110	0.9299	0.8556	0.8341	0.8744
λ_2	0.9625	0.9555	0.9684	0.9115	0.8917	0.9279
λ_3	0.9797	0.9740	0.9841	0.9103	0.8805	0.9329
λ_4	0.9888	0.9841	0.9922	0.9254	0.8873	0.9510
λ_5	0.9924	0.9874	0.9954	0.9431	0.8959	0.9692
λ_6	0.9968	0.9932	0.9985	0.9528	0.8853	0.9810

*Source:*Author's computation using R (package wavemulcor)

Table B.5: Wavelet multiple correlation: The Periphery

scale	$wavemulcor_0$	$lower_0$	$upper_0$	$wavemulcor_1$	$lower_1$	$upper_1$
λ_1	0.6505	0.6133	0.6848	0.7139	0.6751	0.7487
λ_2	0.8465	0.8197	0.8696	0.7754	0.7293	0.8145
λ_3	0.9401	0.9239	0.9530	0.7504	0.6766	0.8094
λ_4	0.9660	0.9518	0.9760	0.7295	0.6114	0.8158
λ_5	0.9824	0.9709	0.9894	0.8281	0.7005	0.9044
λ_6	0.9912	0.9815	0.9958	0.7452	0.4623	0.8905

*Source:*Author's computation using R (package wavemulcor).

Table B.6: Wavelet multiple correlation: the states outside the Eurozone

scale	wavemulcor_0	lower_0	upper_0	wavemulcor_1	lower_1	upper_1
λ_1	0.4847	0.4359	0.5306	0.7402	0.7044	0.7723
λ_2	0.7003	0.6528	0.7423	0.8019	0.7606	0.8368
λ_3	0.8327	0.7903	0.8671	0.8048	0.7447	0.8520
λ_4	0.9012	0.8620	0.9297	0.8340	0.7551	0.8891
λ_5	0.9316	0.8886	0.9584	0.8618	0.7559	0.9238
λ_6	0.9700	0.9380	0.9856	0.8953	0.7557	0.9571

Source: author's computation via R(wavemulcor).

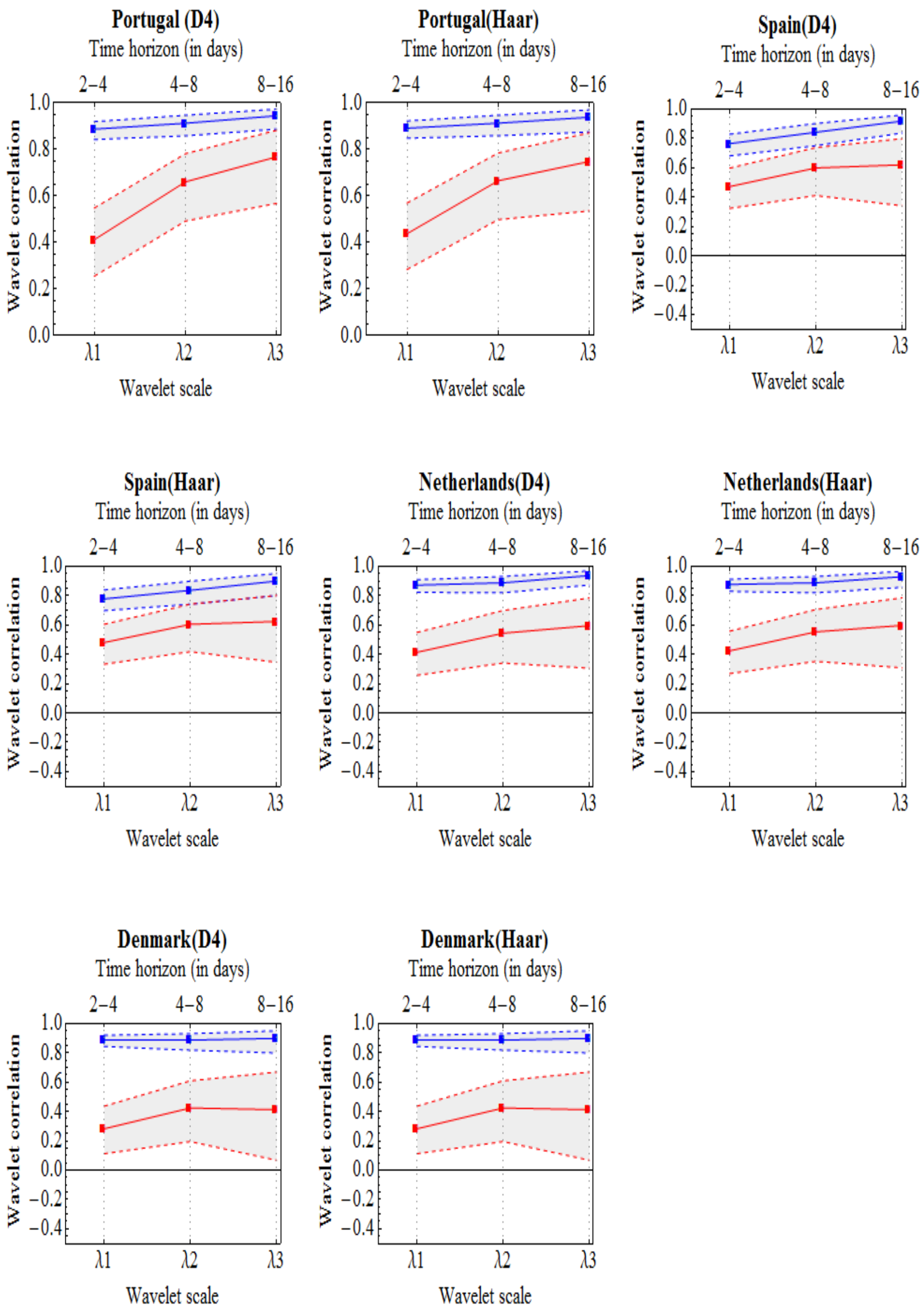
Table B.7: Wavelet multiple correlation: all states together

	wavemulcor_0	lower_0	upper_0	wavemulcor_1	lower_1	upper_1
λ_1	0.9415	0.9340	0.9481	0.9209	0.9086	0.9315
λ_2	0.9697	0.9640	0.9745	0.9554	0.9451	0.9638
λ_3	0.9826	0.9778	0.9864	0.9484	0.9308	0.9616
λ_4	0.9909	0.9871	0.9936	0.9546	0.9308	0.9703
λ_5	0.9932	0.9887	0.9959	0.9660	0.9372	0.9817
λ_6	0.9974	0.9945	0.9988	0.9711	0.9288	0.9884

*Source:*Author's computation using R (package wavemulcor)

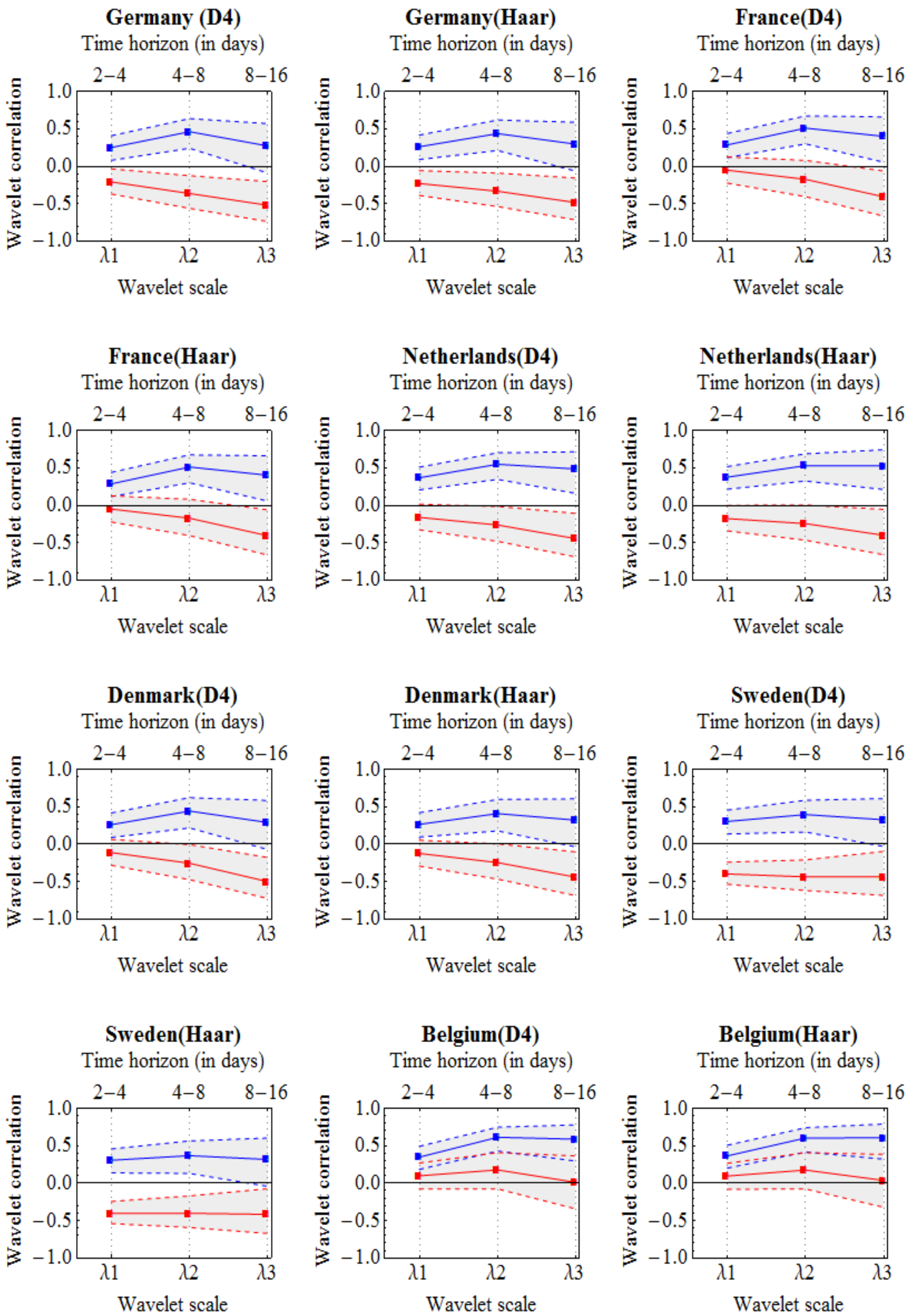
B.3 Chapter 7

Figure B.1: Lehman Brothers: Robustness Check



Source: Author's computation via R (package waveslim)

Figure B.2: 20.10.2009: Robustness Check

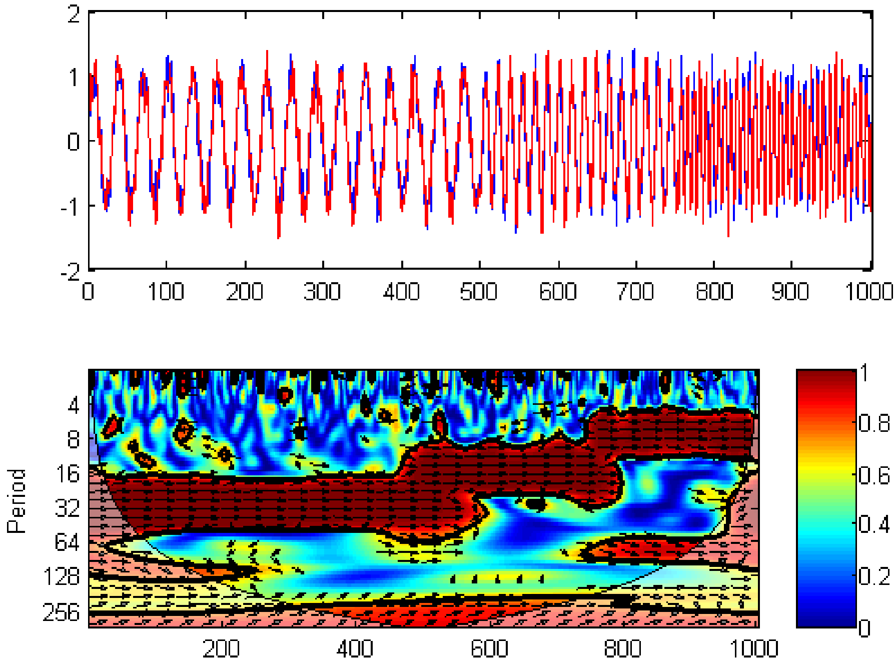


Source: Author's computation via R (package waveslim)

Appendix C

Coherence analysis in detail

Figure C.1: Change of wavelet coherence through time and frequency - example



Own computation, based on code from *http://www.alivelearn.net/?p=1169*.

Figure C.1 demonstrates ability of wavelet coherence to catch changes of co-movement across scales (frequencies). Firstly there is high coherence on the scales 16-64, then 8-16 and at last on 4-8.

Appendix D

Material uploaded in SIS

Following additional materials were uploaded to SIS:

1. **Data**(csv,xls,mat)
2. **Chapter 5** - timescale maps in detail, Matlab code.
3. **Chapter 6** - **R** code.
4. **Chapter 7** - numerical results, **R** code.
5. **Chapter 8** - **R** code, *Mathematica* code.