The goal of this work is to characterize all norm-euclidean quadratic extensions of $\mathbb{Q}$. The work treats completely the part of imaginary quadratic extensions. In the case of real quadratic extensions, we give a list of such discriminants $D$ that the field $\mathbb{Q}(\sqrt{D})$ is norm-euclidean. Furthermore, we prove an estimate $D<2^{14}$ for all norm-euclidean fields $\mathbb{Q}(\sqrt{D})$. Subsequently, the case $D \not \equiv 1(\bmod 4)$ is discussed in detail. For the case $D \equiv 1(\bmod 4)$ we mention references to the results of other authors.

