Instituto de Matemáticas

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Referee's report on the thesis "Homeomorphisms in topological structures" by Mgr. Benjamin Vejnar

Benjamin's work is on the area of Continuum Theory. Benjamin has worked on several different topics of Continuum Theory. He has been able to answer important published problems. His thesis includes an impressive collection of counterexamples. On the other hand, he also has been able to develop some important mathematical tools to obtain interesting and deep results.

I consider this thesis has a high quality and its mathematical content is original, creative and interesting for people in the area of Continuum Theory. So, I have no doubt that it is a work that certainly exceeds the usual requirements that one expects from a doctoral dissertation. This thesis indeed shows that the author has the ability for creative scientific work. Thus, I strongly recommend that the thesis be accepted for obtaining the PhD degree.

Benjamin's results are presented in seven papers. All of them contain strong results. I will comment three of them, which are nearest to my interest. In these papers Benjamin and its coauthors solved problems I have attacked for some time. It is important to mention that all the results of the thesis are very well described in its introduction.

A *continuum* is a nondegenerate compact connected metric space.

A. The *disconnection number* of a continuum X means the least cardinal (finite or countable) number n such that X\N is disconnected whenever cardinality of N equals n.

Since 1993, Sam B. Nadler, Jr. conjectured that there are exactly 26 continua (finite graphs) with disconnection number 4. This problem was solved by Benjamin in 2010, who also gave a method to construct, inductively, the continua having disconnection number n+1, when one knows those continua with disconnection number n. This problem was independently solved by Gladdines and van de Vel, in 2012. I consider that Benjamin's method is elegant and can be used to solve other problems on the topology of graphs (when they are seen as topological spaces).

B. A subset of a continuum X is called a *shore set* if there is a sequence of continua in the complement of this set converging to X. The notion of shore sets was introduced by Luis Montejano and Isabel Puga in 1992. In 1993, Isabel Puga and Victor Neumann used this notion to characterize locally connected dendroids. In 1993, in the Continuum Theory Seminar of the Autonomous National University of Mexico, Isabel Puga asked if in dendroids, the union of two shore one-point sets is a shore set. For some years, several specialists in this area tried to solve this problem until I found the solution in 2001 by proving that, in a dendroid, the union of a finite family of closed, pairwise disjoint shore sets is a shore set. I also gave an example showing that the hypothesis that the sets are pairwise disjoint is necessary. However, I was not able to solve the problem: in dendroids, is the union of two closed disjoint shore sets a shore set? I posed this problem as a question in my paper in 2001. In 2007, Van C. Nall studied the relationships of shore sets with other notions defined for dendroids, and asked if my theorem can be extended to lambda-dendroids.

In chapter 4 of Benjamin's thesis, it is solved Van C. Nall's question in the negative and in chapter 5, my original question is answered in the negative.

To finish, I want to mention that I have a very high opinion of Benjamin, since he and his coauthors have been able to solve problems that I and several reputed colleagues of mine were unable to solve.

Please, do not hesitate in contact me if you need something else.

Sincerely yours,

Alejandro Illanes,
Full Professor.