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Referee's report on the doctoral thesis *Homeomorphisms in Topological Structures* presented by Benjamin Vejnar

Contents.
This doctoral thesis consists of seven chapters, each of which is prepared as a separate journal article.

The first article *A note on a disconnection number* has been published in Topology and its Applications, 157 (2010), 2873-2875. It shows an elegant proof of a theorem stating that there are precisely 26 continua with disconnection number four. The author shows an inductive procedure thanks to which one can classify all continua with disconnection number *n+1* if continua with disconnection number *n* are known. This answers a question by Sam B. Nadler, Jr. – the author of the concept of the disconnection number.

The second article, *A topological characterization of the Sirpiński triangle,* published in Topology and its Applications, 157 (2010), 2873-2875, gives topological characterizations of four curves: the Sierpiński triangle, the modified triangle, the Apollonian gasket, and the modified gasket. The topological properties of the considered spaces are compared and their embeddings into the plane are investigated.

The third article, *Waraszkiewicz spirals revisited,* coauthored by Pavel Pyrih, has been published in Fund. Math. 219 (2012), no. 2, 97–104. It gives a simple proof of the result by Waraszkiewicz that there is an uncountable family of plane continua no one of which can be continuously mapped onto any other. The authors were able to reformulate the original problem into the language of real functions, and then, using the language of real functions, solve the problem.

The fourth article, *A lambda-dendroid with two shore points whose union is not a shore set,* coauthored by Pavel Pyrih, has been published in Topology Appl. 159 (2012), no. 1, 69–74. It presents a counterexample to a conjecture by Van C. Nall about shore sets. Precisely, the union of two disjoint shore continua in a λ-dendroid does not have to be a shore set. It shows differences between λ-dendroids and dedroids: for dendroids such examples are not possible by a theorem by Alejandro Illanes.

In the fifth article, *Union of shore sets in dendroids,* written together with Josef Bobok, Radek Marciňa and Pavel Pyrih, the investigation about shore sets is continued, this time for dendroids rather than λ-dendroids. The authors answer, by giving a respective counterexample, a question by Alejandro Illanes if the union of two

disjoint shore sets has to be a shore set in the planar dendroid. On the other hand they show that such example is not possible if additional assumption of smoothness of the dendroid is added. Several other interesting examples of the shore sets are presented. The investigation seems to be well organized and exhaustive. In the article I found one of very few editorial mistakes: the first reference should be in the fourth place, in alphabetical order.

The last two chapters of the dissertation are devoted to half-homogeneous continua. Chapter 6 is the article *Half-homogeneous indecomposable circle-like continuum*, Topology Appl. 160 (2013), no. 1, 56–58, written together with Pavel Pyrih. An example of a continuum as in the title is constructed, this answers a question asked by Victor Neumann-Lara, Patricia Pellicer-Covarrubias, and Isabel Puga in their article from Topology and its Applications. Chapter 7, the article *Half-homogeneous chainable continua with end points*, written together with Josef Bobok and Pavel Pyrih, gives a characterization of half-homogeneous continua with two end points, namely the authors show that there are only two such continua: the arc and the arc of pseudo-arcs with the two terminal layers shrank to points.

Comments.
The results presented in this thesis answer several questions posed by known mathematicians: Sam B. Nadler, Jr., Van C. Nall, Alejandro Illanes, Victor Neumann-Lara, Patricia Pellicer-Covarrubias, Isabel Puga, and Pavel Pyrih. Even if some of the presented articles have several coauthors, knowing Benjamin Vejnar I can suspect that he was a leading person in the investigation. Five of the articles have been published in the best topological journals: Topology and its Applications and Fundamenta Mathematicae. Benjamin Vejnar also gave solutions to two of my problems during the Continuum Theory Open Problem Workshop, July 29 - August 6, 2011 in Prague, it was very impressive.

The figures included in the papers really help understanding the proofs. As a reader, I am truly thankful to the authors for them.

The language is clear and understandable, English grammar is correct, I wish all mathematical papers were written this way.

Besides the articles presented in the thesis, Benjamin Vejnar is an author or coauthor of three published articles, according to MathSciNet: Benjamin Vejnar, *Topological compactifications*, Fund. Math. 213 (2011), no. 3, 233–253, Martin Doležal, Petr Pošta, Pavel Pyrih, Martin Rmoutil, Benjamin Vejnar, *Chain of dendrites without monotone supremum*. Questions Answers Gen. Topology 29 (2011), no. 2, 131–133, Wiesław Kubiś, Benjamin Vejnar, *Covering an uncountable square by countably many continuous functions*, Proc. Amer. Math. Soc. 140 (2012), no. 12, 4359–4368, and several others are under preparation. This shows that he is a very active mathematician.

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Conclusion.
The presented Ph.D. dissertation is a very good one, it answers several questions by distinguished mathematicians. I have no doubts that the thesis proves the authors ability for creative scientific work, and thus I strongly recommend Benjamin Vejnar to obtain the title of doctor of mathematics.

Sincerely,

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