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To: Professor Jaroslav Nešetřil
Director of doctoral committee I4 - discrete
models and algorithms
Charles University in Prague
Faculty of Mathematics and Physics

Report on the doctoral thesis of Josef Cibulka

Dear Professor Nešetřil,

Dear Doctoral Committee,

I am writing you on behalf of Josef Cibulka concerning his thesis entitled "*Extremal combinatorics of matrices, sequences and sets of permutations*" submitted to your Committee. I have read the thesis and it is an excellent work, it satisfies the highest expectations and could be acceptable in *any* leading university in the world. I strongly recommend you to award Cibulka the PhD degree.

The subject of this thesis is Extremal Combinatorics, where the author proves a number of interesting results. The importance of this field, the study of finite discrete structures, codes, designs, and algorithms can be hardly overestimated in the age of computers.

The central problem of the thesis deals with excluded submatrices and subpermutations. Let me make some definitions. By a matrix we always mean a matrix with exclusively 0 and 1 entries. A matrix N is said to contain matrix M if N possesses a submatrix such that at its entry is a 1 at every position where M has a 1 (it might contain 1's at positions where M has 0's). The function $\mathrm{ex}_M(n)$ denotes the maximum possible number of 1's in an $n \times n$ matrix not containing M. This harmlessly looking function has proved to be both extremely useful and is very difficult to compute. The investigation of permutations avoiding a certain subpermutation (e.g. 231) goes back to Knuth (1968). The famous Stanley-Wilf Conjecture states that for every fixed permutation P the total number of permutations of length n and avoiding P is less than $(s_P)^n$ for appropriate constant s_P. This conjecture was proved by Marcus and Tardos using excluded submatrices. They proved that $\mathrm{ex}_P(n)$ is linear in n and used it to settle the Stanley-Wilf

Conjecture. Let c_P denote $\lim_{n\to\infty} \mathrm{ex}_P(n)/n$. The author proves the following strong connection between these two quantities (Theorems 2.1 and 2.2)

$$s_P \leq 2.88(c_P)^2, \quad c_P \leq \alpha(s_P)^{9/2}.$$

Along these results both Chapter 1 and 2 contain several important new bounds and interesting constructions. Cibulka is a real master of recursive constructions and he makes clever technical proofs which require deep understanding of the finite structures.

Let me briefly mention in less details two of the strongest results.

"VC-dimension of permutations"
In Theorem 3.2 on page 41 Cibulka (with a coauthor) gives a lower bound for $r_k(n)$, the size of the largest set of permutations of length n that is of dimension k, thus disproving a conjecture of Raz (2000). This bound uses the inverse of the Ackerman function, which is an extremely slowly growing function and to handle it the authors must have profound insight and knowledge of the field.

"Reverse-free sets of permutations"
This topic was initiated and investigated by such leading mathematicians as J. Korner (Rome) and Noga Alon (Tel Aviv). Both the lower bounds in Theorem 4.1 (page 59) and in Claim 4.3 are very beautiful constructions, and the proof of the matching upper bound in Chapter 4.3 is very clever thus opening up new venues for further research.

I recommend Jozef Cibulka to you for serious consideration, he is an excellent candidate for the PhD award.

Sincerely yours,

Zoltán Füredi

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