Title: Extremal combinatorics of matrices, sequences and sets of permutations

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Abstract: This thesis studies questions from the areas of the extremal theory of $\{0,1\}$-matrices, sequences and sets of permutations, which found many applications in combinatorial and computational geometry. The *VC-dimension* of a set $\mathcal{P}$ of $n$-element permutations is the largest integer $k$ such that the set of restrictions of the permutations in $\mathcal{P}$ on some $k$-tuple of positions is the set of all $k!$ permutation patterns. We show lower and upper bounds quasiexponential in $n$ on the maximum size of a set of $n$-element permutations with VC-dimension bounded by a constant. This is used in a paper of Jan Kynčl to considerably improve the upper bound on the number of weak isomorphism classes of complete topological graphs on $n$ vertices. For some, mostly permutation, matrices $M$, we give new bounds on the number of 1-entries an $n \times n$ $M$-avoiding matrix can have. For example, for every even $k$, we give a construction of a matrix with $k^2n/2$ 1-entries that avoids one specific $k$-permutation matrix. We also give almost tight bounds on the maximum number of 1-entries in matrices avoiding a fixed layered permutation matrix.

Keywords: extremal theory, forbidden substructure, set of permutations, $\{0,1\}$-matrix