Report on "On the Power of Weak Extensions of V^0" by Sebastian Müller

Results

The bulk of the thesis consists of two papers, included as Appendices A and B.

The first shows that random propositional formulas, chosen with a certain distribution, have polynomial-size refutations in the TC^0-Frege proof system. While not especially deep, this is a new and significant result, which addresses a question of interest in theorem proving as well as in proof complexity. Such random formulas are known to require exponential-size refutations in the resolution proof system. Resolution is widely studied and widely used in automated theorem proving, and it has even been conjectured that, for "naturally occurring" formulas, it is as good as any other proof system. This result is evidence that this is not the case.

The method of proof is a very nice example of how to use careful formalization in first-order logic to construct propositional proofs, and I hope that this paper will help lead to such methods being used more widely. In particular it shows how non-constructive arguments in combinatorics using real linear algebra can be made to work even in these relatively weak systems.

The second paper shows that small initial segments of models of bounded induction satisfy a stronger theory, in which every treelike circuit has a computation. Hence it shows a uniform subexponential simulation of the Frege proof system by constant-depth Frege, improving and simplifying a 2011 paper showing a non-uniform simulation. The result is not unexpected and the proof is relatively straightforward, but it is nevertheless a useful contribution to the area.

Form of the thesis

Appendix A, the paper on random propositional formulas, was written jointly with Iddo Tzameret and appeared in the conference LICS 2012. It is well-presented, clearly written and shows mathematical maturity, framing some interesting problems and then solving them.

Appendix B, the paper on polylogarithmic cuts, is not of such high quality and the presentation can be difficult to follow, but the author evidently has a good working understanding of the subject, including some complex ideas.

The rest of the thesis consists of introductory sections and summaries of the two papers. This is of mixed quality. The background material is generally fine, but the summaries seem to me to have been hastily written.

Overall assessment

The thesis shows that the author is able to carry out creative scientific work.

Neil Thapen Prague, December 19, 2012