December 18, 2012
Prague

Advisor's opinion of *Sebastian Muller's* PhD Thesis:
On the power of weak extensions of V^0

The thesis is divided into 5 chapters and 3 appendices. Chapter 1 contains preliminaries and Chapter 5 contains conclusions. The mathematical contributions are in the remaining 3 chapters.

In my view the mathematically most interesting part, and best written, is Chapter 2 based on a joint published article with Iddo Tzameret. I. Tzameret was a postdoc in Prague overlapping for some time with S. Muller and the work was done during this time. I was quite close to it and I can attest that S. Muller's contribution equaled to that of his co-author.

They showed a surprising power of a weak proof system - the so called TC^0-Frege - by formalizing a construction of Feige et.al. in a very weak fragment of arithmetic. The result is that a random sufficiently dense 3CNF has a short refutation in the proof system. This work contains some technically quite subtle steps but they are well explained. The original paper is attached as Appendix A.

In Chapter 3 S. Muller proved that short (poly-logarithmic) cuts in models of theory V^0 are themselves models of a stronger theory VNC^1. This has as a corollary a new and model-theoretic way how to prove a sub-exponential simulation of Frege systems by AC^0-Frege systems originally proved differently by Filmus et.al.; in my view this new proof is more elegant and conceptual than the original one.

This chapter is written somewhat more brusquely than the previous one and it is more difficult to read. The paper on which it is based is attached as Appendix B. In addition to the material contained in the paper, the chapter reports on some work in progress pursued jointly with A. Kolokolova about a generalization of the earlier construction to a stronger theory $VTNC$. I find this last part too sketchy to be able to understand it.

Chapter 4 is not based on any published paper but reports on a work in progress concerning the definability of various functions in theory VTC^0. This is approached axiomatically: one introduces symbols for new functions and axioms codifying their properties and then proves various reductions among them. This is apparently meant to single out the key problems in the original formalizations considered and any theory that could prove the axioms about some formalization of the new symbols would work for the original problems too.

I think this is an interesting approach but its presentation here contains various imprecisions and small errors and lacks enough details, and my view is that all in all it would be better not to include this chapter at all.

Finally, Appendix C offers to a reader not familiar with various logical or complexity-theoretic concepts and facts a brief introduction. This may be useful as a source of literature but the presentation of the topics is uneven so the reader who really needs to learn the background would have to look somewhere else as well.

My overall opinion is that S. Muller clearly proved his ability to do independent research in mathematical logic and obtained several very interesting results. Somewhat more careful write-up would do them more justice. *I do recommend* that the thesis is successfully defended.

Sincerely yours,

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