

Title: Selfdistributive Algebras and Knots

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Abstract: In the first part of this thesis we summarize the basics of knot theory, a part of algebraic topology that studies mathematical knots, introduce algebraic structures called quandles and briefly describe how they are used in knot theory. In the main part of this thesis we derive some properties of affine quandles, a class of quandles associated with abelian groups. We introduce new terminology that allows us to describe affine quandles from a new perspective, and to prove a theorem that gives us a full characterization of finite affine quandles. Using this terminology, we give new detailed proofs of known results that fully describe the situation when two affine quandles are isomorphic.

In the end, we present an algorithm which decides from the Cayley table of a quandle if the quandle is affine. Again, it is based on the terminology and the claims from the previous sections, and significantly improves the previously known results.

Keywords: Alexander invariant, knot quandle, affine quandle, Cayley table, algorithm

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