Referee report on the work

“Chaotic motion around Black Holes”

submitted by Mgr. Petra Suková for the degree PhD

The submitted Thesis summarises author’s study of test particle motion in perturbed black hole space-times with emphasis on distinction among regular and chaotic trajectories. Beside this main topic there is also a chapter on chaotic behaviour of damped mechanical oscillators described by nonstandard constitutive relations.

There are two major parts of output of the author’s work: First, she investigated several different methods of detection of chaotic patterns in the motion of test particles. She discussed their suitability in the framework of general relativity and compared their efficiency which is especially important in the presence of weakly chaotic orbits and/or in the case when large volume of the phase and parameter space needs to be examined automatically. Second, the properties of phase space with respect to the regularity and chaos were studied for systems with known analytic form of metric. Parameters of the orbits (e.g. energy) and of the axisymmetric (perturbing) source of gravity were discussed in the context of onset of chaos. The work done is quite extensive and has led to several new findings, e.g. that the chaotic domain grows with compactness and mass of the perturbing source and energy of the particle. On the other hand, this trend was found not to be general and the regular islands may reappear when the parameters exceed some limit.

The amount of work done, together with the new results presented in the Thesis qualify the author as an perspective scientist and I recommend the Thesis to be accepted as doctoral. Below I rise a couple of questions and some critical comments which are intended to direct the author to possible improvements of her work.

One thing which I miss is brief summary evaluation of the considered methods of detection of chaos. I suspect that it may be quite a difficult task to formulate robust statements in this point. Nevertheless, I’d like to see a discussion saying either that (at least in the case of studied systems) one method clearly outer-performs another one or in which situations one method is better than the others. There are actually several statements in this regard found in the Thesis. Nevertheless, they are often quite fuzzy. In particular, in the beginning of Section 4.1.3, the reader gains an impression that the FLI method is able to detect chaos on very short time, i.e. it is very effective. This feeling however changes in subsequent paragraphs where an example of weakly chaotic orbit is presented. Similar situation is with MEGNO which is first presented as a break through (if one overlooks the fuzziness in determining the limiting value), but then it is put back to reality with the statement that it can misdetect chaotic behaviour if it occurs on time-scales longer than the integration time. Generally speaking, this is probably common limitation of all the methods considered, but if I understand well, MEGNO has an advantage for automatic evaluation of chaos. Such an information should be pointed out and put in context with other methods.

Regarding the study of the onset of chaos itself I recommend to try to look for possible consequences. Some qualitative behaviour was described as I have already mentioned above. Would it be possible to formulate a framework which would give also quantitative results? For example to evaluate the volume of the phase space region which is occupied by states that will reach the black hole horizon due to the chaotic motion. I think that such a quantitative study could be then used for comparison with other effects (e.g. non-stationarity, two-body relaxation, etc.) or for evaluation how important are relativistic corrections (the author write that non-linearity of Einstein equations is a natural 'source' of chaos, however, chaotic motion could be obtained also in analogical systems described with Newtonian gravity).

I didn't like much the formal layout of the Thesis. Neither it is a standalone text as the reader is forced to skip forward and backward in order to see what figures he is reading about, neither it is a set of papers with a brief foreword, putting them into a more general context. Even in the current setup, the author could have had payed a bit more attention to how the references to figures are given. The same figure is once referenced as A.4, second just figure 4 and as figure 4 in Appendix A at another place. Some figures have labels which are hard to read; I miss description of the colour palette in figure C.6. The text in general is quite readable and I particularly appreciate author's ability to briefly introduce the theory of chaos in dynamical systems and methods of its detection. I don't feel qualified to criticise author's English, I only think that the she often violates correct English sentence word order (subject – verb – object – manner – place – time).

Finally, let me ask one explicit question: The accuracy of used numerical integrator is not discussed thoroughly. I have only found some notes on this issue on p. 38. First, it is not written for which parameters the accuracy of the four-velocity normalisation of the order of 10^{-14} was reached. Second, I'm quite surprised by that value as it exceeds the resolution of 64-bit float only by two orders of magnitude. Were the calculations performed in more bit variables? What was the error for other conserved quantities which the integrator is not aware of (energy and angular momentum)?

Praha, 25.8.2013 Ladislav Šubr