Institute of Theoretical Physics
Friedrich-Schiller-University Jena
Max-Wien-Platz 1
D-07743 Jena
Germany

August 17, 2013

The Faculty of Mathematics and Physics
Charles University Prague
Student Affairs Department-Doctoral study programmes
Ke Karlovu 3
12116 Prague 2
Czech Republic

Dear Madam or Sir,

The doctoral thesis

"Chaotic Motion around Black Holes"
written by **Mrs. Petra Suková**

is a novel work, which studies chaotic geodesic motion in static axisymmetric spacetime backgrounds. These spacetimes are produced by superimposing, in a self-consistent manner, the Schwarzschild back hole spacetime with disks or rings. These spacetime backgrounds are modeling systems lying at the centers of galaxies, where supermassive black holes are surrounded by accreting matter. Inspiraling compact objects in such systems will follow approximately geodesic orbits. The study of these geodesic orbits is the main topic of Mrs. Suková's thesis, and this study by itself is of astrophysical interest. However, these inspiraling objects emit electromagnetic and gravitational wave signals, and the characteristics of the inspiraling motion are encoded in these signals. We expect that in the near future we will be able to detect these signals. The fact that Mrs. Suková explores various methods to study geodesic chaos is not only interesting from the aspect of orbital and non-linear dynamics, but also provides the means to analyze the signals coming from the aforementioned astrophysical systems, for example by applying the recurrence analysis on these signals.

From the work done for this thesis two articles (Appendices A & B) have already been published in a high impact factor peer reviewed journal, and a third (Appendix C) is probably ready to follow. The published articles have acquired 13 citations (excluding self-citations) according to NASA ADS database.

By examining Mrs. Suková's thesis, the following main questions arose:

1. The method of two nearby orbits for calculating mLCE in the Appendix C uses only the divergence in the configuration space. However, the mLCE is the maximum Lyapunov characteristic exponent in the phase space. How can we be sure that we have found the mLCE when we study only a subspace, i.e. the configuration space, of the phase space?

2. Is it possible to find an index which reveals the chaotic nature of orbits in time smaller than $t = 1/\text{mLCE}$? Are, for instance, recurrence analysis and WADV methods faster than Lyapunov-like indices?
3. In order to achieve a more realistic approximation of an extreme mass ratio inspiral, the radiation reaction could be included in the evolution of orbits in static axisymmetric spacetime backgrounds. What would be the means and the difficulties to apply and investigate such a setup?
4. What is the upper limit of the noise to signal ratio for which the recurrence methods could discern the signal of a chaotic deterministic orbit from a background noise in an EMRI system?

Below I quote some further minor remarks:

1. pg. 7, eq. (1.3): Can all the integrable Hamiltonian systems be written in the form of eq. (1.3) ? For instance, has the Hamiltonian function of the Kerr spacetime been written in the eq. (1.3) form?
2. pg. 8, parag. 2: The diffusion of chaotic orbits in systems with more than two degrees of freedom is called Arnold diffusion, when it is slow. Hence, does this paragraph refer only to nearly integrable systems?
3. pg. 31, parag. 2: "For the purpose...of the disc", pg. 41, parag. 1: "Therefore...dust torus". What are the exact limitations of the approximation applied to model the motion in a galactic center in the study?
4. The term of the curve of zero velocity (CZV) usually refers to a boundary on which the velocities are zero. Is the term CZV appropriate for a surface of section, e.g. Fig. 4.1, where one of the axes is a velocity?
5. pg. 77, last paragraph: "Contrary to...bifurcation diagram,"
   In conservative systems periodic orbits bifurcate and produce the so called characteristic curves, in which way this differs in dissipative systems?
6. Appendices A to C: Wouldn't it be more appropriate to consider frequency analysis methods for chaos detection, as proposed by Laskar (e.g. Laskar, CMDA **56**, 19 (1993)), when comparing them with the other methods?
7. section 2.6.1 & Appendix B: If adaptive step integration schemes are applied, can we still use recurrence analysis and WADV methods?
8. Appendices B & C: When comparing the different methods of chaos detection, which are the more demanding in computational resources? For instance, for the same orbit what is the CPU time required for each method to find out whether an orbit is chaotic or regular?

In conclusion, Mrs. Suková's thesis proves her ability to conduct creative scientific work, and shows a candidate worthy to acquire the doctoral degree.

Yours Sincerely,

Dr. Georgios Loukes-Gerakopoulos