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Volume - volatility relation across different volatility estimators

Author: **Tomáš Kvasnička**

Supervisor: **PhDr. Ladislav Krištoufek**

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Declaration of Authorship

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Abstract

The main objective of this thesis is to analyze whether traded volume increases predictive power of volatility. We are mostly focused on Garman-Klass volatility estimator, which is more efficient than squared returns. Both univariate (AR, HAR, ARFIMA) and multivariate models (VAR, VAR-HAR) are used to find out if traded volume improves volatility forecasting. Furthermore, GARCH(1,1) both with and without traded volume is carried out and forecasted. All these methods are estimated on a basis of rolling window and during each step 1-day ahead forecast is computed. Final assessment is based on MAPE, RMSE and Mincer-Zarnowitz test of the out-of-sample forecasts, which are compared with the realized volatility. It turns out that traded volume slightly improves predictive power of the scrutinized models in case of FTSE 100 and IPC Mexico, contrary to Nikkei 225 and S&P 500 when a decrease of the predictive power is detected. Moreover, we observe that only HAR and VAR-HAR models are able to produce an unbiased forecast. As the evidence of the improvement is not conclusive and to maintain model parsimony, HAR model fitted by Garman-Klass volatility appears to be the best alternative in case of missing the realized volatility.

JEL Classification C32, C53, C58, G17
Keywords traded volume, volatility, forecast, Garman-Klass estimator, VAR, HAR, ARFIMA, GARCH

Author's e-mail tomkvas@email.cz
Supervisor's e-mail kristoufek@gmail.com

Abstrakt

Cílem této bakalářské práce je zhodnotit, zda-li obchodované množství zlepšuje predikční schopnosti volatility. Převážně se zaměřujeme na Garman-Klassův odhad volatility, který je vydatnější než čtvercové výnosy. Jak jednorozměrné modely (AR, HAR, ARFIMA) tak vícerozměrné modely (VAR, VAR-HAR) jsou použity k zjištění, zda-li obchodované množství zlepšuje predikci volatility. Dále je použit GARCH(1,1), ke kterému je také přidáno obchodované množství, a následná predikce je počítána. Všechny tyto modely jsou odhadovány na základě posuvného okna, kdy během každého posunu je vypočítána jednodenní předpověď' volatility. Konečné zhodnocení je založené na MAPE, RMSE a Mincer-Zarnowitz testu predikčních hodnot poměřených s realizovanou volatilitou. Ukazuje se, že obchodované množství zlepšuje predikční schopnosti v případě FTSE 100 a IPC Mexico a zhoršuje predikční schopnosti v případě Nikkei 225 a S&P 500. Navíc je zjištěno, že pouze HAR a VAR-HAR modely jsou schopny produkovat nevychýlené předpovědi. Jelikož prezentované důkazy zlepšení predikce nejsou přesvědčivé a kvůli zachování jednoduchosti modelu, HAR model obsahující Garman-Klassův odhad volatility se jeví jako nejlepší varianta v případě nedostupnosti realizované volatility.

Klasifikace JEL	C32, C53, C58, G17
Klíčová slova	obchodované množství, volatilita, předpověd', Garman-Klassův odhad, VAR, HAR, ARFIMA, GARCH

E-mail autora	tomkvas@email.cz
E-mail vedoucího práce	kristoufek@gmail.com

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Acronyms

AIC	Akaike Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedasticity
ARFIMA	Fractionally Integrated Autoregressive Moving Average
BIC	Bayesian Information Criterion
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedasticity
FTSE 100	Financial Time Stock Exchange 100
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GK	Garman-Klass volatility
HAR	Heterogenous Autoregressive
IPC	Index of Prices and Quotations
IRF	Impulse Response Function
MAPE	Mean Absolute Percentage Error
MDH	Mixture of Distributions Hypothesis
PACF	Partial Autocorrelation Function
RMSE	Root Mean Squared Error
RV	Realized Volatility
RW	Random Walk
SaP 500	Standard and Poor's 500
SIAH	Sequential Information Arrival Hypothesis
VAR	Vector Autoregressive

Chapter 1

Introduction

Volatility, as a measure of price variability over some time period (Taylor 2005), is a key variable in several areas of finance such as option pricing, risk management or portfolio selection. Measuring volatility can become very tricky as it is not directly observable variable (we call it latent variable). Therefore, several basic proxy variables such as squared or absolute returns were developed. Since Black & Scholes (1973) introduced formula for option pricing, where volatility is one of the inputs, both researchers and practitioners started to scrutinize its estimation and spent past decades developing more efficient estimators.

The efficiency is crucial in several ways. First, if we want to determine the relationship between volatility and some other variable such as macroeconomic announcement or trading volume, its inaccurate estimation can lead to misleading conclusions. Second, it is supposed that the more efficient estimator contains better information set on the future volatility. Third, the evaluation of out-of-sample forecast is based on comparison with volatility proxy and if this proxy is not efficient, final evaluation of the scrutinized models can lead to the wrong model selection. Andersen & Bollerslev (1998) show that traditional ARCH and stochastic volatility estimators does not perform as bad as was considered when the comparison is made with the more efficient 5-minute realized volatility instead of the squared returns. Furthermore, Hansen & Lunde (2006) use both real and simulated data to confirm that evaluation of forecasting models based on squared returns leads to inaccurate ranking and hence extreme value estimators or realized volatility should be used. Moreover, they emphasize that this problem arises whenever forecasts are compared with a proxy variable.

The implied volatility is considered to have the best predictive power of the future volatility (Poon & Granger 2003). Besides implied volatility, many researchers use various models fitted by realized variance, which is generally considered as the most efficient measure of volatility. Andersen *et al.* (2003) uses VAR fitted by realized volatility to outperform traditional ARCH models.

However, the problem can arise in case of lack of necessary data. First, not all of the assets are traded through financial derivatives and hence concept of the implied volatility is not feasible. Second, to compute the realized volatility we have to possess intraday returns, which are mostly not publicly available or their acquisition is very costly. Therefore, we will concentrate on one of the most commonly used extreme value estimator developed by Garman & Klass (1980), which uses daily open, low, high and close prices. Moreover, we will focus on the relationship with traded volume, which is also publicly available information for most of the financial time series.

There are several papers suggesting that volume and volatility are correlated (Karpoff 1987; Harris 1987; Gallant *et al.* 1992; Charles *et al.* 1994). We will apply both univariate (RW, AR, HAR, ARFIMA) and multivariate (VAR, VAR-HAR) forecasting models to traded volume and Garman-Klass volatility to reveal if the detected relationship improves the out-of-sample forecasts. Furthermore, GARCH(1,1) both with and without traded volume will be estimated and compared with the forecasted Garman-Klass estimator. The final evaluation and appropriateness of traded volume will be based on RMSE, MAPE and Mincer-Zarnowitz test. As a volatility proxy we will use the realized volatility, which is publicly available for the highly traded stock indices. All of this research will be made on the basis of rolling window, which will provide insight how the relationship between volume and volatility differs across time.

The thesis is organized as follows: Chapter 2 focuses on literature review and compile the research on this topic, Chapter 3 provides explanation of the methodology used to assess the relationship, Chapter 4 describes our data, Chapter 5 presents results together with the out-of-sample evaluation and Chapter 6 concludes the whole thesis.

Chapter 2

Literature review

Extreme value estimators utilize historical daily open, low, high and close prices reported by many financial sources. The first estimator introduced by Parkinson (1980) takes into account open and close prices. He proves that his estimator provides better results in comparison with the traditional measures such as the squared returns. Other estimators using the range prices are developed by Garman & Klass (1980), Rogers & Satchell (1991) and Yang & Zhang (2000). Several papers show that extreme value estimators are multiple times more efficient than traditional estimators (Wiggins 1991; Rogers *et al.* 1994; Bali & Weinbaum 2005). Taylor (2005) points out that Parkinson estimator is accurate approximately as the sum of five intraday squared returns and Garman-Klass estimator is accurate as the sum of eight squared intraday returns. The problem with the extreme value estimators is the use of daily low and high prices, which tends to be sensitive to outliers. Hence, 5% quantiles might be used instead of the highest and lowest price. However, if we had the quantiles, we could already use the realized measures.

There are two major theoretical approaches explaining relationship between stock price changes and traded volume. The first theory called Mixture of Distributions Hypothesis (MDH) is introduced by Clark (1973) and further developed by Epps & Epps (1976) and Tauchen & Pitts (1983). It assumes that arrival of new information forms the contemporaneous relationship among variables. After release of a new information both volume and volatility increase since investors renew their positions on what they believe is the new true value of their assets. It entails that price changes are sampled from mixture of conditional distributions, where as mixing variable is considered to be the rate at which information appears in the market. In this case, volume is a proxy variable for the information arrival. The other proxies can be, for example, number of transactions or number of information announcments.

The second theoretical framework called Sequential Information Arrival Hypothesis (SIAH) is introduced by Clark (1973) and further developed by Jennings *et al.* (1981). It states that new information arrival is spread sequentially to traders. Therefore, only the well informed traders can adjust their position. This results in a series of sequential equilibria until the information is known to each trader and final equilibrium is reached. Consequently, the sequential arrival of information from trader to trader generates the sequential movements of trading volume and price movements, both increase as the rate of arrival of information to market increases.

Karpoff (1987) is one of the first who provides an extensive summary on the positive relation between stock volatility (measured both as absolute and squared returns) and trading volume. Harris (1987) confirmed the positive correlation between volume and changes in squared returns, Gallant *et al.* (1992) and Charles *et al.* (1994) find positive relationship between volume and volatility. Moreover, they emphasize that number of trades is even more important and has higher influence on volatility. Bessembinder & Seguin (1993) find relationship between volume and absolute returns and show that it is possible to extract even more information from the volume when it is decomposed into expected and unexpected part. They state that the unexpected volume has significantly larger impact on volatility than the expected part.

Besides measuring correlation of squared returns and traded volume, we can also observe the relationship of traded volume with conditional variance through ARCH models. Lamoureux & Lastrapes (1990) assume trading volume to be exogenous variable and included it in the conditional variance equation. After implementation of the contemporaneous trading volume, GARCH effect remains only in four out of twenty stocks, while the volume is significant in all stocks. Futher research on volume and GARCH type models is carried out by Kim & Kon (1994); Andersen (1996); Gallo & Pacini (2000). Most of the studies agree on the fact that volume significantly decreases or even eliminates the persistence in the GARCH equation. Gallagher & Kiely (2005) follow similar pattern of their predecessors on less liquid Irish stocks and find out that in case of low traded stock, the reduction in persistence is less conclusive. They states that the liquidity of market plays an important role and underline it by the fact that the stocks without significant decrease of persistence has over twice as many days with no price change. Similar results are observed on the less liquid Polish market by Bohl & Henke (2003) and the Taiwan market by Huang & Yang (2001).

Lee & Rui (2002) examine dynamic relations between stock market trading volume and returns and find out that volume does not cause return. However, they point out that there exist a positive relationship between trading volume and return volatility in all examined markets. To detect causality between returns, volume and volatility Vector Autoregressive model (VAR) is used. The similar procedure with employing VAR to detect causality can be found in Chen *et al.* (2001); Mestel *et al.* (2003); Wang (2004); Medeiros & Doornik (2006); Pisedtasalasai & Gunasekarage (2007). Chen *et al.* (2001) find positive causality between trading volume and absolute returns on nine observed markets during period from 1973 to 2000. Mestel *et al.* (2003) find strong contemporaneous correlation between trading volume and return volatility. Additionally, they show that past volatility contains information about future trading volume. American, Japanese and Chinese stock markets are scrutinized by Wang (2004) who supports the contemporaneous correlation between trading volume and volatility. On the contrary to preceding evidence, he further finds negative causality between volume and subsequent volatility. Medeiros & Doornik (2006) examine Brazilian stock market Bovespa and find positive both contemporaneous and dynamic relationship between return volatility and trading volume. Pisedtasalasai & Gunasekarage (2007) find evidence from emerging markets in South-East Asia that trading volume has very limited impact on the future dynamics of stock returns, but might contain information that is useful for predicting the future dynamics of volatility.

All of these papers find either contemporaneous or dynamic relationship between trading valume and volatility. However, nearly none of the above mentioned authors show whether their models provide accurate forecast and whether utilization of trading volume improves forecasting ability of a given model. Moreover, they mostly use absolute or squared returns as the volatility proxy. We will try to improve the traditional procedures by implementing more efficient Garman-Klass estimator into several predictive models. Furthermore, we will compare the predictive power of the scrutinized models with GARCH(1,1) to find out whether Garman-Klass estimator together with volume is superior to the traditional GARCH. The influence of traded volume will be based on the accuracy of the rolling out-of-sample forecasts.

Chapter 3

Theoretical background

This Chapter is dedicated to the methodology used throughout this thesis. The first part is focused on different volatility estimators and the second part deals with the models used for forecasting the Garman-Klass volatility estimator.

3.1 Volatility estimators

Realized volatility is generally deemed to be the best volatility estimator since it actually measures the volatility as it continuously tracks the changes of price of a given asset. However, the realized volatility is not published for all financial assets. Therefore, one of the most efficient range estimators, Garman-Klass volatility estimator, is used to substitute the realized volatility. In this section, the assumptions and derivation of Garman-Klass estimator are discussed. Moreover, traditional GARCH model is presented to further compete with the Garman-Klass estimator. Hansen & Lunde (2006) emphasize that realized variance should be used instead of squared returns to maintain consistent ranking of volatility models. Therefore, the realized volatility is used as volatility proxy and hence also further presented.

3.1.1 Garman-Klass estimator

In order to find more efficient volatility estimator than the simple squared returns Garman & Klass (1980) introduce estimator, which takes into consideration open, high, low and close prices. To derive the final estimator, they assume that a diffusion process governs security price as

$$P(t) = \phi(B(t)), \tag{3.1}$$

where P represents security price, t is time, ϕ is a monotonic, time independent transformation and $B(t)$ is a diffusion process with the differential equation

$$\mathrm{d}B = \sigma \mathrm{d}z, \tag{3.2}$$

where dz is the standard Gauss-Wiener process and σ is an unknown constant to be examined. This definition is general enough to fulfill the usual hypothesis of the geometric Brownian motion of stock price.

There are several limitations to the proposed assumptions. First, the price evolves discretely even if we posses high-frequency data. Second, there is certain time interval when the market is closed and the price is not observable. Third, the price evolution does not take into consideration covariance among securities. Finally, it is assumed that the price evolves without drift. Although the proposed model has several pitfalls, Garman & Klass (1980) argue that the theoretical assumptions are necessary to derive the required estimator. They find out that discrete evolution of price causes downward bias, and as the number of trades increases the bias decreases.

Let us denote the opening price at day t as O_t, the highest price as H_t, the lowest price as L_t and the closing price as C_t then we can calculate open-to-close, open-to-high and open-to-low returns as

$$c_t = \ln(C_t) - \ln(O_t) \tag{3.3}$$

$$h_t = \ln(H_t) - \ln(O_t) \tag{3.4}$$

$$l_t = \ln(L_t) - \ln(O_t). \tag{3.5}$$

Then, the minimum variance analytical estimator looks as follows:

$$\hat{\sigma}^2_{GK,t} = 0.511(h_t - l_t)^2 - 0.019(c_t(h_t + l_t) - 2h_t l_t) - 0.383c_t^2 \tag{3.6}$$

Garman & Klass (1980) prove that their estimator is theoretically 7.4 times more efficient than commonly used squared returns. Bali & Weinbaum (2005) compare several extreme-value estimators (including squared returns) with realized volatility and show that Garman-Klass estimator performs best both in bias and efficiency across all tested estimators. The gain in bias and efficiency against squared returns is noticeable even at weekly and monthly frequencies. Molnár (2012) further compares several range-based estimators and GARCH models to show that returns normalized by adjusted[1] square root of Garman-Klass estimator follow approximately standard normal distribution. Despite the fact that not all of the assumptions hold, the Garman-Klass estimator appears to be the best low-frequency estimator.

3.1.2 GARCH(p,q)

Engle (1982) developed the first model capturing basic characteristics of financial returns such as heavy-tailed distribution and volatility clustering. His ARCH(p) model decomposes volatility into constant unconditional and time-varying conditional part where conditional volatility is dependent on the past information set. The basic idea is to model volatility by past squared shocks. However, high number of the shocks is often required which might lead to improper model specification and loss of degrees of freedom. Bollerslev (1986) improves the model by adding q lags of conditional volatilities which makes GARCH an infinite order ARCH with geometrically declining set of weights. This extension makes the model more parsimonious which also leads to higher usefulness in a wide range of data. The general GARCH(p,q) looks as

$$r_t = \mu_t + \epsilon_t \tag{3.7}$$

$$\epsilon_t = z_t h_t^{1/2}, z_t \sim N(0,1) \tag{3.8}$$

$$h_t = \omega_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}, \tag{3.9}$$

where r_t stands for return at time t, μ_t is any stationary/weakly dependent leading process and ϵ_t is a residual term in time t distributed according $N(0, h_t)$ and following equation (3.8) where $\{z_t\}$ is a sequence of iid random variables of zero mean and variance of 1. Furthermore, h_t is conditional variance at time t which is dependent on p lags of squared residuals ϵ_t^2 and q lags of conditional variance h_t. Poon & Granger (2003) suggest in their extensive volatility comparison that GARCH(1,1) is one of the most popular models from GARCH family. To see if the traded volume increases the predictive power of the GARCH model, we add logarithm of traded volume into conditional

[1] If $\hat{\sigma}^2_{GK,t}$ is unbiased estimator of σ^2 then $\hat{\sigma}_{GK,t}$ is not unbiased estimator of σ since $E(X^2) \neq E(X)^2$. Molnár (2012) stresses that $\hat{\sigma} = \sqrt{\hat{\sigma}^2_{GK,t}} * 1.034$ should be used as unbiased estimator.

variance equation of the simple GARCH(1,1) as

$$r_t = \epsilon_t \tag{3.10}$$

$$\epsilon_t = z_t h_t^{1/2}, z_t \sim N(0,1) \tag{3.11}$$

$$h_t = \omega_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + \delta \ln (volume_{t-1}), \tag{3.12}$$

where $\delta \in \mathbb{R}$ measures the influence of lagged trading volume on contemporaneous conditional variance. We omit the mean equation μ_t since Cont (2001) points out the stylized fact that autocorrelations of financial returns are insignificant. Moreover, returns are heavy-tailed and not exactly normally distributed and hence also confidence intervals of the autocorrelation function might be misspecified and overvalued.

Bollerslev (1986) states the following conditions for GARCH(1,1) to be stationary: $\omega_0 > 0$, $\alpha_1 \geq 0$, $\beta \geq 0$ and $\alpha_1 + \beta < 1$. If the latter condition does not hold and $\alpha_1 + \beta = 1$, then GARCH(1,1) becomes nonstationary IGARCH(1,1) developed by Engle & Bollerslev (1986). The problem with IGARCH is that the unconditional volatility is not defined which becomes an issue in forecasting, since the stationary GARCH converges to its unconditional volatility (Starica 2004).

The one-step ahead forecast is computed in order to compare the performance of GARCH with the other forecasting methods taking into consideration logarithm of traded volume. The one-day ahead conditional variance look as follows:

$$\widehat{\sigma_{t+1}^2} = E(h_{t+1} \mid I_t) = \hat{\omega_0} + \hat{\alpha_1} r_t^2 + \hat{\beta}_1 h_t + \hat{\delta} volume_t \tag{3.13}$$

Since conditional variance, squared returns and traded volume are included in information set I_t available at time t, obtaining the forecast is straightforward.

3.1.3 Realized volatility

Taylor (2005) points out that daily volatility can be estimated more precisely as the frequency of squared intraday returns increases. Realized volatility utilize high-frequency intraday returns and therefore the information extracted from data has superior value-added compared to the classical low-frequency data. The concept is appealing mainly as it does not use any parametric method to estimate volatility, but it measures volatility through the sum of N intraday squared returns as

$$\hat{\sigma}_t^2 = \sum_{i=1}^{N} r_{t,i}^2, \tag{3.14}$$

where $\hat{\sigma}_t^2$ stands for realized variance and $\sqrt{\hat{\sigma}_t^2}$ for realized volatility. Early examples of realized volatility can be found in Zhou (1996), Taylor & Xu (1997) and Ebens (1999). The formalized theory together with statistical properties and poofs can be found in Andersen *et al.* (2002) and Andersen *et al.* (2003). Bandi & Russell (2006) and Bandi & Russell (2008) emphasize that as $N \to \infty$ the realized variance estimator becomes biased due to microstructure noise. They study the optimal sampling and find out that 5-minute data brings optimal trade-off between loss of information and bias of the estimator. Furthermore, it is shown by Andersen *et al.* (2001), Areal & Taylor (2002) and Andersen *et al.* (2003) that natural logarithm of realized volatility is approximately normaly distributed which entails that it is more useful in modeling and forecasting. Therefore, logarithmic standard deviations of the realized variance and Garman-Klass variance will be used throughout this thesis. Based on the efficiency of this estimator, it will be used as the volatility proxy which will be further compared with the forecast of Garman-Klass volatility.

3.2 Forecasting methods

We apply several forecasting methods to Garman-Klass volatility estimator and find out whether trading volume improves results of the scrutinized models. We begin with introducing basic models and finish with the more advanced approaches. Finally, all the methods, including GARCH(1,1) as a benchmark, are compared based on their MAPE, RMSE and Mincer-Zarnowitz test. All the estimations and forecasts are carried out on the basis of rolling window when each sample consists of 500 observations and will be moved forward by one observation. This approach provides us with information how the performance of each model evolves in time.

3.2.1 Random walk

Concept of random walk has been used for several decades with the major growth in seventies of the twentieth century. The basic definition provided by Tsay (2005) is as follows.

A time series $\{p_t\}$ is called random walk if it satisfies

$$p_t = p_{t-1} + \epsilon_t, \tag{3.15}$$

where $\{\epsilon_t\}$ is a white noise series. If ϵ_t has a symmetrical distribution, then p_t conditional on p_{t-1} has a 50-50 chance to go up or down which implies that p_t evolves unpredictably. The equation (3.15) resemples AR(1) process with the only difference of the coefficient of p_{t-1} not being less than one in modulus but being unity. It does not satisfy the weak stationary condition of AR(1) and therefore it is often called a nonstationary unit-root process (Tsay 2005).

The model does not use any advanced technique, but only the simple idea that the future l-step ahead forecast is the same as the todays value of p_t:

$$E(p_{t+1} \mid p_t, p_{t-1}, \dots) = E(p_t + \epsilon_{t+1} \mid p_t, p_{t-1}, \dots) = p_t \tag{3.16}$$

$$E(p_{t+l} \mid p_t, p_{t-1}, \dots) = E(p_{t+l-1} + \epsilon_{t+l} \mid p_t, p_{t-1}, \dots) = \dots = p_t \tag{3.17}$$

However, $Var(p_{t+l}) = l\sigma_\epsilon^2$ diverges to infinity as $l \to \infty$. Therefore, the usefulness of point forecast p_{t+l} diminishes for sufficiently large l since it can assume nearly any real number. For the purpose of basic comparison whether Garman-Klass volatility estimator is possible to forecast it is useful to apply the random walk to Garman-Klass volatility estimator as

$$GK_t = GK_{t-1} + \epsilon_t \tag{3.18}$$

and compare the forecasts with the other more advanced techniques.

3.2.2 Autoregressive model

Autoregressive process of length p express conditional expectation of r_t as a function of past values of r_{t-i} where $i = 1, \dots, p$. This process is defined for example by Tsay (2005) as

$$r_t = \phi_0 + \phi_1 r_{t-1} + \ldots + \phi_p r_{t-p} + \epsilon_t, \tag{3.19}$$

where p is a non-negative integer and $\{\epsilon_t\}$ is assumed to be a white noise with mean zero and variance σ_ϵ^2. The condition for series $\{r_t\}$ to be stationary is that all solutions of characteristic equation (3.20) of the model (3.19) must be greater than one in mudulus.

$$1 - \phi_1 x - \phi_2 x^2 - \ldots - \phi_p x^p = 0 \tag{3.20}$$

Or the other way round, series $\{r_t\}$ is stationary if characteristic roots, expressed as inverses of the solutions to (3.20), are less than one in modulus.

The selection of p, number of lags, is usually based on PACF and information criteria of whom the most used are Akaike information criterion (AIC) (Akaike 1973) and Bayesian information criterion (BIC) (Schwarz 1978). However, we choose the p to equal 5, since the measurement of minimal information criteria on each rolling window would be cumbersome. Our model with 5 lags of natural logarithm of Garman-Klass volatility estimator looks as follows:

$$\ln(GK)_t = \phi_0 + \phi_1 \ln(GK)_{t-1} + \ldots \phi_5 \ln(GK)_{t-5} + \epsilon_t \tag{3.21}$$

We use the logarithm of Garman-Klass volatility estimator rather than the absolute values due to its normality. This problem is further described in Data description.

3.2.3 Heterogeneous Autoregressive model

Muller *et al.* (1993) formulate Heterogenous Market Hypothesis which declares that different actors in the market have different time horizons and dealing frequencies. They support this statement by considering FX dealers on the side of high dealing frequency, and central banks and pension funds on the side of low dealing frequency. The different frequencies result in different reactions of market participants to the same news. These findings are summarized and further developed in Fractal Market Hypothesis proposed by Peters (1994). Based on these theories, Muller *et al.* (1997) develop HARCH model which takes into consideration different time horizons of market participants by aggregating squared returns in conditional volatility equation of the classic GARCH model. Corsi (2009) further develops this theory and divides market participants into three categories: short-term traders with daily frequency, medium-term traders with weekly trading frequency and long-term traders who typically rebalance theirs positions in one month or later. He applies this theory to realized volatily and derives the model as

$$RV_t = \alpha + \beta_1 RV_{t-1}^{(d)} + \beta_2 RV_{t-1}^{(w)} + \beta_3 RV_{t-1}^{(m)} + \epsilon_t, \tag{3.22}$$

where $RV_{t-1}^{(d)}$, $RV_{t-1}^{(w)}$ and $RV_{t-1}^{(m)}$ stands for 1-day, 5-day average and 20-day average of realized volatility in time $t-1$, respectively. This decomposition allows to study dynamics of different volatility components. Corsi (2009) shows on several simulations that although this simple model does not belong to long memory models, it captures volatility persistence well and even outperforms long memory ARFIMA process on his out-of-sample forecast. Moreover, he shows that volatility simulated through this model exhibit leptokurtic distribution in the similar way as the real data. We adjust the original model by implementing the logarithm of Garman-Klass estimator instead of the realized volatility. Then, the model looks as

$$\ln(GK)_t = \alpha + \beta_1 \ln(GK)_{t-1}^{(d)} + \beta_2 \ln(GK)_{t-1}^{(w)} + \beta_3 \ln(GK)_{t-1}^{(m)} + \epsilon_t, \tag{3.23}$$

where $\ln(GK)_{t-1}^{(d)}$, $\ln(GK)_{t-1}^{(w)}$ and $\ln(GK)_{t-1}^{(m)}$ stands for 1-day, 5-day average and 20-day average of logarithm of Garman-Klass volatility at time $t-1$, respectively.

3.2.4 Vector Autoregressive model

Vector autoregressive model is developed by Sims (1980) who originally used it for structural inference and policy analysis on macroeconomic level. As this model becomes popular among researchers its usefulness spreads also into microeconomic level and finance. Its popularity stems from assumption that everything may depend on everything and moreover all the relationships are measured in time. For example, Tsay (2005) defines VAR(p) for multivariate time series r_t as

$$\boldsymbol{r_t} = \boldsymbol{\phi_0} + \boldsymbol{\phi_1 r_{t-1}} + \ldots + \boldsymbol{\phi_p r_{t-p}} + \boldsymbol{\epsilon_t}, \tag{3.24}$$

where $p > 0$ is number of included lags, $\boldsymbol{\phi_0}$ is a k-dimensional vector of intercepts, $\boldsymbol{\phi_i}$ is a $k \times k$ matrix of slope coefficients for i-th lag with k variables in the VAR system and $\{\boldsymbol{\epsilon_t}\}$ is a sequence of serially uncorrelated random vectors with zero mean and covariance matrix $\boldsymbol{\Sigma}$. In practice, the covariance matrix $\boldsymbol{\Sigma}$ should be positive definite otherwise the dimension of $\boldsymbol{r_t}$ can be reduced. The VAR(p) model in the equation (3.24) is called the reduced-form model and it measures dynamic dependence of $\boldsymbol{r_t}$. To derive stationarity condition we have to rewrite the equation (3.24) into the form as

$$\boldsymbol{r_t} - \boldsymbol{\phi_1}\boldsymbol{r_{t-1}} - \ldots - \boldsymbol{\phi_p}\boldsymbol{r_{t-p}} = \boldsymbol{\phi_0} + \boldsymbol{a_t} \tag{3.25}$$

$$(\boldsymbol{I} - \boldsymbol{\phi_1}B - \ldots - \boldsymbol{\phi_1}B^p)\boldsymbol{r_t} = \boldsymbol{\Phi}(B)\boldsymbol{r_t} = \boldsymbol{\phi_0} + \boldsymbol{a_t}, \tag{3.26}$$

where $\boldsymbol{I}$ is the $k \times k$ unity matrix, B is lag operator and $\boldsymbol{\Phi}(B)$ is a matrix polynomial of lag operators. Then the necessary and sufficient condition for VAR(p) model in the equation (3.24) is that all roots of determinant $|\,\boldsymbol{\Phi}(B)\,|$ lay outside the unit circle.

The selection of the number of lags p is based on generalized partial autocorrelation function described by Johnson & Wichern (2007) and the information criteria mentioned in section 3.2.2 describing AR model. We again choose the p to be fixed and equal 5 to measure relationship within one week.

After implementing logarithms of volume and standard deviation of Garman-Klass estimator into VAR(5) the model looks as

$$\ln(GK)_t = \phi_{10} + \sum_{i=1}^{5} \phi_{1i} \ln(GK)_{t-i} + \sum_{i=1}^{5} \beta_{1i} \ln(volume)_{t-i} + \epsilon_t^{GK} \tag{3.27}$$

$$\ln(volume_t) = \phi_{20} + \sum_{i=1}^{5} \phi_{2i} \ln(GK)_{t-i} + \sum_{i=1}^{5} \beta_{2i} \ln(volume)_{t-i}) + \epsilon_t^{vol}, \tag{3.28}$$

where ϕ_{1i} and β_{1i} measure relationship between logarithm of Garman-Klass volatility and its i-th lag and i-th lag of logarithm of traded volume. The same logic applies for influence of ϕ_{2i} and β_{2i} on natural logarithm of traded volume.

The huge advantage of VAR methodology is that it brings two important tools for further analysis of causality and effects between the variables, Granger causality and impulse-response functions.

Granger causality

Granger causality is introduced by Granger (1969) and further described in Granger (1980). This approach is commonly used to find out whether there is any causal relationship from one variable to another variable. Considering our VAR system of equations 3.27 and 3.28, the whole concept can be rewritten in terms of the null and the alternative hypothesis as

$$H_0: \quad \beta_{1i} = 0 \quad \text{for all } i = 1, 2, \ldots, 5$$

$$H_1: \quad \beta_{1i} \neq 0 \quad \text{for some } i = 1, 2, \ldots, 5,$$

where under the null hypothesis H_0 logarithmic volume does not Granger-cause logarithm of Garman-Klass volatility and under the alternative hypothesis H_1 logarithmic volume does Granger-cause logarithm of Garman-Klass volatility. The same logic also applies for reverse Granger-causality from Garman-Klass volatility to traded volume.

Impulse Response Function

In the similar way as any autoregressive univariate process can be rewritten as a linear function of past error terms, we can rewrite VAR(p) model into the form as

$$\boldsymbol{r_t} = \boldsymbol{\mu} + \boldsymbol{\epsilon_t} + \boldsymbol{\Psi_1}\boldsymbol{\epsilon_{t-1}} + \boldsymbol{\Psi_2}\boldsymbol{\epsilon_{t-2}} + \dots, \tag{3.29}$$

where $\boldsymbol{\mu} = [\boldsymbol{\Phi}(B)]^{-1}\boldsymbol{\phi_0}$ provided that the inverse matrix exist. Coefficient matrices $\boldsymbol{\Psi_i}$ can be obtained from the equation

$$(\boldsymbol{I} - \boldsymbol{\phi_1}B - \dots - \boldsymbol{\phi_p}B^p)(\boldsymbol{I} + \boldsymbol{\Psi_1}B + \dots + \boldsymbol{\psi_p}B^q) = \boldsymbol{I} \tag{3.30}$$

with $\boldsymbol{I}$ being the identity matrix. Tsay (2005) describes $\boldsymbol{\Psi_i}$ as the impulse response function of $\boldsymbol{r_t}$ since $\boldsymbol{\Psi_i}$ measures influence of past shock $\boldsymbol{\epsilon_{t-i}}$ on $\boldsymbol{r_t}$. However, $\boldsymbol{\epsilon_t}$ and $\boldsymbol{\epsilon_{t-i}}$ are correlated and hence a unit shock of $\boldsymbol{\epsilon_{t-i}}$ has also impact on the shock $\boldsymbol{\epsilon_t}$. Therefore, the impact of the shock to a response variable can not be fully described by the IRF as suggested in (3.29) and further adjustments must be performed. In order to avoid this problem, we use orthogonalized IRFs.

3.2.5 VAR - Heterogenous Autoregressive model

As Corsi (2009) suggest using VAR model to exploit the features of both VAR and HAR models described in the previous sections, we decide to use trading volume as the second variable in the VAR model. We use VAR(1) model with variables $\ln(GK)$ and $\ln(volume)$ and add the 5-day and 20-day averages to each equation as exogenous variables. Then, the proposed model written in matrix form looks as

$$\begin{bmatrix}\ln(GK)_t \\ \ln(vol)_t\end{bmatrix} = \begin{bmatrix}\alpha_1 \\ \alpha_2\end{bmatrix} + \begin{bmatrix}\beta_{11} & \delta_{11} \\ \beta_{21} & \delta_{21}\end{bmatrix}\begin{bmatrix}\ln(GK)_{t-1}^{(d)} \\ \ln(vol)_{t-1}^{(d)}\end{bmatrix} + \begin{bmatrix}\beta_{12} & \delta_{12} \\ \beta_{22} & \delta_{22}\end{bmatrix}\begin{bmatrix}\ln(GK)_{t-1}^{(w)} \\ \ln(vol)_{t-1}^{(w)}\end{bmatrix}$$
$$+ \begin{bmatrix}\beta_{13} & \delta_{13} \\ \beta_{23} & \delta_{23}\end{bmatrix}\begin{bmatrix}\ln(GK)_{t-1}^{(m)} \\ \ln(vol)_{t-1}^{(m)}\end{bmatrix} + \begin{bmatrix}\epsilon_{1t} \\ \epsilon_{2t}\end{bmatrix}, \tag{3.31}$$

where $\ln(vol)_{t-1}^{(d)}$, $\ln(vol)_{t-1}^{(w)}$ and $\ln(vol)_{t-1}^{(d)}$ stands for 1-day, 5-day average and 20-day average of natural logarithm of traded volume at time $t-1$, respectively.

3.2.6 Fractionally Integrated Autoregressive Moving Average model

The most important object in forecasting is to recognize and correctly interpret the autocorrelation pattern of data. If autocorrelation function declines relatively slowly, we can apply any of the ARMA family models. If the autocorrelation pattern does not decline nearly at all, which is the case of random walk model, our data suffer from nonstationarity and further adjustments such as detrending or first differencing should be carried out (Nelson & Plosser 1982). The classification of processes to be integrated of order zero or one stems from the autocorrelation pattern. However, there might appear situations when autocorrelation declines towards zero but the declining speed is not sufficient and the first differencing is too much. The time series which exhibit this persistency is called the long memory process. The comprehensive theory on the long memory processes can be found in Beran (1994). He defines the long memory process $\{x_t\}$ as the one with the feature in time domain as

$$\sum_{k=-\infty}^{+\infty} \rho(k) = +\infty, \tag{3.32}$$

where $\rho(k)$ is the autocorrelation at lag k. This entails that also the autocorrelations at higher lags are also significant. Granger & Joyeux (1980) suggest using fractionally integrated process ARFIMA of order (p,d,q) to model a series with the long memory feature. They define the model as

$$\phi(L)(1-L)^d X_t = \theta(L)\epsilon_t, \tag{3.33}$$

where L is lag operator $\phi(L)$ is autoregressive polynomial, $\theta(L)$ is moving average polynomial, ϵ_t is white noise and d is the fractional differencing parameter. The expansion of $(1-L)^d$ looks as follows:

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \dots \tag{3.34}$$

The process is mean reverting for $d < 1$, is covariance stationary with long memory for $d \in (0, 0.5)$ and is covariance stationary with short memory for

$d \in (-0.5, 0\rangle$. There are several estimators of parameter d with the most widely used log-periodogram estimator proposed by Geweke & Porter-Hudak (1983) and local Whittle estimator proposed by Künsch (1987). In this thesis, we use maximum likelihood estimator developed in R-Project under package fracdiff.

The long memory feature of volatility is reported in various papers. Taylor (1986) analyzed 40 stock returns and finds long memory in both absolute and squared returns with the absolute returns having slower decay of autocorrelations than the squared returns. Ebens (1999) finds long memory in realized volatility based on 5-minute intraday returns of Dow Jones Industrial Average index and estimates its order to be significant between 0.37 and 0.40. Further evidence of long memory of realized variance can be found in Areal & Taylor (2002) and Andersen *et al.* (2003). However, Ashley & Patterson (2011) emphasize that ARFIMA model does not necessarily imply existence of long memory and argue that autocovariances are inconsistent in case of the presence of any time variation of the population mean. They suggest that the apparent presence of long memory could rather signal structural changes of the observed data.

We utilize only the feature of long memory without further modeling of short memory AR or MA processes. The estimated ARFIMA(0,d,0) model of natural logarithm of Garman-Klass then look as

$$(1-L)^d \ln(GK)_t = \epsilon_t, \tag{3.35}$$

which turns out to be an infinite AR process. Bhardwaj & Swanson (2004) use Monte Carlo simulation to show that the significance of higher lags decreases[2] with d.

3.3 Error statistics

We estimate all the models on a basis of rolling window and during each step the one-day ahead forecast is computed. This procedure provides us many out-of-sample forecasts which need to be assessed. Different error measures are chosen to find out if the traded volume improves the predictive power. The range of the error statistics includes relative measure (MAPE), absolute measure (RMSE) and correlation based measure (Mincer-Zarnowitz test)

[2]For example, there are 361 significant lags for $d = 0.05$ and 78 significant lags for $d = 0.75$.

3.3.1 Mean Absolute Percentage Error

We define MAPE measure for the one-day ahead forecast as

$$MAPE = \frac{1}{n}\sum_{i-1}^{n}\frac{|RV_{t+1} - \widehat{GK}_{t+1}|}{RV_{t+1}}, \tag{3.36}$$

where RV_{t+1} and $\widehat{GK}_{t+1}$ stand for the true realized volatility and exponentially transformed one-day ahead forecast of $\ln(GK)$, respectively. The advantage of this measure is its intuitiveness and possible comparison across different sample sets. The disadvantage is that for smaller true values the percentage error corresponds to smaller absolute error than in case of the higher true values.

3.3.2 Root Mean Squared Error

We define RMSE measure for the one-day ahead forecast as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(RV_{t+1} - \widehat{GK}_{t+1})^2}{n}}, \tag{3.37}$$

where the term under the square root equals mean squared error. We choose RMSE rather than MSE since the magnitude of the squared errors range from approximately 10^{-5} to 10^{-10} and the final evaluation might be cumbersome. Since the square root is an increasing linear transformation of MSE, it does not have any influence on the final ranking. The only problem with this measure is that the squaring exaggerates higher errors and diminishes smaller errors.

3.3.3 Mincer-Zarnowitz test

The Mincer-Zarnowitz test is first introduced by Mincer & Zarnowitz (1969) and is based on the regression of true values on forecasted values as

$$RV_{t+1} = \alpha + \beta\, \widehat{GK}_{t+1} + \epsilon_t, \tag{3.38}$$

where α and β coefficients can be interpreted as bias and efficiency of the forecasts, respectively. Under the joint null hypothesis $H_0 : \alpha = 0 \wedge \beta = 1$ the produced forecasts are both unbiased and efficient and the true value becomes composed of only the forecast and the unpredictable error ϵ_t. Further individual tests of $\alpha = 0$ and $\beta = 1$ can be carried out. The advantage of this test is the comparison of bias and efficiency produced by different forecasting methods.

The disadvantage is that it does not provide any information about the accuracy of the forecasting model as the model with higher bias can produce lower MSE (Schwartz 1999).

Chapter 4

Data description

Open, low, high and close prices together with traded volume are freely downloadable from finance.yahoo.com. However, traded volume is reported only for the most traded financial indices and the length of its time series also severely differs across different stocks. The close prices are used to compute continuously compounded returns in order to model GARCH volatility. The formula used for the returns is

$$r_t = \ln(p_t) - \ln(p_{t-1}), \tag{4.1}$$

where p_t is the close price at time t. Garman-Klass volatility estimator is computed from the data according to formula (3.6). The web page realized.oxford-man.ox.ac.uk/data/download provides several measures of intraday realized volatility for the most traded indices. Based on the availability of realized volatility and traded volume we chose four indices, each from different continent. The observed financial indices are FTSE 100, IPC Mexico, Nikkei 225 and S&P 500 providing 2576 observations from 4th December 2002 to 8th March 2013, 2101 observations from 27th October 2004 to 8th March 2013, 2597 observations from 11th June 2002 to 8th March 2013 and 3295 observations from 4th January 2000 to 8th March 2013, respectively. We encountered a problem with several missing observations on traded volume, even though the realized volatility was nonzero. Therefore, the market was inevitably open and some errors on yahoo.fiance must have occurred. FTSE 100 reports 1 missing value on 19th April 2010, IPC Mexico reports 2 missing values on 1st November 2004 and 13th January 2005, Nikkei 225 reports 5 missing values on 30th June 2005, 2nd June 2006, 29th March 2010, 9th June 2010 and 15th February 2011 and S&P does not report any missing value. Due to this inconvenience we were forced to fill in the empty spaces by linearly interpolated values computed by the formula defined by Davis (1975) as

$$y_t^* = y_{t-1} + (y_{t+1} - y_{t-1})\,\frac{x_t - x_{t-1}}{x_{t+1} - x_{t-1}}, \tag{4.2}$$

where y_t^* is the unknown value of traded volume at time t and $\{x_t\}$ is the time series of Garman-Klass volatility estimator.

Natural logarithms of realized volatility, Garman-Klass volatility and traded volume are used throughout the whole thesis. Therefore, all the initial data analyses are also focused on the logarithmic values. Plots of non-logarithmic values are reported in figure A.1 and are presented for informative purposes rather than a subject to analysis. Plots of logarithmic values together with returns can be found in figure A.2. Both the plots of logarithmic and non-logarithmic realized volatility and Garman-Klass volatility exhibit very similar pattern. The only difference is in the thickness of these two lines when the plot of Garman-Klass volatility is thicker and does not follow as straight line as the realized volatility. This confirms the fact that Garman-Klass volatility is more noisy than the realized volatility, but still enough efficient. The descriptive statistics of logarithms is reported in table 4.1 and the descriptive statistics of non-logarithmic values can be found in table A.1.

Stock Index	variable	mean	st. dev.	skewness	kurtosis
FTSE 100	ln(RV)	-4.991	0.517	0.509	3.038
	ln(GK)	-4.956	0.574	0.303	3.085
	ln(volume)	20.932	0.415	-1.148	5.788
IPC	ln(RV)	-4.860	0.517	0.605	3.476
	ln(GK)	-4.863	0.574	0.261	3.357
	ln(volume)	18.743	0.415	-1.120	7.497
Nikkei 225	ln(RV)	-4.792	0.421	0.525	4.072
	ln(GK)	-4.958	0.503	0.254	3.581
	ln(volume)	20.830	0.412	-0.423	3.060
S&P 500	ln(RV)	-4.764	0.521	0.509	3.369
	ln(GK)	-4.945	0.574	0.286	3.273
	ln(volume)	21.597	0.605	0.017	1.924

Table 4.1: Descriptive statistics of logarithms

The kurtosis of nearly all of the logarithmic values, except traded volume, is approximately 3 and the skewness is not far from zero. This is in contrast with the non-logarithmic values with kurtosis ranging from 10.358 to 34.747 and skewness mostly about 3. These statistics confirm the fact that logarithmic values are closer to the normal distribution in terms of skewness and kurtosis than non-logarithmic values and therefore are also more appropriate to modeling. Two mojor facts arise from table A.1. First, FTSE 100 is the less volatile index and second, S&P 500 exhibits the highest average of daily traded volume.

Since the relationship between traded volume and volatility is the subject to scrutiny, we also report contemporaneous correlations of logarithmic realized volatility, Garman-Klass volatility and traded volume in table 4.2.

variable	ln(RV)	ln(GK)	ln(volume)
ln(RV)	1	.	.
ln(GK)	0.8747	1	.
ln(volume)	0.1181	0.0954	1

(a) FTSE

variable	ln(RV)	ln(GK)	ln(volume)
ln(RV)	1	.	.
ln(GK)	0.8234	1	.
ln(volume)	0.4333	0.3610	1

(b) IPC Mexico

variable	ln(RV)	ln(GK)	ln(volume)
ln(RV)	1	.	.
ln(GK)	0.8635	1	.
ln(volume)	0.1156	0.0960	1

(c) Nikkei 225

variable	ln(RV)	ln(GK)	ln(volume)
ln(RV)	1	.	.
ln(GK)	0.8795	1	.
ln(volume)	0.2183	0.1870	1

(d) S&P 500

Table 4.2: Correlation matrices

The evidence of Garman-Klass volatility being highly correlated with realized volatility in all four indices underlines the fact that it is reasonably good substitution for realized volatility if it is not available. Furthermore, the lower correlation between traded volume and Garman-Klass volatility, in comparison with correlation between realized volatility and traded volume, supports the fact that it is still more noisy estimator as the relationship with more noisy variable is harder to detect. Nonetheless, the difference is rather small.

As the stationarity of data is important to correctly determine the true process, we used Augmented Dickey-Fuller test, first developed by Dickey & Fuller (1979), to test for unit root in our time series:

$$\Delta y_t = \alpha_0 + \theta y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \epsilon_t \tag{4.3}$$

The test is based on comparing the null hypothesis of unit root $H_0 : \theta = 0$ against the alternative hypothesis $H_0 : \theta < 0$. The differences Δy_{t-i} are included to capture autocorrelation of the series $\{\Delta y_t\}$ in order the significance of coefficient θ would not be overvalued. The optimal number of lags p is chosen based on the Akaike information criteria proposed by software JMulTi. The results of the test can be found in table 4.3

Stock Index	variable	No. of lags	t-statistics	p-value	Null hypothesis
FTSE 100	ln(RV)	9	-4.114	0.0009	rejected***
	ln(GK)	28	-3.021	0.0330	rejected**
	returns	54	-8.008	0.0000	rejected***
	ln(volume)	32	-2.634	0.0862	rejected*
IPC	ln(RV)	11	-4.583	0.0001	rejected***
	ln(GK)	12	-4.946	0.0000	rejected***
	returns	12	-11.808	0.0000	rejected***
	ln(volume)	33	-3.097	0.0268	rejected**
Nikkei 225	ln(RV)	11	-4.646	0.0001	rejected***
	ln(GK)	12	-4.749	0.0001	rejected***
	returns	26	-8.842	0.0000	rejected***
	ln(volume)	33	-3.124	0.0248	rejected**
S&P 500	ln(RV)	11	-5.117	0.0000	rejected***
	ln(GK)	14	-4.890	0.0000	rejected***
	returns	0	-62.701	0.0000	rejected***
	ln(volume)	57	-1.306	0.6263	not rejected

Significance levels : $***$: 1% $**$: 5% $*$: 10%

Table 4.3: Dickey-Fuller test results

Except ln(volume), unit root of nearly all of the other series is rejected at 1%. The problem arises in case of FTSE ln(volume) and S&P ln(volume), where we can reject the null hypothsis at only 10% and can not reject the null hypothesis, respectively. However, if we chose optimal number of lags based on Schwarz information criteria, which is less strict towards the autocorrelations, we would obtain p being equal 19, 9, 13 and 9 for FTSE 100, IPC Mexico, Nikkei 225 and S&P 500, respectively. This would also lead to rejecting unit root for FTSE and IPC at 1% and for Nikkei and S&P at 5%.

The fact of not rejecting the null hypothesis is not as crucial as it could seem since the coefficient θ is closely connected to d, the order of integration, mentioned in ARFIMA section 3.2.6. In fact, not rejecting the unit root does not imply d to be equal one. If d is lower than one, the time series is mean reverting and hence still proper to forecasting. As we implement HAR and ARFIMA models, whose assumption is the order of integration to be higher than zero and lower than one, further differencing or detrending is not necessary. Moreover, the goal of this paper is not to determine the true process, but to compare performance of different models on different sample sets.

Chapter 5

Results

This chapter contains the estimates of the selected models for the whole period of each stock index and discuss their significance. The estimates obtained on a rolling basis, which serve for computing the one-day ahead forecasts, are plotted in appendix. The ordering of the models follows the pattern as in the methodological part, from the less complicated to the more advanced models. Finally, the comparison of MAPE, RMSE and Mincer-Zarnowitz test of the out-of-sample forecasts is presented in order to assess which model provides the most accurate forecasts and whether traded volume increases the predictive power.

5.1 Model results

Autoregressive model

The selection of this model comes naturally due to its simplicity, intuitivness and the wide spread of use. The coefficients together with standard errors and significance levels are presented in table 5.1. Furthermore, the plots of the coefficients computed with the rolling window including 500 observations can be found in figure B.1.

All the slope coefficients are statistically significant at 1%. Regarding their economic significance, the magnitude is always positive and slightly decreasing with higher lags. These results stem from two theoretical facts. First, Cont (2001) describes the volatility clustering as one of the stylized facts which results in high autocorrelation. Second, volatility usually exhibit long memory feature, as described in section 3.2.6, which makes the autocorrelation function slowly decaying. Hence, it is not surprising that all the autocorrelation terms

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Intercept	-0.60*** (0.078)	-0.92*** (0.103)	-1.01*** (0.097)	-0.66*** (0.073)
$\ln(\text{GK})_{t-1}$	0.27*** (0.020)	0.32*** (0.022)	0.27*** (0.020)	0.22*** (0.017)
$\ln(\text{GK})_{t-2}$	0.20*** (0.020)	0.16*** (0.023)	0.19*** (0.020)	0.24*** (0.018)
$\ln(\text{GK})_{t-3}$	0.14*** (0.020)	0.12*** (0.023)	0.15*** (0.020)	0.16*** (0.018)
$\ln(\text{GK})_{t-4}$	0.14*** (0.020)	0.08*** (0.023)	0.11*** (0.020)	0.14*** (0.018)
$\ln(\text{GK})_{t-5}$	0.12*** (0.020)	0.13*** (0.022)	0.07*** (0.020)	0.11*** (0.017)
R^2	0.5540	0.4231	0.4007	0.5193

Significance levels : ∗∗∗ : 1% ∗∗ : 5% ∗ : 10%
Standard errors in parentheses.

Table 5.1: AR model

are significant. As the plots of the rolling coefficients suggest, the magnitude of the coefficients and their ordering does not change dramatically. The only notable exception is the rise of the first autoregressive term after year 2008 in case of all four indices. This implicates that volatility clustering even magnified in the financial crisis in 2008.

Vector Autoregressive model

The vector autoregressive model is an extension to the univariate autoregressive model and explores the dynamic relationship across more variables. The traded volume is added as the second endogenous variable and its statistical significance, Granger-causality test and Impulse-response functions will be presented. Finally, the economic influence of the traded volume will be based on the comparison of MAPE and RMSE of AR and VAR models. The results of both ln(GK) and ln(volume) equation are presented in table 5.2. The plots of coefficients from ln(GK) equation evolving in time can be found in figures B.2 and B.3.

Considering the Garman-Klass volatility equation, all the autoregressive terms remain statistically significant and even their magnitude is similar to that of the coefficients from the AR model mentioned earlier. The coefficients of lagged ln(volume) are mostly statistically insignificant with the exemption of the third lag of FTSE, IPC and S&P. As for the magnitude of ln(volume), it appears that the influence on volatility is lower than the volatility itself. However, one has to take into consideration that ln(volume) is several times

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Garman-Klass volatility equation				
Intercept	0.44 (0.440)	0.08 (0.429)	-0.55*** (0.038)	-0.62** (0.028)
$\ln(\text{GK})_{t-1}$	0.26*** (0.021)	0.31*** (0.024)	0.27*** (0.021)	0.21*** (0.029)
$\ln(\text{GK})_{t-2}$	0.20*** (0.021)	0.15*** (0.025)	0.19*** (0.022)	0.24*** (0.019)
$\ln(\text{GK})_{t-3}$	0.17*** (0.021)	0.14*** (0.025)	0.15*** (0.022)	0.18*** (0.019)
$\ln(\text{GK})_{t-4}$	0.15*** 0.021)	0.08*** (0.025)	0.11*** (0.022)	0.13*** (0.019)
$\ln(\text{GK})_{t-5}$	0.11*** (0.020)	0.14*** (0.024)	0.08*** (0.021)	0.10*** (0.019)
$\ln(\text{volume})_{t-1}$	0.05 (0.032)	0.02 (0.023)	-0.003 (0.046)	0.04 (0.042)
$\ln(\text{volume})_{t-2}$	0.02 (0.036)	0.01 (0.024)	0.01 (0.051)	0.01 (0.047)
$\ln(\text{volume})_{t-3}$	-0.15*** (0.036)	-0.06** (0.024)	-0.02 (0.051)	-0.12*** (0.047)
$\ln(\text{volume})_{t-4}$	-0.04 (0.036)	0.01 (0.024)	0.05 (0.051)	0.05 (0.047)
$\ln(\text{volume})_{t-5}$	0.07** (0.032)	-0.03 (0.023)	-0.06 (0.045)	0.02 (0.042)
R^2	0.5589	0.4262	0.4015	0.5204
Granger test volume ⇒ GK	0.000	0.048	0.696	0.169
Traded Volume equation				
Intercept	2.17 (0.281)	3.43*** (0.487)	0.76*** (0.195)	0.30** (0.126)
$\ln(\text{GK})_{t-1}$	0.01 (0.013)	0.07*** (0.024)	-0.01 (0.009)	0.02 (0.008)
$\ln(\text{GK})_{t-2}$	-0.003 (0,013)	0.02 (0.025)	0.004 (0.010)	0.01 (0.008)
$\ln(\text{GK})_{t-3}$	-0.0005 (0.013)	0.01 (0.025)	-0.003 (0.010)	-0.02* (0.009)
$\ln(\text{GK})_{t-4}$	0.02 (0.013)	-0.03 (0.025)	-0.01 (0.010)	-0.02** (0.008)
$\ln(\text{GK})_{t-5}$	-0.05 (0.013)	-0.04 (0.024)	-0.03*** (0.009)	-0.01* (0.008)
$\ln(\text{volume})_{t-1}$	0.52*** (0.020)	0.33*** (0.024)	0.51*** (0.021)	0.50*** (0.019)
$\ln(\text{volume})_{t-2}$	0.11*** (0.023)	0.11*** (0.025)	0.16*** (0.023)	0.13*** (0.021)
$\ln(\text{volume})_{t-3}$	0.14*** (0.023)	0.05* (0.025)	0.05** (0.023)	0.10*** (0.021)
$\ln(\text{volume})_{t-4}$	-0.02 (0.023)	0.13*** (0.025)	0.08*** (0.023)	0.13*** (0.021)
$\ln(\text{volume})_{t-5}$	0.14*** (0.029)	0.20*** (0.024)	0.15*** (0.021)	0.12*** (0.019)
R^2	0.6561	0.3977	0.8147	0.9140
Granger test GK ⇒ volume	0.001	0.021	0.000	0.000

Standard errors in parentheses. Results of Granger-test provide p-value.
Significance levels : $***$: 1% $**$: 5% $*$: 10%

Table 5.2: Vector Autoregressive model

higher in absolute terms than ln(GK). The goodness of fit R^2 is only slightly higher than in case of AR model. Moreover, the plots of rolling autoregressive coefficients show nearly the same pattern as the coefficients from the autoregressive model, which indicates that adding traded volume does not decrease the volatility persistence.

Regarding the volume equation, nearly all of the autoregressive terms of ln(volume) are significant at 1%. Most of the coefficients of ln(GK) are both statistically and economically insignificant. The exemption is the case of IPC with the first lag of ln(GK) being significant at 1% and implying that higher volatility today means higher traded volume tomorrow. The second exemption is S&P with the negative third, fourth and fifth lag being significant at 10%, 5% and 10%, respectively. The plots of rolling coefficients do not indicate any stable pattern as the individual coefficients of lagged traded volume fluctuate around zero. Moreover, the coefficients are not statistically significant most of the time and hence confirmation of any pattern would be unjustified.

Results of the Granger-causality test described in section 3.2.4 are mentioned both for volatility and volume. It turns out that ln(volume) Granger-causes ln(GK) only in case of FTSE and IPC at 1% and 5%, respectively. Furthermore, strong Granger-causality from ln(GK) to ln(volume) is detected in all four indices. In order to asses the relationship across those variables in detail, Impulse-response functions, as described in section 3.2.4, are carried out to find out the influence of a unit shock of an impulse variable to a response variable. The plots of IRFs with 95% confidence intervals for ln(volume) to ln(GK) can be found in figure B.4 and for ln(GK) to ln(volume) in figure B.5.

It appears that a unit shock of ln(volume) in short term causes increase in ln(GK). However, the zero value is contained in the 95% confidence interval and hence the positive influence is not statistically significant. The interesting finding is that the unit shock has negative influence on level of volatility in long term. In case of FTSE and IPC this feature is even statistically significant. This suggests that if volume unexpectedly increases the, level of volatility decreases in approximately two weeks. This might correspond to a rush day when the abnormal trading activity is triggered by a negative information release after which markets remain in tension with higher volatility for several days. In the longer term, as the markets stabilize, the level of volatility returns to its initial value. Furthermore, a unit shock in ln(GK), on the contrary, has significant positive influence on ln(volume) and diminishes as the time passes and becomes insignificant for higher lags.

Heterogenous Autoregressive model

Heterogenous autoregressive model developed by Corsi (2009) is straightforward, intuitive and provides surprisingly good results outperforming even some ARFIMA models. However, in comparison with ARFIMA models it does not appear in research articles as often as one could expect. The selection of this model also stems from the fact that its vector modification with other variables is possible and the influence of those variables can be additionally measured. The results for the whole periods are presented in table 5.3 and the plots of coefficients evolving in time are in figure B.6.

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Intercept	-0.31*** (0.024)	-0.51*** (0.115)	-0.49*** (0.108)	-0.36*** (0.078)
$\ln(\text{GK})_{t-1}^{(d)}$	0.13*** (0.046)	0.21*** (0.026)	0.16*** (0.023)	0.07** (0.021)
$\ln(\text{GK})_{t-1}^{(w)}$	0.42*** (0.041)	0.31*** (0.049)	0.31*** (0.045)	0.48*** (0.041)
$\ln(\text{GK})_{t-1}^{(m)}$	0.39*** (0.083)	0.37*** (0.047)	0.44*** (0.044)	0.37*** (0.037)
R^2	0.5682	0. 4361	0.4179	0.5308

Significance levels : $***$: 1% $**$: 5% $*$: 10%
Standard errors in parentheses.

Table 5.3: Heterogenous Autoregressive model

All the slope coefficients are both statistically and economically significant. From the results arises that medium and long term traders influence level of volatility more than the short term traders as the coefficients of weekly average and monthly average of ln(GK) are generally higher than daily volatility. This is confirmed by the plots of rolling coefficients where both weekly and monthly averages dominate the daily value in case of all four indices. Regarding goodness of fit R^2, although the model contains less variables in comparison with AR and VAR, it is higher by approximately 2%. Therefore, the model better describes the evolution of volatility and might be more appropriate for forecasting.

Vector Heterogenous Autoregressive model

As described in section 3.2.5, heterogenous terms of traded volume are added to the initial HAR equation in order to asses the influence of traded volume on volatility. The model is compiled on a basis of VAR(1) with additional exogenous variables of weekly and monthly averages of ln(GK) and ln(volume). Moreover, joint significance of ln(volume) coefficients in volatility equation and joint significance of ln(GK) coefficients in volume equation are measured and computed on the same basis as Granger-causality test in section 3.2.4. Results of the model together with p-value of the Granger-causality test are presented in table 5.4. Furthermore, plots of the coefficients of ln(GK) equation evolving in time can be found in figures B.7 and B.8.

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Garman-Klass volatility equation				
Intercept	-0.41 (0.445)	0.22 (0.533)	-0.61 (0.442)	-0.41 (0.284)
$\ln(\text{GK})^{(d)}_{t-1}$	0.19*** (0.025)	0.32*** (0.029)	0.15*** (0.025)	0.05** (0.023)
$\ln(\text{GK})^{(w)}_{t-1}$	0.44*** (0.047)	0.34*** (0.057)	0.25*** (0.050)	0.46*** (0.047)
$\ln(\text{GK})^{(m)}_{t-1}$	0.40*** (0.043)	0.37*** (0.054)	0.51*** (0.049)	0.41*** (0.043)
$\ln(\text{volume})^{(d)}_{t-1}$	0.12*** (0.038)	0.04 (0.027)	0.04 (0.053)	0.12** (0.049)
$\ln(\text{volume})^{(w)}_{t-1}$	-0.11** (0.045)	-0.06 (0.056)	0.20** (0.087)	-0.01 (0.083)
$\ln(\text{volume})^{(m)}_{t-1}$	-0.004 (0.011)	-0.02 (0.055)	-0.23*** (0.070)	-0.10 (0.067)
R^2	0.5700	0.4382	0.4207	0.5321
Granger test volume ⇒ GK	0.013	0.208	0.006	0.036
Traded Volume equation				
Intercept	2.22*** (0.285)	1.51*** (0.542)	0.38* (0.203)	0.11 (0.126)
$\ln(\text{GK})^{(d)}_{t-1}$	0.02 (0.016)	0.10*** (0.029)	0.003 (0.011)	0.02** (0.010)
$\ln(\text{GK})^{(w)}_{t-1}$	-0.03 (0.030)	-0.03 (0.058)	-0.04* (0.023)	0.03 (0.021)
$\ln(\text{GK})^{(m)}_{t-1}$	-0.02 (0.028)	-0.10* (0.055)	-0.01 (0.022)	-0.08*** (0.019)
$\ln(\text{volume})^{(d)}_{t-1}$	0.42*** (0.024)	0.20*** (0.028)	0.42*** (0.024)	0.39*** (0.022)
$\ln(\text{volume})^{(w)}_{t-1}$	0.47*** (0.028)	0.24*** (0.057)	0.31*** (0.040)	0.29*** (0.037)
$\ln(\text{volume})^{(m)}_{t-1}$	-0.01 (0.007)	0.47*** (0.056)	0.24*** (0.032)	0.31*** (0.030)
R^2	0.6561	0.4070	0.8143	0.9159
Granger test GK ⇒ volume	0.029	0.002	0.000	0.000

Standard errors in parentheses. Results of Granger-test provide p-value.
Significance levels : *** : 1% ** : 5% * : 10%

Table 5.4: Vector Heterogenous Autoregressive model

The autoregressive terms in volatility equation remain both statistically and economically significant with the nearly same magnitude as in the HAR model. Moreover, the plots of rolling coefficients of daily, weekly and monthly averages of volatility exhibit nearly the same pattern as the rolling coefficients produced by HAR model. The influence of volume on volatility is higher, compared to the VAR model, as there are several terms of volume in FTSE, Nikkei and S&P significant at 1% and 5%. This is also reflected in Granger-causality test with resulting causality from ln(volume) to ln(GK) in FTSE, Nikkei and S&P being significant at 5%, 1% and 5%, respectively. Moreover, the rolling coefficients of traded volume suggest that the influence of daily and weekly values increased in the time of financial crisis during 2008-2010 in case of FTSE, Nikkei and S&P contrary to the decrease of monthly average. Additionally, R^2 is higher by approximately 0.2-0.3% in comparison with HAR model.

Regarding the volume equation, nearly all of the volume terms are significant at 1%. Their economic significance is rather higher in short term as is in the case of FTSE, Nikkei and S&P. Strong influence of ln(GK) to ln(volume) is detected through the Grange-causality test since the p-value is close to zero for nearly all the indices.

Fractionally Integrated Autoregressive Moving Average model

It is reported by many studies that volatility exhibit long memory feature. For simplicity, none of AR and MA terms are included, the only parameter to estimate is the order of integration. Formally, the estimated model can be written as ARFIMA(0,d,0) with d being the order of integration. Maximum likelihood estimator developed in R-project under package fracdiff is used to estimate the order of integration. The only variable modeled by ARFIMA is ln(GK), but for the informative purposes the order of integration of ln(RV) and ln(volume) is also presented. The results can be found in table 5.5 and the dynamic evolution of d coefficient of ln(GK) is plotted in figure B.9.

variable	FTSE 100	IPC	Nikkei 225	S&P 500
ln(RV)	0.49^{***} (2.4×10^{-8})	0.39^{***} (6.2×10^{-6})	0.47^{***} (5.1×10^{-8})	0.46^{***} (6.0×10^{-6})
ln(GK)	0.36^{***} (1.2×10^{-5})	0.35^{***} (1.2×10^{-5})	0.33^{***} (1.3×10^{-5})	0.35^{***} (1.8×10^{-5})
ln(volume)	0.48^{***} (2.1×10^{-4})	0.34^{***} (1.2×10^{-5})	0.50^{***} (8.5×10^{-6})	0.50^{***} (1.1×10^{-5})

Significance levels : $***$: 1% $**$: 5% $*$: 10%
Standard errors in parentheses.

Table 5.5: ARFIMA - Order of integration

All the variables exhibit long memory since the order of integration is statistically significant at 1%. Although the estimation was written in order to allow d to be from 0 to 1, all the coefficients are included in the interval $(0, 0.5\rangle$ which makes all the sets of data stationary. The order of integration of ln(RV) and ln(volume) is approximately the same, and with the exemption of IPC index, is slightly bellow 0.5 for all other indices. The rolling plots of the order of integration of ln(GK) suggest that it fluctuates around 0.35 with the notable increase after year 2008 and further decrease in year 2010. This confirms the theory mentioned in results of autoregressive and vector autoregressive models that volatility persistence increases during financial crisis.

Generalized Autoregressive Conditional Heteroskedasticity model

GARCH model developed by Bollerslev (1986) and described in section 3.1.2 is one of the most cited models in financial econometrics. Therefore, it is natural to compare our results with this model. Disadvantage of this model against the Garman-Klass volatility estimator is that it uses squared returns (or squared residuals) which are generally considered as less efficient than the realized volatility. Therefore, even the in-sample comparison with realized volatility might produce poor results. For simplicity, only GARCH(1,1) without any mean process is considered. Cont (2001) mentions no autocorrelation of returns as one of the stylized facts. Having assumed this stylized fact to hold, we are not obliged to model the mean equation and the estimation reduces only to GARCH coefficients. The results of the whole periods are presented in table 5.6 and the coefficients evolving in time can be found in figure B.10.

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Conditional variance equation				
Omega	$9.6\times10^{-7***}$	$2.1\times10^{-6***}$	$3.6\times10^{-6***}$	$1.5\times10^{-6***}$
	(2.9×10^{-7})	(6.1×10^{-7})	(9.7×10^{-7})	(3.2×10^{-7})
Alpha	0.085^{***}	0.094^{***}	0.101^{***}	0.086^{***}
	(0.010)	(0.012)	(0.012)	(0.009)
Beta	0.901^{***}	0.896^{***}	0.885^{***}	0.904^{***}
	(0.011)	(0.012)	(0.013)	(0.009)

Significance levels : $***$: 1% $**$: 5% $*$: 10%
Standard errors in parentheses.

Table 5.6: GARCH

All the coefficients including intercept are significant at 1%. One of the key measures of GARCH(1,1) models is the sum $\alpha + \beta$ to be less than one in order to avoid nonstationary IGARCH. For all indices this summation is close to one. This might be caused by the long sample sets including both volatile and calm periods, which makes the model nonstationary. The plots of coefficients computed on a basis of rolling window suggest that the sum approaches one nearly for all the time in case of all four indices. The interesting finding is that although the sum is still close to one, the contribution of alpha and beta changes during 2008-2010 as the alpha coefficient rises and beta coefficients decreases. This indicates that influence of the error made yesterday (in our case return) increases, and the influence of conditional variance from yesterday decreases in financial crisis.

Generalized Autoregressive Conditional Heteroskedasticity model with volume

In order to measure the influence of traded volume on the predictive power of GARCH model, lagged traded volume is added to the conditional variance equation. The results are presented in table 5.7 and plots of coefficients evolving in time can be found in figure B.11.

variable	FTSE 100	IPC	Nikkei 225	S&P 500
Conditional variance equation				
Omega	2.1×10^{-7}	4.1×10^{-7}	1.1×10^{-6}	3.5×10^{-7}
	(5.5×10^{-6})	(2.0×10^{-5})	(3.0×10^{-5})	(6.4×10^{-6})
Alpha	0.079***	0.091***	0.101***	0.080***
	(0.009)	(0.012)	(0.013)	(0.008)
Beta	0.919***	0.901***	0.884***	0.915***
	(0.009)	(0.012)	(0.014)	(0.008)
ln(volume)	1.6×10^{-8}	7.4×10^{-8}	1.3×10^{-7}	3.8×10^{-8}
	(2.6×10^{-7})	(9.6×10^{-7})	(1.4×10^{-6})	(2.9×10^{-7})

Significance levels : $***$: 1% $**$: 5% $*$: 10%
Standard errors in parentheses.

Table 5.7: GARCH with volume volume

The expected effect is the sum of $\alpha + \beta$ to decrease. However, it appears that nothing significant happened to α and β coefficients. Even the rolling alpha and beta coefficients does not exhibit any notable difference compared to the simple GARCH(1,1) without the traded volume The significant difference is in ω coefficient which is now insignificant. Moreover, the magnitude of traded volume and its statistical insignificance suggest that traded volume does not bring any additional information in the model. The out-of-sample forecasts will serve as the final measure of the appropriateness of traded volume in the conditional variance equation.

5.2 Rolling forecasts

Each model is estimated on a basis of rolling window containing 500 observations and in each step one-day ahead forecast is computed and compared with the true realized volatility. The number of forecasts that is generated by this procedure equals the length of the volatility time series minus the length of the rolling window. Several error measures described in section 3.3 are applied to those forecasts to find out which model performs best.

Ouf-of-sample evaluation

All the plots of forecasts (red line) compared to the true realized volatility (black line) are presented in figure B.12. It appears that most of the forecasts follow the similar pattern as the realized volatility. The exemptions are both GARCH and GARCH with volume which apparently overestimate the true volatility. The accuracy of the forecasts is based on MAPE and RMSE and is presented in table 5.8.

Stock Index	measure	RW	AR(5)	VAR(5)	HAR	VAR HAR	ARFIMA	GARCH	GARCHv
FTSE 100	MAPE	32.27%	22.47%	22.81%	21.63%	21.71%	22.18%	55.17%	55.57%
	RMSE	0.00385	0.00285	0.00286	0.00281	0.00281	0.00284	0.00525	0.00530
IPC	MAPE	34.83%	24.85%	26.31%	24.20%	24.37%	24.52%	54.68%	54.52%
	RMSE	0.00485	0.00418	0.00436	0.00412	0.00414	0.00413	0.00572	0.00573
Nikkei 225	MAPE	28.94%	20.98%	20.96%	20.41%	20.24%	20.55%	76.56%	76.20%
	RMSE	0.00394	0.00369	0.00367	0.00359	0.00345	0.00369	0.00752	0.00753
S&P 500	MAPE	31.59%	23.92%	23.91%	23.49%	23.37%	23.42%	39.35%	39.17%
	RMSE	0.00465	000452	0.00452	0.00442	0.00442	0.00459	0.00457	0.00460

Table 5.8: Comparison of out-of-sample forecasts

It appears that both GARCH and GARCH with volume perform poorly compared to the models including Garman-Klass volatility estimator. If the sum of $\alpha + \beta$ equals one, and GARCH becomes IGARCH, the forecast of conditional volatility is not defined any more. From the rolling plots of GARCH coefficients in figures B.10 and B.11 it is clear that the sum is close to one. Starica (2004) proves that in case the sum is close to one, GARCH(1,1) significantly overestimates the true volatility and points out that this is caused by unstable GARCH unconditional volatility. This might be prevented by using shorter rolling window since the shorter sample set of observations might include less structural shocks. Considering the improvement of predictive power, GARCH with volume in conditional variance equation is better in case of IPC, Nikkei and S&P, by 0,16% 0.36% and 0,18% in terms of MAPE, respectively.

Therefore, it seems that volume improves the accuracy of GARCH. However, in terms of RMSE GARCH without volume is better in case of all four indices.

Taking into consideration only the models with Garman-Klass volatility estimator, random walk turns out to produce larger errors by approximately 8% - 11% in terms of MAPE compared to other models. Therefore, volatility forecasting is meaningful and further modeling is appropriate. It appears that models considering the long memory feature dominate the other models. HAR model produces the lowest errors in case of FTSE and IPC and VAR-HAR in case of S&P and Nikkei. The similar results are provided by comparison of RMSE which evaluate HAR and VAR-HAR as equally good. Interestingly, HAR and VAR-HAR models outperforms the widely cited ARFIMA model in all four indices.

Regarding the influence of traded volume on predictive power of the multivariate VAR model compared to the univariate AR model, there is evidence that the included traded volume even worsen the accuracy of forecast as the AR model is better by 0.34% and 1.46% for FTSE and IPC, respectively. The improvements registered in Nikkei and S&P are rather negligible, 0.02% and 0.01%.

From comparison of HAR and VAR-HAR models we observe the similar results as in case of AR and VAR models. Traded volume has negative impact in case of FTSE and IPC when MAPE of HAR is lower by 1.08% and 0.19%, respectively. The percentage improvement in case of Nikkei and S&P is 0.17% and 0.12%. Therefore, there does not seem to be any conclusive evidence whether traded volume increases the predictive power of the scrutinized models.

Mincer-Zarnowitz test

The Mincer-Zarnowitz test does not serve for assessing the accuracy of the forecasts, but is rather useful in revealing features such as bias and efficiency. The goal of the testing is not to reject the null hypothesis of unbiasedness and efficiency, $H_0 : \alpha = 0, \beta = 1$. The results of the Mincer-Zarnowitz regression together with the test results are presented in table 5.9.

The results suggest that except FTSE, none of the models produce both unbiased and efficient forecast as p-values of the F-test approach zero. In case of FTSE, the desired unbiasedness and efficiency is reached under VAR, HAR and VAR-HAR models. To further explore the individual features two singe tests are carried out, $H_0 : \alpha = 0$ and $H_0 : \beta = 1$. It appears that except HAR

Index	Model	α	β	F test	R^2	Index	Model	α	β	F test	R^2
FTSE	RW	0.0022*** (0.0001)	0.66** (0.012)	0.000	0.6108	IPC	RW	0.0035*** (0.0001)	0.64*** (0.017)	0.000	0.4648
	AR(5)	-0.0003** (0.0001)	1.04*** (0.015)	0.031	0.6915		AR(5)	0.0035*** (0.0003)	1.22** (0.028)	0.000	0.5385
	VAR(5)	-0.0003* (0.0001)	1.03* (0.015)	0.180	0.6882		VAR(5)	-0.0003 (0.0003)	1.16*** (0.030)	0.000	0.4877
	HAR	0.0000 (0.0001)	1.00 (0.014)	0.940	0.6993		HAR	-0.0003 (0.0002)	1.14*** (0.026)	0.000	0.5434
	VAR-HAR	0.0000 (0.0001)	1.00 (0.014)	0.496	0.7006		VAR-HAR	-0.0002 (0.0002)	1.14*** (0.026)	0.000	0.5382
	ARFIMA	-0.0006*** (0.0001)	1.08*** (0.016)	0.000	0.6980		ARFIMA	-0.0006** (0.0001)	1.18*** (0.027)	0.000	0.5405
	GARCH	0.0011*** (0.0001)	0.61*** (0.010)	0.000	0.6573		GARCH	0.0009*** (0.0002)	0.67*** (0.016)	0.000	0.5077
	GARCHv	0.0011*** (0.0001)	0.60*** (0.010)	0.000	0.6554		GARCHv	0.0010*** (0.0002)	0.66*** (0.016)	0.000	0.5066
Index	**Model**	α	β	**F test**	R^2	**Index**	**Model**	α	β	**F test**	R^2
Nikkei	RW	0.0032*** (0.0001)	0.72*** (0.015)	0.000	0.5275	S&P	RW	0.0027*** (0.0001)	0.84*** (0.013)	0.000	0.5888
	AR(5)	-0.0013*** (0.0002)	1.42*** (0.023)	0.000	0.6387		AR(5)	-0.0003*** (0.0001)	1.35*** (0.017)	0.000	0.6891
	VAR(5)	-0.0012*** (0.0002)	1.39*** (0.023)	0.000	0.6307		VAR(5)	-0.0005*** (0.0001)	1.34*** (0.017)	0.000	0.6839
	HAR	-0.0005*** (0.0002)	1.30*** (0.021)	0.000	0.6466		HAR	-0.0002 (0.0001)	1.30*** (0.016)	0.000	0.6974
	VAR-HAR	-0.0002 (0.0002)	1.25*** (0.020)	0.000	0.6511		VAR-HAR	-0.0003** (0.0001)	1.30*** (0.016)	0.000	0.6965
	ARFIMA	-0.0015*** (0.0002)	1.44*** (0.023)	0.000	0.6442		ARFIMA	-0.0011*** (0.0002)	1.41*** (0.018)	0.000	0.6855
	GARCH	0.0016*** (0.0001)	0.50*** (0.009)	0.000	0.5949		GARCH	0.0006*** (0.0001)	0.79*** (0.011)	0.000	0.6653
	GARCHv	0.0016*** (0.0001)	0.50*** (0.009)	0.000	0.5954		GARCHv	0.0007*** (0.0001)	0.78*** (0.010)	0.000	0.6651

Standard errors in parentheses. Results of F-test provide p-value of $H_0 : \alpha$=0 and β=1. Significance of β is reported against $H_0 : \beta$=1.
Significance levels : $***$: 1% $**$: 5% $*$: 10%

Table 5.9: Mincer-Zarnowitz test results

and VAR-HAR in case of FTSE none of the models produce efficient forecasts since p-values of $H_0 : \beta = 1$ approach zero. Part of the efficiency loss might be caused by the comparison of the less efficient Garman-Klass volatility estimator with the more efficient realized volatility. Regarding only the unbiasedness, it turns out that only HAR and VAR-HAR models are able to produce unbiased forecast as the null $H_0 : \alpha = 0$ can not be rejected in case of FTSE, IPC and S&P for HAR and in case of FTSE, IPC and Nikkei for VAR-HAR. As for the magnitude of the bias, both GARCH and GARCHv produce relatively high positive bias, which confirms the theory of GARCH models to overestimate the true volatility. Except RW, all the other models produce negative bias.

The coefficient of determination R^2 of each equation explains how much of variation of the true volatility is explained by the forecast. Therefore, this measure can also be used to asses the predictive power of the given models. It turns out that again, HAR and VAR-HAR dominates other models as HAR is the best in case of IPC and S&P and VAR-HAR is the best in case of FTSE and Nikkei. Surprisingly, the GARCH models does not performs as bad as in the case of RMSE and MAPE statistics, but still worse than most of the models.

5.3 Possible extensions

Several models are discussed in this thesis ranging from the simple random walk to the long memory ARFIMA model. Therefore, there are many possible extensions and improvements to the current procedure. The plots of the rolling coefficients mentioned in appendix suggest that the dependence of the variables is not constant and evolve in time. Therefore, the length of the rolling window might influence both the coefficients and the forecasts, and further analysis of the optimal length that minimize the forecasted error could be carried out. Second, the order of AR and VAR models was determined to be 5 and hence the autocorrelation pattern was not always fully exploited. Based on the information criteria, an algorithm to recognize the optimal order in each rolling window could be developed.

Regarding the poor performance of GARCH, it could be enhanced by implementing t-distribution of innovations rather than the normal distributions. As was also pointed out earlier in this thesis, volatility does not respond to positive and negative shocks in the same way. Therefore, other GARCH models such as TGARCH or EGARCH could used. Moreover, different multivariate GARCH models could be applied to find out how the individual stock markets are linked together.

Considering the forecasting horizon, not only the 1-day ahead forecasts but also the 5-day or 20-day ahead forecasts could be done. However, this procedure would also require to model the traded volume several days ahead which might finally result even in higher forecasted error. This might be avoided be using weekly or monthly averages where only the 1-step ahead forecast would be computed and the further modeling of traded volume would not be necessary.

As was mentioned earlier, both volume and volatility has the long memory feature as the order of integration varies from 0.35 to 0.5. To exploit this feature in higher detail both the variables could be first modeled by an ARFIMA model. The produced residuals would be then the subject of interest to the analysis through a VAR model.

As the results of Mincer-Zarnowitz test suggest, there are several methods producing negatively biased forecasts and several methods producing positively biased forecasts. It would be useful to find out whether is it possible to extract some information from different forecasting methods and different volatility estimators to increase the predictive power and produce both unbiased and efficients forecasts.

Finally, the real impact on a portfolio selection could be measured. It might be interesting to find out whether the covariance matrix obtained from both historical covariances and traded volume would produce a better portfolio in terms of final profit.

Chapter 6

Conclusion

The most precise volatility estimator is considered to be the realized volatility as it directly measures changes of price in each second. However, this data is available only for the mostly traded shares. Usually, stock exchanges and other institutions charge fees for providing this data. Therefore, we substituted the realized volatility by publicly available Garman-Klass volatility estimator taking into account daily open, low, high and close prices. Moreover, we were interested whether the second publicly available statistics, traded volume, might improve the predictive power of Garman-Klass volatility estimator.

We used natural logarithms of both Garman-Klass volatility estimator and traded volume as these measures appeared to follow less leptokurtic distribution and hence were more suitable to forecasting. We implemented the logarithms into both univariate models (AR, HAR) and multivariate models (VAR, VAR-HAR) to asses the influence of the volume on the predictive power. Additionally, we compared those models with the widely used GARCH(1,1) both with and without volume. Moreover, we were interested how these models perform in comparison with the long memory ARFIMA model. We found out that both GARCH models provide significantly worse results than the simple random walk of Garman-Klass volatility. HAR and VAR-HAR models, despite their simplicity, outperformed all the other models and compared to the second best model, ARFIMA, the gain in accuracy is about 0.05% - 0.6%. Moreover, Mincer-Zarnowitz test shows that only HAR and VAR-HAR models are able to produce unbiased forecasts.

To measure the impact of traded volume on volatility, we ran Granger-causality test in VAR and VAR-HAR models. In case of VAR, the results are ambiguous as the the Grange-causality is rejected for Nikkei and S&P and can not be rejected for FTSE and IPC. Regarding Granger-causality test in VAR-HAR model, the results seems to be more in favor of Granger-causality as it can not be rejected for FTSE, Nikkei and S&P. The reverse causality from volatility to volume is stronger, it can not be rejected in case of all four indices both for VAR and VAR-HAR models. These findings are supported by impulse-response functions which show that a unit shock in volume does not have any significant influence on volatility in short term compared to the significant influence of shock of volatility to volume.

Considering the influence of traded volume on the predictive power of the scrutinized models, the results do not provide evident proofs of improvement. In case of AR against VAR, the improvements for Nikkei and S&P are negligible and even drop in accuracy for FTSE and IPC by 0.34% and 1.46% was detected. In case of HAR against VAR-HAR, the traded volume decreases predictive power for FTSE and IPC by 1.08% and 0.17%, and increases the predictive power for Nikkei and S&P by 0.17% and 0.12%.

Although we revealed that traded volume might Granger-cause volatility and several volume coefficients in VAR and VAR-HAR models were significant, the analysis of the out-of sample forecasts did not provide sufficient evidence to confirm that traded volume improves volatility forecasting. Therefore, to maintain model parsimony we recommend using HAR model with Garman-Klass volatility estimator as it outperformed nearly all of the models under scrutiny.

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Appendix A

Data description

Stock Index	variable	mean	st. dev.	skewness	kurtosis
FTSE 100	return	0.0002	0.0125	-0.196	11.013
	RV	0.0079	0.0049	2.651	15.519
	GK	0.0084	0.0058	2.734	15.662
	volume	1 330m	469m	-0.006	2.087
IPC	return	0.0002	0.0125	-0.196	11.013
	RV	0.0089	0.0049	3.003	18.911
	GK	0.0090	0.0058	2.798	16.598
	volume	156m	79.3m	2.645	21.944
Nikkei 225	return	0.0000	0.0156	-0.534	10.874
	RV	0.0091	0.0048	3.373	23.474
	GK	0.0080	0.0050	3.866	34.747
	volume	1 210m	477m	0.980	5.805
S&P 500	return	0.0000	0.0135	-0.164	10.358
	RV	0.0099	0.0065	3.002	19.038
	GK	0.0085	0.0060	3.327	22.909
	volume	2 860m	1 700m	0.933	3.530

Table A.1: Descriptive statistics of absolute values

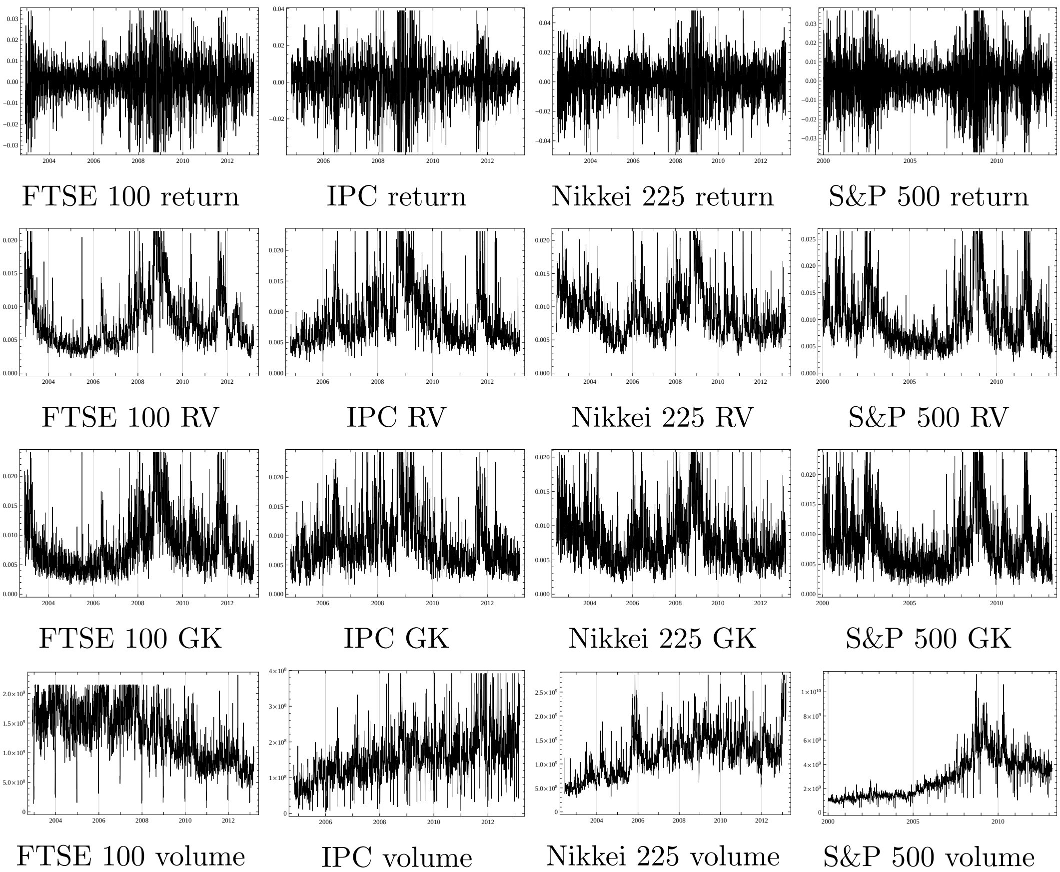


Figure A.1: Plots of absolute time series

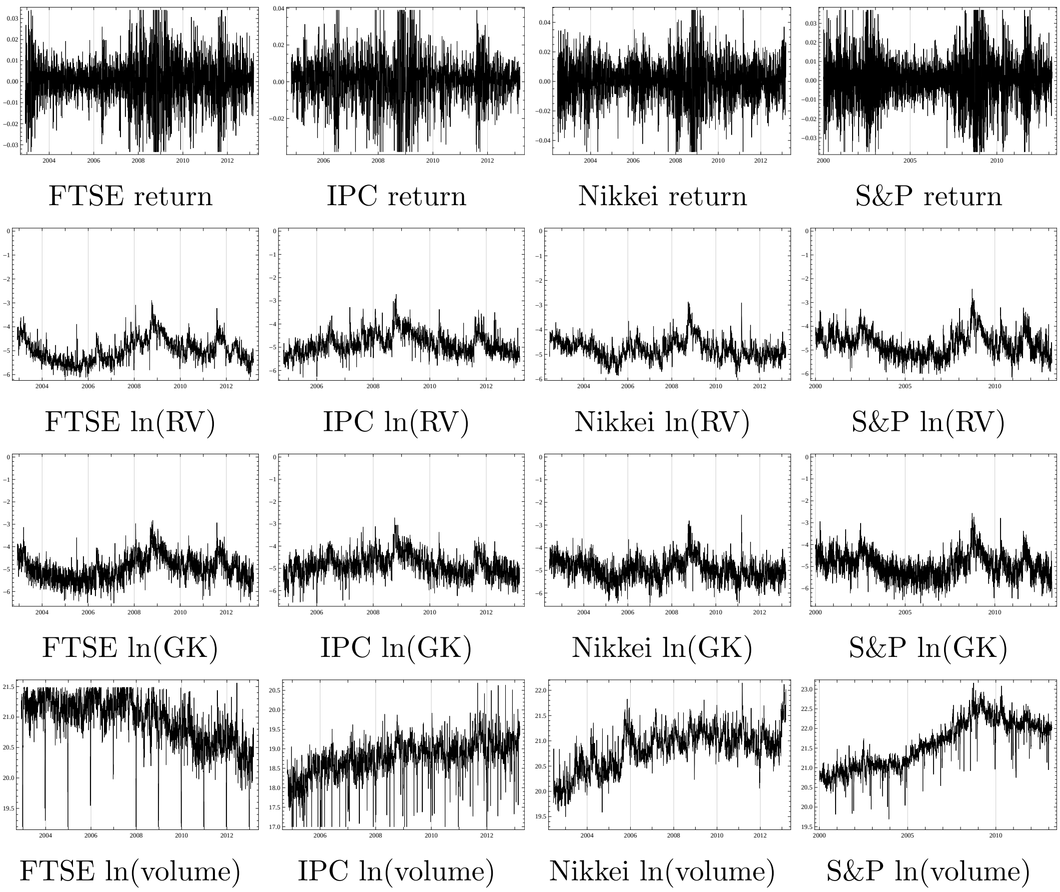
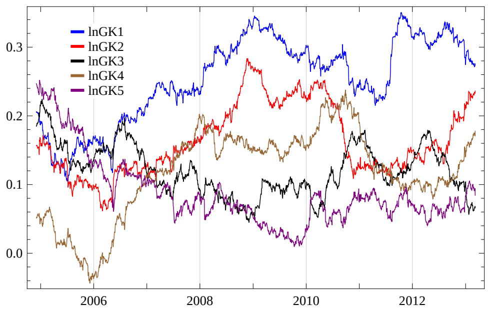


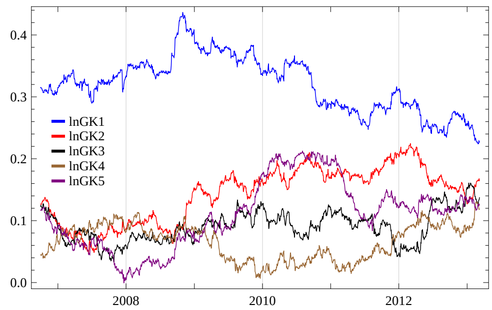
Figure A.2: Plots of logarithmic time series

Appendix B

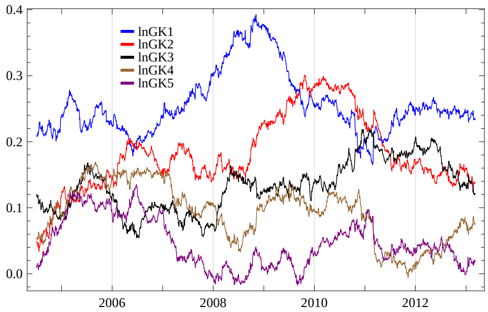
Results



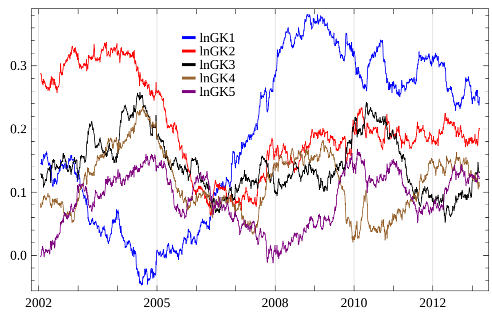
FTSE 100



IPC



Nikkei 225



S&P 500

Figure B.1: AR(5) rolling coefficients

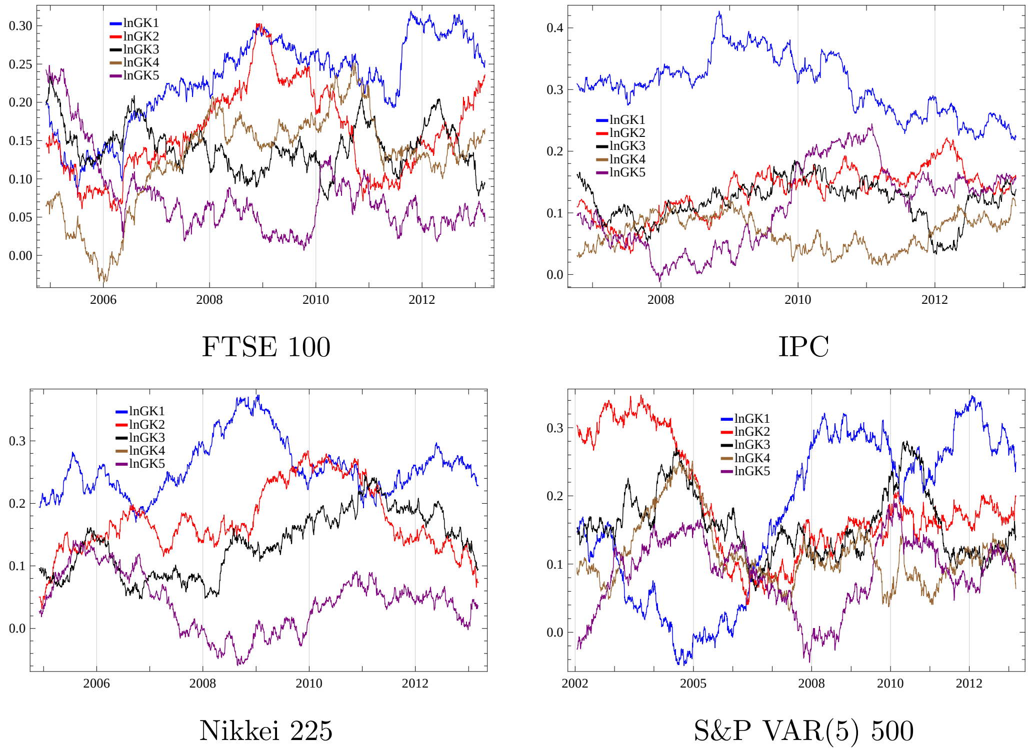


Figure B.2: VAR(5) volatility equation: rolling ln(GK) coefficients

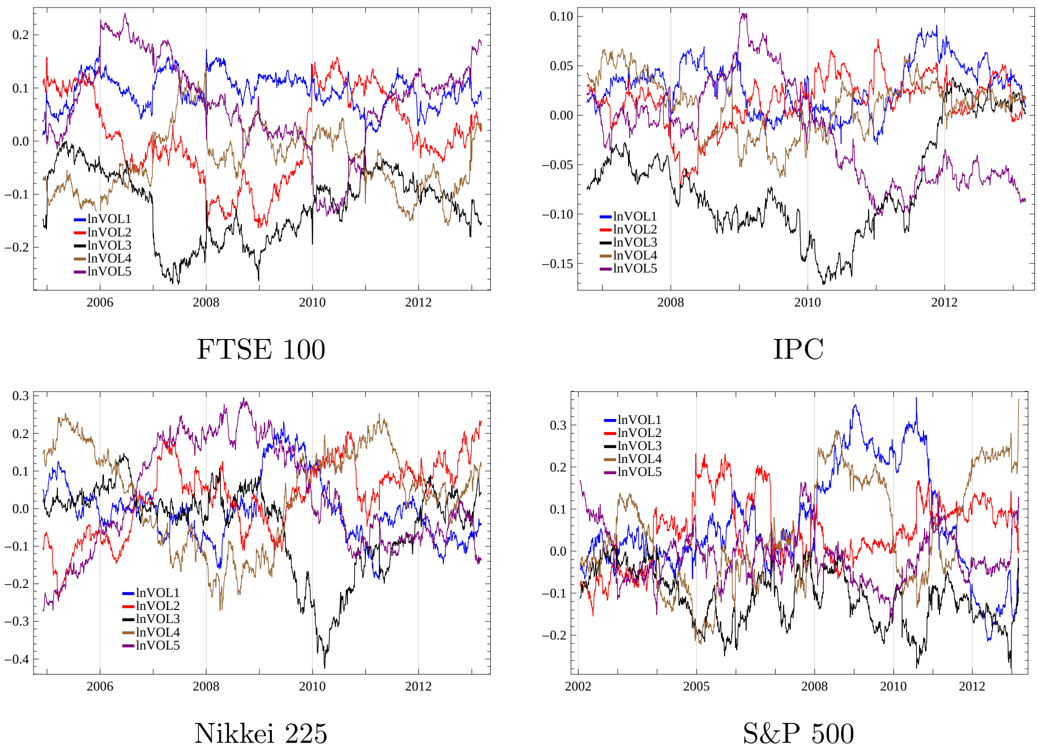


Figure B.3: VAR(5) volatility equation: rolling ln(volume) coefficients

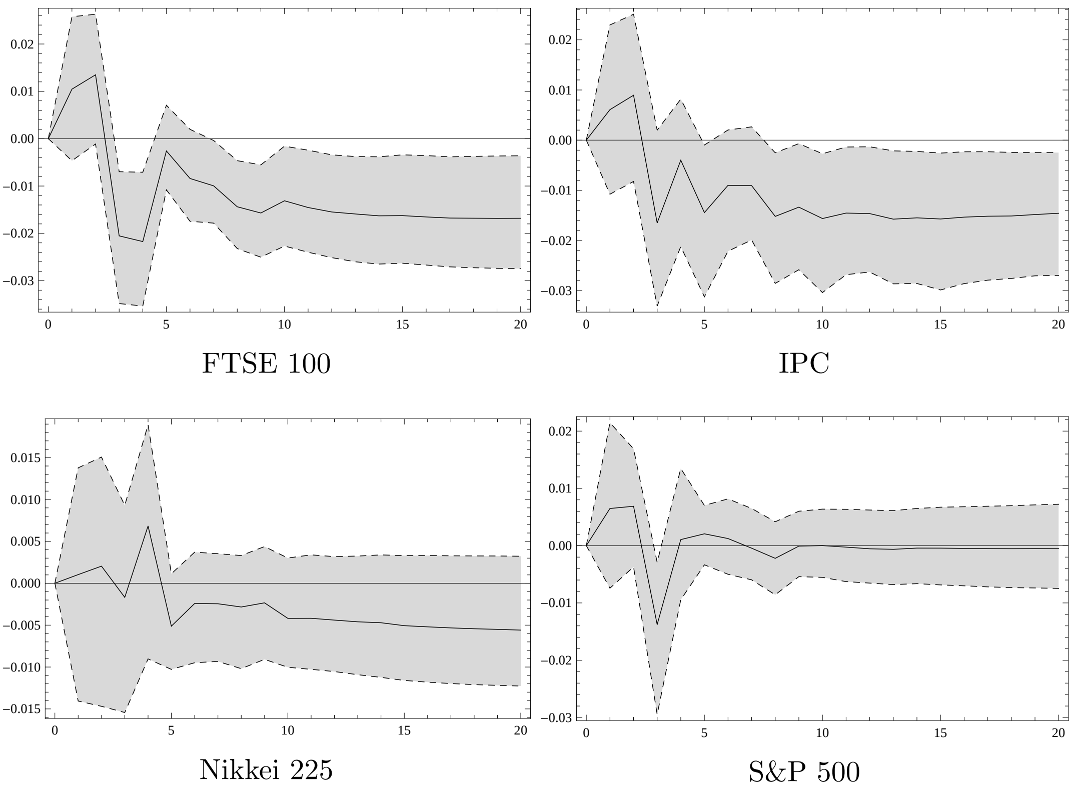


Figure B.4: IRF volume to volatility

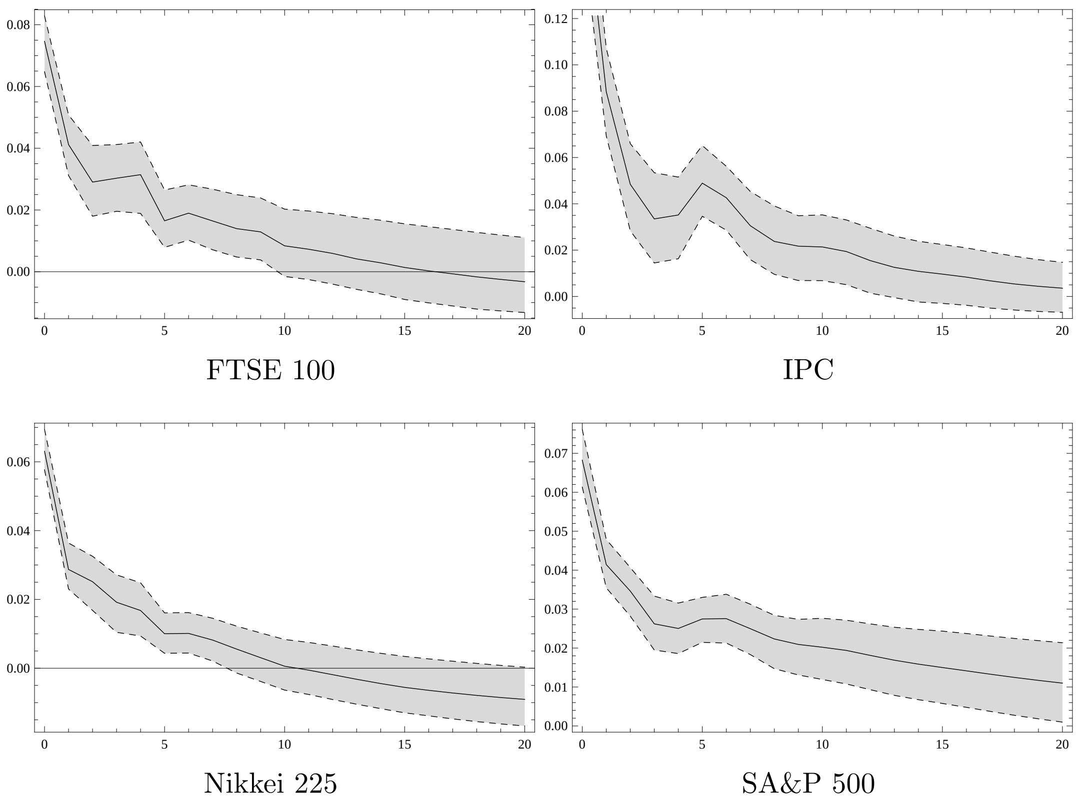


Figure B.5: IRF volatility to volume

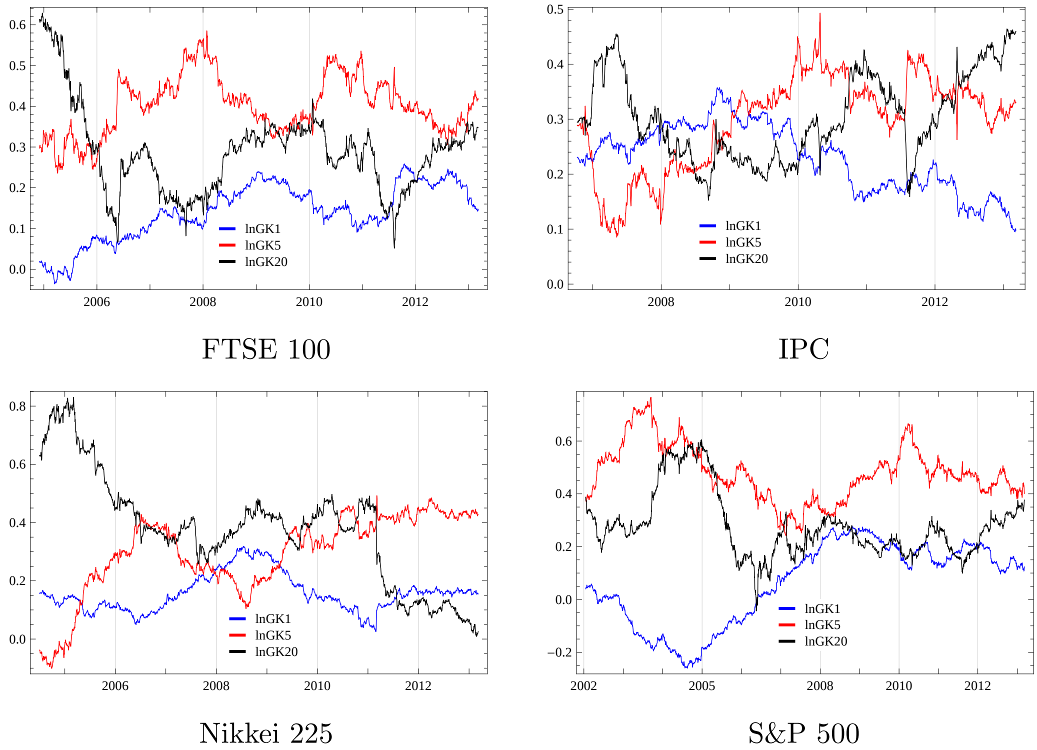


Figure B.6: HAR rolling coefficients

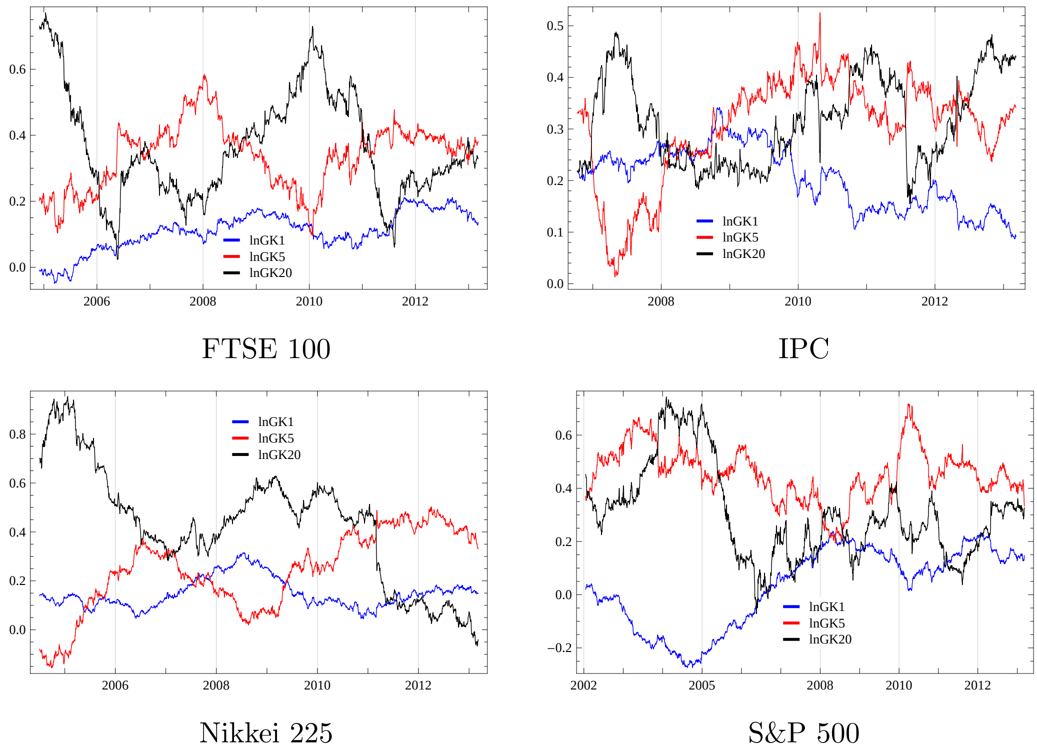


Figure B.7: VAR-HAR volatility equation: ln(GK) rolling coefficients

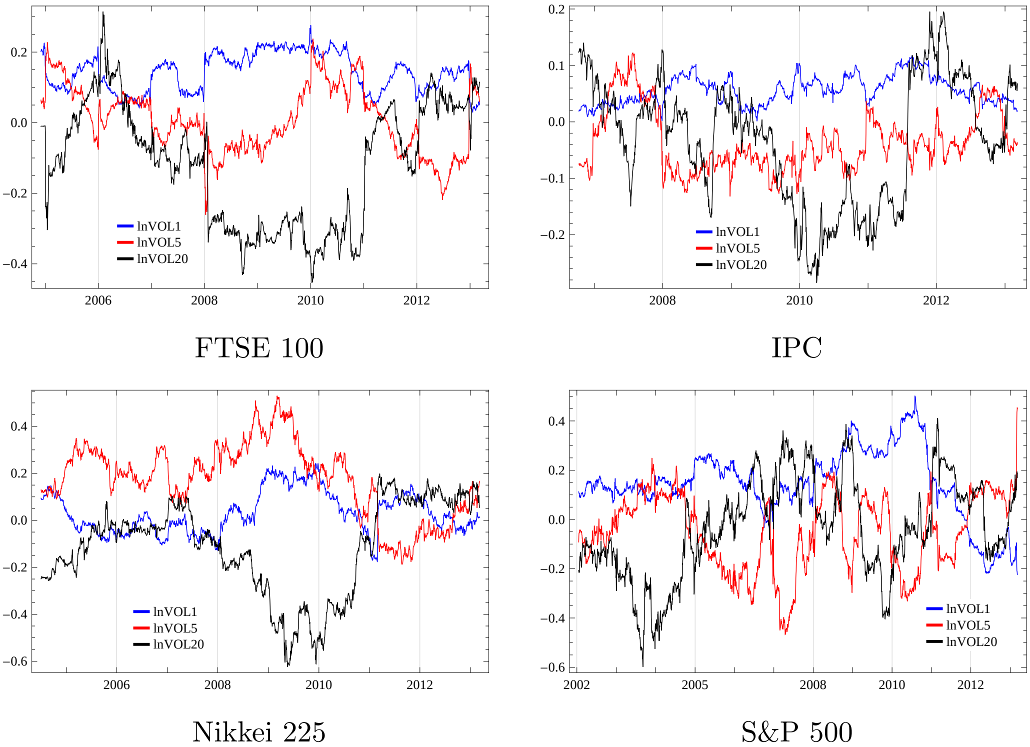


Figure B.8: VAR-HAR volatility equation: ln(volume) rolling coefficients

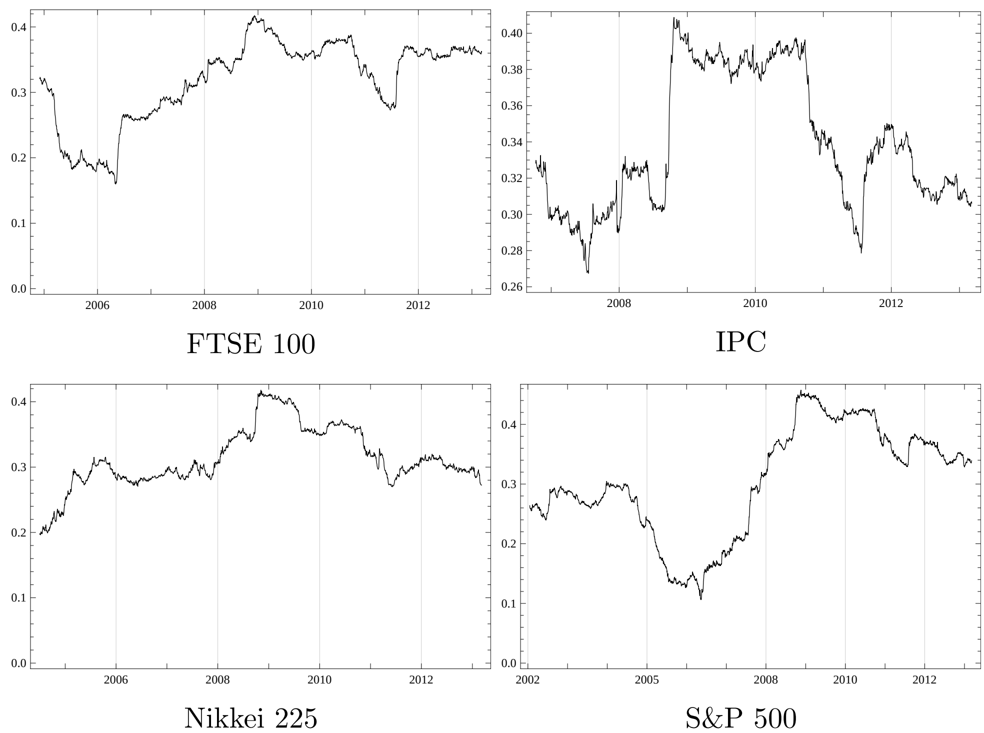


Figure B.9: Order of integration of ln(GK)

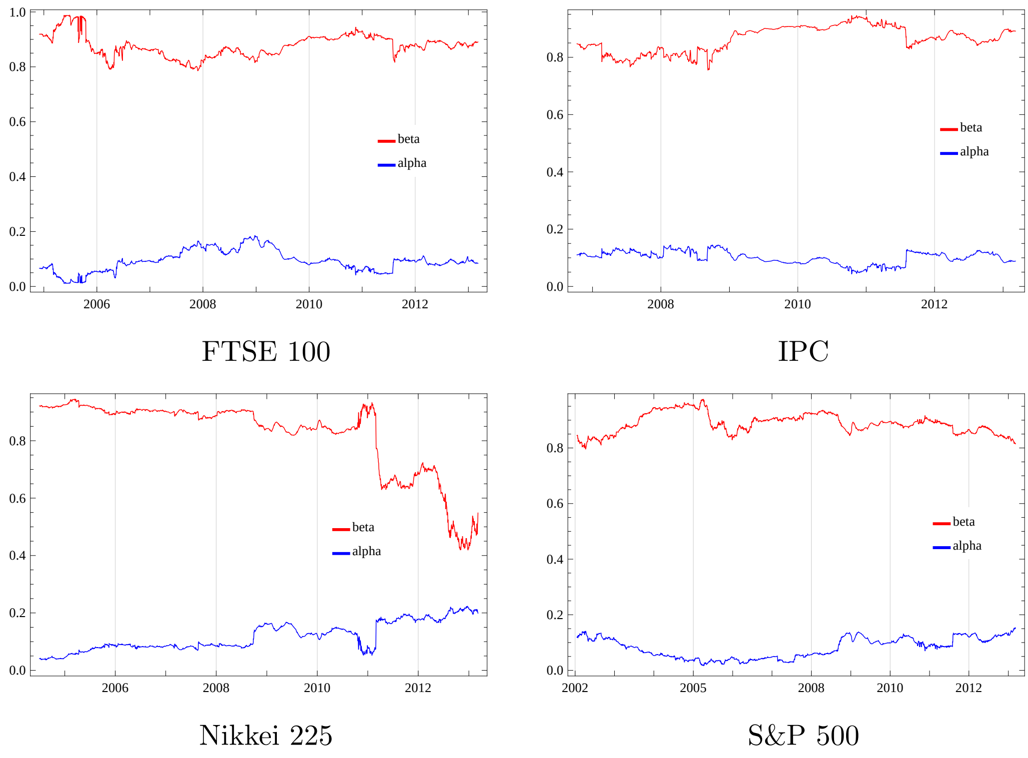


Figure B.10: GARCH rolling coefficients

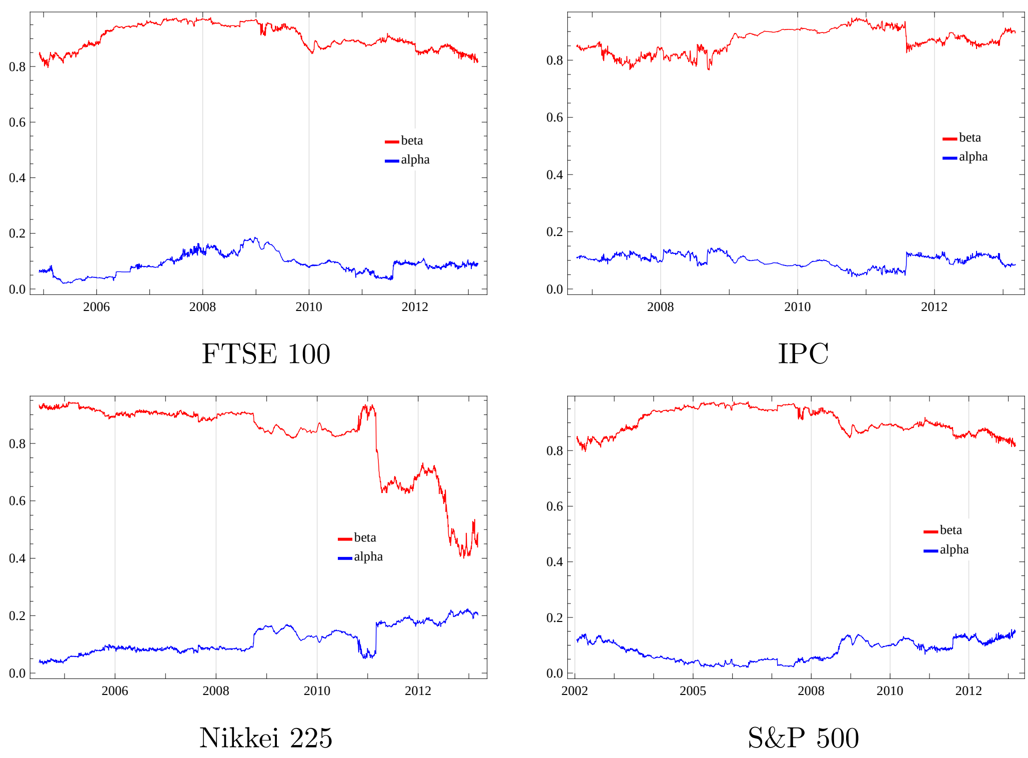


Figure B.11: GARCH with volume rolling coefficients

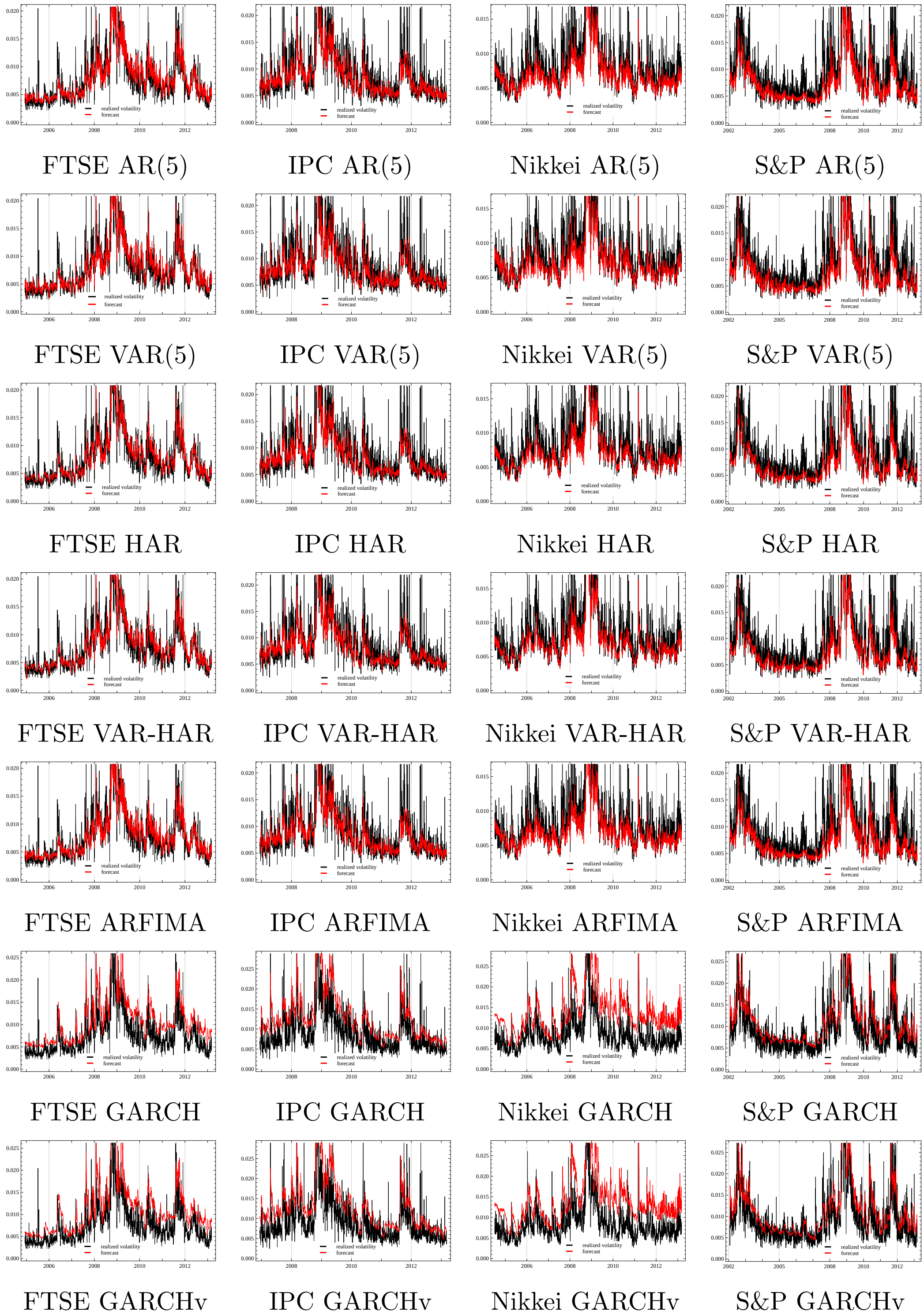


Figure B.12: Forecasts

Bachelor Thesis Proposal

Author	Tomáš Kvasnička
Supervisor	PhDr. Ladislav Krištoufek
Proposed topic	Volume - volatility relation across different volatility estimators

Preliminary scope of work Volatility, as a latent variable, is one of the most important aspects in asset allocation, risk management and pricing of financial instruments. Recent research has proved that considering intraday data (open/low/high/close) leads to more efficient range-based volatility estimators. As a consequence of that, application of range-based volatility estimators in financial sector has become more frequent. However, there are still some disputes among researchers about implications regarding trading volume and volatility.

This paper takes into consideration both classical volatility models (squared returns, absolute returns) and range-based volatility models (Garman-Klass, Parkinson) and explores their relations to traded volume. Data from different financial assets are used to scrutinize the implications. The causality regarding volume and volatility is obtained using vector autoregression approach. Subsequently, impulse-response functions are made to determine influence of given variables in time. Predictability of scrutinized models including volume is measured by mean squared error approach.

Main goals of this paper:

1. To present basic range-based volatility estimators and underline their advantages.
2. To find and compare relations among different volatility estimators and traded volume.
3. To explore significance of traded volume across selected financial assets.

Předběžná náplň této práce v češtině Volatilita, jakožto latentní veličina, je jednou z nejdůležitějších proměnných v risk managementu, alokaci aktiv a oceňování finančních derivátů. Nedávné výzkumy ukazují, že využití intraday dat k výpočtu range-based odhadů přináší vydatnější výsledky, v důsledku čehož jejich využití ve finančním sektoru roste. Co však zůstává nevyřešené a na čem se ekonomové neshodují, je vztah mezi volatilitou a obchodovaným množstvím.

Tato práce vezme v úvahu jak klasické modely volatility (čtvercové výnosy, absolutní výnosy), tak i range-based modely (Garman-Klass, Parkinson), a bude zkoumat jejich vztah s obchodovaným množstvím. Jako vzorek dat poslouží různá finanční aktiva (burzovní indexy, akcie, bondy). Ke stanovení daných kauzalit bude využita metoda VAR. Následně budou sestaveny impulse-response funkce, které budou popisovat vliv daných proměnných v čase. Vydatnost jednotlivých modelů bude měřena na základě střední čtvercové chyby.

Cílem této práce je:

1. Prezentovat range-based modely a popsat jejich výhody.
2. Nalézt a porovnat vztah mezi jednotlivými modely volatility a obchodovaným množstvím.
3. Zkoumat, zda-li se vliv obchodovaného množství liší napříč vybranými finančními aktivy.

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Author

Supervisor