Charles University in Prague

Faculty of Social Sciences
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MASTER THESIS

Asset Price Bubbles:
Housing Markets Data

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, May 17, 2013

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Acknowledgments

Abstract

This study assess Prague real estate market from price level bubble existence point of view. For this purpose construct loglinear regression models for estimating price level of purchase and renting for several segments on the real estate market. These estimated price levels afterwards compare to theoretical price levels set by user cost concept, which employs fundamental factors such as capital costs, depreciation rates, personal income and property taxes, additional asset risk and expected capital gain. Study concludes that Prague real estate market does not currently experience positive price bubble, even one of the segments assess as underpriced.

Abstrakt

Tato studie se zabývá hodnocením pražského realitního trhu z pohledu existence cenové bubliny. K tomuto účelu sestavuje loglineární regresní modely pomocí kterých je odhadována cenová hladina nájmů a cen vybraných segmentů nemovitostí. Tyto odhadnuté cenové hladiny následně porovnává s teoretickými cenovými hladinami určených za využití konceptu nákladů užívání nemovitosti. Tento concept zohledňuje cenu kapitálu, míru znehodnocení, daně z nemovitosti a z příjmu, dodatečné riziko aktiva a očekávané zhodnocení aktiva. Ze závěrů studie vyplývá, že pražský realitní trh s nemovitostmi aktuálně neprochází pozitivní cenovou bublinou, naopak jeden sledovaný segment trhu dokonce hodnotí jako podhodnocený.

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Acronyms

CDO   Collaterized Debt Obligation

NPV   Net Present Value

OLS   Ordinary Least Squares

Master Thesis Proposal

Author:	Bc. Jakub Mrhal
Supervisor:	Prof. Ing. Oldřich Dědek, Csc.
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Proposed Topic:

Asset Price Bubbles: Housing Markets Data

Topic Characteristics:

Based on global economic crisis 2008 – 2010 and possibly upcoming another one, is more and more obvious that not only our economic system is repetitively and continuously creating and bursting asset price bubbles, but also is considerably vulnerable to such bursts. Understanding how asset price bubbles are generated and how to recognize them in real time is essential to avoid devastating impacts of asset price bubbles bursts in the future. Housing market is due to its characteristics highly price bubbles prone and in combination with high-level involvement of wide population in developed countries is one of the most interesting markets for this topic. These are main reasons why I would like to focus on topic of assets price bubbles with real estate markets extension in my thesis. More specifically provide reader with short sum of nowadays theory and some historical examples. Then introduce and discuss several approaches of identification price bubbles in real estate assets and afterwards employ these models on empirical data.

Hypotheses:

1. Significant dependence of housing prices development on fundamental variables chosen for the model
2. Verification of real estate assets price bubbles existence
3. Identification of housing markets with possible ongoing price bubble

Methodology:

1. Basic descriptive analysis and time series comparison
2. Linear and loglinear regression models using ordinary least squares method
3. Stationarity and unit root tests

Outline:

1. Introduction
2. Theory and Historical examples of Asset Price Bubbles
3. Real Estate Assets Prices & Fundamental Factors
   a. Housing Pricing Methodology
   b. Fundamental Factors
4. Empirical modeling of Real Estate Price Bubbles
   a. Data
   b. Model
   c. Results
5. Conclusion

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1   INTRODUCTION

The topic of asset price bubbles is definitely not the brand new one. Many well-known economists paid a lot of attention on this topic for a long time and studied cases from our history even almost 400 years old, such a Dutch tulip mania from the 17th century or South Sea bubble from the 18th century. Despite all the time and interest, the topic is still not understood very well. At least in a practical manner, that could be used in common financial matters. Even though, it is getting more and more obvious, how important this topic really is. Two worst economic crises in modern mankind history, Great Depression 1929 and Global Financial Crisis 2007 that still more or less lasts, were actually triggered by asset price bubbles bursts. Economical knowledge about asset price bubbles could be compared to knowledge about earthquakes. We have theories what cause them, we can ex post identify[1] and somehow measure them, we know that consequences could be disaster, but still we cannot predict them with certainty in time, location nor magnitude.

In my thesis, I will focus on real estate bubbles on housing markets. Real estate assets capitalization in developed countries is estimated to 54 % of all assets and around 40 % out of all assets residential properties only[2]. In developed countries, there are also from 40 % to 80 % (little over 60 % in average) population living in their own properties[3]. These facts combined with statistics that roughly 9 out of 25 most famous asset price bubbles were actually real estate bubbles, makes this particular field very interesting. We could state that the real estate market is the biggest market with highest level of wide population involvement, prone to asset price bubbles on the world. These are the main reasons that in this thesis we try to identify not only past price bubbles, but also potential bubble that could be present on selected housing markets in these days.

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[1] Actually there could be academic discussion about this as for example Kroszner (2003) argues that 1929 Great Depression was not actually positive asset price bubble as following gains justified top price but did not justified lower prices so market was actually underpriced.

[2] http://www.housingfinance.org/publications/, Homeownership Trends Worldwide, Nasser Munjee

[3] dtto

The thesis is structured as follows: after introduction, second chapter provide some most famous historical examples of price asset bubbles, then present several valuable definitions and general theoretical concepts. Third chapter introduces main theoretical concept of user costs that this study utilize for empirical assessment of market price level. In next chapter study deals with estimation of loglinear models needed for determining current market price levels for renting and purchase real estate property. Chapter five presents empirical testing of chosen real estate market segments price levels employing theoretical concepts from third chapter in combination with practical results from chapter four. Last chapter summarizes our main findings and conclusions.

2 THEORY AND HISTORICAL EXAMPLES

2.1 HISTORY

"Those who does not know history are destined to repeat it."

Edmund Burke

In order to better understand the asset price bubbles nature and topic in general, I will firstly introduce several famous historical cases of impressive price growths and declines. I find useful to observe what is topic about in real world and what consequences could follow at the very beginning of this work. Also the message of this part is that, asset price bubbles exist all the time along with existence of the free markets. The time era when it appears only influence the transmission mechanism but not basic merit of the bubbles. Lets starts with chosen cases in chronological order.

2.1.1 TULIPMANIA

First chronicled case ever and maybe also historically most famous one was in Continental Europe in the first half of seventeenth century. More specifically it was situated in Holland from 1634 to 1637. It begun during 16th century, when tulip flowers were introduced by Ottoman Empire in the Western Europe. They became very popular in Netherlands during Dutch Golden Age for its fascinating bright colors. Uprising popularity in combination with the fact that it takes 7 years to grow the flower from the seed caused that recently there was an exceed of demand over supply making the price grow. At the price top in late 1636 and the beginning of 1637, the growth from 1634 was above 5 900 %. People even mortgaged their houses just to buy flowers in order to resell them for higher price.

According to Charles Mackay's book "Extraordinary Popular Delusions and the Madness of Crowds" following prices were paid for one single tulip bulb: two lasts of wheat, four lasts of rye, four fat oxen, eight fat swine, twelve fat sheep, two

hogsheads of wine, four tons of beer, two tons of butter, one thousand lbs. of cheese, a complete bed, a suit of clothes, a silver drinking cup. They were also signing contracts at the end of the season for the next season harvest. That could be considered as first future contracts in history. The bubble burst once one buyer denied to pay for such future contract he signed and all the market eclipsed very quickly after.

2.1.2   THE SOUTH SEA BUBBLE

The South Sea Company was formed in 1711 by Robert Harley to convert £10 million of government war debt from war with Spain into its own shares. In exchange company obtained a monopoly on trade with South America and South Seas and also received annual interest payments on the debt. Even though the fact that company never started to trade with oversees properly, it continued with several other debt exchanges and as company price was rising they finally introduced the plan to take over whole British national debt in exchange for guaranteed 5 % annual interest payments. When the plan was publicly announced, the company shares price just rocked up. Every public offer was sold out within hours and subscribers were from all social classes.

Such success was inspiration for many new joint-stock companies, which were created by these times. These companies were known as "bubble" companies. The absurdity of the situation well describes the name of one newly founded enterprise entitled: "A company for carrying on an undertaking of great advantage, but nobody to know what it is." The subscription for this company was sold out very quickly as well and the lucky entrepreneur showed the advantage when promptly disappeared to the Continental Europe as says notes of Charles Mackay.

In 1720 the South Sea Company owners were scared out of competition from these new companies and in order to sustain high share price convinced the government representatives to pass act that prevented from establishing new companies without government permission and allowed existing companies only to carry out those activities that were prescribed by their charters. In the late June 1720 the stock price of South Sea Company reached its peak at £1050 that made 750 % growth from initial prices. In August 1720 firstly foreign investors lost confidence in the company model and sold out their shares. That made domestic credit stretched out the limits when finally in September the shares lost 75 % in four weeks and whole

market collapsed. At the end there was investigation and several politicians and businessmen were imprisoned for corruption and over £2 million was confiscated from South Sea Company directors.

2.1.3 THE BULL MARKET OF THE ROARING TWENTIES

In the second decade of the 20th century, there were taking place many important changes to the United States markets. 1913 was established Federal Reserve institution. There were introduced new policies that helped to improve free trading, anti-inflation and anti-trust measures took place. Also due to new production improvements workers productivity was growing and allowed higher focus on research and development field. There was one more factor important for rapid growth. That was debt and thanks to it allowed leverage. 1920s was the decade of massive spread of consumer as well as corporate credit. Americans used borrowed money for new mass production goods purchases such as radios and automobiles. But consumers also used borrow credit to stocks investments. So it happened that this credit support demand for stocks as well as for goods made by companies traded on stock markets. The wheels of stock market growth were rolling on maximum, especially in new technologically branches. Radio stocks raised 400 % during 1928.

On September 3rd 1929 the Dow Jones index reached its top for the decade and started to decline slowly. But on October 24th known also as "Black Thursday", things speed up rapidly and started the period known as "Crash of 1929". Markets were affected by panic and the trading of 13, resp. 16.5 millions shares per day with loss of 38 %, resp. 30 % in following days took place. Markets reached its first bottom in spring 1930, but than in early 1930s there was another decline and second bottom. This era was followed by well known years of the "Great Depression" that was considered as world biggest crisis until recent Global Financial Crisis 2007.

2.1.4 THE JAPANESE "BUBBLE ECONOMY"

Let us make a box: After Second World War, Japanese experienced one of the highest economy growths on the world during 1960s to late 1980s. In the 1970s the Japanese government deregulated financial markets, which allowed very cheap credit

on low interest rates. There was also important invention of "zaitech" or "financial engineering" what simply meant that speculative trading on stock market became inseparable part of corporate earning strategy. After the firms got the cheap credit and invested it on stock market, thanks to permanent growth of Nikkei index, companies raised funds easily. And then very often used the funds to reinvest on stock markets again since the market was going nowhere else than up. These profits were reported as earnings growth, what made investors even more wanting to invest in these success-full companies and again driving the market to growth and providing more new funds to companies speculative actions. At the end of 1980s there was estimated that speculative investment became major source of earnings even for most of non-financial companies.

There is also one important driver for Japanese bubble economy and that is the land. In Japan, the land is traditionally highly valued because of scarcity of quality land due to presence of mountains everywhere. Thanks to its high value, banks often accepted properties with lands as collateral for credit easily. It was one of the key drivers for cheap credit and as economy and markets grew, the value of lands also grew. At the peak it made around 500 % growth from initial position and in Tokyo city center the price was estimated a million dollars per square meter. That made the Imperial Palace in Tokyo worth more than all properties in state California combined.

In 1989, Japanese government representatives became worried about massive growth of Nikkei index and land values as well. As response they tighten up monetary policy and raised interest rates in May 1989 in order to stop or at least slow the land values growth. During 1990 they raised interest rate several more times. From January 1990 Nikkei stock market price kept falling as well and recession was unavoidable. Japan then during 1990s had slower growth than any other important industrial country on the world.

2.1.5   FLORIDA 1920S LAND BOOM

Miami in state Florida is actually favorite location for real estate price bubbles. First of them took place in 1920s. Location thanks to warm weather and beautiful nature scenery became very popular among the tourists and investors and thanks to that experienced great economic prosperity and development. Sunshine state, as called, even had the light billboards on New York's Times Square advertising this Eden on

earth. Prices were going nowhere than up so speculative trading came into place. At the peak there was 400 % growth from initial steady price. Then came a series of unpleasant incidents that caused the bubble to burst. First, on January 1925, Forbes magazine printed out an article that warned about ridiculously high prices of Florida real estates based on speculative expectations. Suddenly the growth disappeared. Adding the rising railroad shipping prices, two hurricanes in 1926 and 1928 and Wall Street Crash 1929 followed by the Great Depression, the bubble hit its bottom.

2.1.6   URANIUM SURGE

After long times that uranium commodity price slowly but steadily rose, in 2007 prices just rocked up exponentially. Possible explanation could be combination of following factors. In October 2006 was flooded the biggest undeveloped source of uranium on the world, Cigar Lake Mine of Saskatchewan. Strengthen of global warming phenomenon through beginning of the 21st century, where Nuclear Plants could be part of the answer of growing demand for electricity consumption and decreasing "greenhouse gases". And finally last factor, depleting of huge stockpiles from 1980s that were created for planned new nuclear power plants that were actually never built for political and public pressures.

At the top of the bubble, beginning 2007, price per kilogram of the uranium was 300 $. In comparison, for the same amount in June 2002, it was only 20 $. Final growth against initial price was at the peak 1 844 %. But when flood of Cigar Lake Mine of Saskatchewan ended and was resumed to works to deploy this source, new nuclear plants could not be built in that short time and actually most of power plants had long-term supply contracts signed in advance, it was more than obvious that real demand will not meet the expected one. Prices plummet during one year under 100 $ per kilogram and the bubble burst.

2.1.7   DOT-COM BUBBLE

Dot-com bubble last roughly from 1995 to 2000 and it is closely connected to the fascination with new amazing technology called Internet in those times. All companies specializing in this or related industry were growing on stock markets

with fascinating pace. Sometimes was enough to add e- prefix or .com suffix to the company name and market price doubled over-night. Also new interesting business model strategy was invented during this dot-com frenzy. Strategy was called "get large or get lost" and was based on aggressive capture as big market share as possible and as soon as possible no matter costs. Also quickly spread company awareness with belief that these assets will be profitable later in the future[4]. Investors believed this strategy as well and also believed to the bright future of this new Internet technology. This particular combination allowed conditions, where firms that never made a profit and not even single revenue in some cases, were able to obtain incredible funds through initial public offers. This fact combined with very low interest rates in 1998-1999 encouraged massive growth of new companies in this new industry, even the absurd ones comparable to those from South Sea bubble. For example investors were able to invest their money to firm called "Digiscents" that specialized for development of computer peripheral that would make the web sites smell.

During 1999 and early 2000, U.S. Federal Reserve raised six times interest rates and as a result economy finally slowed down. The NASDAQ index peaked on March the 10th 2000 with price more than double last year and more than 5 times higher than 1995. But already on the March 20th, which means in only ten days, NASDAQ lost 10 % out of peak and by 2001 was Dot-com bubble deflating at full speed. Many of dot-com companies just burned out the remaining venture capital during one year and vaporized out of the market never made a single profit. Actually it is somehow surprising how was the end of dot-com companies surprising, because the high mortality existence is built-in feature in "get large or get lost" business strategy. It is easily understandable that only a few companies could reach high market share position for not that large number of web services available at the time. So it was an unavoidable and possibly predictable destiny for most companies just to "get lost". But what makes it remarkable is the market value loss that was caused by that part of "get large or get lost" business model strategy and that is more than 5 trillion $ from March 2000 to October 2002. But in fact, that bubble burst did not cause too big economy crisis as could be guessed out of single loss number. There are also companies that survived through stock price turmoil and already broke the maximum prices from year 2000, for example well known company named Amazon.

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[4] In fact, Google and Amazon grew among biggest companies on the world using this business strategy.

2.1.8 REAL ESTATE AND LEVERAGE BUBBLE

The last example of historical price bubbles I am going to describe is also the most recent experienced one and we could say that we cope with its consequences until these days. It is originated in United States of America in year 2007 and during only one year spread to most of developed countries around the world. I will pay attention only for the initial part that took place in U.S., often also called sub-prime mortgages bubble.

The fundamentals were laid down during year 2000 when some essential aspects of way how industry worked so far were changed. To the system where lender institutions were supposed to hold the debt on their very own books until final maturity was introduced distribution mechanism that shifted actual debts from original lenders away. At the end, whole chain was actually quite long. Basic and simplified backbone included: homeowners – mortgages buyers, brokers – mortgages dealers, lending institutions – actual mortgages originators, investment bank institutions – mortgage packages buyers using leverage and after application of financial engineering magic seller of collateralized debt obligations (CDO) and last but not least pension funds, hedge funds etc. as investors buying CDO tranches[5]. And this chain was working pretty well and everybody was happy. Homeowners had their houses, brokers had their commissions for dealing, lending institutions had earnings from sold mortgage packages (and even more money to lend), investment banks had pretty high earnings on sold CDOs tranches that they built from mortgage packages they bought on high leverage for very cheap credit (as Federal Reserve set very low interest rates) and finally investor institutions had their relatively high return investment (relatively as bonds were no more interesting for very low interest rates). The real turning point came in the moment, when thanks to very cheap credit from low interest rates and huge inflow of foreign resources, there were so much money to lend but market of responsible homeowners with sufficient income was completely saturated. Supply exceeded demand and the only solution in this situation, besides slow down, was to create new demand. Because nobody in the chain wanted to give up his or her profit or commission, subprime mortgages time came. Subprime

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[5] In this basic chain I do not mention institutions as Insurance companies, Government-sponsored enterprises or Rating agencies... These institutions played their part in whole mechanism as well, but are not essential to understand the main driving forces that allowed bubble to be created so I will just omit them. The same stays for CDOs creation mechanism.

mortgages actually meant free money, because there was no initial payment needed, no proof of income what so ever, property as collateral was enough. In fact, many American families took credit and collateralized their own houses just to use gained money for boosting their consumption. Nobody in that moment thought that there could be any problem, because even if homeowner default on his or her interest payments, there still was collateral to sell and as house prices were uprising almost forever, nothing bad could really happened. And even if something bad happened it really was not problem for the debt originator, because he was selling all the risks very quickly away, even though he was really the only one who knew who he was selling mortgages to. So the bomb was ticking and it was only a matter of time when it is going to explode. As more and more homeowners default, there were more and more foreclosed properties to be sold on the market making supply exceeding demand. With supply over demand prices were after long time finally decreasing and as result some of the homeowners who were able to pay their regular payments get to the situation, where value of their mortgage was far over the market price of the house. So it was irrational to continue with the payments and they stopped. This created even more pressure on supply and prices were just falling. Suddenly whole chain stopped working. Lending institutions did not want any new homeowners from broker as they had low quality mortgages on their hands that nobody wanted to buy. Investment banks had on one hand the CDOs that in fact became nothing else than boxes with worthless houses and surprisingly nobody wanted to buy these anymore. But on the other hand they still had to repay borrowed credit, which they used for the leverage. And finally investors that did not want to buy anything new from investment banks, but already had full pockets of toxic CDOs losing its value as well. Market was frozen and toxic CDOs were actually spread all over the world.

2.2 TERMS AND DEFINITIONS

Let's start this part with introduction of two basic terms that needs to be clear for understanding next parts of the work properly.

Basic Term 2.1 Price is the value that will purchase or transfer ownership of a definite quantity, weight or other measurement of good or service.

Basic Term 2.2 Intrinsic value is the inherent worth of something independent of its value to anyone or anything else.

So the price is the amount of money for which the asset or anything else is sold on the market. The intrinsic value or just a value is the pure internal value of the asset if all psychological, emotional or other influences on final price that are not directly connected to ability of the asset to make cash flows in the future are deducted. It may seem very easy, but it is of essential importance to be able to distinguish between those two terms in order to understand what price bubbles means in their basics.

" Price is what you pay. Value is what you get."

Warren Buffet

This quote of the famous investor Warren Buffet implies on the fact that the price of asset does not has to be always matched with its real value. These two very important characteristics could be in fact oscillating in time one around another, but never diverge in probability or long-run period, at least for most assets on the markets. It could be seen as cointegration case, both variables could be considered as random walks but the difference between them is linked one to another[6]. In short time periods, due to information asymmetry, psychological aspects etc., there are gaps opened. Longer the gaps last and bigger they get means higher probability that they will be closed or at least became smaller.

Such case inequilibrium provide some opportunities as well as threats. Some might make fortune some might lose it. Redistribution of wealth certainly takes the

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[6] Michael P. Murray (1994), A drunk and her dog: An illustration of cointegration and error correction.

place. The question is, if this aspect is really the problem or just an application of Darwin concept – stronger (cleverer) survives (get richer). If one puts it in that easy and straightforward way, it is really difficult to argue with natural order. But in real world the regulations and bailouts comes in to the picture and as they may ease the economical impact of the bubble burst, they definitely damages the responsibility chain and breaks significantly upper mentioned Darwin concept. Another important thing is redistribution of political power. Bubbles bursts are often accompanied by social tensions that provide soil for political instability. But as far as these aspects are both very important and complicated, they are also out of main framework of this thesis. I find important to mention them, but I will not discuss them in details or pay any further attention to them in this work.

Another interesting thing about these gaps is that there are out of logic two basic kinds, negative and positive one. Negative gap is ended with growth closing it and positive vice versa. The amazing thing about this is, that if we speak about asset price bubble in general, we only mean the positive one terminated with negative drop.

I think that it is caused by two main reasons. First is the way our generation is used to think. We are living in the "growth" generation[7] that is expecting growth everywhere and in everything. The growth is the only thing that could interest us and draw our attention. So in case of negative bubble, because there is no growth on the beginning, we just do not pay attention to it. And afterwards, when the growth phase takes place, attention is somehow shifted because it was not important before. That means that we do not notice the first phase and without this part it could not be bubble, it is just growth upon average during second phase. And anyway, growth is what we are expecting, so nothing strange here.

If we just switch the phases, this is what happens. During the first growth phase, everybody is interested by growth higher than the normal usual average one. That makes us pay attention to it and praise it. When second phase of correction and decrease takes place, everybody is scared and worried what happened to that upon average growth we all liked and admired. It has all the attention for both phases and that allows us to look at it as on the complete bubble with both phases.

Second reason is that it is generally accepted that people are more sensitive to declines than to growths, for the very same reason I just mentioned above I guess. Bigger decline means more sensitive reaction. And we all know that if you want to

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[7] Tomas Sedláček, Good and Evil Economy (2009)

experience big deep fall, you should climb pretty high at first. This allows only the scenario with growth phase first and fall phase second.

Now finally lets get to definitions of "Price Bubble" itself. Firstly in 18th century was used name "Mania" according to first recorded bubble "Tulipmania" in Netherlands. During the 19th century become more popular name "Bubble" and it lasts till nowadays. This name is actually very precise, because if we imagine natural bubble such as soap or gum bubble, it steadily grows when air inflows as far as pressure reach threshold and the bubble pops. That is, figuratively meant, exactly the very same that happens to asset price bubbles. In the second half of 20th century there comes first economically serious definitions on this phenomena.

Definition 2.1 (Kindelberg, 1978). Bubble is an upward price movement over an extended range, that than implodes.

This one of the first definitions on economically literature is in fact very close to the example of natural bubble pop mentioned above. It only cares about price change itself and in positive direction with afterwards drop.

Definition 2.2 (Kindelberg, 1987). A bubble may be defined loosely as a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the assets rather its use or earnings capacity.

Nine years later the same author comes with a little bit extended definition. This time is definition more specific in the way of the growth nature. It implies the presence of internally unjustified price growth due to speculative pressure on demand. The question remaining is how to identify this substantial speculative growth.

Definition 2.3 (Stiglitz, 1990). If the reason the price is high today is only because investors believe that the selling price will be high tomorrow—when 'fundamental' factors do not seem to justify such a price—then a bubble exists.

In 1990 the Fed adopted the definition of "bubble" offered by Joseph Stiglitz. This definition suggests existence of fundamental factors that are important for backing up asset price development. That means, we should be able to compare these fundamental elements with asset price movements and find out, whethet there is sufficiently strong co-movement or if there is not. Of course, we have to know what fundamentals are at the first.

Second part included in this definition is also important one, that is introduce and this is investor beliefs and expectation about future. Does not suggest that it has to be necessarily speculating pressure. We allow here possibly mispriced market due to its expectations about future.

Definition 2.4 (Rosser, 2000). A speculative bubble exists when the price of something does not equal the market fundamentals for some period of time for reasons other than random shocks. Fundamental is usually argued to be long-run equilibrium consistent with a general equilibrium.

Even though this definition does not include any mathematical equation that could be used for analytical identification of price bubble, it contains interesting idea how to approach this topic. It suggests we should be able to separate price of the asset to three parts. One is fundamental price that captures solid and justifiable asset value, second is random shocks that exists for only short periods of time and tha last part would be component capturing residuum between first two parts sum and market asset price. We could call it asset price bubble indicator and if this particle grows to some extent, we might proclaim that the asset is in bubble position.

The question then is how to determine first and second part. Random shocks are relatively easy to identify, as they are short-term variances that does not have long-term effect. Shocks also should not represent significant factor so it is definitely more important how to set fundamental value. One way is using other fundamental variables and derives from them this part, though that could be very difficult or even impossible in some cases. But there is also another widely accepted concept how to look at intrinsic value determination. Net present value (NPV) of future cash flows of the asset. This concept is very easy and straightforward to employ for determination of the real value of the asset and then with simple comparison we could say if the market price is in bubble position or not. But of course there are some shortcomings in this approach as well, otherwise there may not be any more price bubbles anymore as every investor could apply this approach.

The first problem is the uncertainty of future cash flows as well as discounting factor. Let say for stocks, dividends as well as inflation and cost of capital may vary in time and even a small change could have considerable influence on the results and thus real value. Another problem is, that high percentage of investors buy asset with expectation of future sell. That makes infinite perpetuity NPV formula useless. Instead we are supposed to use the finite one, where the cash flow from selling the asset should be calculated as well. But then it is absolutely rational to buy asset even though the price is already on upward phase of the bubble, because there is still

expected, that the price of sell will be higher and the investor will make the profit. NPV would not be any lower than market price. The problem is on the edge where the growth breaks to decline. Any further NPV will be probably lower than the market price and that makes asset purchase irrational anymore. But still, it is based only on expectations and even the buyer right before the edge could be rational in the way of expected growth. But this is already too far from intrinsic value we started with for now.

2.3 THEORY

Ever since markets exist, there is also possible to trade on such markets not only for reasons to get what people need in exchange for what they do not need that much, but also for simple profit itself. One of ways how to make this profit is speculative trading. It is the most straightforward possibility and simply it means to buy asset, commodity or etc. and sell after the price growth. Profit is relevant and understandable reason for trade and everyone could use one. Problem occurs when this speculative trading becomes a significant share of market volume. This is the point, where self-fulfilling prophecy comes on the scene and where the price growths is not out of rational reasons anymore but for only one reason left and that is the price growth itself. Speculative traders pushes out the rational demand out of game, what is problem in longer horizon, because speculative traders buys the asset in order to sell it, not to use it or employ it any other way. And at the particular moment, when some speculative traders (or anybody else on demand side) breaks the chain and do not buy anymore, market participants are in a really bad position. Because speculative buyers does not have any other plan for their purchase than resell, and nobody is buying anymore as price is too high, they just hold asset and lose money as the price returns to the rational value path. Often the drop is even under the rational level resulting from the consequences impact of this scenario on the economy as a whole.

Very interesting about this speculative cycle is that even though from general macroeconomic point of view, final aftermath is definitively negative and thus is desirable to avoid these situations to happen or lets say that it is generally irrational to support such development. From microeconomic individual point of view, it is also opportunity soil as huge amount of wealth is redistributed. So in case individual know when enter and when to quit, it could be very rational and desirable to be part

of bubble market and even actively participate on its formation. As interesting this topic area is, it is not important matter of our focus in this work.

So speculative forces could be one of the possible motivations and life-cycles of the price bubble. In fact there could be more initiations and variations, but the merit is always the same. Significant price growth that is not backed-up sufficiently by growth of important fundamentals and afterwards plunge of the prices. Following problems as ineffective allocation of resources due to price mechanism distortion, economy turmoil as credit system crushes due to collateral mechanism breaks etc.. These consequences are definitely worth to try identify price bubbles at the beginning of their life-cycle and undertake appropriate steps and actions to prevent bubble creation at first place or at least minimize its impacts. There is ongoing discussion between economists whether there should be taken actions against forming bubbles or if it is generally more effective to let it burst and cope with bubble aftermath. However this is not aim of this work to analyze these approaches, to make this discussion relevant we need to be able to identify forming price asset bubble at first place.

If we take into considerations all written above, we end up with three or maybe four basic possible approaches to asset price bubble identification. First is simply measurement of excessive changes over given level in asset price through given short period of time. Next approach is to compare asset price development with fundamental variables development and assess whether asset price is backed up by those fundamentals. Third general approach is to calculate (or estimate if there is no sufficient data available) future cash flows and thus net present value of such asset. Last is kind of additional one that could be added for the second and third one and it is to care about investors future beliefs and their expectations. Case, Shiller (2004) used this approach, combined with extensive fundamental analysis to point out upcoming real estate subprime bubble in three years advance. But as this more like sociological approach so we would not discuss it further. Now lets have a closer look at those three main general approaches and discuss their advantages and shortcomings.

2.3.1   THEORETICAL CONCEPTS

First and most simple one is derived from Kindelberg definitions and cares about significantly excessive asset price changes in short time period, mainly growths. The

idea is based on elementary notion that for every state of the nature in the economy, there is some natural threshold of possible growth beyond that must be some speculative forces added to overcome such values. And these speculative forces push the asset price value to bubble position where burst is inevitable when speculators run out of fuel.

Definitely considerable advantage of this approach is its simplicity and straightforward. But the question is, where to set that threshold that would indicate, which price changes are still in tact and which are not. It definitely would not be the same for all assets classes, also there should be different thresholds for different states of economical realities. Kroszner (2003) also shows, that asset mispricing position does not have to be always accompanied by the change of that price asset. Sometimes it could be also situation that asset price remains on some level even though there no longer last fundamentals that would support such price. In situations like these this approach fails completely. But actually for open markets with high trading volumes, this could be thanks to its simplicity valuable signaling instrument, for situations where we should pay more attention to and care about them starting more complicating procedures indicating whether the price change is justifiable or not. For all our historical examples, this signaling mechanism would probably works.

Second is the comparison of asset price development to its fundamental factors as Stiglitz definition suggests. The logic here is that price growth without some fundamentals employing explanation that would back up such price development is in time unsustainable. Thus for every asset we care about, we are supposed to find out those significant fundamental factors, estimate or derive long-run equilibrium relation and assess deviations from that equilibrium. For case of real estate markets are these fundamental factors very often represented by household income, rent level, construction costs, new construction level, population growth, mortgages rates, etc..

Advantage of this concept is its applicability in real time under condition we have relevant current data of fundamental factors. We could be even able to relatively accurately predict asset price development if we would be able to estimate future development of its fundamentals. But as fundamental factors could be advantage of this approach, they also could be the problem for this method. Sometimes it could be very difficult to determine relevant fundamental factor as well as there could be just too many of them and it could be virtually impossible to compare them all together. Another thing is what happens when these factors are mispriced as well. We could

end up in situation where we assess fundamental factors for fundamental factors to find out if they are priced precisely, which is strange idea.

The last concept, which is deduced from Rommer definition, is to determine assets intrinsic value or long-run general equilibrium consistent price. As we already mentioned, this could be done by calculating asset net present value of all its related cash flows.

This approach is very sound in its economical theory background. It is also generally applicable for every kind of asset as its value should be sum of its all incomes in the future. This is also very understandable and straightforward concept. As long as we have sufficient and relevant information, it is very easy to take historical data, count all operational and capital gains and costs and discount them by realized inflation and compare obtained result with price of asset a the beginning of tested period. If backward looking analysis, it is definitely the most precise method, no matter any fundamental factor at all. Kroszner (2003) even calculated that 1929's Great depression actually was not price bubble, because even very high prices at the top before great price drop, were in terms of capital costs justified by future earnings.

Problems emerge when trying to utilize this concept for assessing current price of asset. Dimension of future expectations could be very deceiving and what more, whole concept is highly sensitive for even small changes in expectations for important factors. Even if we are able to predict very precisely future cash-flows, only a little change in discount factor would provide us with completely different results.

3 REAL ESTATE ASSETS PRICE ASSESSMENT USING USER COSTS

In order to empirically test, if there is a possibly ongoing price bubble on Prague real estate market, we need some methodological concept that will allow us to compare fundamental prices with current price level. For this we are going to employ method that is in some sense combination of fundamental factors comparison approach as well as estimating intrinsic value of the asset itself. It is based on exploring mutual relation between three main pillars, which are market price of purchase, market price of renting and user cost for the real estate asset. First element is simply the price of the property for which it is offered on the market, lets say it is the real exercised price on the nowadays market. The same holds for the second element, but for the case of long-term renting real estate property. So this element should stand for the price of renting the real estate property on the nowadays market. Last pillar is the most complicated one, and we are calling it "user cost" of housing. We could look at it as imputed rent or as sum of all cash-flows, costs as well as benefits, that are connected with owning a real estate property in present state of economic reality.

We will denote:

P_t … purchase price level on real estate market in time t

R_t … renting price level on real estate market in time t

u_t … as user cost level for real estate properties in time t

Then the basic idea is standing on theoretical background, that equilibrium state should be the one, where the renting market price as fraction of the market price of purchase is equal to the user cost of the owning the real estate property.

(1) $\dfrac{R_t}{P_t} = u_t$

Or in other words, market price of renting should be equal to market price of purchase multiplied with user costs.

(2) $\qquad R_t = u_t * P_t$

If there is inequality between market price of renting and user cost, there will be arbitrage opportunity for cheaper renting on the one side or cheaper owning on the other side. This arbitrage opportunity should naturally force market to shift the demand in favor of cheaper alternative and thus on long run close the gap and set equilibrium. Alternatively we could employ user costs variable as discounting factor for calculating perpetuity of holding real estate asset for infinity. This relation provides us with fraction determining willingness to pay for owning the property in rent price terms.

This approach is quite straightforward and should give very clear conclusions whether the real estate market is or is not in equality state. Also thanks to user cost structure, in comparison to simple rent/price ratio for example, it takes into account interest rates level, effective taxes and other important factors neglected by other ratios and thus it provides more precise and sensitive information about cost of owning a real estate property. But again, the simplicity and clear logic structure is probably the biggest advantage of this approach, on the other hand, there are also some challenging issues contained. For results and their validity, it is very crucial, how precise and valid those three main elements are. In case of user costs element, its underlying fundamentals are relatively easy to obtain more important is the way we adjust and employ them to the calculation itself, but I pay more attention to that later in the next chapter. The real problem as well as challenge is other two elements. It is of highest importance, that price of real estate property purchase and price of renting will be as precise as possible and also independent in structure of those two estimates. On the same hand it is intended to keep same level of quality and other properties of real estate for reason of estimation price levels of purchase and renting. This is also very important, as we need to remain consistent in these estimations, because this ratio is the key factor to be compared with user costs. Despite those really high demands, it is very difficult to gain data fulfilling these tight assumptions and requirements, more for Prague region only.

These are reasons why I decided to estimate price of purchase and renting indices on my own, using hedonic price models. Great advantage of estimation on our own is the fact, that models should provide very precise price of purchase and price of renting levels for detailed selection of parameters describing the property and also that this parameters selection would hold very same for both purchase and renting

price levels. With those two pillars estimated separately, we could also state that these estimates are independent on each other and thus provide unbiased results. It also offers us opportunity for testing about price bubbles in different segments of real estate properties that could be possibly estimated by our models. Drawback is absence of time-series data as information dataset needed for hedonic pricing models estimation is available only as static offer advertisements, but we do not lose forecast possibilities completely. We still could have expectations about some underlying variables for user cost influencing its development and thus predict potential change in price levels as well, expecting that price levels follow user cost development in order to close potential gap and fulfilling "no arbitrage" condition.

3.1 USER COSTS

As we already stated, user cost should incorporate all cash-flows, costs as well as benefits, that are connected to owning a property in present state of economic reality. In some perspective we could look at it as on imputed rent calculated for the owner if occupied the property herself. Through this pillar we should be able to project influence of important factors as cost of invested capital, differences in risks, property and income taxes as well as tax deductibility benefits, maintain expenses and expected capital gains from owning the real estate property. These influences that many of other price bubble assessment instruments neglect or fail to integrate to valuation process, will take important part in our approach.

We will start with formula used for estimating of imputed rent (Potreba 1984) that we will drop index *i* distinguishing individual real estate property cases as we are only interested in market price level as aggregate:

$$(3) \qquad R_t = P_t - \frac{1-\delta_t}{1+r_t^m+\gamma_t} E[P_{t+1}] + (1 - \tau_t)\omega_t P_t - \tau_t r_t^m P_t$$

Where following symbols stands for:

R_t … the imputed price of renting at time *t*

P_t … the market price of purchase at time *t*

$E[P_{t+1}]$ … expected price of purchase at time *t+1*

δ_t … reflects maintenance cost as fraction of the property value at time *t*

r_t^m … stands for mortgage rate at time *t*

γ_t ... represents additional risk compensation for property owning against renting

τ_t ... stands for income taxes rate for property owners at time t

ω_t ... tax property rate at time t

u_t ... real estate property user cost at time t

In order to obtaining more suitable form for straightforward explanation of individual particles meaning, we slightly re-arrange the original formula and obtain following form:

$$(4) \qquad R_t = P_t - \frac{E[P_{t+1}]}{1+r_t^m+\gamma_t} + \frac{\delta_t E[P_{t+1}]}{1+r_t^m+\gamma_t} + \omega_t P_t - \tau_t \omega_t P_t - \tau_t r_t^m P_t$$

If we apply on this equation relation (1), we derive real estate property user cost as:

$$(5) \qquad u_t = 1 - \frac{E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} + \frac{\delta_t E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} + \omega_t - \tau_t \omega_t - \tau_t r_t^m$$

Now we could separate six main sections that user cost consists of and assign their proper meaning to them:

$1 + r_t^m + \gamma_t$ ... this sum stands for discounting factor employing cost of capital with additional risk compensation

$1 - \frac{E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t}$ ... section captures discounted expected capital gain (loss) from real estate asset price change

$\frac{\delta_t E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t}$ ... section stands for discounted maintenance (depreciation) costs

ω_t ... reflects property tax cost

$\tau_t \omega_t$ ... express property tax deductibility benefit on income taxes

$\tau_t r_t^m$ ... represents mortgage rate deductibility benefit on income taxes

The last modification we will exercise in this part is presentation, how we could employ equation (5) for deciding if there is price equality state on the real estate market or not. As we know from formula (1), if the market is balanced, user cost is equal to fraction of price level of renting and price level of purchase. Let us denote r_t^i to be rent income put as percentage out of property price. Then we could substitute this rent income to equation (5) instead of user cost. For easier interpretation, we multiply equation by discount factor and neglect insignificantly small parts. Then we obtain following:

$$(6) \qquad r_t + (E[P_{t+1}]/P_t - 1) + \tau_t \omega_t + \tau_t r_t^m = \delta_t \, E[P_{t+1}]/P_t + \omega_t + r_t^m + \gamma_t$$

Where benefits, are on the left side, and costs are on the right side. To clear the market this equality must hold. Otherwise if the left side is greater than the right one, income exceeds costs and market is due to underlying fundamentals underpriced. Then with unchanged fundamentals, property price should grow or alternatively price of renting should decline. This situation holds for vice versa situation as well.

3.2 FUNDAMENTAL FACTORS AND USER COST SENSITIVITY

Although for further calculations we will employ formula (5), for now we accept negligible loss of accuracy and simplify equation (5) to even simpler form than is equation (6), where individual components will be as separated as possible:

$$(7) \qquad u_t = \gamma_t + r_t^m + \delta_t + \omega_t - \tau_t(\omega_t + r_t^m) - g_{t+1}$$

With this type of formula, where g_{t+1} holds for expected capital gain (loss) from holding asset, we can easily observe, how sensitive user costs are for changes in individual parameters. All cases described are for ceteris paribus situation. With one percentage point move of additional risk compensation, maintenance cost or expected capital gain (loss), user cost move together with this change for one percentage point as well. For the first two parameters move is in the same direction, for the third one in opposite one. For the case of changes in mortgage rate as well as property tax rate, the change in of user cost moves in the same direction, but in proportionally smaller amount depending on income tax rate. Higher income taxes cause smaller impact on

user cost and vice versa. And finally income taxes itself, changes go in opposite direction but the magnitude is decided by sum of property tax and mortgage rate. As it is usual for these two parameters to be relatively low, the proportion of change cause by income tax change is generally low as well. On the other hand it is also true that income taxes could change significantly in time, even two digit percent differences, which makes it not negligible part of user cost calculation.

In matter of user cost sensitivity topic, we should also emphasize that this sensitivity to fundamental changes relatively grows for situations with relatively low user costs. For example, one percentage point growth in interest rate cause one percentage point growth in user cost, which could be 33% growth if original user costs were 3 % or alternatively 20% growth if original user costs were 5 %. This might not seem on first sight that crucial, but let us remind that user cost also could be used as discount factor for perpetuity determining price level of property or simply price to income ratio determined by user cost. Following chart demonstrate this relation between user cost size and its impact on property price level if changed.
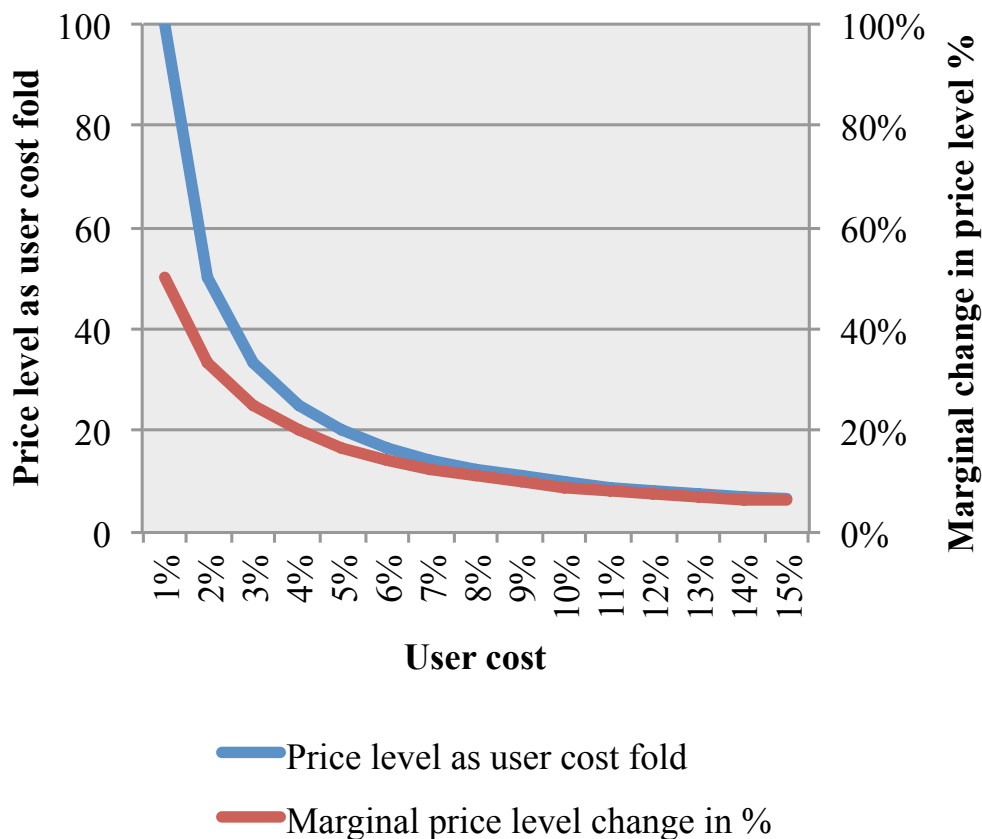

Figure 3.1: User cost and price level relation

For example if original user costs were 1 % and they grow on 2 %, property price level is supposed to lose 50 % of its value from 100 fold of user costs to only 50 fold (fall of price to rent ratio from 100 to 50). Of course that this is extreme example, but this monumental price level drop was caused only by a little higher than one percentage point growth in interest rates (mortgage rate) or alternatively one percentage point decline of expected capital gain from real estate asset, which are definitely not that extreme changes.

This property of user cost sensitivity for fundamental factors changes makes extremely interesting situations, with relatively low levels of user costs. As additional risk premium and property taxes could be considered as relatively low. Income taxes and maintenance costs are not the highest components as well, but more importantly we could consider them as relatively stable in time. That leaves us with two fundamentals of high interest. Especially for situations with low interest rates and high expected capital gains from real estate assets. These situations pushes on lowering user costs and thus creates environment of high sensitivity of real estate prices on changes in fundamental factors via user costs.

3.2.1   INTEREST RATES

The real interest rates are one of two key factors as already mentioned. But it is also important to stress out to keep in mind, that its importance holds only for real rates. It is because the inflation part of nominal rate is offset by presence of the very same inflation in expected capital gain determinant, that goes in opposite direction against interest rates. Real interest rates thus represent capital costs for the property owner. Lower rates reduce overall user cost because of making debt financing cheaper and opportunity costs for investing to real estate is lower as well.

There is still one question which interest rates best suit for our purpose. Poterba (1984) employs mortgage rate, as more practical explanation of capital cost is preferred. This suggests that price of capital for real estate property investment is best estimated by direct mortgage rate. On the other hand Himmelberg, Mayer and Sinai (2005) argue for favor of long-term interest rates rather than short-term ones[8]. Firstly

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[8] Of course we mean here mortgage rates that are derived from short-term rates. For Czech republic generally in recent years holds that average mortgage rate index is approximately 2 percent points over real short-term rates. Source: cnb.cz, hypoindex.cz, own calculations.

they look at this determinant more in way of opportunity costs for invested capital. So they prefer risk-free rate instead, actually long-term interest rates to be accurate, as there is important to consider practical impact of expected future real interest rates on expected appreciation rate of future real estate prices. This argument could be demonstrated on example, when real short-term interest rates are well below real long-term interest rates, bond market then anticipate real short-term interest rates to rise in the future. As real-short-term rates are expected to grow, user costs are expected to grow as well and thus lower future real estate prices. That will be projected to expected capital gain (loss) factor by lowering these expectations. Decline in future expectations should roughly offset the real long-term and real short-term rates spread. So it should be indifferent if using which rates we use if taken in consideration influence on future expectations. On the other hand we could argue, that spread between real long-term and real short-term rates might in some special cases prevail for a very long time and what is even more important, some gap will, because of basic economic logic about time risk premium, remain unclosed forever. Another interesting fact about decline in real short-term interest rates, which influence mortgage rates most, is that it does not only make debt financing cheaper and thus more affordable, but also cause growth of maximal debt limit borrower could reach. We demonstrate this on example, where household have gross income EUR 2,000 per month. That makes EUR 24,000 per year and if we estimate the solvency clarification process simply by multiplying their income by ratio 0.5 we obtain maximal possible installments EUR 12,000 per year. Now if we derive from that fix installments maximal debt limit as dependent on mortgage rate, we conclude significant growth. Let say, when mortgage rate is 4.5 %, maximal debt limit for such household is little over EUR 156,000, but if mortgage rate drop to 3.5 %, maximal debt exceed EUR 170,500 limit standing for 9.3 % growth. Another effect of this limit shift is that some originally insolvent applicants become solvent now. This effect of real short-term rate definitely should have important influence on property price level. But it is also very important to care about stability of this effect. Because it works more like a swing effect multiplying overall effect of real short-term rates change in both directions. Because if rates grow again, limit drops, but more importantly some of borrowers could become insolvent and thus quit repaying their debt. Foreclosed properties would boost supply and lead another force to market price level contraction. At the end, this effect might support choice of both rates. Long-term one in case of unstably low short-term interest rates situations, where it is expected return to higher and more stable level and this correction might result to significant price level drop. Or in case of possibly stable real short-term interest rates,

when forces on price level resulting from short-term rates are more decisive than the long-term ones and then it is more accurate to employ those short-term rates.
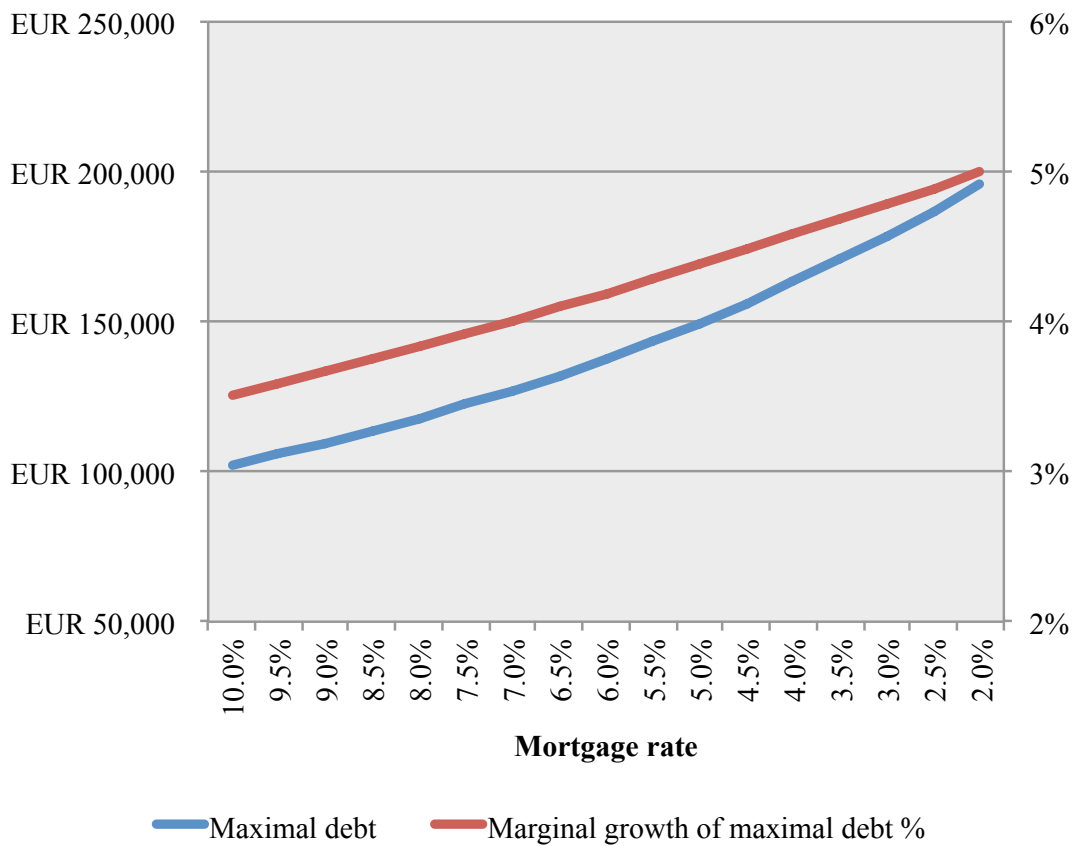

Figure 3.2: Mortgage rate and maximal debt limit

Source: author's computations.

We conclude here, that appropriate choice of interest rates depends mainly on their stability and spread. If the spread is high, we could expect short-term rates to be less stable and because of reasons mentioned above using short-term rate will provide us with overpriced estimations. On the other hand using long-term interest rate could underestimate effect of more affordable debt financing for cases of small and stable spread.

Appropriate choice of interest rate for calculating of user cost is especially crucial for situations with low interest rates, as we are currently experiencing. We decided then to use weighted average of long-term interest rates and current market mortgage rates index, with growing weight of long-term rates for cases with higher spread.

3.2.2   EXPECTED CAPITAL GAIN

The second key factor, influencing user cost most, is expected capital gain. Combination of its high influence on overall user cost and the fact of uncertainty included in future expectations dimension makes this fundamental central point of debate when constructing user cost estimation for particular markets. We should keep in mind that expected changes of all fundamental factors should influence expectations about future price of real estate asset and thus is incorporated already before expected change takes place. Another important aspect about this fundamental factor is that expectations construction subject to some basic psychology aspects such as memory effect or negative news asymmetry impact.

Another interesting thing is, if we follow basic economic theory, appreciation expectations should be especially high in locations with inelastic supply of real estates. This relation is also empirically proved for example by Himmelberg, Mayer and Sinai (2005). This is very common in cities or other locations with some concentration factor, where inelasticity is mainly the product of bounded land supply and construction regulations. These locations are highly interesting for us because of these supply inelasticity supported high expectations about future appreciation pushes user cost down and makes the market price level more sensitive to fundamental changes. Completely other story is on the market, where supply is highly elastic. On such market, mechanism of user cost needs to introduce other important fundamental factor, which is construction costs. Then two general scenarios could emerge. First, price to user cost ratio suggest lower price level than construction costs allows, so the assumption about elastic supply will not be employed in practice, because no one should pay price for newly constructed properties. So this scenario is generally similar in other aspects to situation with inelastic supply. Second and more interesting scenario is, when price to user cost ratio suggest higher price level than construction costs and if the market is sufficiently competitive thus arisen difference is not exploited, construction costs should be predominant price level setting factor. On such market, no one has to pay purchase price that price to user cost ratio suggest. In this kind of situation, user cost approach fail to assess rational price level correctly. It is permanently signaling underpriced situation, but market does not converge to equality state as construction costs does not let this happened. But we do not have to care about this shortcoming much, because these situations are in practice very rare, definitely when it comes to heavily populated locations such as big cities, which is the case of our empirical testing.

3.2.3 OTHER FUNDAMETALS

Let us start wit depreciation rate or maintenance costs. Depends on point of view, because we could perceive this fundamental as both as value lost because of using the asset in time or as costs we need to pay for keeping the asset in 100 % condition state. In practice, is most often both explanations combination observable. If these costs are high, it causes real estate to lose its value quickly and makes holding asset more costly in time. And if rising, results in renting relatively cheaper to owning. This factor is perceived to be relatively stable in time and its annual value, is by Harding, Rosenthal and Sirmans (2004) estimated as 2.5 %. We will accommodate this estimate with several adjustments for our analysis as well.

Next factor is income tax rate. Its growth lower user cost and makes owning relatively less expensive than renting. This is caused because of owner-occupied real estate provide higher tax deductions.

Property taxes are simply taxes imposed on the ownership of the property for owning the asset itself. So it is cost added to user costs and as property taxes rise, overall user cost rise as well. This makes owning of property more costly and if other remains unchanged, makes renting cheaper than owning. The little confusing about this fundamental is that these taxes are not only different internationally, but very often also regionally, for type and usage purpose of real estate property etc.

Last fundamental factor for user costs is additional risk premium. This is again very straightforwardly cost factor for capital invested in asset and thus its growth makes user cost to grow as well as investor demand to be compensated more for risk included in such investment. According to Flavin and Yamashita (2002) this additional risk premium is calculated as 2 %. However this estimate seems a bit too high. Real estate market, if considered long run horizon, is not most risky asset as well as we could look at property owning as kind of insurance against future undesirable changes in renting price levels.

4   EMPIRICAL MODELING OF REAL ESTATE PRICE LEVELS

In this part we are going to empirically model price levels of purchase and renting on Prague real estate market. More precisely in medium flat segment excluding new built structures. We need those two price levels to be able to assess whether is ongoing bubble on Prague real estate market. As we already mentioned, significant advantage of estimation on our own is the fact, that models should provide very precise price levels for detailed selection of parameters describing the property and also that these parameters will be same for both price levels and thus will provide consistent relation between those two levels.

4.1   DATA

For purposes of estimation price of purchase and renting by hedonic models, I use as data source public website sreality.cz with the widest range of offers for both, purchases as well as renting real estate properties in Czech Republic. On this server are very easily accessible data for every single offered real estate property as price of purchase or alternatively price of rent, floor size, GPS location, condition of property, if garage or other services are included in the offer etc. These are the data we will employ in our models, but there is also one inconvenience we should not forgot about. These data are mainly representing supply side of the market. Some advertisement offers could not be exercised for very long time as they are overpriced, some of them even never. The general estimation by CZSO is that this difference moves in time around 5 – 10 percent[9]. But as it is very problematic to obtain realized data and the main aim of this work is not price estimation itself, but its relation with price of renting, which is probably dealing with the very same, and user costs that are linked to estimated price as well, it is not a matter of great concern for us. For strengthen the consistency of estimated models I decided to put some restrictions on data selection. These restrictions are imposed in order to keep as much heterogeneity

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[9] http://www.czso.cz/csu/redakce.nsf/i/ceny_bytu

as possible only within model variables, but exclude all other factors that are not covered in model or are against main model logic. These restrictions on offer advertisements are:

- No older than 7 days from date of data gathering – 09.05.2013 – for data time consistency.

- Only offers located in Prague, for flat segment with floor size from 50 to 80 squared meters in inhabitable condition. Because of property type consistency.

- We exclude new buildings, built in 2012 and later.

- We include only personal owned properties to leave out ownership type influence.

- Rents offered only for long term rent possibility. Short-term accommodation has totally different price level and aspects influencing price of rent, so we strictly exclude those offers.

At the end, after data trim and preparation, there left 172 cases for price of purchase estimation model and 380 cases for rent price estimation. Now, I will describe individual explanatory variables, as they are with one exception common for both models.

4.1.1 CONDITION

This variable captures quality condition of property as whole. Mainly the state of wear rate as well as quality of equipment with special importance paid to kitchen and bathroom. This variable is graded in percent scale with following levels:

- 100 % means after overall reconstruction with well looking kitchen and bathroom and none or almost none wear rate.

- 75 % is still pretty good shape, well looking kitchen and bathroom with some noticeable rate of wear.

- 50 % inhabitable, but with considerable rate of wear and prevailing older equipment.

- 25 % properties with these grading are those with really high rate of wear and generally needs some kind of reconstruction or other construction treatment.

4.1.2   DISTANCE

This variable is, as its name suggests, distance measure from property location to city center indicated by Old Town Square chosen as historical, geographical and cultural central point of the Prague city. Logic of this variable is based on Von Thunen property location theory that concludes two important features we employ:

- Land and property values tend to decrease with increasing distance from city center due to growing transportation costs.

- The land value is one of concentric zones pattern from the city center.

For measuring distance itself I employ great-circle distance method. This method measures shortest distance between two points on a sphere based on GPS location. For data itself it means that firstly we have to convert GPS coordinates to decimal degrees, than calculate angle c in radians between two points using formula:

$$c = arccos * (sin(latitude\ A) * sin(latitude\ B) + cos(latitude\ A) * cos(latitude\ B) * cos\ (|longtitude\ A - longtitude\ B|)$$

To obtain final points distance we need to multiply angle c with sphere radius. In our case is sphere Earth, which is actually due to its rotation oblate ellipsoid with radius from 6,357 km on poles to 6,378 km on equators. Around latitude 50 degrees where Prague lies, radius should be approximately around 6,369 km and this is also number we use to multiply angle c. But the precise radius of Earth is not that much important for us anyway, because for us is important difference between measures for individual cases and for that great-circle formula provide us very good estimation. For models itself we use distances in meters.

4.1.3   FLOOR SIZE AND DUMMY VARIABLES

Rest of explanatory variables is very easy and straightforward. Floor size stands for internal floor size of the property and is expressed in squared meters. All other

variables are dummy type, that means they signal whether individual property has this characteristics or not. These dummy variables are:

- Garage – signaling if the garage place is part of the property, which could have significant positive impact on total price as garages have their own value itself.

- Prefabricated structure – shows if the building in which is individual property is build from prefabricated panels parts. Other buildings are mostly built from bricks or other materials. Generally is viewed at panel structures as less quality buildings with shorter life expectancy. So it should have negative effect on total value.

- Ground level – this variable stands for the ground level or first lower floor level properties. Flats on these lowest floor levels provide less intimacy and provide generally provide almost none view. Also often there is less sun-time during the day and other factors as more often higher humidity etc. These are the reasons why properties with this characteristic should have slightly lower price or rent than other equivalents on higher floor levels.

- Furniture – this last dummy variable is employed for rent price model only. We expect positive effect of such variable on price of renting property, as owner has to be compensated for equipment usage

4.1.4   PRICE OF PURCHASE

Price of purchase is dependent variable in our first type model. It is generally the price demanded for selling of the ownership rights for real estate property. But there were two important adjustments that had to be done. Firstly, some of the prices were listed with and some without real estate agency fees. For those prices with real estate agency fees we subtracted them as estimated 5 % from listed price. Second adjustment is because prices distribution has naturally right skewness. That is because prices could not be negative on the left side, but on the other side, there is generally no limit. This characteristic needs to be dealt with because of using ordinary least square estimation. This skewness might and actually cause heteroskedasticity to model residuals, which is unpleasant. Natural logarithm of the variable could help us significantly as it modifies data in wanted direction. Logarithm

regression cause loss of simple and straightforward results explanation, but that is reasonable price for more precise estimations.

4.1.5   PRICE OF RENT

Price of rent is dependent variable in our second type model. This is the very same case as price of purchase, it is the price listed in offers as price for renting the property. For easier application in further analysis we multiply rent values that are listed as month figure by 12 to obtain rent price for a year. The other adjustments are the same as for purchase price. Some of the prices were listed including with fees for energy and other service payments. Luckily in almost all cases, where rent prices were not listed alone, information about value of these fees was included in description. So it was easy to deduct them directly. And again as for purchase price, distribution of rents has right skewness property. Natural logarithm again should solve this problem for us.

4.2 MODELS

In this chapter we describe the methodology that lies behind modeling three main pillars that we are going to compare at last. A estimation approach for market price of purchase level is the same as for market price of renting level and this is linear and loglinear regression by ordinary least square method. For user cost we are going to built formula that consider and process important macroeconomic fundamentals influencing costs and benefits resulting from real estate property ownership.

4.2.1   ASSUMPTIONS

As already mentioned, we are going to apply linear and loglinear regression models using ordinary least square method allowing us to identify decisive factors influencing real estate price levels and finding out their coefficients that we will be

able to employ later in next steps. But first to apply ordinary least square method correctly, following assumptions needs to be fulfilled:

I.   Model is *linear in parameters*

II.   The data are a *random sample* of the population. The *errors are statistically independent* from one another

III.   The expected value of the errors is always *zero*

IV.   The independent variables are measured *precisely*

V.   The independent variables are not too strongly *collinear*

VI.   The residuals have *constant variance*

VII.   The errors are *normally distributed*

First and third assumptions are fulfilled from model execution. With second assumption, for selected segment of data, we generally used all cases available, so we cover all population. Also other part of second assumption is no concern for us as we estimate cross-sectional data. I already mentioned problem with fourth assumption (supply data), but unfortunately there is nothing to do about it and also for the cause we are going to need models results, it is not a big deal. From fifth to seventh assumptions we will test ad hoc for each model.

4.2.2   PRICE MODEL 1

First model is defined by following equation:

$$priceCZK = \alpha + \beta_1\ condition + \beta_2\ distance\_m + \beta_3\ floor\_size\_m2$$
$$+\ \beta_4\ garage\_D + \beta_5\ prefabricated\_structure\_D$$
$$+\ \beta_6\ ground\_level\_D + u$$

Below we could observe results for model 1. We have obtained very high R-squared over 68 %. This high number indicates that this model covers great deal of variability of dependent variable. Also p-value of ANOVA testing suggests that model covers variability well. We also observe significance on 99% significance

level for all coefficients excluding coefficient for variable garage and ground level, which is the only insignificant variable in the model. Interesting is negative value for constant, but when we take closer look at the results, we could observe that the role of constant for model appropriated dummy variables with variable describing condition of property. The coefficient for the last named variable suggests over 13,000 CZK change in total price for every one percent point of condition score. That makes more than 1.3 million CZK constant for property with 100 % condition. For distance variable resulting coefficient inform us, that every meter step from Old Town Square will cost us almost 76 CZK on the total price. Price for every square meter also seems very reasonable, as 42,290 CZK more or less corresponds with current market price. Price for garage place is approximately 225,000 CZK. Discount for prefabricated panel building is more than half of million and discount for ground floor almost ninety thousands. In fact, for all variables there is direction of slope as we expected and also price tags seem very reasonably as well. But we should check the assumptions as we might have a problem with skewness of explained variable.

Model 1: OLS, using observations 1-172

Dependent variable: priceCZK

Table 4.1: Model's 1 results

	Coefficient	Std. Error	t-ratio	p-value	
const	-279546	320285	-0.8728	0.38404	
condition	13815.8	2002.27	6.9001	<0.00001	***
distance_m	-75.811	15.0743	-5.0292	<0.00001	***
floor_size_m2	42290.8	3625.2	11.6658	<0.00001	***
garage_D	225175	131535	1.7119	0.08879	*
prefabricated_structure_D	-575217	83534.1	-6.8860	<0.00001	***
ground_level_D	-87095.8	85437.4	-1.0194	0.30950	

Mean dependent var	2838313	S.D. dependent var	737385.7
Sum squared resid	2.90e+13	S.E. of regression	419000.1

R-squared	0.688451	Adjusted R-squared	0.677122
F(6, 165)	60.76857	P-value(F)	2.78e-39
Log-likelihood	-2467.132	Akaike criterion	4948.264
Schwarz criterion	4970.296	Hannan-Quinn	4957.203

Source: author's computations, Gretl SW

As we tested assumptions number 5 to 7. Results are unfortunately as we expected negative. Both tests for heteroskedasticity, White's test (A) as well as Breusch-Pagan (B) gives us information, that null hypothesis of homoskedasticity is rejected. Also null hypothesis of normally distributed residuals is strongly rejected (C). Only collinearity test (D) gives us positive information of none collinearity presence. Heteroskedasticity and not normally distributed residuals could be problem for estimated coefficients precision as well for their validity. We need to solve this situation before we use estimated model for determination of price of purchase level.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 41.4258

with p-value = P(Chi-square(23) > 41.4258) = 0.0105818

(B)

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 16.0314

with p-value = P(Chi-square(6) > 16.0314) = 0.0135866

(C)

Test for normality of residual −

Null hypothesis: error is normally distributed

Test statistic: Chi-square(2) = 11.2207

with p-value = 0.00365973

(D)

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition   1.127

Distance_m   1.543

Floor_area_m2   1.103

Garage_D   1.100

Prefabricated_structure_D   1.676

Ground_level_D   1.083

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.3   PRICE MODEL 2

Second model is very similar to the first one. In Fact it is defined with the same equation with only one slight difference, on explained variable. As almost all assumptions tested for previous model were violated, we need to pay attention to that topic. Residuals suffered from unstable variance and non-normal, right skewed, distribution. We try to repair these shortcomings by introducing natural logarithm of explained variable. So the equation that holds this time is following:

$$ln\_priceCZK = \alpha + \beta_1\, condition + \beta_2\, distance\_m + \beta_3\, floor\_size\_m2$$
$$+ \beta_4\, garage\_D + \beta_5\, prefabricated\_structure\_D$$
$$+ \beta_6\, ground\_level\_D + u$$

Again, in below table, we obtained results of OLS method for equation (3). We have even higher R-squared values with adjusted over 70 %. This number suggests that we have explained a great deal in modeled variance. ANOVA test result also provides us with positive information about model ability to capture variability of dependent variable. Even all variables strengthen their significance values including garage variable, which is now significant on higher significance level, and ground level variable, that remained insignificant, but came closer to significance threshold.

In comparison to first model, there is one important difference that this time constant regained its natural position of relatively high positive base number. That happened mainly due to rescaling of explained variable. All other coefficients roughly remained same in direction as well as their relative size. This time is for us more important if the model accomplished to stick to assumptions better than the one before and thus we could declare its results consistent.

Model 2: OLS, using observations 1-172

Dependent variable: ln_priceCZK

Table 4.2: Model's 2 results

	Coefficient	Std. Error	t-ratio	p-value	
const	13.7727	0.102338	134.5814	<0.00001	***
condition	0.00478237	0.000639766	7.4752	<0.00001	***
Distance_m	-2.63842e-05	4.81655e-06	-5.4778	<0.00001	***
floor_size_m2	0.0142644	0.00115833	12.3146	<0.00001	***
garage_D	0.0871041	0.042028	2.0725	0.03977	**
prefabricated_structure_D	-0.194845	0.0266909	-7.3000	<0.00001	***
ground_level_D	-0.0400001	0.0272909	-1.4653	0.14474	

Mean dependent var	14.82740		S.D. dependent var	0.247876
Sum squared resid	2.957400		S.E. of regression	0.133879
R-squared	0.718523		Adjusted R-squared	0.708287
F(6, 165)	70.19880		P-value(F)	6.98e-43
Log-likelihood	105.3764		Akaike criterion	-196.7528
Schwarz criterion	-174.7203		Hannan-Quinn	-187.8137

Source: author's computations, Gretl SW

We tests assumptions 5-7 about collinearity, constant error variance and normal distribution of errors. First assumption tested by (D) was not problem in previous model and is not a problem now as well. According to two executed tests for assessing constant variance in errors, White's test (A) as well as Breusch-Pagan (B), we could not reject null hypotheses of homoscedasticity. That is definitively desirable change against model 1. Also last assumption about normally distributed errors holds. Test (C) cannot reject null hypothesis about normality. All the tests came out as we desired, so we ended up with estimated coefficient that are consistent with used method of estimation and thus we could employ them in further analysis.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 29.0815

with p-value = P(Chi-square(23) > 29.0815) = 0.177622

(B)

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 8.24299

with p-value = P(Chi-square(6) > 8.24299) = 0.220836

(C)

Test for normality of residual -

Null hypothesis: error is normally distributed

Test statistic: Chi-square(2) = 3.25606

with p-value = 0.196316

(D)

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition   1.127

Distance_m   1.543

Floor_area_m2   1.103

Garage_D   1.100

Prefabricated_structure_D   1.676

Ground_level_D   1.083

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.4   RENT MODEL 3

With this model we will try to gain instrument for price level of renting modeling. Model 3 is defined by following equation:

$$rent/YCZK = \alpha + \beta_1 \, condition + \beta_2 \, distance\_m + \beta_3 \, floor\_size\_m2$$
$$+ \beta_4 \, garage\_D + \beta_5 \, furniture\_D$$
$$+ \beta_6 \, prefabricated\_structure\_D + \beta_7 \, ground\_level\_D + u$$

The results are listed in table below. R-squared value suggests that this model explains less variability of dependent variable than previous models, but in general it is still very good model with over 55 % of variance explained. Loss of some variance explanation power could be caused by the fact, that landlords could have slightly different tactics about pricing in case of rent and fees separation. They in some cases input some rent fraction to fees, in order to make the rent price more competing and others just announcing rent price as whole, fees included, with no need to bias rent and fees values. But ANOVA p-value also provides us with positive information about model competence. Most of the explanatory variables show themselves to be significant on highest level, which is also positive information.

Similarly to model 1, it is a bit surprising that constant is negative, but there is the same explanation, as before. Constant natural position in model took over dummy variables and condition variable. Condition coefficient concludes that for every percentage point in condition grade score tenant should pay almost 900 CZK per year, this makes 100 % condition property base of 89,490 CZK rent per year. Distance from city center is expressed with relation of rent decline around 3,300 CZK for every kilometer in year terms. Another coefficient suggests that for every rented square meter we should be yearly paying extra 1,239 CZK. Furniture and garage variable indicate that for extra services such as renting furniture or a garage parking place will approximately cost you extra 1,000 CZK, respectively 2,000 CZK per month. For renting a flat in panel prefabricated building or on ground floor level we should get discount as expected, but for those two variables, influence is not significant. Again as in model 1, we obtain reasonable results with directions as expected, but we still need to test assumptions whether the model is consistent with estimation method employed.

Model 3: OLS, using observations 1-380

Dependent variable: Rent/YCZK

Table 4.3: Model's 3 results

	Coefficient	Std. Error	t-ratio	p-value	
const	-7344.39	9493.8	-0.7736	0.43966	
condition	894.908	66.9954	13.3578	<0.00001	***
distance_m	-	0.44940	-7.3617	<0.0000	***

	3.30842	9		1	
floor_size_m2	1239.26	111.711	11.0934	<0.00001	***
garage_D	25681.2	6060.08	4.2378	0.00003	***
furniture_D	13442.5	2115.99	6.3528	<0.00001	***
prefabricated_structure_D	-4106.85	3200.28	-1.2833	0.20019	
ground_level_D	-158.414	3047.24	-0.0520	0.95857	

Mean dependent var	134713.7	S.D. dependent var	30313.22
Sum squared resid	1.49e+11	S.E. of regression	20012.97
R-squared	0.572178	Adjusted R-squared	0.564128
F(7, 372)	71.07442	P-value(F)	9.45e-65
Log-likelihood	-4298.726	Akaike criterion	8613.451
Schwarz criterion	8644.973	Hannan-Quinn	8625.959

Source: author's computations, Gretl SW

If we observe results for assumptions tests, the only one we have no problem with is collinearity testing one (D). Both test (A) and (B) strongly rejects null hypothesis of heteroskedasticity not present. Also test for normally distributed residuals strongly rejects this assumption and we have to deal with those violations in following model for the second time.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 47.1748

with p-value = P(Chi-square(30) > 47.1748) = 0.023923

(B)

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 24.9642

with p-value = P(Chi-square(7) > 24.9642) = 0.000769954

(C)

Test for normality of residual -

Null hypothesis: error is normally distributed

Test statistic: Chi-square(2) = 13.0186

with p-value = 0.00148955

(D)

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition   1.090

Distance_m   1.337

Floor_area_m2   1.077

Garage_D   1.066

Furniture_D   1.037

Prefabricated_structure_D   1.476

Ground_level_D   1.041

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.5   RENT MODEL 4

In this model we are trying to get rid of assumptions violations by using loglinear regression. Model 4 is thus defined by following equation:

$ln\_rent/YCZK = \alpha + \beta_1\ condition + \beta_2\ distance\_m + \beta_3\ floor\_size\_m2 + \beta_4\ garage\_D + \beta_5\ furniture\_D + \beta_6\ prefabricated\_structure\_D + \beta_7\ ground\_level\_D + u$

The results of OLS method for equation above are listed in following table. We have even higher R-squared values with adjusted over 56 %. This number suggests that we have explained over half of modeled variance. ANOVA test result is also positive as it suggests that model has high ability to capture variability of dependent variable. Even all variables strengthen their significance values including prefabricated structure and ground level variables, which remained insignificant, but came closer to significance threshold. But it is needed to say, that the second named variable remained very strongly insignificant.

For case of constant, there is very similar development as for the first adjustment from the model 1 to model 2, where within loglinear regression model intercept regained its natural position of relatively high positive base number. That again happened mainly due to rescaling of explained variable. In comparison with model 3, all other coefficients roughly remained the same in direction as well as their relative size. To the comparison of results for model 2 and model 4 we will pay more attention in next chapter. Now more importantly for us we need to check assumptions if the model is consistent this time.

Model 4: OLS, using observations 1-380

Dependent variable: ln_rent/YCZK

Table 4.4: Model's 4 results

	Coefficient	Std. Error	t-ratio	p-value	
const	10.7248	0.068225	157.1972	<0.00001	***
condition	0.00665246	0.000481447	13.8177	<0.00001	***
distance_m	-2.28193e-05	3.22957e-06	-7.0657	<0.00001	***
floor_size_m2	0.00924848	0.000802782	11.5205	<0.00001	***

garage_D	0.154413	0.0435494	3.5457	0.00044	***
furniture_D	0.0915367	0.0152061	6.0198	<0.00001	***
prefabricated_structure_D	-0.0311787	0.0229981	-1.3557	0.17601	
ground_level_D	0.00632776	0.0218983	0.2890	0.77277	

Mean dependent var	11.78689	S.D. dependent var	0.218032
Sum squared resid	7.694363	S.E. of regression	0.143819
R-squared	0.572936	Adjusted R-squared	0.564900
F(7, 372)	71.29485	P-value(F)	6.82e-65
Log-likelihood	201.7432	Akaike criterion	-387.4864
Schwarz criterion	-355.9650	Hannan-Quinn	-374.9786

Source: author's computations, Gretl SW

The tested assumption (D) about linear independence of explanatory variable is fulfilled as in all other models. We mainly care about other two assumptions and their tests. The assumption about constant variance is tested by White's (A) and Breusch-Pagan (B) tests. This time we could not reject null hypotheses in both tests by far. These results suggest residuals to be heteroskedastic. In last test (C) we also could not reject null hypothesis about normality of residuals and that result concludes that for this model are all assumptions fulfiled as we desired.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 26.9989

with p-value = P(Chi-square(30) > 26.9989) = 0.62333

(B)

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 6.67566

with p-value = P(Chi-square(7) > 6.67566) = 0.463413

(C)

Test for normality of residual -

Null hypothesis: error is normally distributed

Test statistic: Chi-square(2) = 1.08284

with p-value = 0.581922

(D)

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition   1.090

Distance_m   1.337

Floor_area_m2   1.077

Garage_D   1.066

Furniture_D   1.037

Prefabricated_structure_D   1.476

Ground_level_D   1.041

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.6   MODELS COMPARISON

Now we are about to compare differences between models 2 and 4, as we count with employing these models for estimating price levels of chosen real estate segments where we want to find out if there is possibly forming price bubble. These differences could have important influence on distinction and explanation of differences of total price levels estimated.

Starting with most important difference between models, we compare prefabricated structure variable as it was highly significant in price model, but it is not in rent model. This difference suggests, that the fact if the flat is located in panel prefabricate structure has negative impact if we are purchasing this property, but it has no significant effect if we are renting it. This could be explained by shorter life expectancy of the panel structures and thus introducing additional risk in case of ownership, owner wants to be compensated for. But in case of renting, tenant is not bearing such risk of shorter life expectancy, she just could move away if such situation happens so there is no need to be compensated for this risk. Similar situation is in case of garage variable where changed significance level in favor of higher importance is for renting case. This is a bit more difficult difference to explain. Probably it would be caused by the fact that renting garage is in some way luxury service and thus it stands for luxury aspect of property and is price as one. But when we are purchasing property with garage, this decision stands before more rational consideration if we are really willing to pay the sum for this feature. Panel prefabricated structure and garage where two main differences in pricing models, but also other factors have relatively different impact on price level estimation in case of purchase and renting, as condition of property has relatively higher impact in case of renting than purchasing, this could be explained with the fact that in case of owning the property its reconstruction is possible in most cases for relatively smaller fraction of purchase price so it has relatively smaller importance than in case of renting, when it is practically unthinkable to reconstruct the premises as we are not the owners. Also in case of renting the condition state is more important for us, because we will be probably living in the rented property as long run re-renting of noncommercial properties is very scares, because there is very small value added from agent side giving only small space for service premium. But in case of purchase, we could alternatively rent the property to tenant, makes the condition state personally less important for us.

In the case of distance, the relative impact on price is more or less the same, which is interesting, because one could expect that there might be in different locations different expectations about capital gains from owning the property and thus different compensation for owning the property reflecting in rent, but this effect could be offset by relatively higher rents to purchase price on high distance location, where is expected capital gain from property lower as there is more free space for new constructions. Also this approximately equality is consistent with Von Thunen location theory about price differentiation in distance due to transport costs, that are the same for owning or renting case. The last variable to compare is floor size in squared meters. This variable is more relevant for purchase pricing case. Again the explanation will be in logic similar to prefabricated panel structure but vice versa case. If we are renting floor size, our motivation is only usage of this space and we also needs to pay for whole floor size no matter if we utilize it or not, so more floor space than needed is undesirable. On the other hand in purchase case we could expect additional capital gains from more floor size. From that point of view, all floor size is utilized at least for that one reason for the owner. So more floor size is desirable for the owner.

5   EMPIRICAL ANALYSIS

Now as we have estimated models for price of purchase and price of renting levels. And we also have final formula for calculating user costs. We could finally employ these procedures to estimate values of those three pillars in order of comparing them and assess whether market is in equilibrium or is not.

We will process estimations, calculations and comparisons for three different Prague real estate market segments, which we could cover them with our models:

A. "standard quality periphery flat"

B. "low quality, medium city small flat"

C. "high quality inner city flat"

Fundamentals for user cost would be same for all segments with two exceptions. First is depreciation rate or maintenance cost, which is set as 2.5 %[10], but for prefabricated panel structures is this cost 0.5 % higher as we consider shorter life expectancy. Second is expected appreciation level. We are going to apply most recent data from CZSO about Prague flat prices development[11]. These data suggest average annual appreciation from 2nd quarter 2005 to 2nd quarter 2012 to be 4.8 %. This number consist of average 2.6 %[12] inflation in the same time interval and 2.2 % left as real annual appreciation. But we should include psychology factor as recent information has bigger impact on expectations than historical ones. We achieve this by employing weighted average where every one period older information loss 10 % of its importance for overall expectations. As we have 7 observations, this is given by equation:

$$E[g_{t+1}] = \frac{\sum_{i=0}^{7} 0.9^i (g_{t-i})}{\sum_{i=0}^{7} 0.9^i}$$

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[10] Harding, Rosenthal and Sirmans (2004) estimation.

[11] http://www.czso.cz/csu/redakce.nsf/i/ceny_bytu

[12] http://www.czso.cz/csu/redakce.nsf/i/mira_inflace

Now we obtain expected appreciation 3.29 %, which is much more realistic as market is experiencing stagnation or even prices decline for last four years. We did not care about negative news asymmetry impact, because it is due to time-series development naturally included in weighted average formula, as bad news years are the last four with highest weight.

Then we try to take in consideration individual differences between segments, so we add 1 % premium appreciation for high quality city center segment, because the supply should be strongly inelastic for them. On the other hand we will subtract 1 % for standard quality periphery segment, as they have to compete to new higher quality developers projects in their area. Last segment we leave as it is with no premium or discount.

Additional risk premium we are going to set at 1.0 % level, as we lowered original Flavin and Yamashita (2002) estimation for reason of considering owning real estate property as insurance against undesirable growth in rent prices. Income tax is set according to current personal income tax in Czech Republic, which is flat tax of 15 %[13]. Another taxing fundamental factor of property tax would be the same for all segments, but in numerical value. Because this tax is for personal housing real estate properties calculated as multiple of regional coefficient, size of city coefficient, tax coefficient, tax tariff per square meter and floor size of property in squared meters[14]. As all coefficients are the same for all segments, differences will be observable only due to floor size. Also we wish to express this tax as fraction of overall price per year, se we need to calculate it when we are going to know the price levels for individual segments.

Last fundamental is interest rate. As we already discussed, probably most suitable would be weighted average of long-term rate and current mortgage rate, where weights depends on spread between those two rates. Weights should move in favor of long-term rate as spread grows and short-term rates become unstable. In our case, as we calculate this average on only one date, it should be sufficient arithmetic

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[13] Under tax law n. 586/1992 modified 1.1.2010

[14] http://www2012.mfcr.cz/cps/rde/xchg/mfcr/xsl/dc2_dane.html

average of these two rates. According to Fincentrum Hypoindex[15] average mortgage rate in 1st quarter 2013 was 3.2 %. For long-term rate we put 3.875 % as it is coupon on 10 year Eurobond 6th emission in February 2012[16]. Average is then equal to 3.54 %.

For final assessment whether there is considerable inequality on the market segment, we calculated approximately 90% confidence interval boarders by adding and subtracting 1.5 of rents standard deviation. These standard deviations were calculated by standard procedure for around 40 most similar segment type cases in dataset for rents modeling. We consider this confidence interval as signaling of price inequality if user cost is outside it.

5.1 STANDARD QUALITY PERIPHERY FLAT

This is the type representing most common segment of real estate housing property in Prague. It is defined as medium size – 68 m2, medium quality – 75 % condition state, in high distance from city center – 10.000 meters, this distance roughly includes huge residence settlements as Háje, Letňany, Zličín, Černý Most and similar, where lives highest share of Prague population. We also set this type of building as prefabricated panel structure without garage and furniture.

Table 5.1: Type's A results

Real Estate Flat Price & Rent Level Calculator - 9.05.2013	
data	Inputs
CONDITION	75%
FLOOR (50-80 m2)	68
DISTANCE (metres)	10000
GARAGE (dummy)	0
FURNITURE (dummy)	0
PREFABRICATED STRUCTURE (dummy)	1
GROUND LEVEL (dummy)	0
	Results

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[15] Weighted average of personal mortgage rates during current month. Weights are total debt volumes for individual banks included to calculation.

[16] http://www2012.mfcr.cz/cps/rde/xchg/mfcr/xsl/podle_splatnosti.html#3-2013

	CZK
PRICE Estimation	2,286,558
RENT per year Estimation	CZK 108,384
RENT per month Estimation	CZK 9,032
Rent to Price ratio	4.74%
RtP + 1.5 standard deviation	5.95%
RtP - 1.5 standard deviation	3.78%
Price to Rent ratio	21.1
User Cost Calculator	Inputs
Depreciation rate or Maintenance cost	3.0%
Expected appreciation level	2.29%
Additional risk premium	1.5%
Personal income tax	15.0%
Property tax	0.04%
Cost of capital	3.54%
	Results
User cost	5.04%
Price to User cost ratio	19.8
(Price to Rent/Price to User cost) - 1	6.3%

Source: author's computations

From results in table 5.1 we conclude that this market segment for chosen input parameters is 6.3 % overpriced, but generally priced well. User cost fit within confidence interval boarders so we conclude there is no significant mispricing.

5.2 LOW QUALITY INNER CITY SMALL FLAT

This segment is defined as small size – 55 m2, lower quality – 50 % condition state, in medium distance from city center – radius 4.500 meters including locations as Vinohrady, Dejvice, Žižkov, Smíchov and similar. Flat is in other type of building than prefabricated panel structure and does not include garage or furniture.

Table 5.2: Type's B results

Real Estate Flat Price & Rent Level Calculator - 9.05.2013	
data	Inputs
CONDITION	50%
FLOOR (50-80 m2)	55
DISTANCE (metres)	4,500
GARAGE (dummy)	0

FURNITURE (dummy)	0
PREFABRICATED STRUCTURE (dummy)	0
GROUND LEVEL (dummy)	0
	Results
	CZK
PRICE Estimation	2,368,118
RENT per year Estimation	CZK 95,175
RENT per month Estimation	CZK 7,931
Rent to Price ratio	4.02%
RtP + 1.5 standard deviation	5.00%
RtP - 1.5 standard deviation	3.23%
Price to Rent ratio	24.9
User Cost Calculator	Inputs
Depreciation rate or Maintenance cost	2.5%
Expected appreciation level	3.29%
Additional risk premium	1.5%
Personal income tax	15.0%
Property tax	0.03%
Cost of capital	3.54%
	Results
User cost	3.62%
Price to User cost ratio	27.6
(Price to Rent/Price to User cost) - 1	-10.0%

Source: author's computations

From results in table 5.2 we conclude that this market segment for chosen input parameters is 10 % underpriced, but generally priced well. User cost fit within confidence interval boarders so we conclude there is no significant mispricing.

5.3 HIGH QUALITY CITY CENTER FLAT

Last type of property is defined by larger floor size – 75 m2, highest quality – 100 % condition state and generally located in city center as radius is set to 1.000 metres. Flat is again in other type of building than prefabricated panel structure and does not include garage or furniture.

Table 5.3: Type's C results

Real Estate Flat Price & Rent Level Calculator - 9.05.2013	
data	Inputs

CONDITION	100%
FLOOR (50-80 m2)	75
DISTANCE (metres)	1,000
GARAGE (dummy)	0
FURNITURE (dummy)	0
PREFABRICATED STRUCTURE (dummy)	0
GROUND LEVEL (dummy)	0
	Results
	CZK
PRICE Estimation	4,388,125
RENT per year Estimation	CZK 172,968
RENT per month Estimation	CZK 14,414
Rent to Price ratio	3.94%
RtP + 1.5 standard deviation	5.34%
RtP - 1.5 standard deviation	2.91%
Price to Rent ratio	25.4
User Cost Calculator	Inputs
Depreciation rate or Maintenance cost	2.5%
Expected appreciation level	4.29%
Additional risk premium	1.5%
Personal income tax	15.0%
Property tax	0.02%
Cost of capital	3.54%
	Results
User cost	2.68%
Price to User cost ratio	37.3
(Price to Rent/Price to User cost) - 1	-31.9%

Source: author's computations

From results in table 5.3 we conclude that this market segment for chosen input parameters is 31.9 % underpriced, and thus not priced well. User cost does not fit within confidence interval boarders so we conclude there is significant mispricing situation on this market. That is probably caused by + 1 % premium on expected appreciation. So we should again rethink if this is reasonable assumption and if we conclude that it sticks, we confirm market situation where for this segment is property owner significantly more compensated via rent value than user cost would indicate.

6   CONCLUSION

In time period of poor economical performance for several last years, that was actually triggered by real estate bubble on U.S. market in 2007, appropriate pricing of real estate assets become more important topic than ever before. When housing bubble burst, a great amount of value is lost and directly or indirectly affects generally everybody.

This study focuses on theoretical and practical determination of real estate market price levels for both renting as well as purchase. After we derive theoretical concept of user costs for practical usage and estimate models for determination of current renting and purchase price levels on chosen segments of Prague real estate market, we finally compare those results allowing us to assess if there is equilibrium state on the markets.

For theoretical determination of relation between renting price level and price level of property purchase we accommodate so called user cost concept. This approach is based on calculation of imputed rent, which indicates equality relation of subjected two price levels. If market is in inequality compared to user cost relation, arbitrage opportunity emerges and triggers market mechanisms to reach equilibrium.

In order to compare market price levels we estimate by ordinary least squares method loglinear regression models that allow us to determine structurally consistent price levels for both purchase and renting. As we need those models primarily for estimating price levels. They provide us with several interesting conclusions such as the fact that if the flat is located in prefabricated panel structure has significant negative impact on purchase price, but is insignificant for rent level estimation.

Finally we set three basic market segments to cover different property types distinctions and compared estimated price to rent ratio with equality ratio suggested by user cost formula. Based on this comparison we concluded, that medium sized, standard quality, prefabricated panel structure, periphery located property segment is insignificantly overpriced, approximately 6.5 % above user cost ratio suggests. This conclusion is interesting mainly because of comparable advantage of panel structures of relatively smaller purchase price and comparable price of renting, but at the end higher depreciation rate and lower expected appreciation rate outweighed in favor of overpriced situation. Another example segment of smaller size, low quality, medium

distance around 4.5 km radius from city center is insignificantly underpriced at 10 % below user cost expected price level. Last tested market segment was bigger size, high quality, city center properties. They are indicated to be significantly underpriced around 32 % below user cost suggested level. This result is definitively influenced by expected appreciation 1 % premium that we set on this kind of property because of highly inelastic supply. But as more influencing fundamental factor we call exceptionally low interest rates – mortgage rate as well as long-term interest rate, those positively influence results for all segments.

At last we conclude that Prague real estate market is not overpriced and definitely there is not ongoing positive price bubble. Actually due to very low interest rates, it seems that some segments are rather underpriced. It will be interesting to observe development in the future if interest rates would rise again.

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Appendix A: Model 2
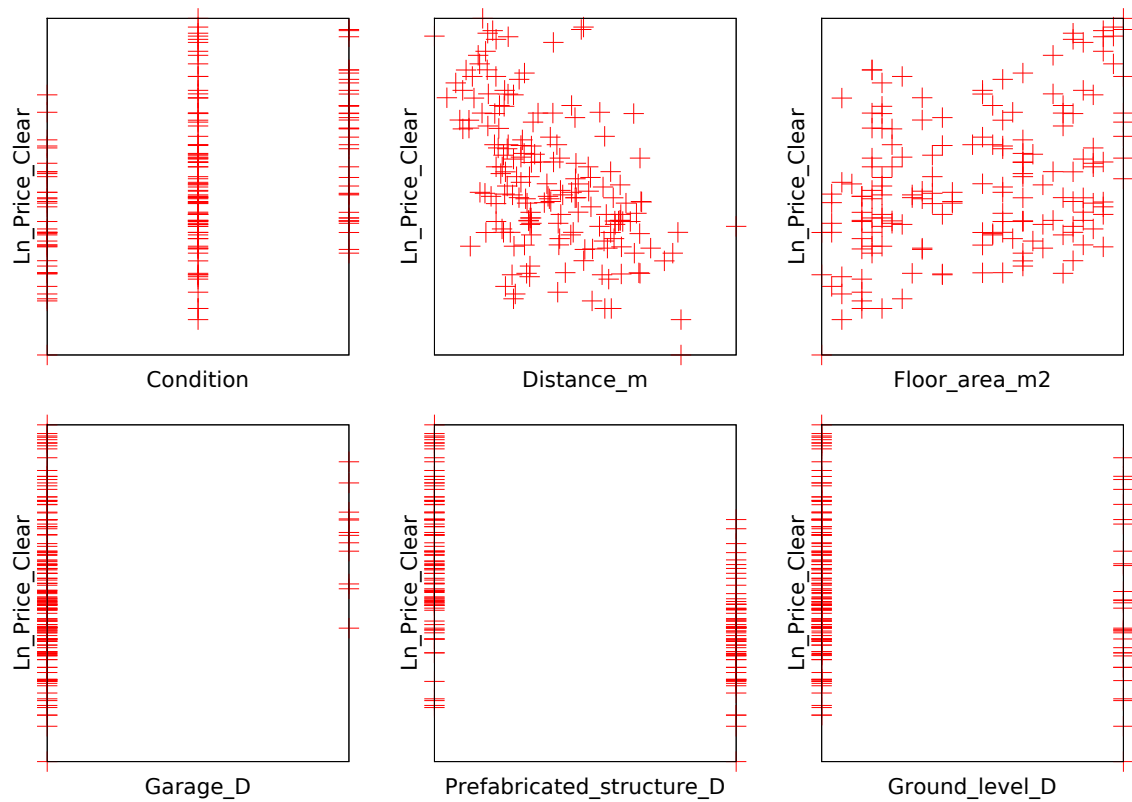
Figure A.1: Dependent vs. Explanatory variable
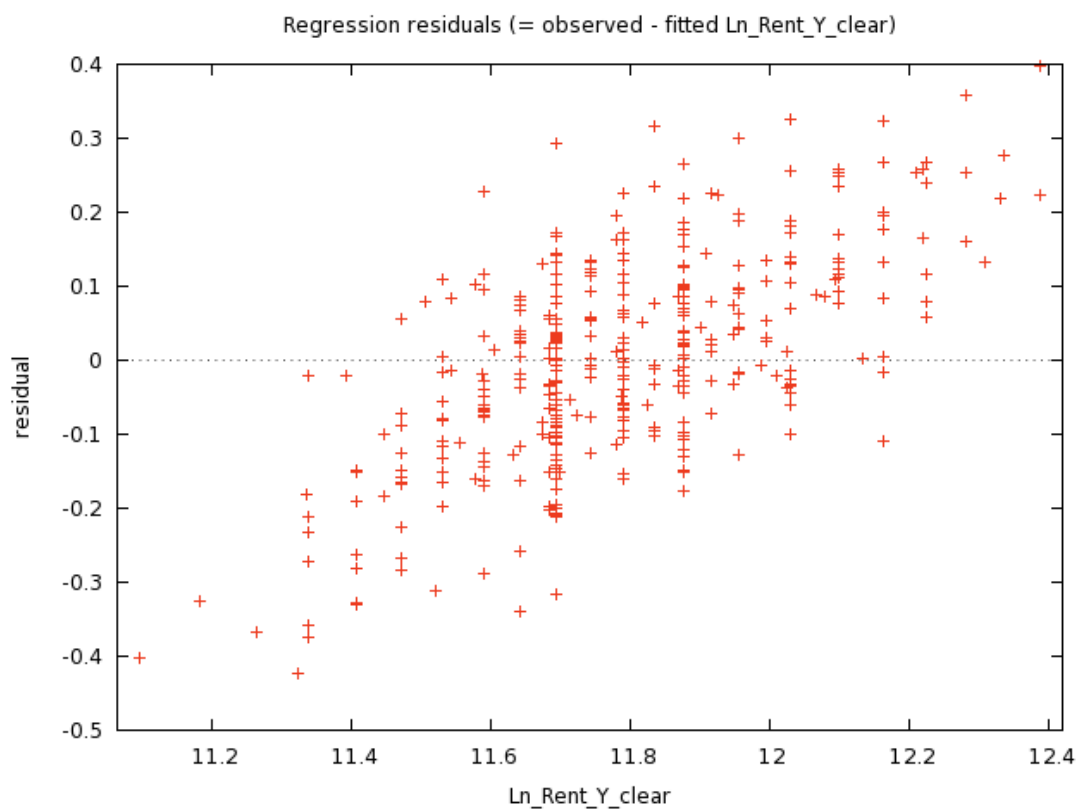
Source: sreality.cz, author's computations. Gretl SW
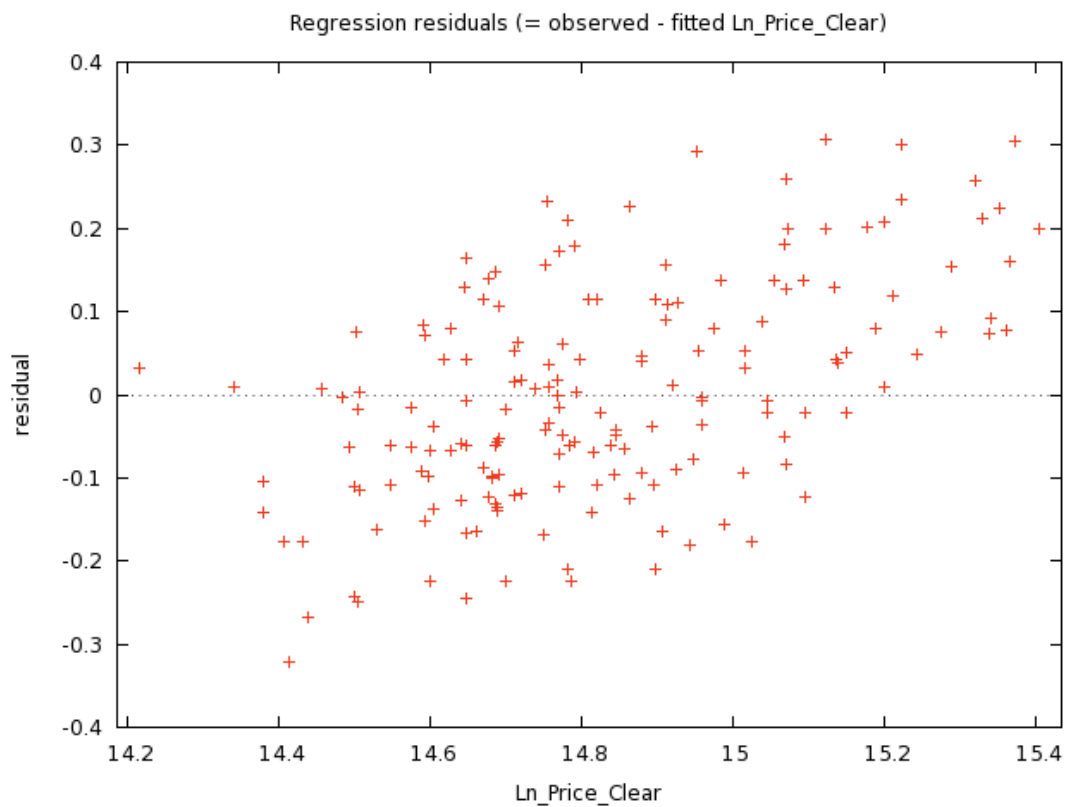
Figure A.2: Reg. Residuals vs. Dependent variable

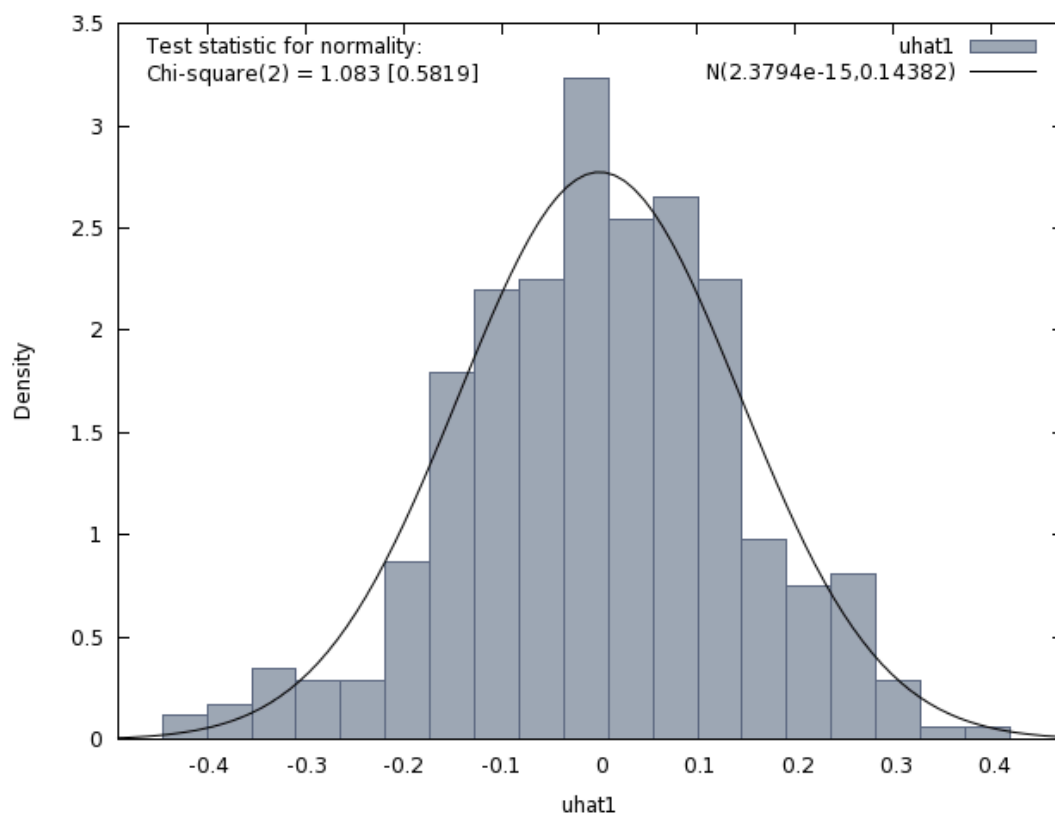*Source*: sreality.cz, author's computations. Gretl SW
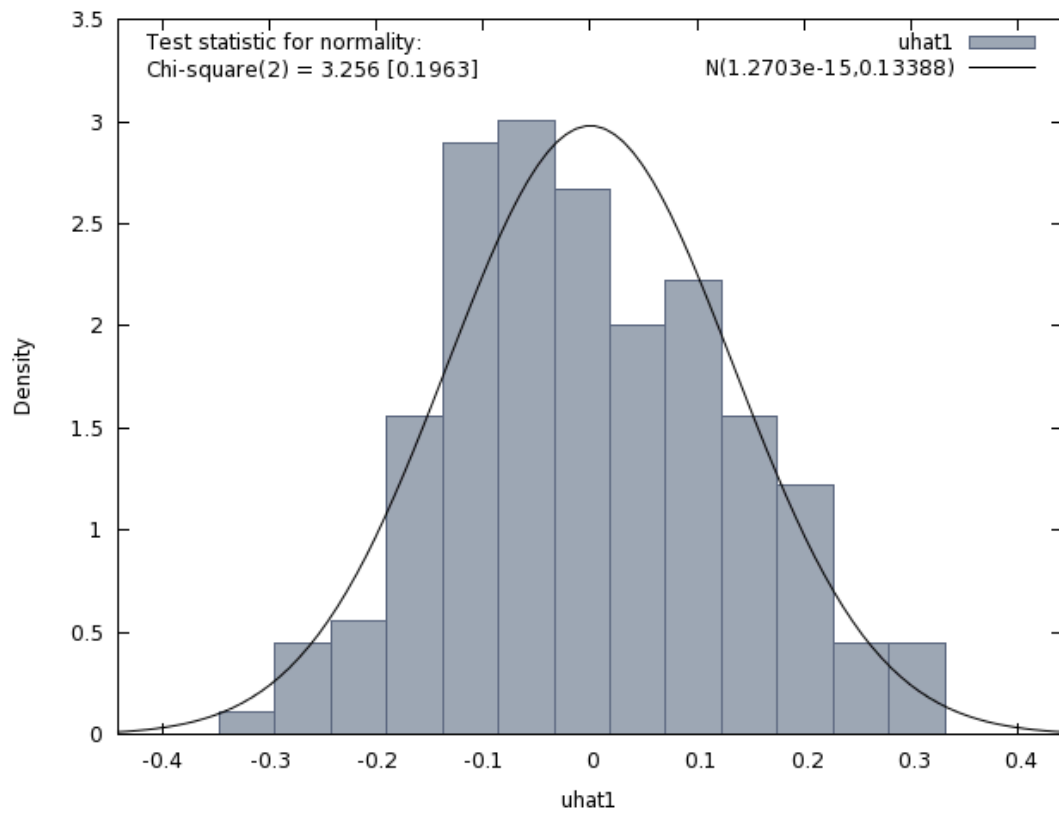
Figure A.3: Reg. Residuals Normality
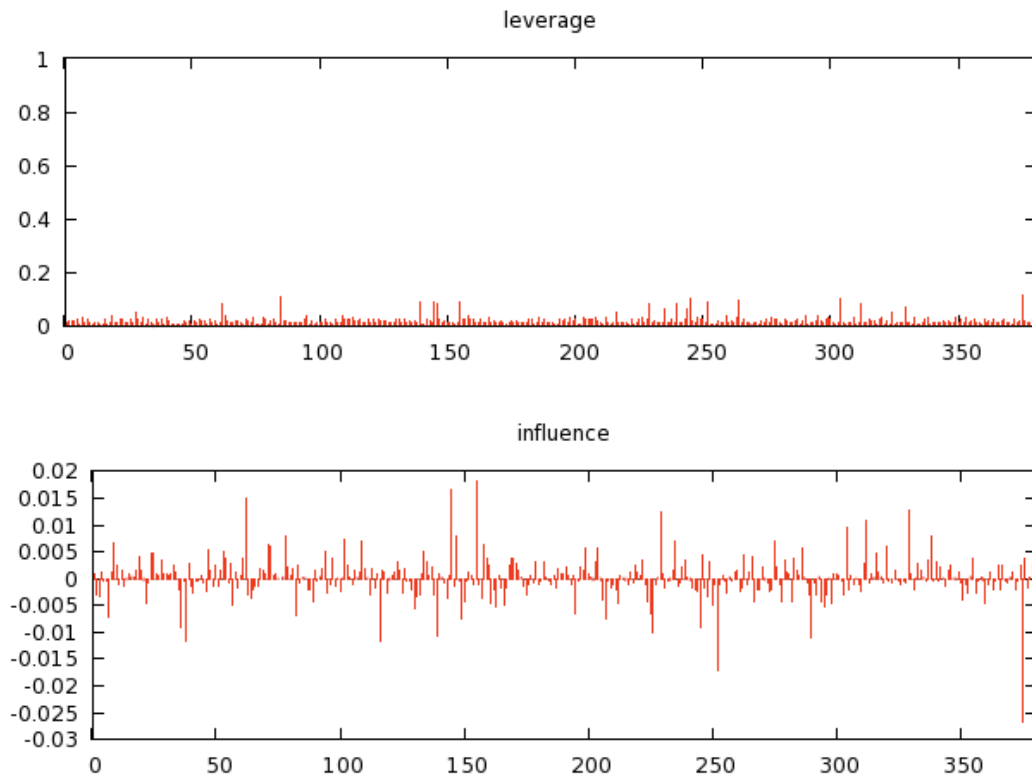
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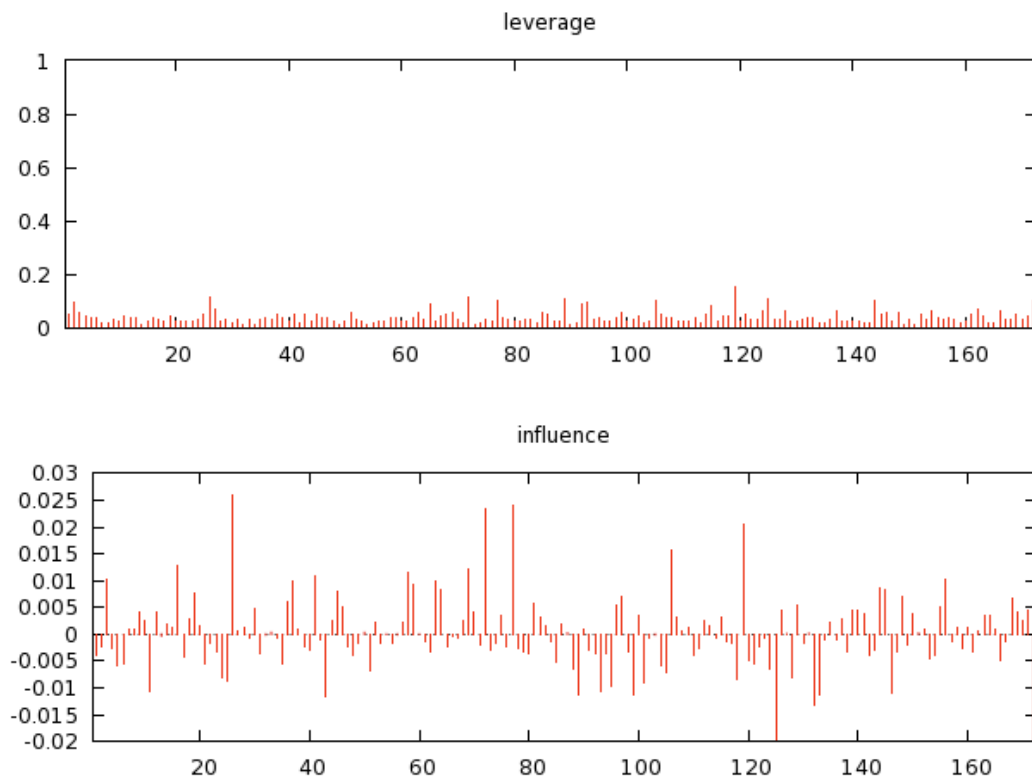
Figure A.4: Individual Cases Influence and Leverage

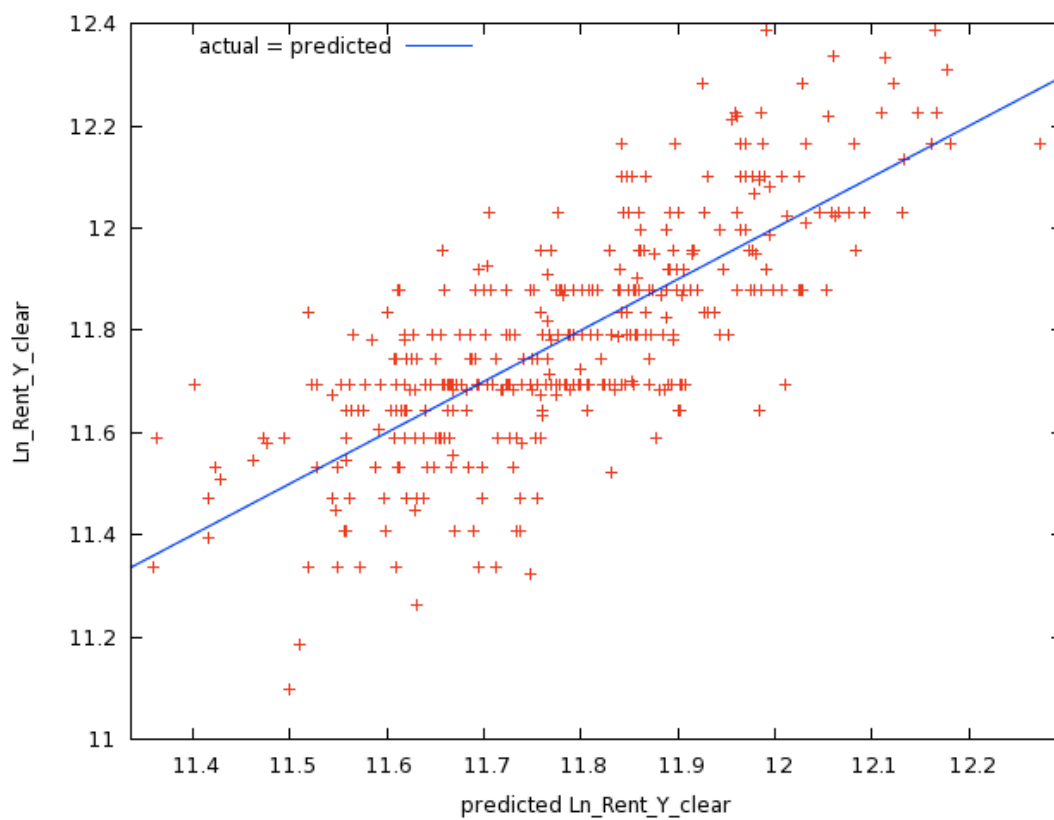*Source*: sreality.cz, author's computations. Gretl SW
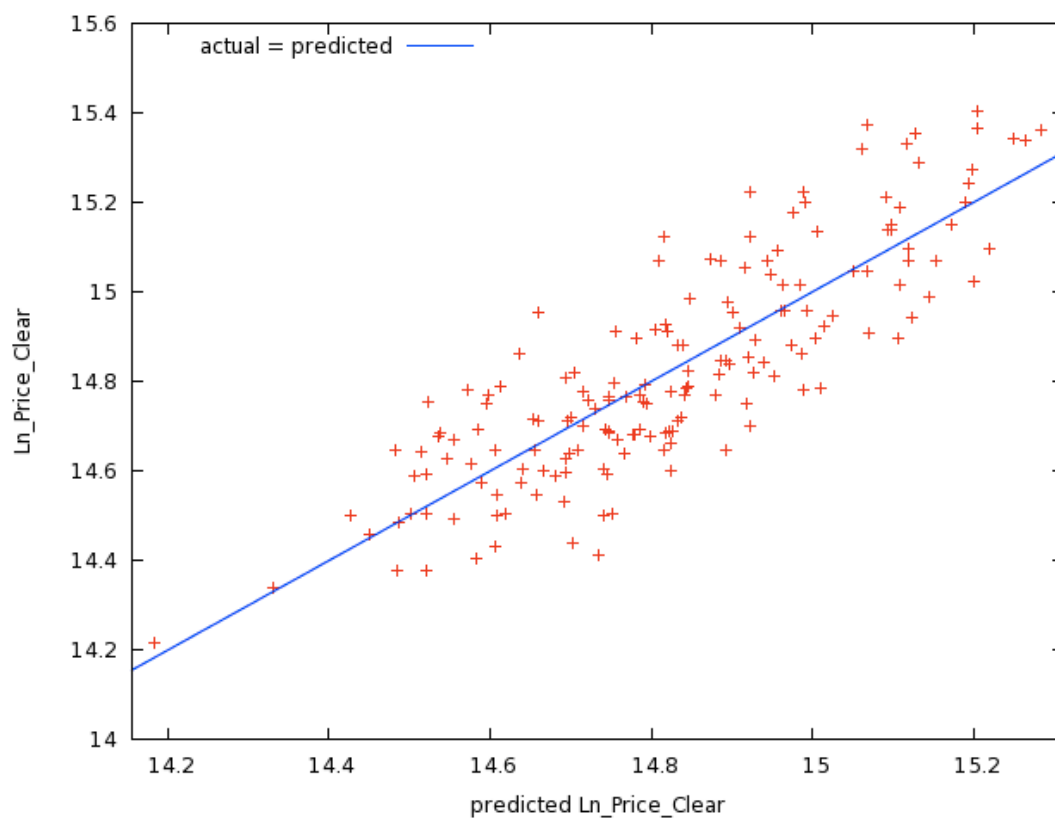
Figure A.5: Actual vs. Fitted, MODEL 2 PRICE

Source: sreality.cz, author's computations. Gretl SW

Appendix B: Model 4
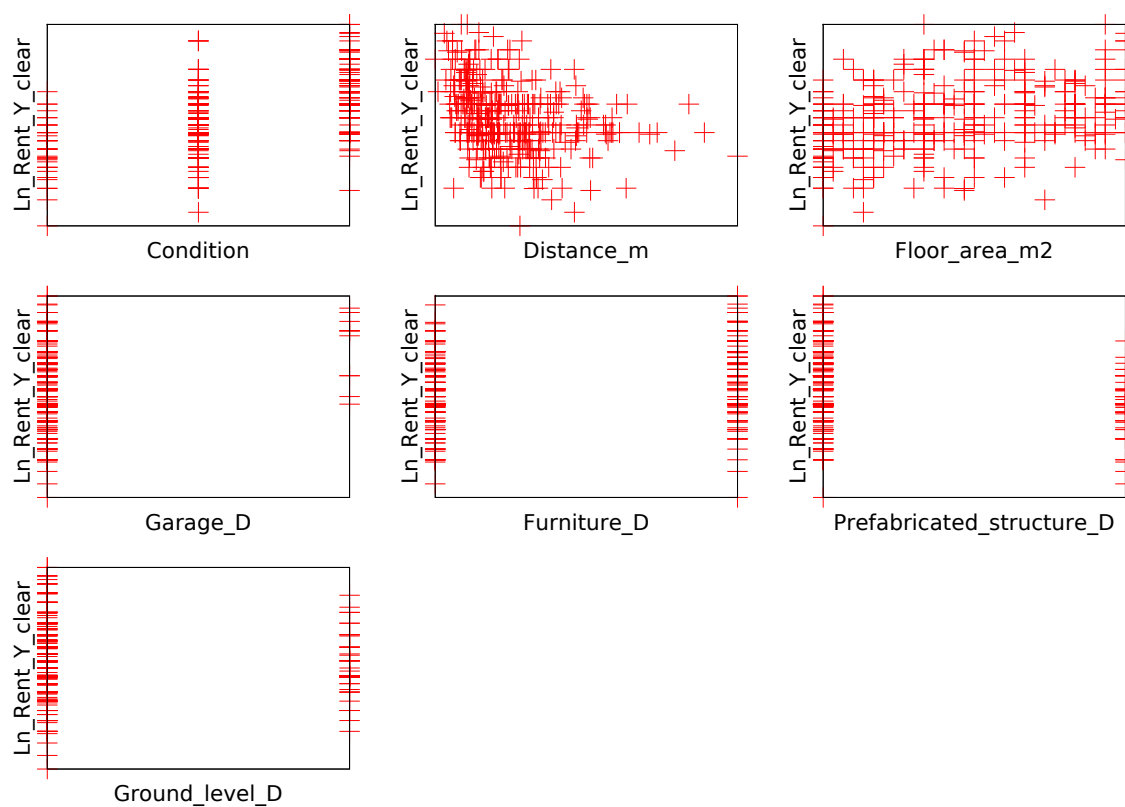
Figure B.1: Dependent vs. Explanatory variable

Source: sreality.cz, author's computations. Gretl SW

Figure B.2: Reg. Residuals vs. Dependent variable

Source: sreality.cz, author's computations. Gretl SW

Figure B.3: Reg. Residuals Normality

Source: sreality.cz, author's computations. Gretl SW

Figure B.4: Individual Cases Influence and Leverage

Source: sreality.cz, author's computations. Gretl SW

Figure B.5: Actual vs. Fitted, MODEL 4 RENT

Source: sreality.cz, author's computations. Gretl SW

Appendix C: Price & Rent Estimated Density Function
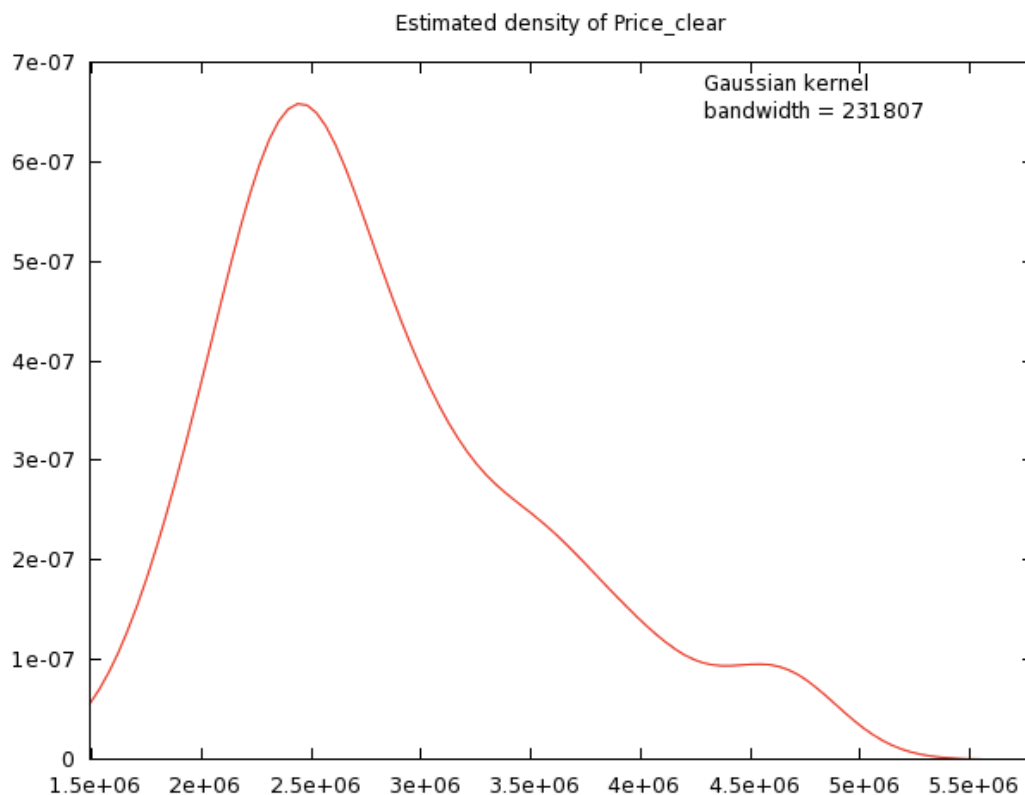
Figure C.1: Estimated density function Price
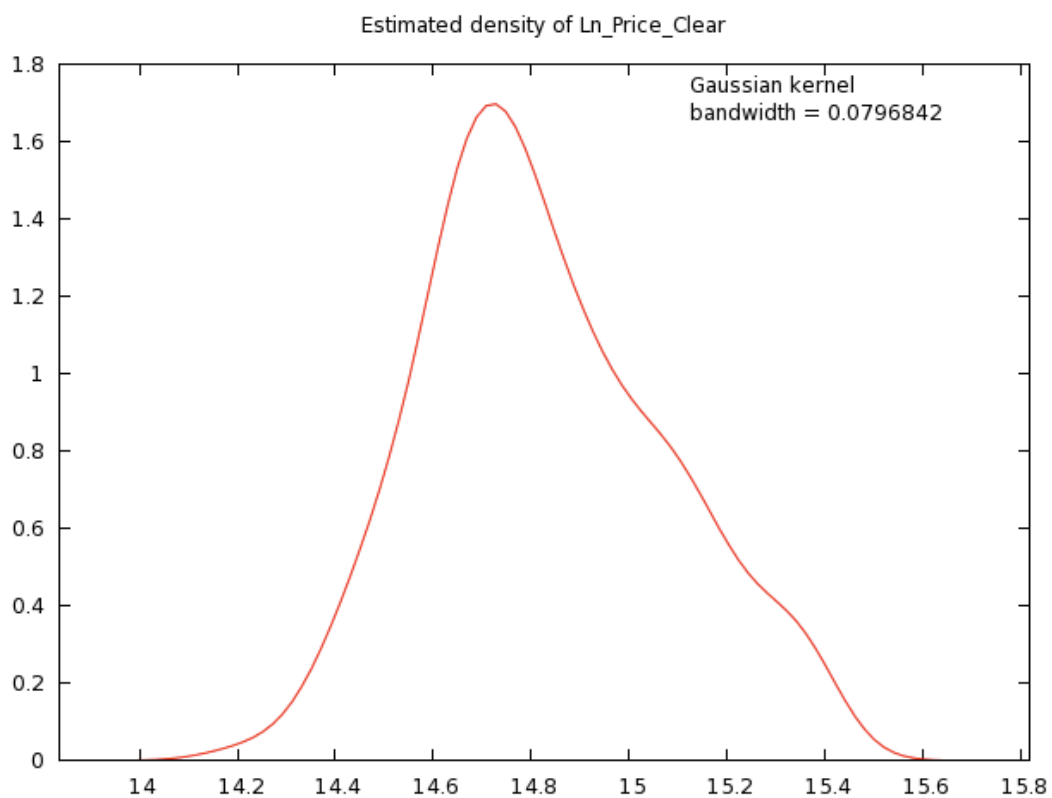
Source: sreality.cz, author's computations. Gretl SW

Figure C.2: Estimated density function Ln_Price

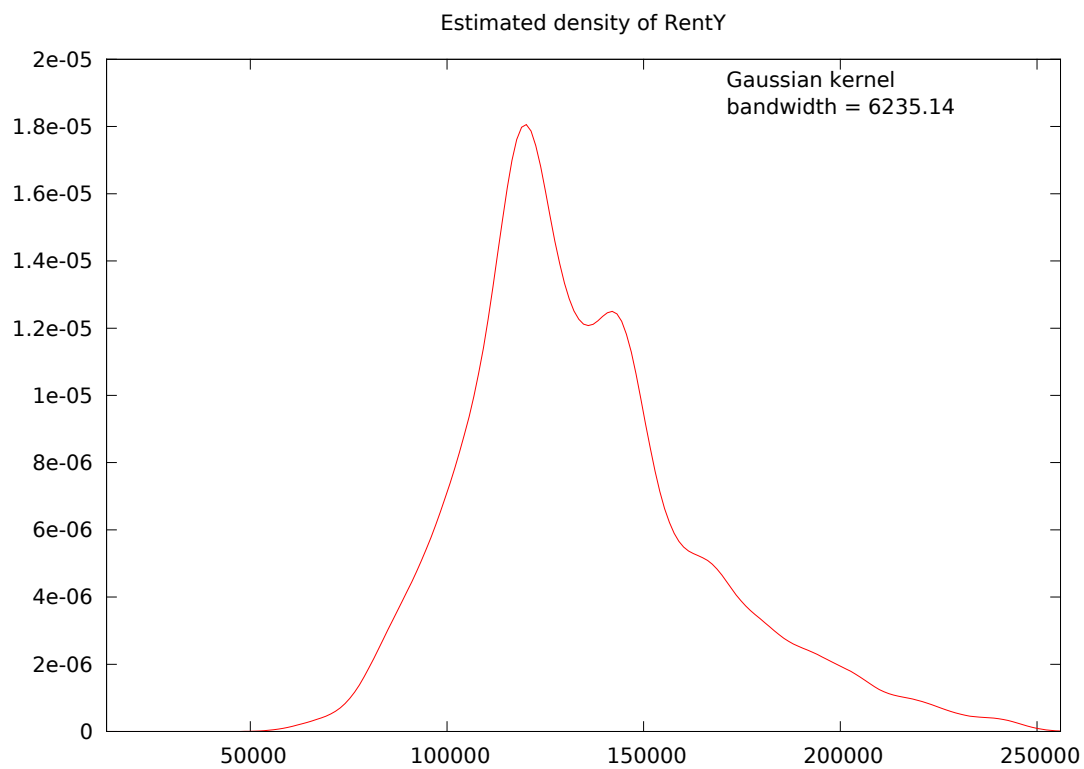*Source*: sreality.cz, author's computations. Gretl SW

Figure C.3: Estimated density function RentY

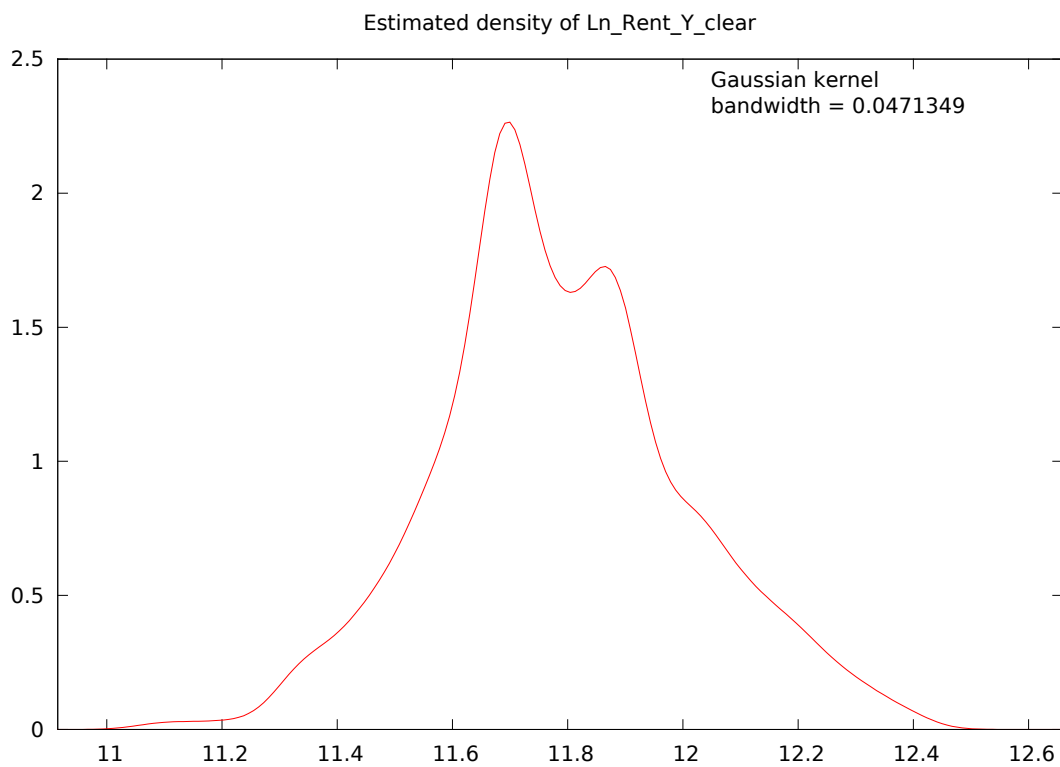*Source*: sreality.cz, author's computations. Gretl SW

Figure C.4: Estimated density function Ln_RentY

Source: sreality.cz, author's computations. Gretl SW