Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

Master Thesis

Modern way of calculation of CAPM coefficient: Beta hedging application

Author: **Bc. Daniel Šopov, MSc.**

Supervisor: **Petra Andrlíková, MSc.**

Academic Year: **2012/2013**

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 15, 2013

Signature

Acknowledgments

I am deeply indebted to my academic supervisor Petra Andrlíková MSc. whose help, stimulating suggestions and encouragement helped me to write and finish this thesis.

Furthermore, I would like to thank my brother PhDr. Boril Šopov, MSc., LLm., whose help and motivation was also very useful during writing this thesis.

And I would like to express my gratitude to all those who gave me the possibility to complete this thesis, especially my family.

Abstract

Capital Asset Pricing Model is considered to be the benchmark when evaluating the systematic risk of assets and its covariance with market returns. This thesis uses this framework and by employing various methods, such as Ordinary Least Square, Dynamic Conditional Covariance Multivariate GARCH and State Space Formulation is trying to find the most suitable method among these to estimate the coefficients of systematic risk. These coefficients are then used to hedge portfolios, which are created from the stocks traded on different stock exchange- NYSE Composite and NASDAQ Composite. According to the results of the hedge performance of each portfolio we will be able to evaluate which method is the most suitable to estimate the systematic risk within CAPM framework.

Keywords	CAPM, Systematic risk, Portfolio risk hedge, OLS, DCC MGARCH, SSF model
JEL Classification	C22, C58, G11, G12, G15

Author's e-mail	danielsopov@email.cz
Supervisor's e-mail	andrlikova@gmail.com

Abstrakt

Model CAPM je považován za základní model při oceňování systematického risku aktiv a jeho provázanosti s výnosností trhu. Tato práce využívá této struktury a použitím různých metod, mezi které patří OLS, DCC MGARCH a SSF modelovaní, se snaží najít nejvhodnější metodu z výše zmíněných, která dokáže nejlépe odhadnout koeficienty systematického risku. Tyto koeficienty jsou dále použity pro zajištění rizika portfolií, které jsou vytvořeny z akcií obchodovaných na různých burzách- NYSE Composite a NASDAQ Composite. Na základě obdržených výsledků o výkonu zajištění rizika v každém portfoliu budeme schopni vyhodnotit, která z metod je nejvhodnější pro odhad systematické risku v modelu CAPM.

Keywords	CAPM, Systematic risk, Portfolio risk hedge, OLS, DCC MGARCH, SSF model
JEL Klasifikace	C22, C58, G11, G12, G15

Author's e-mail	danielsopov@email.cz
Supervisor's e-mail	andrlikova@gmail.com

Contents

List of Tables

List of Figures

Acronyms

coeff. coefficient

CAPM Capital Asset Pricing Model

est. estimate

OLS Ordinary Least Squares

SSF State Space Formulation

DCC MGARCH Dynamic Conditional Covariance Multivariate GARCH

WP Weighted Portfolio

EWP Equally Weighted Portfolio

NYSE New York Stock Exchange Composite

NASDAQ National Association of Securities Dealers Automated Quotations Composite

Chapter 1

Introduction

This master thesis aspires to find the most suitable econometric approach to estimate the systematic risk of different stocks on different stock markets and based on these findings hedges the risk of created portfolios and tests their performance in time. We have focused on the systematic risk as the systemic risk is hard to measure and despite its impact, which may bring down the whole financial system, it would not provide us with necessary information for our research.

There are two main reasons why we have picked this topic. First is the latest financial recession that has caused turmoils on financial markets influencing plenty of companies and households. We assume that finding the most suitable method among those we use in this thesis, for estimating the time-varying systematic risk coefficients and hedging this risk to decrease the loss of the investment might be very useful and may reduce consequences of possible future financial crisis or may give us a fighting chance to avoid severe losses. Secondly, we have found out that there are not plenty of research papers dedicated to systematic risk and time-varying betas applied on the stock markets, therefore we would like to focus on this field in our thesis.

Moreover, we will concentrate on particular stocks from different stock indexes NYSE Composite and NASDAQ Composite and for financial analysis we have chosen popular Capital Asset Pricing Model (CAPM). Based on this model we will try to find the coefficients of systematic risk through couple of econometric approaches. We will employ three approaches in this thesis, which we assume are capable of this task.

First of them is Ordinary Least Square estimation on rolling sample. Even though we assume this approach as being the weakest among the methods we have picked, we suppose that this approach will provide us of important benchmark when evaluating the results later on. Second approach that is employed in our thesis is Dynamic Conditional Covariance Multivariate GARCH, which is powerful tool when analyzing time series. Finally, last approach casts CAPM into State Space Formulation, which should be the most powerful tool for estimating the systematic risk in CAPM. Furthermore, as soon as we obtain the estimates of systematic risk of each stock, we will introduce the $\hat{\beta}$-hedging strategy that will be applied on portfolios Weighted and Equally Weighted composed of stocks from both indexes (NYSE, NASDAQ) and then we will compare their performance in time. Based on these results we will evaluate the best method for estimating the systematic risk.

The structure of this thesis consists of eight chapters. Second chapter summarizes literature that has been published and has concentrated on topics relevant for our research. Thus, chapter 2 is divided into 5 parts which are dedicated subsequently to Capital Asset Pricing Model, Ordinary Least Square, GARCH family of models, State Space Formulation and Systematic risk and Time-Varying Beta. Following part introduces the methodology that was used for our analysis of time series and estimation of systematic risk coefficients. This part is divided into two sections. First goes through assumptions of Capital Asset Pricing Model and second section focuses on all econometric approaches and introduces each of these methods. Chapter 4 is dedicated to a data sets, which have been collected and provides of data statistics and selection of stocks from each index.

In addition, chapter 5 and 6 are both empirical. Chapter 5 treats with the estimation of the betas - systematic risk of each stock, whereas chapter 6 describes our hedging strategy that is used in this thesis and also explains the way how portfolios were designed and built. Moreover, in this chapter we evaluate the results of hedge performance of each portfolio and based on these findings we propose the best method for estimation of systematic risk within CAPM framework. Chapter 7 summarizes findings of this thesis and last part of this thesis is the Related Literature, which is summing up all the literature that was used during writing this thesis.

Chapter 2

Literature review

This chapter will go through the literature that has already been dedicated to this topic and will highlight the findings of this literature. Firstly, we focus on the CAPM and its pros and cons for our research.

Second part concentrates on the econometrics and will describe all of the approaches, we will use in this thesis and their foundation and usefulness for estimating the systematic risk in CAPM.

Last part summarizes the results that have been obtained from the previous research papers dedicated on the hedging and performance of the portfolios.

2.1 Capital Asset Pricing Model - History

Capital Asset Pricing Model (CAPM) was introduced by Treynor (1962). His model was the first, which derives the relation between expected return and covariance with market portfolio. (French (2003))

Despite the fact that his work was cited by plenty of other economists who proceed with development of his asset pricing model and some of them even viewed Treynor's model as the first for asset pricing (Black (1981)), his paper was not officially considered as the initial foundation of the CAPM. Simultaneously, Sharpe (1964), Lintner (1965) and Mossin (1966) have developed concept of CAPM, which was similar to Treynor's. They all were inspired of Markowitz (1952, 1959) and Tobin (1958) theoretical frameworks about diversification and modern portfolio theory, which have become core for foundation of the theory of CAPM.

During the years, this early pricing model has undergone few changes, such as relaxation of some assumptions, for instance the effects of taxation. (Brennan (1970))

Furthermore, Mayers (1972) restricted trading of risky assets, transaction costs and information asymmetries, whereas Rubistein (1973) added moments into model and as well as Mayers created CAPM, where no risk less assets occurred. Black (1972) considered assumption of unrestricted risk free lending and borrowing as unrealistic, and created model without this assumption and by this change proved that market portfolio is mean-variance-efficient under different assumption. The only difference between Sharpe-Lintner and Black CAPM is the way they define expected return on asset.

In addition, some economists argue that CAPM model, even thought it is powerful and provides satisfying expectations how to measure risk and its relation to expected returns, is not a very good empirical tool. (Fama, French (2004)) Fama and French (2004) found the possible reason for this empirical "failure" of CAPM in the simplifying of the assumptions or invalidity of testing the model. Another problem, which was considered by Fama and French (2004) was the misinterpretation of couple of definitions, such as market portfolio.

Moreover, empirical testing of the model has revealed plenty of shortcomings such as imprecise estimates of β when regressing cross-sectional data. Thus some economists such as Blume(1970), Black, Jensen and Scholes(1972) applied CAPM model just on portfolios rather than securities, which has approved as being good approach. Another issue caused by correlation of residuals in this type of regression was solved by Fama and Macbeth (1973).

Jensen in 1968 introduced time-series regression test, which has rejected functionality of the version of CAPM designed by Sharpe and Lintner. The problem was that the intercept, which Jensen included was greater than the average risk free rate and also β is less than market return. Other tests from other authors, such as Miller and Scholes (1972), Blume and Friend (1973) and Fama and French (1992) came out with similar results. Furthermore, CAPM has undergone another wave of testing, which tested whether β can explain expected returns. (Fama, MacBeth (1973))

According to this test, Black version of the CAPM seemed better, as the assumption of his model held during all tests. The main difference between Sharpe-Lintner model and Black model was that Black model expected β to be suffice to explain expected returns and β risk premium is positive. (Fama, French (2004))

Since the testing success of the Black model, many tests had challenged this model as well. Among first, there was Basu (1977), who found out that returns on high earnings price ratio stocks are higher than what CAPM can predict. Another non believer Banz (1981) argued that returns on small stocks are higher, when these are sorted by market cap than CAPM prediction. Lastly, Rosenberg, Reid and Lanstein (1985) proved that stocks with high $\frac{B}{M}$ equity ratios have quite high returns, which are not reflected in their βs.

All of these shortcomings were supported by Fama and French (1992, 1996) they reached the conclusion that any of price ratios have alike information about expected return. Since the evidence from Fama and French in 1992, most of the economists were aware of the fact that CAPM might have plenty of loopholes. (Fama, French (2004))

However, there are two stories that look at that differently. Behavioralists whose investors overreact to either good or bad times, so this sorting companies based on $\frac{B}{M}$ equity ratios is not appropriate. (DeBondt, Thaler (1987)) Second story is the need of more sophisticated asset pricing model which is not based on the unrealistic assumptions, such as investor is keen on mean and variance of portfolio during one period. (Fama, French (2004))

For this purpose, the CAPM model has been extended by Merton (1973), who added to his model assumption of longer period instead of the one period as in CAPM and so become ICAPM, which means inter-temporal capital asset pricing model.

On the other hand, Roll (1977) stood on opposite side and argued that despite these all results, the CAPM model has never been tested properly and it will never be, as there are plenty of proxies used in the empirical testing, which are not sufficient source.

Nevertheless, the CAPM is often used to measure the performance of funds and even though it does produce abnormal returns, we consider this model as being appropriate for the purpose of this thesis. Furthermore, the reason why we have not used other variations of Capital Asset Pricing Model [1] is the applicability of econometric methods on these models and also data collection, where the data are hard to collect for couple of variables.

[1]such as Fama (2006) multi factor model

2.2 OLS Rolling Window

In this thesis we will employ three different econometric approaches to find the coefficients of systematic risk in CAPM model. First of them is OLS and its rolling regression. Generally, this method is very popular but we assume there are better approaches for estimation of systematic risk coefficients. Nevertheless, we involve this method, so we can later on compare the results of the hedge performances of portfolios. Fabozzi and Francis (1978) used this method to show that betas are random and also argued that this can explain the poor performance of the return on assets.

2.3 GARCH family of models

GARCH model belongs to a family of ARCH models. There exist plenty of types of these models, but we will concentrate on some models from GARCH root. At the beginning, we have to mention general information about ARCH model, which was founded by Engle (1982). ARCH model in contrast to another econometric model ARMA (Box, Jenkins (1970)) considers conditional variance dependent on past information (errors and variances), whereas ARMA assumes one-period variance forecast. Furthermore, ARCH model simulates heteroscedastic volatility over time, which is in comparison to ARMA model different and based on that ARCH model can easily explain volatility clustering, which is typical feature of financial market. (Mandelbrot, Taylor (1967))

GARCH model was introduced by Bollerslev (1986) as an extension of ARCH. The rationale behind was to prolong memory and create more flexible structure of lags. The initiative of this model "derivation" was application of long lags empirically in ARCH by Engle (1982, 1983), Engle, Kraft (1983), which generated better results and the negative estimates of variance disappeared. In other words, slow decay in ARCH model needs large number of lags, whereas this decay in GARCH is done through exponential function. (Engle, Kraft (1983)) According to Silvennoinen and Tarasvirta (2009) GARCH family of models is considered as being appropriate in modeling stock and stock index returns, interest rates etc.

Furthermore, in our thesis we will use model from Multivariate GARCH root of models and we will also employ rolling window assumption. First model that was introduced was VEC (Bollerslev, 1988), where each element of matrix was modeled as linear combination of the lagged squared errors and cross-products of errors and lagged values of the elements of matrix. (Bauwens, 2006) Nevertheless, this model had shortcomings, which were the high number of parameters and positive definiteness of matrix was not assured. Engle and Kroner (1995) have imposed new parameter on this model, which has guaranteed the positiveness of matrix, this model was called BEKK.

The model we will use in this thesis was introduced in 1990 by Bollerslev and as well as GARCH was an extension of ARCH, M-GARCH was for GARCH. A typical feature of CCC Multivariate GARCH is its modeling of the conditional covariance matrix. Moreover, the problem of this model (GARCH (1,1)) is the number of unknown coefficients, which also increases as the more time series are involved. (Pagan (1996))

Bollerslev (1990) suggested to solve this complication to set coefficients off-diagonal of the matrix to zero. This have reduced number of coefficient to seven. This is one of the reasons, why we will use this model, which will be introduced in methodology. Second reason is that according to Engle (2001), ARCH/GARCH models are useful wherever there is a volatility of returns a central issue and according to Engle (2001, p.167): "...provides a statistical stage on which many theories of asset pricing and portfolio analysis can be exhibited and tested." This model was used by plenty of economists, who estimated time varying betas. (De Santis and Gerard (1998), Hafner and Herwartz (1998))

On the other hand, this Constant Conditional Correlation Multivariate GARCH and its assumption may cause a problem for practical usage. (Ledoit (2003)) Thus, in our thesis we will use a Dynamic Conditional Correlation MGARCH model, which was proposed by Engle (2002) and it generalizes Bolerslev's model (1990). This model is flexible as any of GARCH models but it combines parsimonious parametric models with it. The core of this model is the estimation based on two stage method using likelihood function. According to Engle (2002) DCC MGARCH performs well empirically and provides of reasonable and sensible results.

2.4 State Space Formulation Model

This method has become very powerful tool for time series analysis since 1960, when Kalman launched his paper. This framework accommodates different specifications of models, which are related to time series. These days this tool is very popular among econometricians, but it has gained its popularity gradually. (Sopov (2010))

Furthermore, SSF model in comparison to ARMA model (Box, Jenkins (1970)) models time series in structural way explicitly. (Durbin, Koopman (2001), pp. 52)

Durbin and Koopman (2001) also provided of theoretical background of this model and advises how to model via SSF and apply Kalman filter on time series. The efficiency of Kalman filter was improved in 2008 Jungbacker and Koopman. In our application, we will use the SSF ability to extract latent path/factor of unobserved nature. Common critique of standard estimation of CAPM (Jagannathan, Wang (1994)) is aimed at the assumption of constant parameter beta. We try to alleviate this issue using the rolling window for OLS estimation, hence introducing some dynamics. In case of SSF, we simply specify the CAPM parameter beta to follow a random walk, thus we estimate its evolution over the whole sample.

This approach is suitable for beta evolution analysis *ex-post*[2], yet for portfolio/hedge evaluation, we cannot use all the information to estimate parameters. This would imply we "know" the future observation at the time of evaluating performance at the beginning observations. In other words, to calculate β_t, we use data only up to time t. This leads to using rolling data-set also for SSF estimation.

2.5 Systematic Risk & Time Varying Beta

There has been written quite a lot of about the estimating of systematic risk in Capital Asset Pricing Model or any other models. Jansky, Adam, Benecká (2012) estimated the systematic risk of banking sector, using similar methods as we will do in our thesis.

Banking sector was a objective of King's (2009) analysis, who estimated costs of capital. Merger and Bulla (2008) has concentrated their research of time-varying betas on the financial sector, which has included also insurance companies.

Similar research was done by Groenewold and Fraser (1999), who aimed on Australian market. Lie (2000) analyzed stock market in Australia and for this purpose he used GARCH and Kalman filter.

[2]The beta evolution paths were created like this.

Wooldridge(1988) and Engle, Lilien and Robins(1987) tested CAPM and for modeling time varying betas they used multivariate GARCH. Furthermore, Schwert and Seguin (1990) use covariances to model time variation in GARCH and also tested CAPM on couple of portfolios.

Adrian and Franzoni (2008) employed Kalman filter to model conditional betas and tested portfolios by $\frac{B}{M}$ ratio and by size. They obtained results, that supported theory of conditional CAPM.

Jostova, Philipov (2005) propose new beta model, which describes and sees systematic risk- beta as general mean reverting stochastic process. Their model puts together stochastic components and time variation in systematic risk process. For the estimation of these betas, they used Bayesian methods. Ang and Chen (2003), Petkova Zhang (2002) and Campbell and Vuolteenaho (2002) have confirmed this mean reversion in systematic βs.

Moreover, Christiansen and Ranaldo (2010) also discussed the mean reverting of betas, and found out that carry trade strategy is more exposed to stock market.

As we can see from the short summary of the literature, there has been a couple of papers written about the systematic risk and time-varying betas in many fields of economics. We lack more papers dedicated on the stock market and its cross market comparison, therefore our thesis is aimed to do this task.

Chapter 3

Methodology

This chapter of thesis will introduce Capital Asset Pricing model and all three econometric approaches that have been employed in empirical parts of this thesis.

3.1 CAPM model

This section will describe the model we use in our thesis. As we know the CAPM is a pricing model for portfolios or securities. So for the purpose of this thesis it is well chosen. This model builds relation between expected returns and systematic risk to security market line, this relation shows what should be the market price of security when considering the risk class of the security. The equation of our model is as follow:

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f) \tag{3.1}$$

where:

$E(r_i)$ is an expected returns of asset

r_f is a risk free rate of interest

β_i is a sensitivity of expected asset returns to market returns

$E(r_m)$ is a market expected returns

$E(r_i) - r_f$ is known as risk premium

$E(r_m) - r_f$ is known as market premium

Moreover, $\hat{\beta}$ is conceived as a measure of systematic risk and can be calculated as:

$$\hat{\beta}_i = \frac{cov(r_i, r_M)}{\sigma_M^2} \tag{3.2}$$

where:

- r_i is a return of asset i
- r_M is a returns of the market
- σ_M^2 is a variance of the returns of the market
- $cov(r_i, r_M)$ is a covariance between asset i and market returns

This equation basically tells us the risk contribution of well diversified portfolio to an asset. Furthermore, all equations and their derivatives will follow us during the empirical parts of this thesis, where we focus on the estimations of the β , which will help us afterwards in finding the best method to hedge the systematic risk. Following part of this chapter, will propose the methods, we use for estimating β.

3.2 Econometric approaches

This section will describe all the methods employed for estimating the β, which we will use later in our thesis for hedging the risk of portfolios. These portfolios will be then used to perform in time under particular beta hedging strategy, which is introduced in chapter 6.

Moreover, to find the best method we have chosen three different econometric approaches that might help us to find the betas and to compare the performances of hedged portfolios. Each of these three methods has its pros and cons and we assume that the estimations of β will not be the same, even though they should behave similarly in time. The methods are OLS rolling window, MGARCH and State Space Formulation Model.

3.2.1 OLS Rolling Window

As mentioned before, this method is very popular among economists, but we consider this method as not being very powerful when analyzing time series. This method's results will provide us with useful benchmark, which will be useful later for comparison with other results.

Rolling regression estimates, in this case time varying betas, are obtained through the OLS regression on a moving window of given time period (number of observations). It is essential to find the right size of the window and also be aware of outliers, which may affect the OLS estimates, as the OLS method is very sensitive for the outliers. In our thesis, the rolling window consists of 250 observations, which is more or less a trading year at stock exchange.

Following equation is the modification of CAPM, which we used for estimating systematic risk coefficients using OLS method:

$$r_{i,t} - r_{f,t} \quad = \quad \alpha + \beta_{i,t}(r_{m,t} - r_{f,t}) \tag{3.3}$$

where

$r_{i,t}$ is a daily return of each stocks

$r_{f,t}$ is a daily risk free rate

$\beta_{i,t}$ is systematic risk coefficient of stock i

$r_{m,t}$ is a daily market returns

3.2.2 DCC MGARCH

This part of this chapter introduces Dynamic Conditional Covariance Multivariate GARCH. But before we will present this model we will introduce CCC MGARCH, which have similar assumptions as DCC except for one.

Moreover, Multivariate GARCH model allows to employ multiple returns of assets in modeling. However, this model have two drawbacks that is number of parameters grow rapidly with the increasing number of assets and the second is the positive definiteness, which cannot be ensured. Constant Conditional Correlation Multivariate GARCH is used to decrease the number of parameters.

But firstly, we need to define model and its disturbances. We assume that y_t is stochastic process of dimension N x 1 and θ is a finite vector of parameters then:

$$y_t \quad = \quad \mu_t(\theta) + \varepsilon_t \tag{3.4}$$

Where $\mu_t(\theta)$ is the conditional mean vector and

$$\varepsilon_t \quad = \quad H_t^{\frac{1}{2}}(\theta) z_t \tag{3.5}$$

where $H_t^{\frac{1}{2}}(\theta)$ is a positive definite matrix and we assume that for vector z_t is:

$$E(z_t) = 0 \quad (3.6)$$

$$Var(z_t) = I_N \quad (3.7)$$

where I_N is a positive definite matrix N x N.

Constant Conditional Covarience MGARCH model was introduced by Bollerslev (1990), who has restricted the conditional correlation, which leads to conditional covariances that are proportional to the conditional standard deviations. Moreover, it does not model matrix H_t directly, but it decomposes H_t on correlation and conditional standard deviation. Despite not very straightforward way to obtain results for stationarity, ergodicity and moments, this model has less parameters and so they are easier to be estimated. Following equations are definition of this model:

$$H_t = D_t R D_t = (\rho_{ij}\sqrt{h_{iit}h_{jjt}}) \quad (3.8)$$

where

$$D_t = diag(h_{11,t}^{\frac{1}{2}} \ldots h_{MM,t}^{\frac{1}{2}}) \quad (3.9)$$

and the off diagonal elements are defined as:

$$[H_t]_{ij} = \sqrt{h_{it}}\sqrt{h_{jt}\rho_{ij}}, i \neq j \quad (3.10)$$

where $R = [\rho_{ij}]$ is positive definite, symmetric matrix, where $\rho_{ii} = 1$ for $i = 1, \ldots, M$. As mentioned in literature review, Multivariate GARCH models can be modeled as GARCH(p,q) and so for the purpose of our thesis we will use GARCH(1,1) to model Multivariate GARCH. The system of equations we obtain is as follows:

$$h_{ii,t} = c_{11} + a_{11}\varepsilon_{i,t-1}^2 + b_{11}h_{ii,t-1} \quad (3.11)$$

$$h_{MM,t} = c_{22} + a_{33}\varepsilon_{M,t-1}^2 + b_{33}h_{MM,t-1} \quad (3.12)$$

$$h_{iM,t} = \rho\sqrt{h_{ii,t}h_{MM,t}} \quad (3.13)$$

According to equation 3.10 -12, we can estimate time varying β. From equation 3.2 we can see that Capital Asset Pricing Model calculates $\hat{\beta}$ as ratio of covariance between return of assets and market return and the variance of the market. As long as the matrix in MGARCH consists of variances and covariances, which are time dependent, it can be used for calculating time-varying $\hat{\beta}$. Following equation explains it:

$$\hat{\beta}_{it} = \frac{cov(r_{it}, r_{Mt})}{\sigma^2_{Mt}} = \frac{h_{iM,t}}{h_{MM,t}} \tag{3.14}$$

For hedging purposes we always use the latest covariance and variance estimates obtained from the rolling window sample and then we will calculate betas. Furthermore, DCC MGARCH model we use in our thesis differs from CCC MGARCH model in definition of correlation matrix H_t, which in DCC MGARCH parametrizes R and so it becomes time-varying in model. So the matrix looks as follows:

$$H_t = D_t R_t D_t$$

Based on the H_t matrix obtained we can easily calculate covariances (by definition) and time-varying estimates β subsequently. To evaluate our hedging performance we again use the latest beta estimates. The reason why we use DCC MGARCH instead of CCC MGARCH is the sensibility of results, which are obtained by using this model.

Moreover, as well as in case of Ordinary Least Square method we employ rolling window, whose size is alike to OLS. As a result of this step we obtain same number of betas as in OLS and application of DCC MGARCH on the window will provide us of more accurate estimates.

3.2.3 State Space Formulation Model

The last method we use in this thesis is SSF model. We use Kalman filter to estimate the parameters of reformulated Capital Asset Pricing Model, this change will give us higher degree of uncertainty in α and β estimation and it also allows us to specify dynamics for α and β explicitly. Furthermore, we specify both coefficients-α and β to follow the random walk. Coefficient α will then stands for the discrepancy in the observed and real risk free rate. The reformulated model is as follows:

$$
\begin{aligned}
r_t - r_{f,t} &= \alpha_t + \beta_t(r_{M,t} - r_{f,t}) + \varepsilon, \varepsilon \sim N(0, \sigma_\varepsilon^2) && (3.15) \\
\alpha_{t+1} &= \alpha_t + \eta_t, \eta_t \sim N(0, \sigma_\eta^2) && (3.16) \\
\beta_{t+1} &= \beta_t + \epsilon, \epsilon \sim N(0, \sigma_\epsilon^2) && (3.17)
\end{aligned}
$$

We can rewrite these equations to a standard way, which is used when defining state space formulation models:

$$
\begin{bmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \eta_t \\ \epsilon \end{bmatrix} \tag{3.18}
$$

$$
r_t - r_{f,t} = \begin{bmatrix} 1 & (r_{M,t} - r_{f,t}) \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \end{bmatrix} \tag{3.19}
$$

Where equation 3.18 is state equation and 3.19 is measurement equation. These equations will be employed when estimating the betas-systematic risk.

Chapter 4

Data Set

This chapter introduces the data that were collected from finance.yahoo.com and used for the estimation of the systematic risk in CAPM model. We have prepared two datasets for each of indexes and this chapter will also introduce stocks we have picked. Furthermore, we will provide of data descriptive statistics and show evolution of daily stocks returns in time.

4.1 Stocks and stock indexes

As mentioned before, we have chosen two different stock indexes, which are NYSE Composite and NASDAQ Composite. The reason why we have chosen these indexes is the structure of stocks traded on each stock index and also because of the high liquidity of each of these indexes. Furthermore, we assume that the domicile might play its role when estimating the betas and also comparing of the performance of the portfolios, thus we have picked the indexes from the same country.

Furthermore, from each stock indexes we have picked 10 companies' traded on particular stock exchange. Each data set consists of daily values of stocks prices and they span from 1991 to 2012 for both indexes- NASDAQ Composite and NYSE Composite. The values of indexes span to the same period. We were very cautious when choosing these companies as we wanted all of these companies to be quoted on the stock exchange since 1991 or earlier.

Following tables 4.1 and 4.2 show companies that have been picked from each index and their abbreviations that are used in following chapters of this thesis. Our choice was quite limited as not many companies have been quoted since 1991. Despite this fact, we tried to find stocks from different industries.

Table 4.1: NYSE Composite

Abbreviations	Stock	Industry or Sector
GE	General Electric Company	Conglomerate
EX	ExxonMobil Corporation	Oil & Gas
PG	Proctor & Gamble Co.	Consumer goods
WM	Wal Mart Stores Inc.	Department Stores
AL	Alcoa Inc.	Aluminium
3M	3M Company	Conglomerate
IBM	IBM	IT
MCK	Merck& Co.Inc.	Pharmaceuticals
AMEX	American Express Company	Consumer Finance
McD	McDonald's Corporation	Fast food
WPNY	Weighted Portfolio NYSE	
EPNY	Equally Weighted Portfolio NYSE	
NYSE	New York Stock Exchange	

Table 4.2: NASDAQ Composite

Abbreviations	Stock	Industry or Sector
APP	Apple Inc.	IT,Electronics
CSC	Cisco Systems Inc.	Networking Equipment
CST	CostCo WholeSale Corp.	Retail
DLL	DELL Inc.	Software, Hardware
MTT	Mattel Inc.	Toys and Games
MCR	Microsoft Corporation	Computer Software
ORC	Oracle Corporation	Software, Hardware
PTC	PTC Inc.	Software
TXI	Texas Instruments Inc.	Semiconductor- Broad line
VD	Vodafone Group P.Ltd.Co.	Telecommunications
WPNQ	Weighted Portfolio NASDAQ	
EPNQ	Equally Weighted Portfolio NASDAQ	
NQ	NASDAQ	

Moreover, according to CAPM definition we had to collect the data for the risk free rate. But there is not any official database of this rate so we have picked the interest rate of 10 years governmental bonds of the USA for both indexes- NYSE & NASDAQ analysis, we assume that this rate is less risky in comparison to Greek Bonds.

4.1.1 Preparation of dataset

Before we begin with estimating and calculation of systematic risk coefficients within CAPM framework we had to prepare datasets. First step we did with our dataset was the transformation of stock prices to daily changes of returns. We used the formula stated below:

$$r_t = ln \frac{P_t}{P_{t-1}}$$

This step was done for both datasets. Moreover, we had to transform also the risk free rate, as we collected daily data with annual return, we had to divide annual rate with number of days. In case of US Treasury the denominator was equal to 360.

Furthermore, because of different approaches used for calculation of betas, we had to prepare two different types of datasets for each index. Based on CAPM framework we had to subtract risk free rate from the stocks returns and also from market return-index return. The result dataset are used for the OLS Rolling Window and State Space Formulation analysis. On the other hand, for DCC MGARCH analysis we subtract mean values of each stock and index from the daily data of relevant stock and index. We assume that this step will provide us with smoother time series more appropriate for the calculation of betas.

4.2 Data properties

This section presents the descriptive statistics of the corresponding time series of daily returns. We involve basic statistical informations such as mean, skewness, kurtosis and standard deviation. Table 4.3 and 4.4 shows the information about our dataset.

According to the tables 4.3 and 4.4 we can see, that most of the data are skewed to the left and are leptokurtic with heavy tails, which is a common feature of financial data. Furthermore, if we look at the figures 4.1 and 4.2 we can see the evolution of daily stock returns on each stock exchange. When we compare the volatility of the stocks, we can see that most stocks traded on New York Stock Exchange are quite volatile. On the other hand there are some stocks, which does not experience with any marginal volatility over time (if omitting few outliers), such as Proctor&Gamble, Merck and Alcoa.

Table 4.3: Descriptive Statistics-NYSE Composite

Stock	Mean	St.Dev.	Min.	Max.	Skewness	Kurtosis
AMEX	-0.062	0.012	-0.1	0.12	-0.384	13.7
AL	-0.1362	0.0239	-0.19	0.187	0.0354	9.8
EX	0.067	0.0247	-0.17	0.2	-0.07	10.6
GE	0.0964	0.0154	-0.15	0.158	0.04	11.4
IBM	-0.1723	0.0188	-0.14	0.18	0.007	10.64
MCD	0.113	0.0186	-0.169	0.12	0.008	9.78
3M	0.438	0.0164	-0.137	0.10	-0.04	7.18
MCK	-0.047	0.0153	-0.10	0.104	-0.036	7.29
PG	-0.1652	0.0185	-0.31	0.12	-1.09	22.5
WM	0.085	0.0153	-0.36	0.09	-2.46	61.38
WPNY	0.009	0.011	-0.09	0.1	-0.12	9.73
EPNY	0.0002	0.012	-0.098	0.104	-0.191	10.44
NYSE	-0.144	0.0178	-0.1	0.104	0.09	6.02

Table 4.4: Descriptive statistics-NASDAQ Composite

Stock	Mean	St.Dev.	Min.	Max.	Skewness	Kurtosis
APP	0.577	0.031	-0.73	0.285	-2.26	64
CSC	0.235	0.022	-0.269	0.17	-0.506	14.8
CST	0.225	0.219	-0.271	0.18	-0.51	14.9
DLL	0.485	0.031	-0.287	0.19	-0.467	8.4
MTT	0.296	0.022	-0.352	0.169	-1.08	27.3
MCR	0.421	0.021	-0.169	0.178	-0.002	8.79
ORC	0.776	0.03	-0.343	0.36	0.17	14.5
PTC	0.129	0.038	-0.673	0.243	-1.2	27.2
TXI	0.435	0.028	-0.21	0.22	0.133	5.99
VD	0.352	0022	-0.139	0.14	-0.035	7.33
WPNQ	0.483	0.017	-0.248	0.121	-0.584	14.1
EPNQ	0.393	0.0138	-0.089	0.093	-0.129	6.11
NASDAQ	0.0002	0.0158	-0.1	0.132	-0.066	8.65

Secondly, if we look at the daily returns of stocks traded on NASDAQ (figure 4.2) we can see that there are some stocks, whose evolution of returns in time is quite smooth (if we omit couple of outliers). These stocks are PTC, Mattel, Apple and Cisco. Obviously, there are some periods where these stocks experienced with decreases or increases of the stock price, but in contrast to Microsoft or Vodafone, which are more like roller coaster, they seem quite smooth.

Based on these figures we expect that the results obtained from hedging portfolios may differ - as the weights of each stock may have significant impact on the portfolio and its returns. We assume that higher volatility will test our hedging strategy and DCC MGARCH may be the best of the methods, but just in case the weighting of the portfolio would create pretty volatile evolution of portfolios returns.

Figure 4.1: NYSE Composite - daily returns of stocks

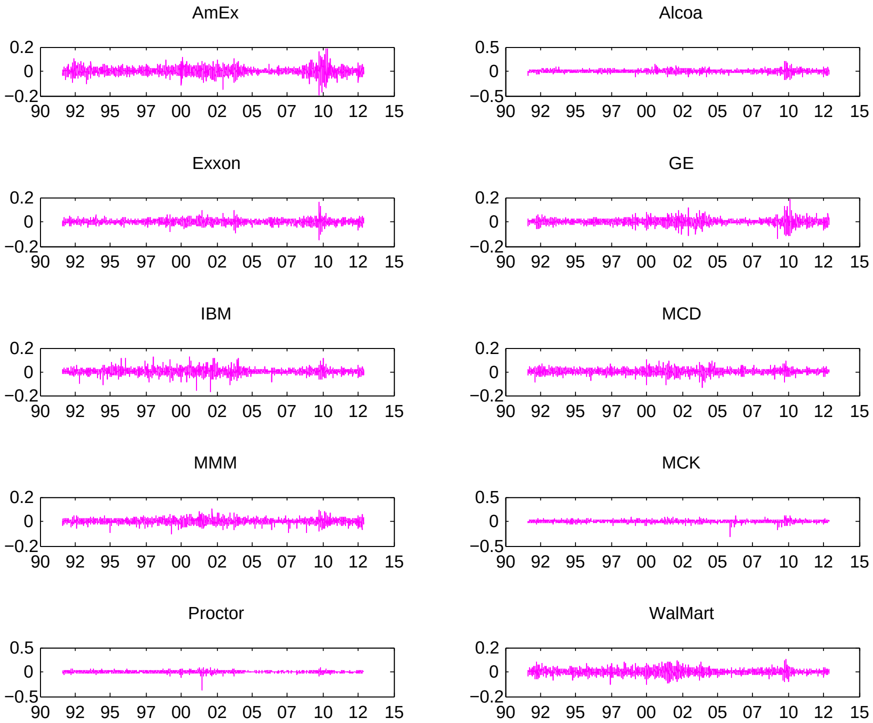
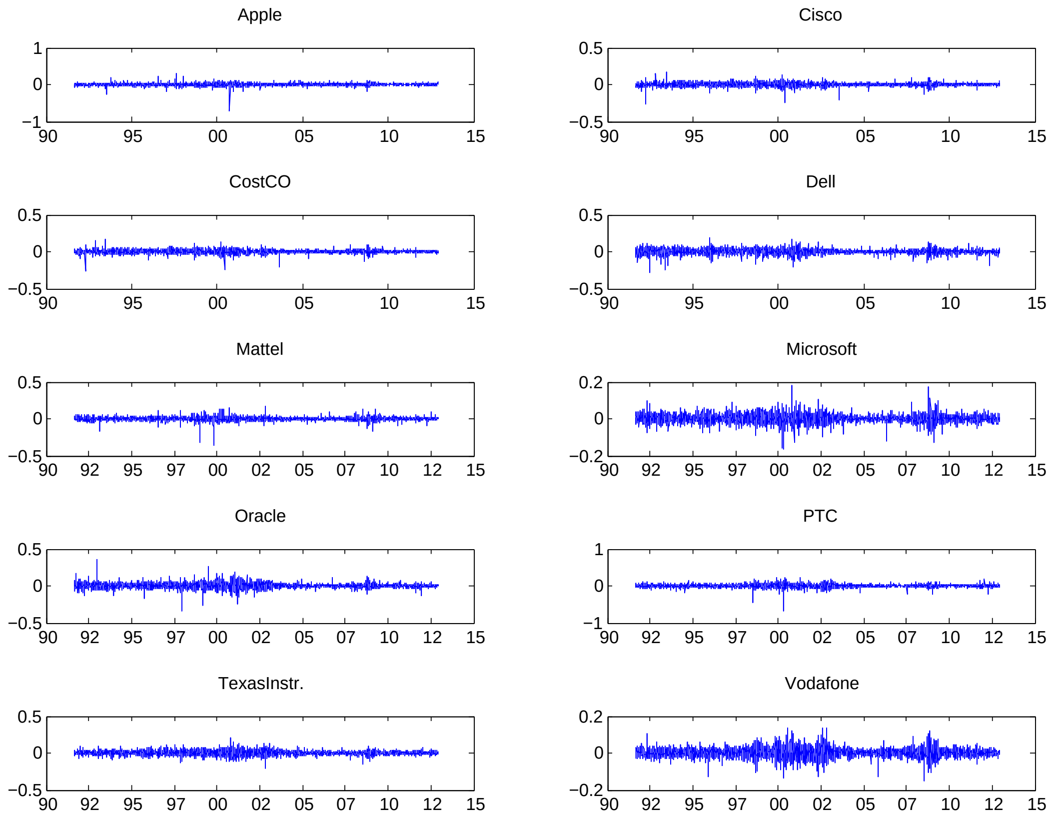


Figure 4.2: NASDAQ Composite - daily returns of stocks



Chapter 5

Estimating of systematic risk

5.1 Introduction to the problem

This chapter is dedicated to empirics. In this chapter we will find the estimates of systematic risk of CAPM via various methods introduced earlier in this thesis. Later in this chapter we will evaluate the results and based on this outcome, we will set the systematic risk-β hedge on the created portfolios.

5.2 OLS rolling window

First method that is employed is Ordinary Least Square applied on the rolling window. In this case the window is 250 observations long as we assume that this is significantly large window for daily data, as this window should cover approximately a trading year on a stock market.

New York Stock Exchange Figures A.1 and A.2 in Appendix show the systematic risk of each stock we have received by using OLS rolling window method. We can see from these pictures that evolution of systematic risk coefficients among the NYSE stocks is more or less smooth (American Express, GE, Proctor& Gamble and 3M). On the other hand IBM, Exxon and Alcoa's coefficients of systematic risk seems pretty volatile, this fact might be caused by the industry and market. These companies are operating on- IT, Oil&Gas and natural resources mining respectively. In case of Merck the progress in time seems as well as in case of IBM quite unpredictable, but we assume as well as in those other companies that it is because of the specification of industry, for instance in case of Merck it can be the expiration of patent on any of their pharmaceutical products.

National Association of Securities Dealers Automated Quotations According to figures A.3 and A.4 included in Appendix, we can see in comparison to NYSE's coefficients of systematic risk that the NASDAQ's stocks coefficients are similar to the size but apart from Vodafone, Oracle and Mattel they are pretty volatile. Furthermore, it can be seen that the industry may influence the pattern of the evolution of coefficients no matter the Stock Exchange, for instance if we compare Microsoft, Cisco and Apple we can see either periods of volatility or quite smooth periods. On the other hand, CostCo as a corporation, which is selling consumer goods experiences with pretty volatile evolution of betas, we assume that the competition on this field is high so it is hard to keep the level of income steady.

If we check other similar companies from different stock exchange such as Dell&IBM and CostCo& Wal Mart the pattern of evolution of coefficients is different. In Dell&IBM case it seems like Dell experiences with smother evolution, but this might be caused by wider IBM's portfolio of services offered. On the other hand, CostCo experiences with much larger volatility than Wal Mart.

5.3 DCC MGARCH

Secondly, for calculating of systematic risk coefficients within CAPM framework we have chosen Dynamic Conditional Covariance Multivariate GARCH. As mentioned in previous chapters we have specified window of 250 observations and as well as in OLS we had been moving it in time. As long as we obtained time varying correlations and standard deviations we used them to calculate systematic risk-$\hat{\beta}$ of each stock in time. We assume that by this step we have obtained more sensitive coefficients of systematic risk than we would in case of running DCC MGARCH on whole time series.

New York Stock Exchange In Appendix, there are figures A.5 and A.6 that show the progress of coefficients of systematic risk in time, which were obtained by using DCC MGARCH method used on rolling window. Generally when looking at the betas, we can say that they are more volatile then the coefficients obtained from OLS regression. In case of OLS we considered American

Express, GE and Proctor as being pretty smooth but in case of DCC MGARCH the results are opposite and the betas seems very volatile. Furthermore, 3M coefficients are similar to those obtained from OLS.

On the other hand, some companies whose OLS coefficients were considered as being volatile- IBM and Merck, has changed and now evolution of coefficients seem to be smooth in contrast with other stocks. Moreover, the volume of coefficients has increased, from the figures A.5. and A.6. we can see that the open interval has in most of the times doubled. All the changes are caused by the fact that DCC MGARCH by definition is more sensible on the volatility, which may provide us of very important results later when we will analyze performance of hedge.

National Association of Securities Dealers Automated Quotations Coefficients of systematic risk that were obtained from DCC MGARCH are showed in figures A.7 and A.8 in Appendix. When looking firstly one may say that coefficients are less volatile than those obtained from OLS or NYSE's coefficients. But when looking closer we can see that the rationale behind the "smooth" evolution are couple of outliers. For instance Apple's, Oracle's, Cisco's experiences with either huge jump of beta or decrease and so the interval of their betas has widen sometimes even five times and so visually made evolution of coefficients "smooth". Nevertheless, the coefficients evolution pattern is alike to the one NYSE stocks have, it means that it is more volatile than OLS estimates of systematic risk.

5.4 State Space Formulation Model

Last method that is used is State Space Formulation model, we assume to be better for estimation of coefficients of systematic risk than rolling window OLS. We also believe that it can challenge DCC MGARCH on the grounds of actual hedge performance. We assume that coefficients from SSF and following hedging of portfolios may result in the best performance in time of of our portfolios, which will be introduced in chapter 6.

New York Stock Exchange Figures A.9 and A.10 involved in Appendix show the coefficients that were obtained from the SSF modeling. Generally, we can see from these figures that the coefficients are not so volatile as the ones obtained from the DCC MGARCH method, which is not surprising given that the DCC GARCH based methods work in conditional volatility rather than "in mean equations". Furthermore, when comparing these coefficients with the ones obtained from Rolling Window OLS, we can see that there is slight difference among them especially when talking about the size and generally their progress in time experiences with very similar behavior. [1]. There is slight difference at the beginning of time series in AmEx, but then the coefficients from SSF smoothen and flow similarly as the ones from OLS.

National Association of Securities Dealers Automated Quotations The coefficients received from SSF are showed in Appendix on figures A.11 and A.12.. We can see that the coefficients are more or less similar to the ones obtained from OLS Rolling window. The size is also similar. The reason is that SSF model as we defined it (see chapter 3) estimates essentially the same thing but in a more complex way resembling extending window GLS, in other words it provides us with more accurate results than OLS.

5.5 Comparison of the Betas

Last section of this chapter will compare the estimates of systematic risk obtained by using various method. We have picked just one stock from each index, to show the difference between systematic risk coefficients, as the pattern of coefficients obtained from various method is alike for any of stocks we have picked for our analysis.

New York Stock Exchange From the NYSE we have chosen American Express, as you can see from the figure 5.1 that the evolution of betas obtained from SSF and OLS are almost the same as we mentioned in previous section of this chapter. On the other hand DCC MGARCH provides us of totally different scenario.

National Association of Securities Dealers Automated Quotations Figure 5.2 compares Mattel's coefficients obtained from different methods. The reason why we have picked Mattel is the volatility of DCC MGARCH, which is in comparison to Apple less volatile and so other estimates (OLS and SSF) are visible on common figure. Once again as you can see SSF coefficients are more or less same as the OLS so they almost cover each other.

[1] See Chapter 3 on methodology.

Figure 5.1: NYSE comparison of coefficients

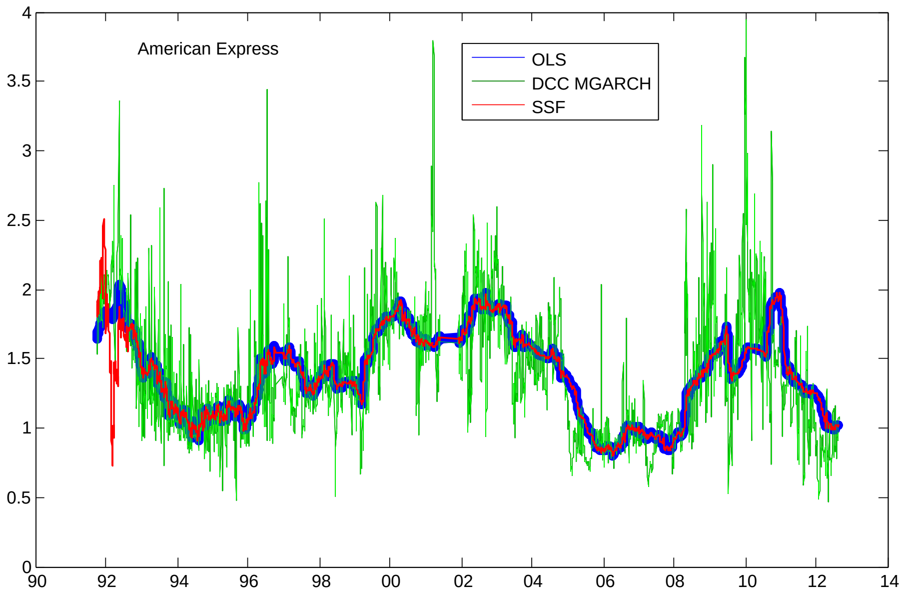
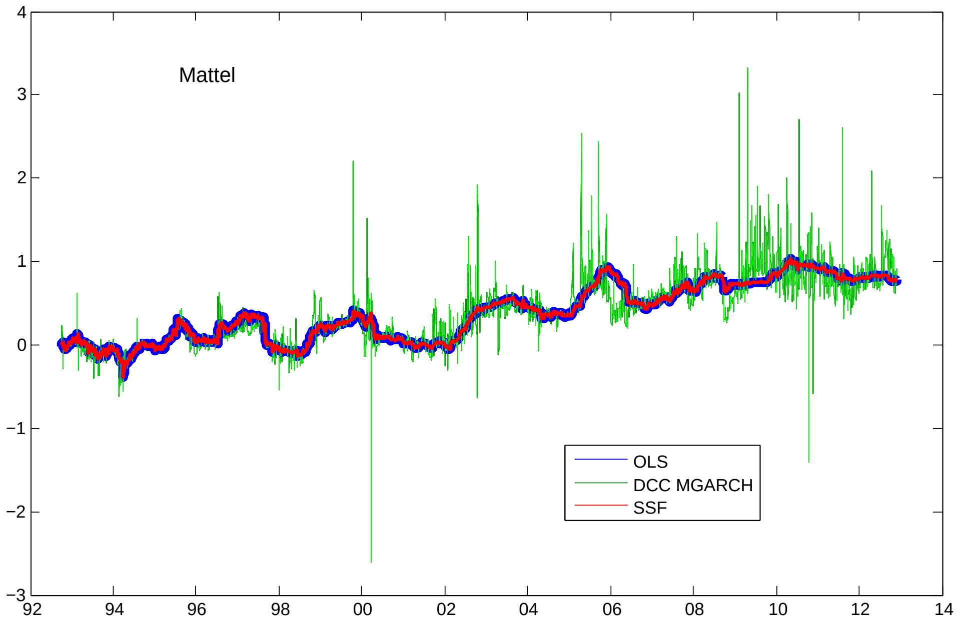


Figure 5.2: NASDAQ comparison of coefficients



Chapter 6

Weighted portfolios and their hedge performance

This chapter is the second empirical part of our thesis and this part is directly related to the previous one as we will use the estimates of betas for hedging portfolios. As soon as we know the systematic risk-$\hat{\beta}$, we would like to offset the loss/gains that may occur. At the beginning of this chapter we will introduce the hedging strategy, which will be applied on the portfolios and then we will briefly explain how we structured each portfolio and at the end of this chapter we evaluate the results of how each of portfolio performs. We assume that both portfolios hedged with coefficients obtained from DCC MGARCH should perform better than portfolios hedged with OLS and SSF coefficients. Moreover, when comparing SSF and OLS coefficients hedge we suppose that SSF hedge will perform slightly better as this method applied on this task works similarly as OLS but the estimates are more accurate.

6.1 Hedging strategy

To see whether we can exploit different approaches for estimation of betas we propose a $\hat{\beta}_t$ hedging strategy. The strategy is simple and it follows simple rule, where we start 1000 dollars long of $\frac{1}{\hat{\beta}_t}$ in each portfolio - Weighted and Equally Weighted and short of particular index - NYSE Composite and NASDAQ Composite. Furthermore, for every period of 250 trading days we evaluate the performance and will rebalance the portfolio to match the initial hedge by using newly estimated $\hat{\beta}_t$. We also keep the track of money borrowed that are needed to rebalance the hedge. We assume that risk free rate is equal to zero, so is does not change over time, thus just effectively keeps track of balances on daily basis.

Moreover, as long as we finish with this moving of the hedge in time, we will obtain the results of each portfolio in time, so we will be able to show how much our beta hedging strategy employed on couple of portfolios gained or lost.

Based on these findings we will be able to conclude, which method from those three might be the best when estimating the systematic risk coefficient within CAPM framework.

6.2 Structure of the portfolios

Apart from hedge, we will use another tool to decrease the risk - diversification of portfolio, where we will structure the portfolio from the stocks we have mentioned in chapter 4. According to Elton and Gruber (1977), the risk related to portfolio, when 10 stocks are included in portfolio, decrease to a half in contrast to one stock risk. We propose two options for structuring the portfolios in this thesis- weighting of portfolios based on the market cap and the second option is equally weighted portfolio, which means that each of the stock will have weight in our portfolio equal to 10%.

6.2.1 Weighted portfolios

This part is dedicated to the weighted portfolio as we need to introduce the weights of each stock. Moreover, each stock included in portfolios will have the different weight, which is derived from the market cap of each stock to the October 2012. We assume that this approach would provide us with better portfolio than we would obtain from equally weighted portfolio. According to this weights we will calculate the returns of the weighted portfolios and also the time varying betas of these portfolios, which we will get as well as the returns by weighting the stocks betas in the same proportion as returns. Weights of each stock are shown in the tables 6.1 and 6.2.

As you can see from the table 6.1 the largest impact on our NYSE analysis will have the evolution of stocks of Exxon Mobile and General Electric, which are followed by IBM and Proctor&Gamble. On the other side, the least influence will have Alcoa, the mining company.

From the table 6.2, we can see that there are two large companies, whose stock evolution will influence the most our analysis of hedging, these are Apple

Table 6.1: NYSE Composite

Stock	Weight
General Electric	14.1%
ExxonMobile	23.23%
Proctor & Gamble	12.1%
Wal Mart	14.1%
Alcoa	0.67%
3M	4.2%
IBM	13.9%
Merck	7.7%
American Express	4.2%
McDonald's	5.8%
Total	100%

Table 6.2: NASDAQ Composite

Stock	Weight
Apple	31.5%
Cisco	9.6%
CostCo	4.1%
Dell	2.3%
Mattel	1.8%
Microsoft	21%
Oracle	13%
PTC	0.9%
Texas Instruments	3.1%
Vodafone	12.7%
Total	100%

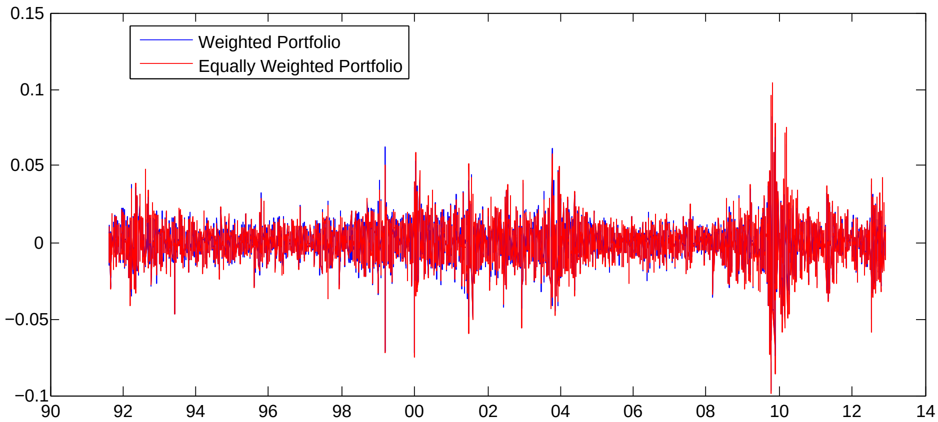
and Microsoft. Apart from the Vodafone, there are more or less similar weights among the rest of the companies included in our portfolio.

6.2.2 Weighted vs. Equally Weighted Portfolios

In this subsection of this chapter, we are going to compare the returns of two portfolios- Equally Weighted and Weighted according to market cap.

NYSE Composite If we look on the figure 6.1 we can see the evolution of returns of NYSE stocks portfolios. We can see that Equally Weighted Port-

Figure 6.1: NYSE-portfolios returns



folio's returns more or less cover the returns of the Weighted Portfolio. That might be caused by the fact that some of the stocks, whose weight is based on the market capitalization was low then has increased significantly, for instance Alcoa's weight increased from 0.67% to 10% and its individual daily returns are quite volatile (around year 2010, according to figure 4.1) and so the returns of

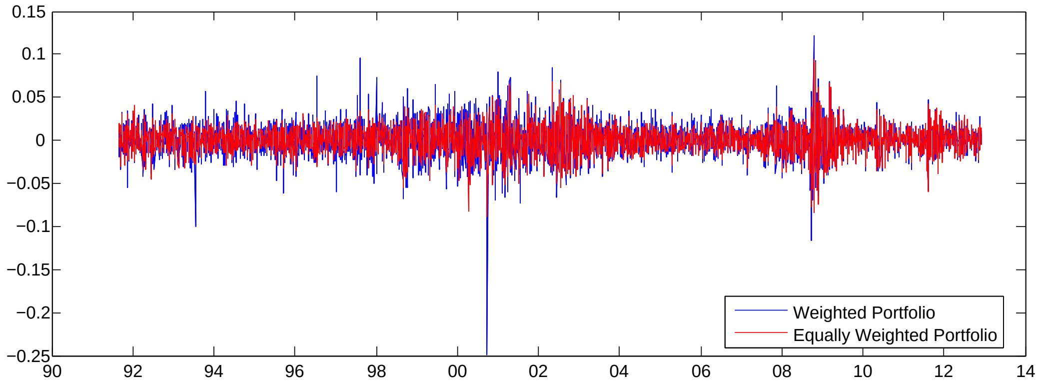
Equally Weighted Portfolio differs. Moreover, other stocks that are responsible of this EWNY's evolution of returns are American Express, McDonald's and 3M, which have also experienced with significant changes in returns over time (figure 4.1) and their weights have increased during creating Equally Weighted Portfolio.

Furthermore, the daily returns of both portfolios span to the interval between 10% and minus 10%. We can see that the market was very volatile around 2010, which could be the influence of the World recession, which has begun in 2007. There is a reaction delay, but this may be because of the rigidity of industry, long term contracts or just uncertainty on the market situation and postponing of the economic situation of the company.

NASDAQ Composite Figure 6.2 shows the returns of both portfolios created from the NASDAQ stocks. When looking closely, we can see that Equally Weighted Portfolio is less volatile than the one with different weights. In comparison to the NYSE's scenario, which is opposite, we assume that this is caused by the choice of the company and the weights respectively. In case of the weighted portfolios, the weight shares leaders are Apple and Microsoft, whose share is approximately 53% (3 companies' share in NYSE portfolio is similar). Therefore, "weakening" of the share of these stocks in Equally Weighted Portfolio decreases also its returns as these companies are worldwide leader in the industry, they are operating on.

Moreover, if we omit the couple of outliers, the returns span to the interval between 10% increase and 10% decrease so it is similar to the NYSE portfolios. When looking closer, we can see, that there is huge decrease in returns of Weighted Portfolio by the end of 2000. If we look at the individual evolutions of returns (figure 4.2), we can see that Apple and Microsoft (highest weights in portfolio) experienced of the large decrease during that period, but not only these two companies but also Vodafone and Cisco, whose weights are also pretty high and together with Apple and Microsoft have approximately 75% of portfolio's weights. The reason of this fall was the burst of the internet bubble in 2000, also known as Dot-com bubble. This can be also seen when looking at the figure 6.3, which shows the evolution of prices of NASDAQ.

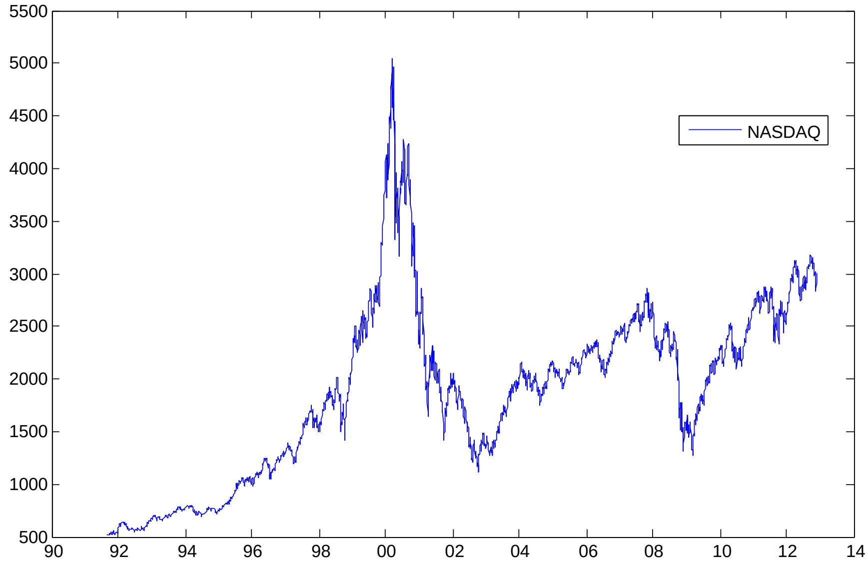
Figure 6.2: NASDAQ-portfolios returns



6.2.3 Systematic risk of portfolios

This part of this chapter will discuss and show the systematic risk coefficients of both portfolios created from two indexes. Furthermore, this subsection is divided into two paragraphs each consisting of one index. You can also find in

Figure 6.3: NASDAQ- Dot-com bubble 2000



Appendix comparison of each portfolios' coefficients according to the method used for their estimation. (Figures A.13-A.18)

NYSE Composite Figure 6.4 shows us the comparison of the systematic risk coefficients of two portfolios. We can see that the evolution of these coefficients is very similar between Weighted Portfolio and Equally Weighted Portfolio. The reason why it is like that is the weighting of stocks, where four stocks' weights are more or less equal to those weights used for Equally Weighted Portfolio. In comparison to portfolios created from stocks from NASDAQ the scenario is different as most of the weight belongs to Apple and Microsoft and so each portfolio's coefficients are different.

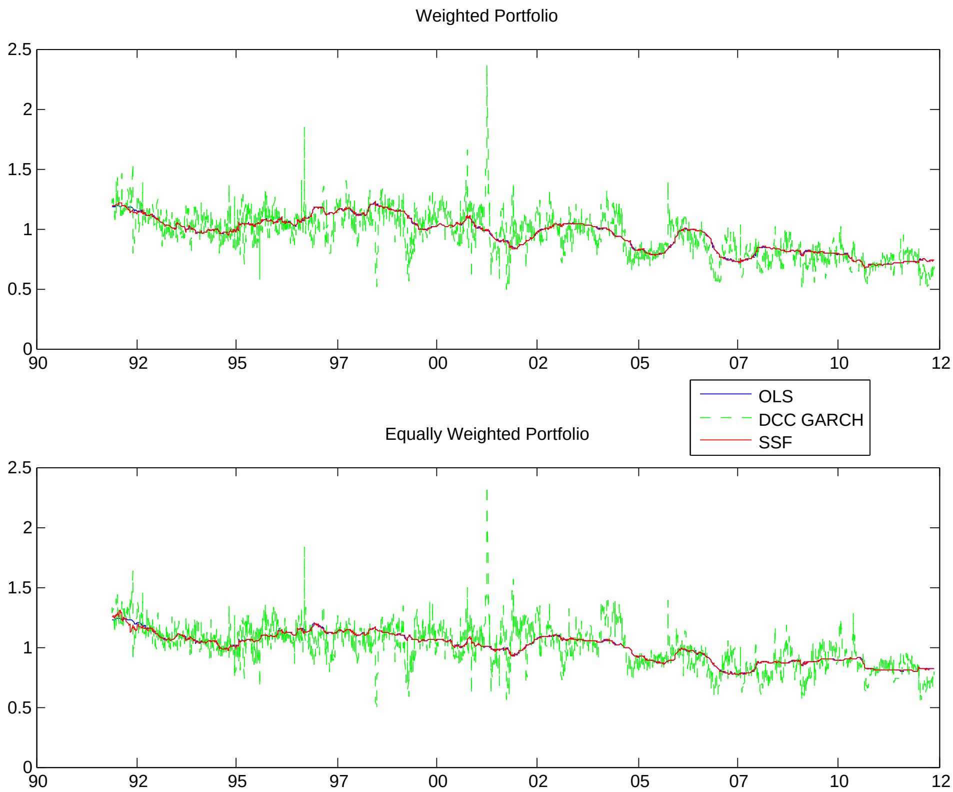
When comparing among the methods, DCC MGARCH estimates of systematic risk are the most volatile, whereas coefficients obtained from the OLS and SSF are alike and quite smooth. In figure 6.5 you can see zoomed period of figure 6.4 to see that OLS estimates are alike to those obtained from SSF. Moreover in Appendix on figures A.13, 14 and 15 you can see that different weighting of stocks in our portfolios has not caused any major change of systematic risk coefficients. Thus, we expect that hedging of both portfolios and its performance in time should provide us of very similar results.

NASDAQ Composite When we look at the figure 6.6 we can see that coefficients of Weighted Portfolio are larger as they are very sensible on Apple and Microsoft changes. In case of Equally Weighted Portfolio, we can see that the evolution of coefficients of systematic risk is more volatile than in case of Weighted Portfolio. As you can see on the figure 6.6 the Dot-com burst of bubble that we mentioned before is also noticeable on the figure.

Moreover, if we compare the coefficients based on the method used for their estimation, we can see that SSF coefficients are very similar to those obtained from OLS rolling window (figure 6.7). On the other hand, coefficients obtained from DCC MGARCH are the most volatile among all and also the largest among others.

In addition, if we look at the figures included in Appendix (A.16,17,18), we can see that there is a difference between Weighted and Equally Weighted Portfolios' systematic risk coefficients. In comparison to the NYSE's figures (A.13-15) it is markable difference especially during nineties of the twentieth century. We can also see that OLS estimates are similar to SSF estimates, whereas DCC MGARCH estimates have totally different pattern.

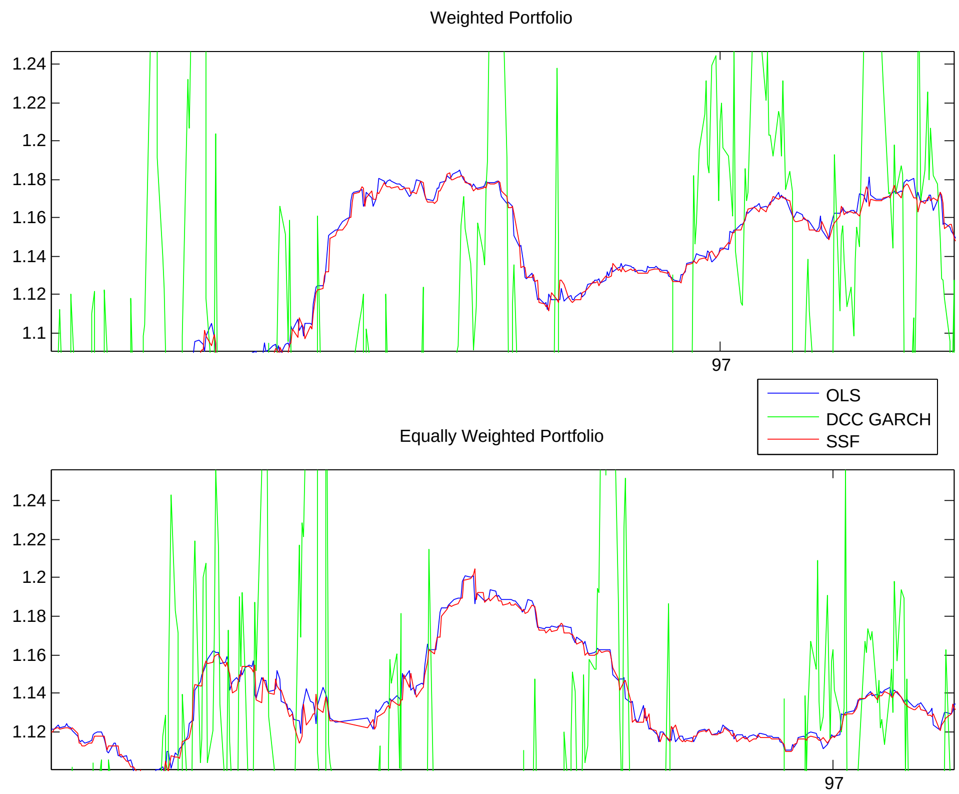
Figure 6.4: Systematic risk coefficients- NYSE Portfolios



6.3 Performance of hedge in time of portfolios

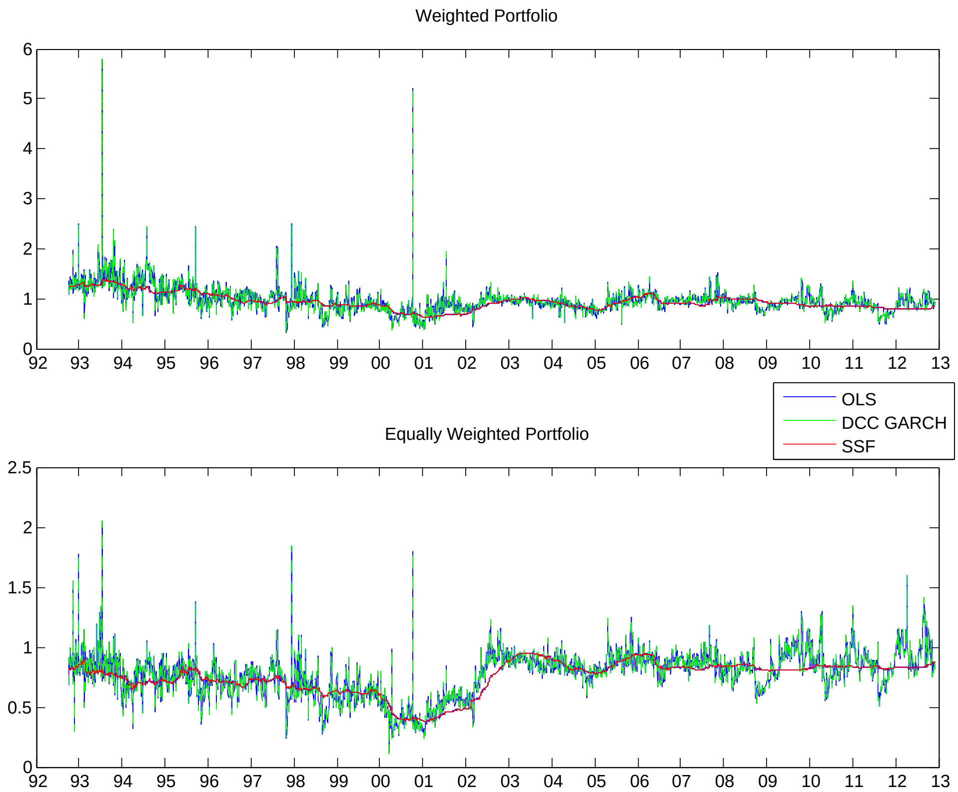
Last part of this chapter is dedicated to the most important part of this thesis - Beta hedging and its performance. The strategy used is explained in previous section of this chapter. This section is divided into two parts, where each

Figure 6.5: Zoom- Systematic risk coefficients- NYSE Portfolios



consist of two paragraphs each dedicated to one portfolio.

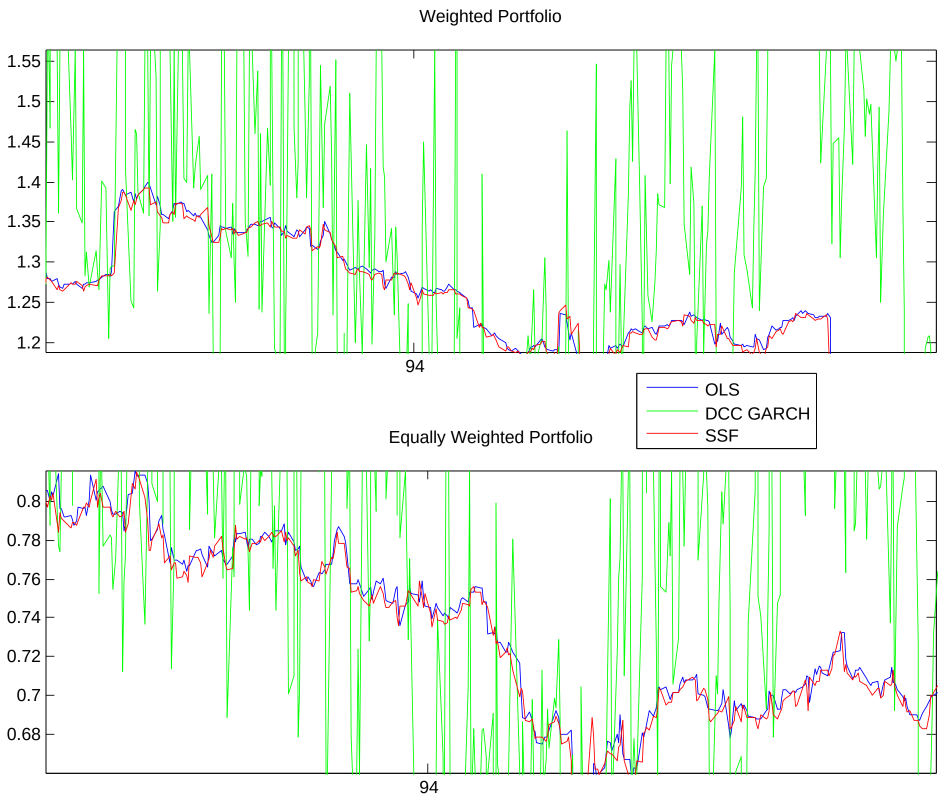
Figure 6.6: Systematic risk coefficients-NASDAQ Portfolios



6.3.1 NYSE Composite

According to the previous analysis, we expect that the results of hedge of each both portfolios will be alike, because of the structure of weights used for creating Weighted Portfolio, which are not so dependent to the change of one

Figure 6.7: Zoom-Systematic risk coefficients- NASDAQ Portfolios



or two stocks. Apart from the ExxonMobile's weight and Alcoa we may say that the weights used for the Weighted and Equally Weighted Portfolios are similar with the same impact.

Weighted Portfolio vs. Equally Weighted Portfolio

Firstly, we will compare the day by day changes of each portfolio based on the method used for estimating the coefficients of the systematic risk. Figure 6.8 and 6.9 shows a daily change of the values of our portfolios in time. We can see that the values of either Weighted or Equally Weighted are very alike among the methods and even more the values of OLS and SSF are very similar.

On the other hand we can see that daily changes of portfolio hedged with DCC MGARCH estimates of systematic risk are the most volatile among all and the size of either decrease or increase are sometimes 10 times higher than those changes of values of portfolio hedged with OLS and SSF coefficients. Moreover, we can see that the changes of hedged portfolios are similar. So in this case we can say that weighting does not changed the performance of hedged portfolios.

Hedge Performance

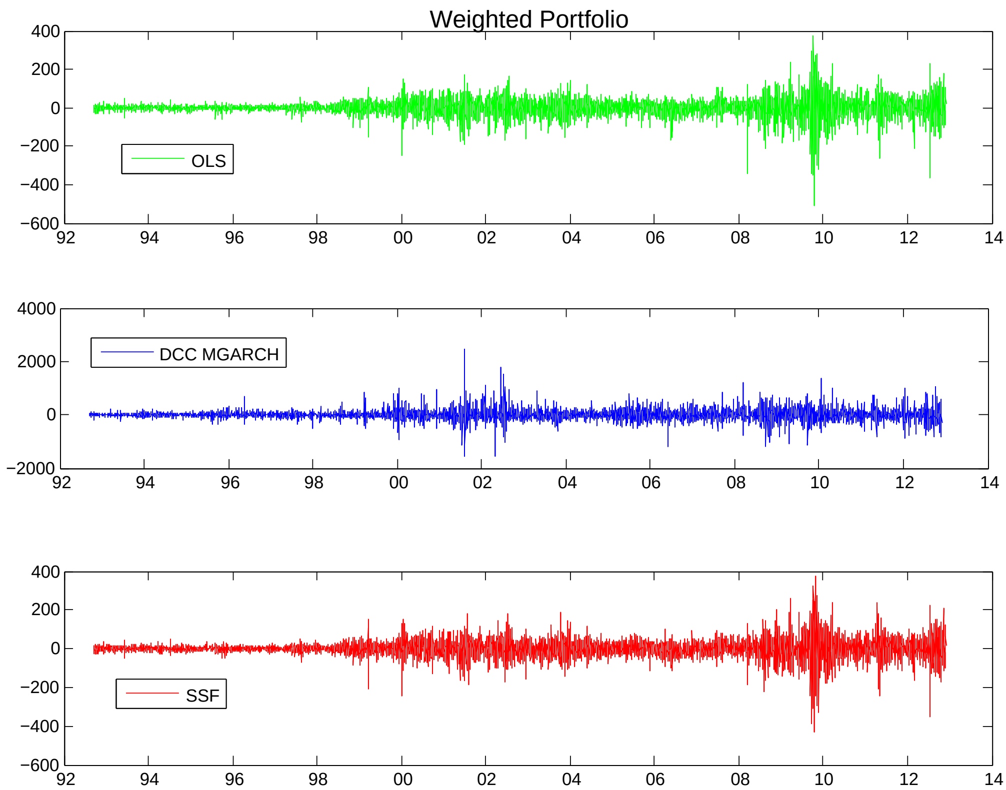
Secondly, based on previous part we will evaluate the gains or loss of each portfolio and later in conclusion we will discuss on the best method to hedge the risk of portfolio. Figure 6.10 and 6.11 show a performances of hedges applied on each portfolio. We can see that the changes between OLS and SSF are more or less equal. On the other hand DCC MGARCH experiences with large volatility, but at the end this volatility results with the highest value of both portfolios.

Furthermore, in table 6.3 are final gains of our hedge applied on both portfolios. We can see that our hedge was successful and no matter the method we did not lose anything. Moreover, DCC MGARCH hedge gained on average approximately 200 dollars more than OLS and SSF hedge. We can also see that Weighted Portfolio performed better than Equally Weighted Portfolio, the reason behind is that the largest weight belongs to Exxon Mobile and when looking at individual performance of Exxon Mobile stocks (figure 4.1) it does not experience with any significant decrease. Annual yield is calculate as follows:

$$y_a = \frac{V_p - V_0}{V_0} * \frac{1}{t} \tag{6.1}$$

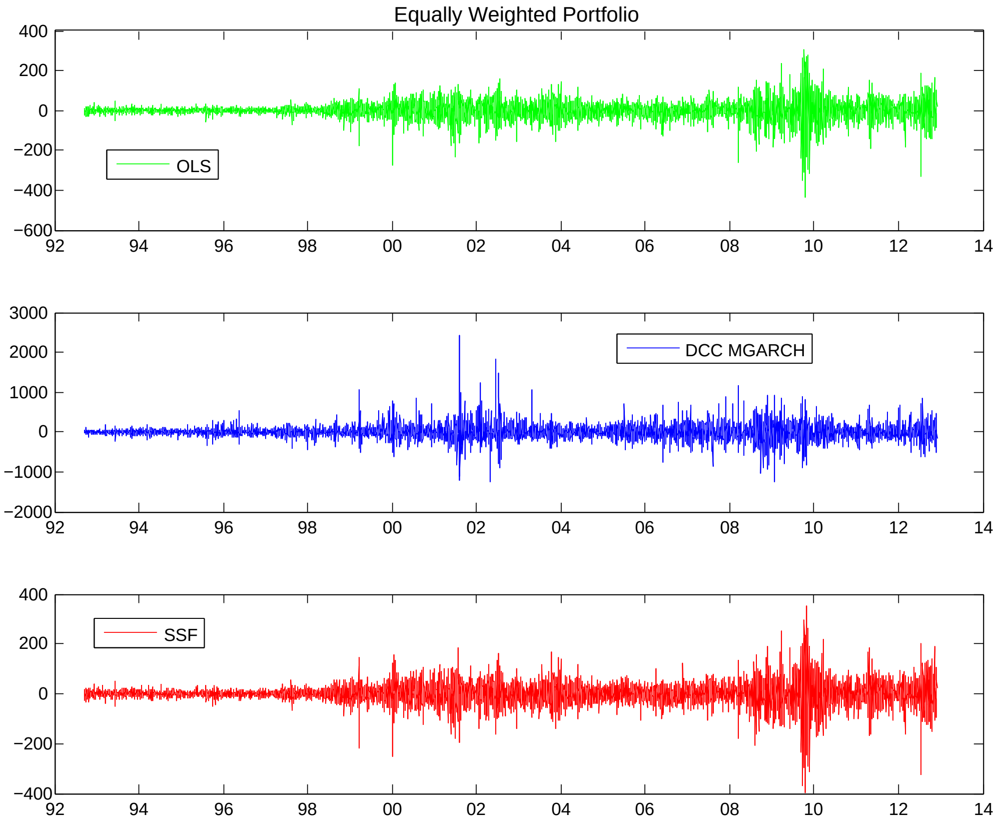
where V_p stands for the final value of portfolio, V_0 is the initial value of portfolio,

Figure 6.8: NYSE- daily changes of value



which equals to 1000 dollars for both portfolios and t is number of years of hedge simulation.

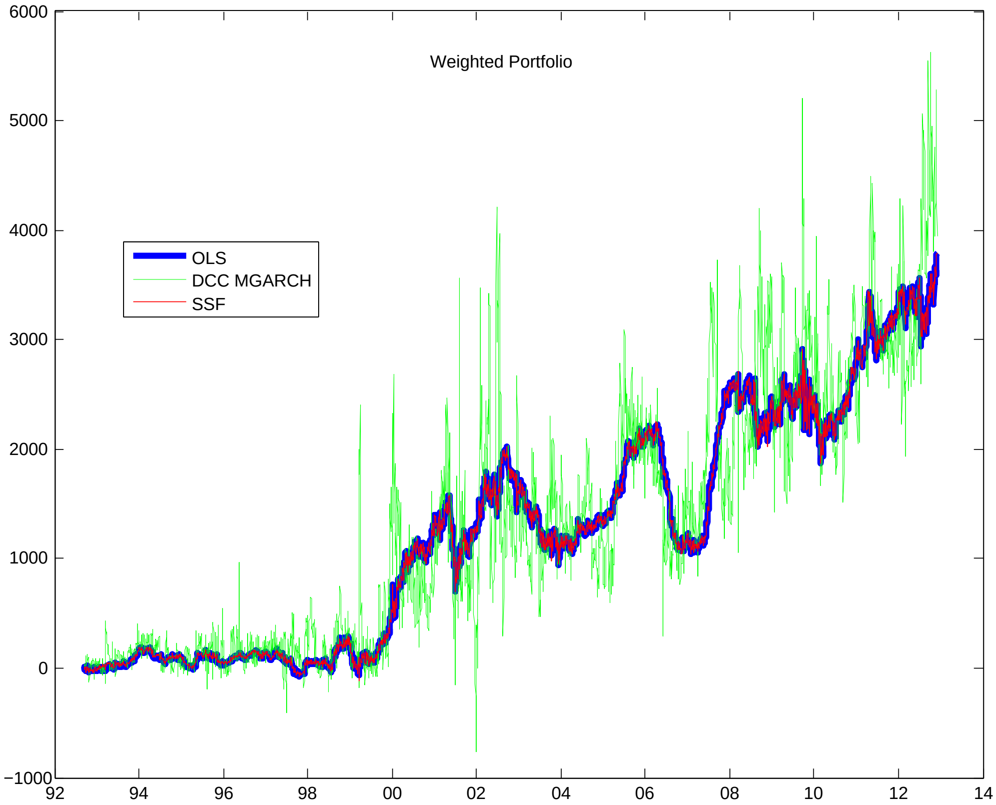
Figure 6.9: NYSE- daily changes of value



6.3.2 NASDAQ Composite

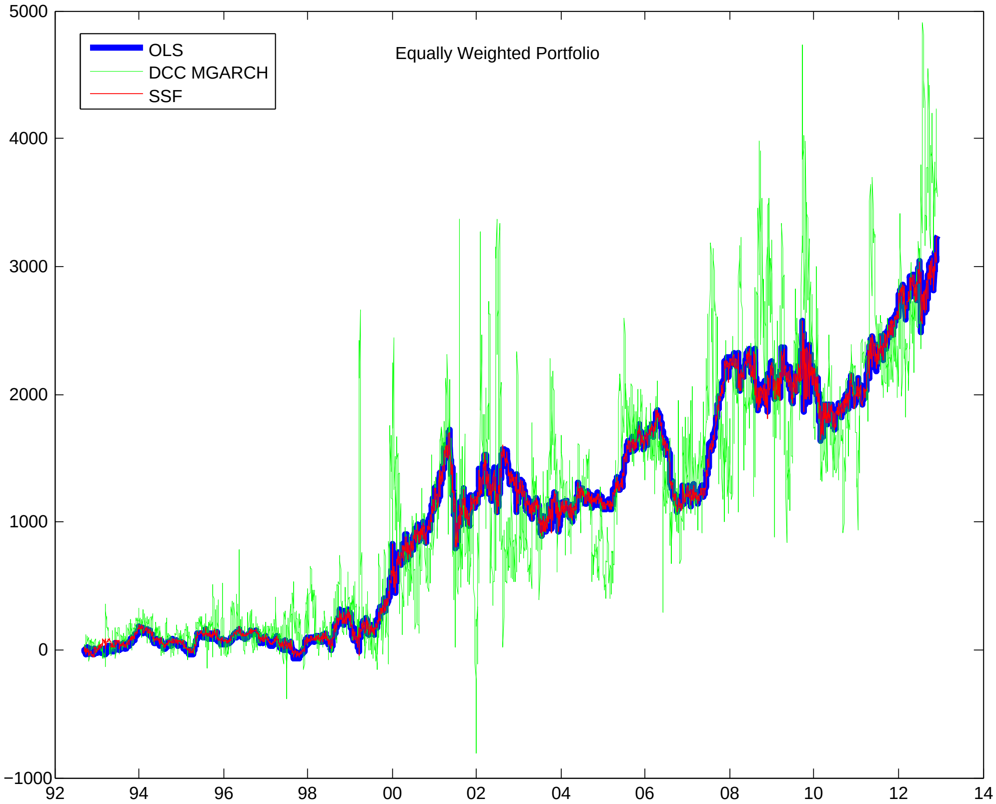
Second index we have picked is NASDAQ Composite. We assume in this case that Weighted Portfolio may be quite biased as the two companies' weight share is around 50%. But on the other hand, we expect that this fact will provide

Figure 6.10: NYSE - Weighted Portfolio hedge performance



us with different results between Weighted and Equally Weighted Portfolios' performances.

Figure 6.11: NYSE - Equally Weighted Portfolio hedge performance



Weighted Portfolio vs. Equally Weighted Portfolio

According to the figures 6.12 and 6.13, which show daily changes of the hedge value we can see that portfolios created from NASDAQ stocks are more volatile than NYSE portfolios. Moreover, the size of daily changes are sometimes

Table 6.3: NYSE- Performance of hedge

Value of:	Weighted Portfolio	Equally Weighted Portfolio
OLS	3794	3235
annual gain/loss	12.59%	10.07%
DCC MGARCH	3949	3544
annual gain/loss	13.29%	11.47%
SSF	3797	3237
annual gain/loss	12.61%	10.08%

100times larger than in case of NYSE portfolios. It is caused by the fact that Apple and Microsoft, both very successful companies, are sharing approximately half of the weights and in case of Equally Weighted Portfolio other very successful IT companies such as Oracle and Cisco influences the portfolio hedge. When looking at the returns of stocks of these four companies (figure 4.2) we can see that all has experienced with expansion and rapid growth since 1991.

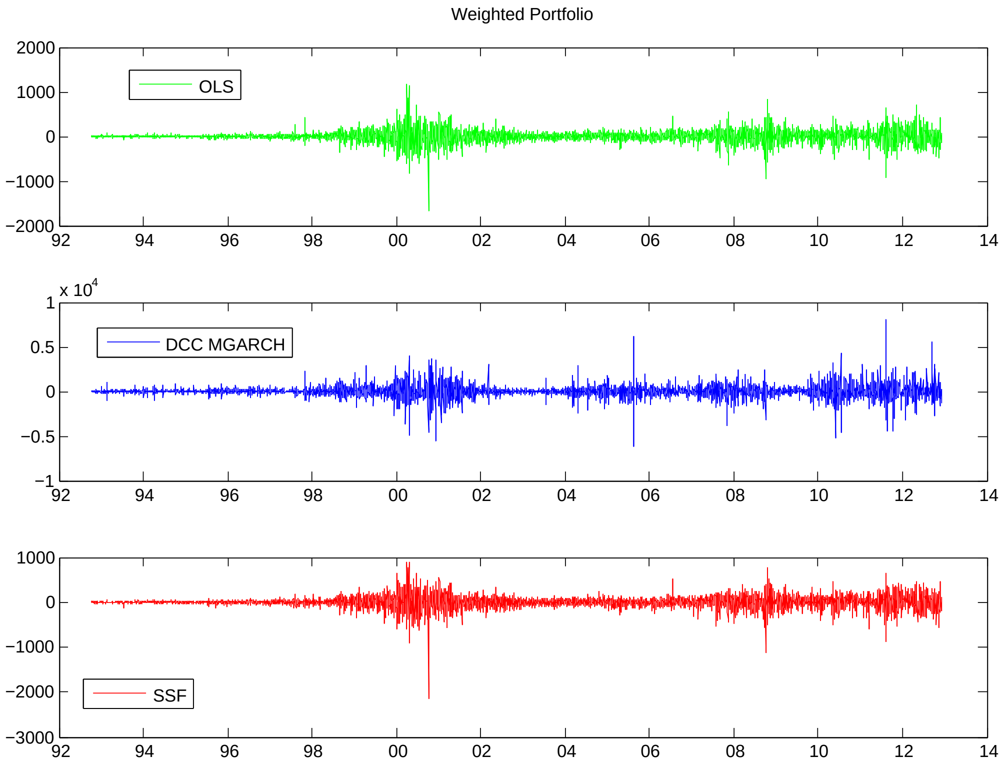
Once again, when looking closer we can see that Dot-com crisis influenced the daily changes of hedged portfolios' values very significantly. In case of OLS and SSF both portfolios experienced with large decrease. In addition, DCC MGARCH hedge performed even worse than OLS and SSF hedge and so the daily change of portfolio value was decrease of approximately 5000 dollars in case of Weighted Portfolio and around 7000 decrease in case of Equally Weighted Portfolio. We assume that this significant decrease may result in worse performance of our DCC MGARCH hedge than OLS and SSF hedge performances.

Hedge Performance

In addition, figures 6.14 and 6.15 show the evolution of the values of hedged portfolios in time. We can see that Weighted Portfolio performed better than the Equally Weighted Portfolio. The reason is high dependence of Weighted Portfolio on Apple and Microsoft stocks returns. Moreover, when comparing performance of hedge according to methods used for estimating the coefficients we can see that OLS and SSF hedge perform better than DCC MGARCH hedge no matter the type of portfolio is.

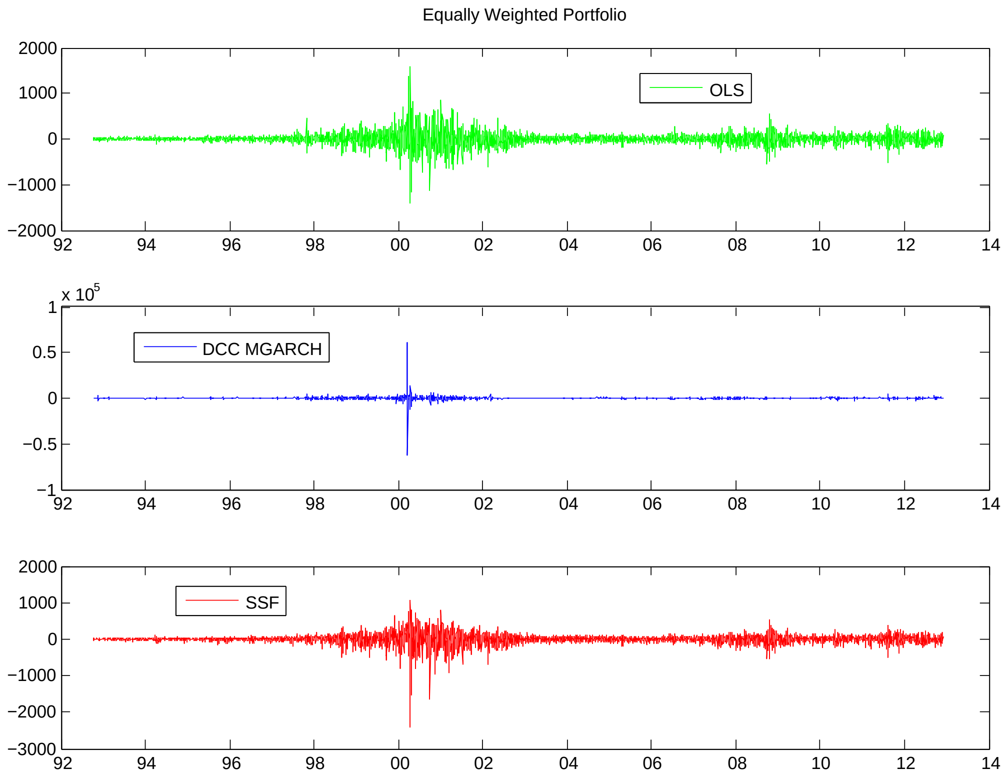
We assume that this is caused by the definition of the DCC MGARCH and

Figure 6.12: NASDAQ daily changes of value



its sensitivity on price changes. Another reason is the burst of bubble and following internet crisis, which had influenced most of the stocks returns of picked companies that mostly operates in industry, which is very related to the internet. Furthermore, table 6.4 show the performance of each hedge sorted by method. As mentioned before, OLS and SSF performed better than DCC

Figure 6.13: NASDAQ- daily changes of value



MGARCH by approximately 2000dollars in case of Weighted Portfolio and 200 dollars in case of Equally Weighted Portfolio. Annual yield is calculated according to formula 6.1.

Figure 6.14: NASDAQ - Weighted Portfolio hedge performance

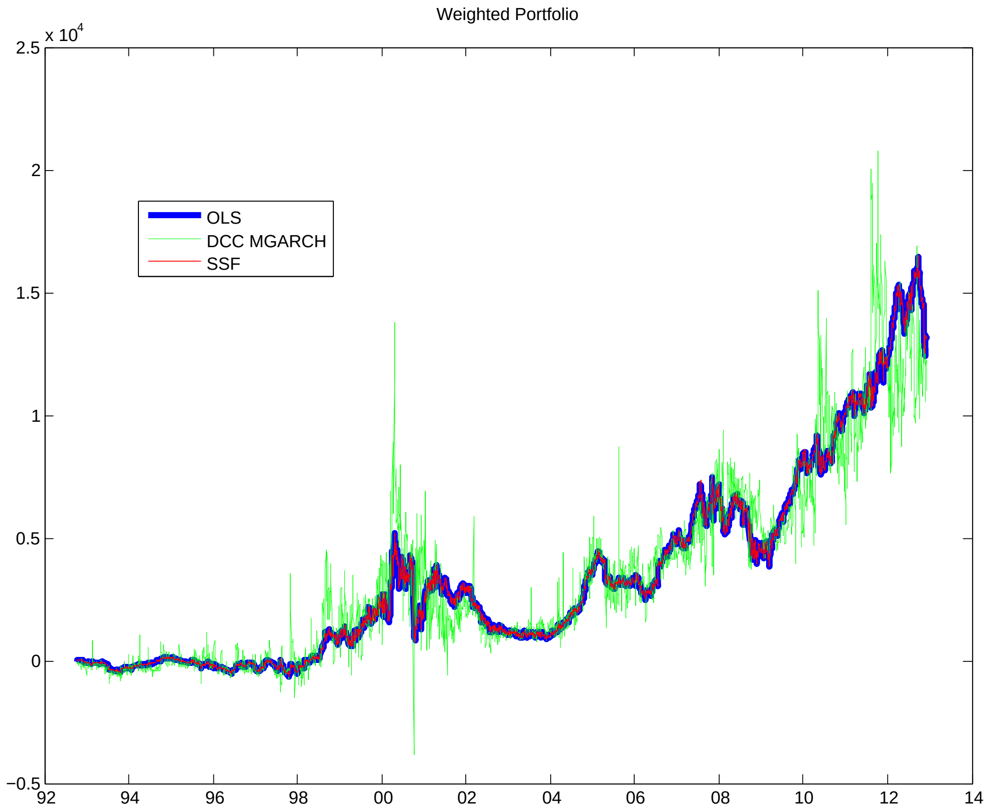


Figure 6.15: NASDAQ - Equally Weighted Portfolio hedge performance

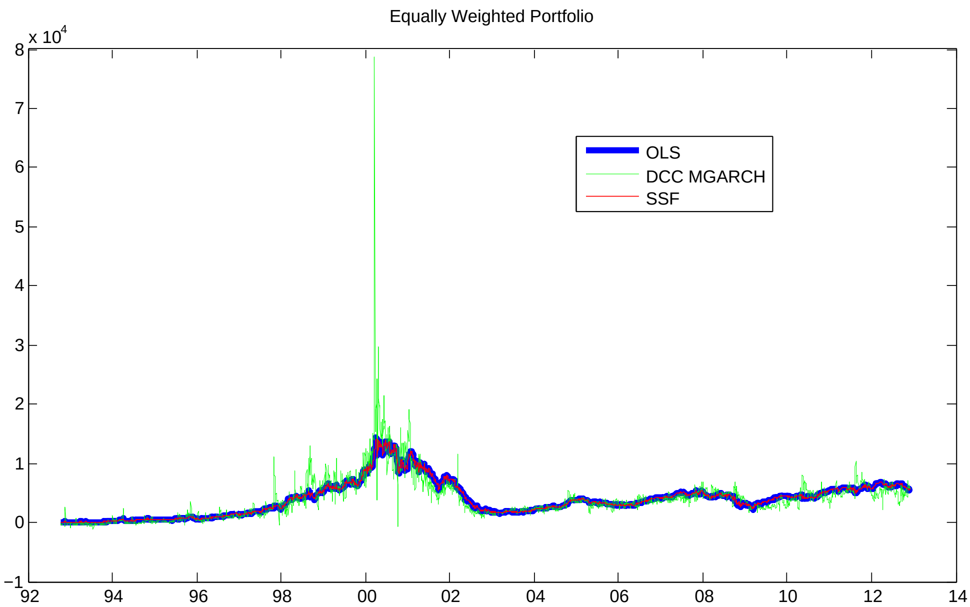


Table 6.4: NASDAQ-Performance of hedge

Value of:	Weighted Portfolio	Equally Weighted Portfolio
OLS	13229	5704
annual gain/loss	54.3%	20.89%
DCC MGARCH	10976	5630
annual gain/loss	44.3%	20.56%
SSF	13203	5706
annual gain/loss	54.19%	20.9%

Table 6.5: Total gains of Portfolios' value based on the method

Method	Total gains
OLS	25962
DCC MGARCH	24099
SSF	25943

Chapter 7

Conclusion

The goal of this master thesis was to find the most suitable method to estimate systematic risk within the CAPM framework. For this purpose we have picked three methods, which we supposed to be the most appropriate to solve this task. These methods were Ordinary Least Square 250 days Rolling Window, Dynamic Conditional Covariance Multivariate GARCH also applied on the window of 250 days and the last method was State Space Formulation model, which as well as two previous methods was applied on the 250 days rolling window. All methods have provided us with relevant results of time varying estimates of systematic risk $\hat{\beta}_t$, which we used for hedging the risk of two portfolios from different stock exchange index- NYSE and NASDAQ. We have designed beta hedging strategy and observed the performance in time of both hedged portfolios -Weighted and Equally Weighted. We discovered that Weighted Portfolios, no matter the stock exchange, perform better than Equally Weighted Portfolios. In case of NYSE there is a slight difference between the portfolios performances but in case of NASDAQ the difference between performance of portfolios is huge, in a sense that Weighted Portfolio value after 50 years of hedge simulation is more than twice larger than value of Equally Weighted Portfolio composed from same stocks.

The reason is weighting of the stocks in Weighted Portfolio as mentioned previously, thus the Weighted Portfolio created from NASDAQ stocks is very sensitive and influenced by the stock price changes of Apple and Microsoft. That also might be the reason of very large annual gains of our hedge applied on NASDAQ Weighted Portfolio as these two corporates have experience with steady growth since 1991 as they have been the ones who have begun the computer boom worldwide. We could have experienced with different scenario if we would added Lehman Brothers stocks into our set of stocks but fortunately this company did not meet our quotation period condition, when we were picking the stocks for our analysis.

Furthermore, the core of our research was to find the most suitable method to estimate the coefficients of systematic risk. We have found out that in case of less volatile stock exchange - NYSE, the best hedge results were obtained from the DCC MGARCH coefficients, whereas hedges built on coefficients of systematic risk obtained from OLS and SSF methods provided us with worse performance, but still our hedging strategy offsets the risk and the values of portfolios increased. Moreover, if we compare the performances of OLS and SSF hedges there is a slight difference between values of portfolios. Nevertheless SSF method and its systematic risk coefficients hedge performed slightly better than OLS.

On the other hand, in case of NASDAQ the results are twofold. We may say that the most suitable methods for estimating systematic risk for hedging purpose according to NASDAQ analysis are OLS and SSF, as their estimated coefficients of systematic risk have been very powerful and outperformed the hedge performance of DCC MGARCH coefficients. That is quite surprising as DCC MGARCH performed very well in case of NYSE. But we blame the Dot-com bubble, which has decreased DCC MGARCH values of hedge by approximately 6000 dollars in 2001, which at the end could had influenced the values of both portfolios composed from NASDAQ stocks. The decrease in 2001 also influenced OLS and SSF hedge performance as well, but the proportion of this decrease was not so large in contrast to the DCC MGARCH rapid decrease. Nevertheless, if we would split period into few smaller time interval, we would see that DCC MGARCH and its coefficients hedge performs better in each interval, while omitting the burst of the internet bubble. In table 6.5 we have summarized the results we obtained in chapter 6. We can see that the difference amongst the results is not so large and hedge built on OLS and SSF systematic risk coefficients performed almost similarly. Also we can see that even though DCC MGARCH coefficients based hedge outperformed OLS and SSF in case of NYSE Portfolios when we added results from NASDAQ it has become the "weakest" method amongst all.

To conclude, based on our findings we may say that the most suitable method to find the systematic risk coefficients and then $\hat{\beta}$ hedge the risk of portfolios is DCC MGARCH, even though the results obtained are not very convincing. Although it may provide us with very volatile evolution of values in time and so may persuade the investors to withdraw from this hedging strategy based on DCC MGARCH coefficients. Moreover, we suppose that in long term this hedge performs better amongst the others. On the other hand, when the stock market is unpredictable and the investor is averse to risk, we would recommend the investor to choose one of the method less sensible on volatility, in our case OLS and SSF to find the correct coefficients to hedge the risk of the portfolios.

Chapter 8

Related Literature

- Adrian T., F.Franzoni, 2008: *Leaarning About Beta: Time Varying Factor Loadings, Expected Returns, and the Conditional CAPM*, SSRN.com, id391562

- Ang, A., J. Chen, 2003 :*CAPM Over the Long-Run: 1926-2001*, Working Paper, Columbia University and University of Southern California

- Banz, Rolf W., 1981: *The Relationship Between Return and Market Value of Common Stocks Journal of Financial Economics. 9:1, pp. 3-18.*

- Bauwens, L., S. Laurent, J. V. K. Rombouts, 2006 :*Multivariate garch models: a survey.* Journal of Applied Econometrics 21(1): pp. 79-109.

- Basu, S, 1977: *Investment Performance of Common Stocks in Relation to Their Price- Earnings Ratios: A Test of the Efficient Market Hypothesis* Journal of Finance. 12:3, pp. 129-156.

- Black, F. , 1981 : *An Open Letter to Jack Treynor* The Financial Analysts Journal July/August. Letters to the Editor, p. 14.

- Black, F., 1972: *Capital Market Equilibrium with Restricted Borrowing* Journal of Business. 45:3, pp. 444-454.

- Black, F., Jensen M. and M. Scholes, 1972: *The Capital Asset Pricing Model: Some Empirical Tests*, in Studies in the Theory of Capital Markets. Michael C. Jensen, ed. New York: Praeger, pp. 79-121.

- Blume, M. , 1970: *Portfolio Theory: A Step Towards Its Practical Application* Journal of Business. 43:2, pp. 152-174.

- Blume, M and I. Friend, 1973: *A New Look at the Capital Asset Pricing Model* Journal of Finance. 28:1, pp. 19-33.
- Bollerslev, T., Engle, R., and Wooldridge, J., 1988:*A capital asset pricing model with time-varying covariances* The Journal of Political Economy, pages 116-131.
- Bollerslev, T., 1990:*Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model* The Review of Economics and Statistics, 72(3):pp. 498-505.
- Bollerslev, T., 1986:*Generalized autoregressive conditional heteroskedasticity* Journal of Econometrics 31(3): pp. 307-327.
- Box, G. E. P. , G. M. Jenkins,1970:*Time series analysis: Forecasting and control.* San Francisco: Holden-Day.
- Brennan, M. J., 1970 :*Taxes, Market Valuation, and Corporate Financial Policy* National Tax Journal 4, 417-427.
- Campbell, John Y., T.Vuolteenaho, 2002: *Bad Beta, Good Beta*, Working Paper, Department of Economics, Harvard University.
- Christiansen Ch., A. Ranaldo, 2010:*The Time-Varying Systematic Risk of Carry Trade Strategies*,Journal of Financial and Quatitative Analysis
- De Santis G, Gerard B., 1998:*How big is the premium for currency risk? Journal of Financial Economics 49: 375?412.*
- Durbin, J., and Koopman, S. J.,:*Time Series Analysis by State Space Methods*, Oxford University Press, 2001, ISBN 978-0-19-852354-3
- Engle, R. , 2002 : *Dynamic conditional correlation.* Journal of Business and Economic statistics 20(3): pp. 339-350.
- Engle, R. F., D. Kraft, 1983: *Multiperiod forecast error variances of inflation estimated from arch models.* Applied Time Series Analysis of Economic Data p. 293?302.
- E. J. Elton, M. J. Gruber,1977: *Risk Reduction and Portfolio Size: An Analytic Solution*, Journal of Business 50 , pp. 415-37

- Fama, E.F., French K.R., 2004 : *The Capital Asset Pricing Model: Theory and Evidence*, SSRN, id 440920
- Engle R., 2001: *GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics*, Journal of Economic p Perspective- Volume 15, No. 4. pp. 157-168.
- Engle R, Kroner FK, 1995: *Multivariate simultaneous generalized ARCH*, Econometric Theory 11: 122-150.
- Engle, R. F., 1982:*Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation*, Econometrica 50(4): pp. 987-1007.
- Engle, R. F., 1983:*Estimates of the variance of u.s. inflation based upon the arch model.* Journal of Money, Credit and Banking 15(3): pp. 286-301.
- Engle, R., D. Lilien, and R. Robins, 1987:*Estimation of Time Varying Risk Premia in the Term Structure: the ARCH-M Model* Econometrica, 55, 391-407.
- Fama, F. and K. R. French, 1996: *Multifactor Explanations of Asset Pricing Anomalies.* Journal of Finance. 51:1, pp. 55-84.
- Fama, E. F. and K. R. French, 1992 : *The Cross-Section of Expected Stock Returns* Journal of Finance. 47:2, pp. 427-465.
- Fama, E., 1996 *Multifactor Portfolio Efficiency and Multifactor Asset Pricing* Journal of Financial and Quantitative Analysis. 31:4, pp. 441-465.
- Fama, E., 2006: *Multifactor Investing*, Dimensional Fund Advisors Inc.
- Fama, E.F. and J. D. MacBeth, 1973 *Risk, Return, and Equilibrium: Empirical Tests* Journal of Political Economy. 81:3, pp. 607-636.
- French C.W., 2003: *The Treynor Capital Asset Pricing Model*, Journal of Investment Management, Vol.1. No.2., pp. 60-72
- Groenewold, N. and Fraser, P.,1999:*Time-varying estimates of capm betas* Mathe- matics and Computers in Simulation, 48(4):531-539

- Hafner C, Herwartz H., 1998:*Time-varying market price of risk in the CAPM. Approaches, empirical evidence and implications. Finance 19: 93-112.*

- Janský I., Adam T., Benecká S., 2012: *Time-varying Betas of the Banking Sector*, IES working paper

- Jagannathan, R., Z. Wang, 1994:*CAPM is alive and well*, Working Paper, Economics Working Paper Archive at WUSTL,

- Jensen, M. C., 1968 *The Performance of Mutual Funds in the Period 1945-1964.* Journal of Finance. 23:2, pp. 389-416.

- Jostova G., Philipov A., 2005: *Bayesian Analysis of Stochastic Betas*, SSRN.com, id545282

- Jungbacker, B. and Koopman, S. J., Wel, van der, M.: *Dynamic Factor Models with Smooth Loadings for Analyzing the Term Structure of Interest Rates* Discussion Paper TI 2009-041/4, Tinbergen Institute, 2009

- Ledoit, O., P. Santa-Clara, M. Wolf , 2003: *Flexible multivariate garch modeling with an application to international stock markets.*Review of Economics and Statistics 85(3): pp. 735-747.

- Lie, F., Brooks, R., and Faff, R.,2000: *Modelling the equity beta risk of australian financial sector companies* Australian economic papers, 39(3):301-311

- Lintner, J. , 1965a: *The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets.* The Review of Economics and Statistics 47, 13-37.

- Mandelbrot, B. , H. M. Taylor, 1967: *On the distribution of stock price differences.*Operations Research 15(6): pp. pp. 1057-1062.

- Markowitz, H. M., 1952: *Portfolio Selection* The Journal of Finance 7(1), 77-91.

- Markowitz, H. M., 1959 : *Portfolio Selection: Efficient Diversification of Investments*, Cowles Foundation for Research in Economics at Yale University, Monograph 6. New York: John Wiley & Sons, Inc. (Second edition, 1991, Cambridge: Basil Blackwell, Inc.)

- Mayers, D., 1972 :*Nonmarketable Assets and Capital Market Equilibrium Under Uncertainty* In Studies in the Theory of Capital Markets. Michael C. Jensen (editor) New York: Praeger, pp. 223-248.

- Mergner, S. and Bulla, J., 2008:*Time-varying beta risk of pan-european industry portfolios: A comparison of alternative modeling techniques*The European Journal of Finance, 14(8):771-802.

- Miller, M., M.Scholes, 1972: *Rate of Return in Relation to Risk: A Reexamination of Some Recent Findings* in Studies in the Theory of Capital Markets. Michael C. Jensen, ed. New York: Praeger, pp. 47-78.

- Mossin, J. , 1966 : *Equilibrium in a Capital Asset Market* Econometrica 34(4), 768-783.

- Pagan, A., 1996: *The econometrics of financial markets* Journal of Empirical Finance, 3(1):15-102

- Petkova, R., L. Zhang, 2002: *Is Value Riskier than Growth?* Working paper, University of Rochester.

- Rosenberg, B., K. Reid, and R. Lanstein, 1985: *Persuasive Evidence of Market Inefficiency* Journal of Portfolio Management. 11, pp. 9-17.

- Rubinstein, M., 1973 :*The Fundamental Theorem of Parameter-Preference Security Valuation* The Journal of Financial and Quantitative Analysis 8, 61-69.

- Schwert, G. W., P. Seguin, 1990 :*Heteroskedasticity in Stock Returns* Journal of Finance, 45, 1129-1155.

- Silvennoinen, A., T. Terasvirta, 2009:*Multivariate garch models* In T. Mikosch, J.-P. Kreiß, R. A. Davis, T. G. Andersen (editors), Handbook of Financial Time Series, pp. 201-229. Springer Berlin Heidelberg.

- Sopov B., 2010:*Alternative Yield Curve Modelling Approach: Regional Models*, doctoral thesis at IES FSV UK in Prague

- Tobin, J. , 1958:*Liquidity Preference as Behavior Towards Risk* The Review of Economic Studies, No. 67, February, 65-86. Reprinted as Cowles Foundation Paper 118.

- Treynor, J. L., 1962: *Toward a Theory of Market Value of Risky Assets* . A final version was published in 1999, in Asset Pricing and Portfolio Performance. Robert A. Korajczyk (editor) London: Risk Books, pp. 15-22.

- Woolgridge J.M., Bollerslev T., R.Engle, 1988: *A capitral asset pricing model with time-varying covariances*, Journal of political Economy, vol 96, no 11

Appendix A

Appendix

Figure A.1: OLS- NYSE stocks systematic risk

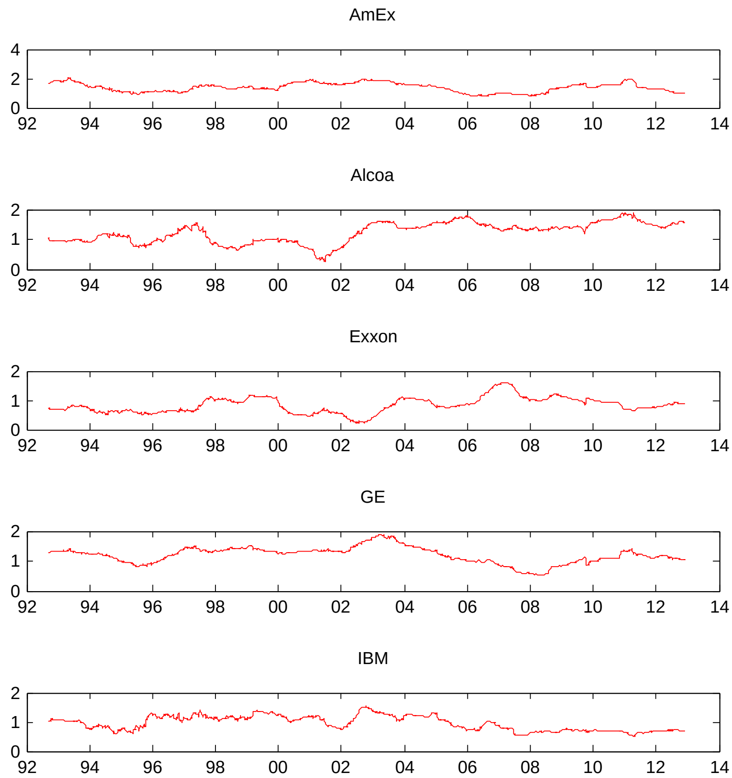


Figure A.2: OLS- NYSE stocks' systematic risk

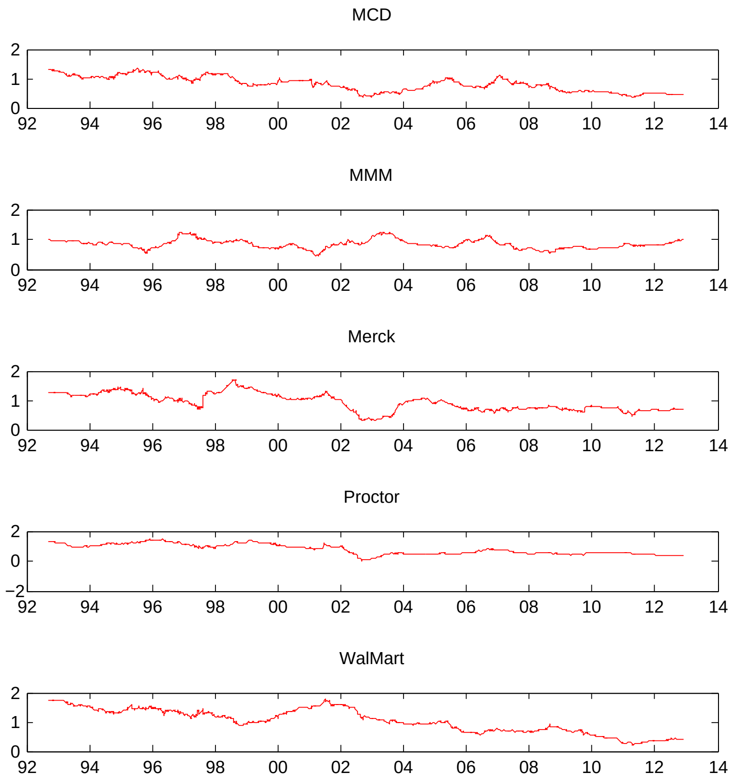


Figure A.3: OLS- NASDAQ stocks' systematic risk

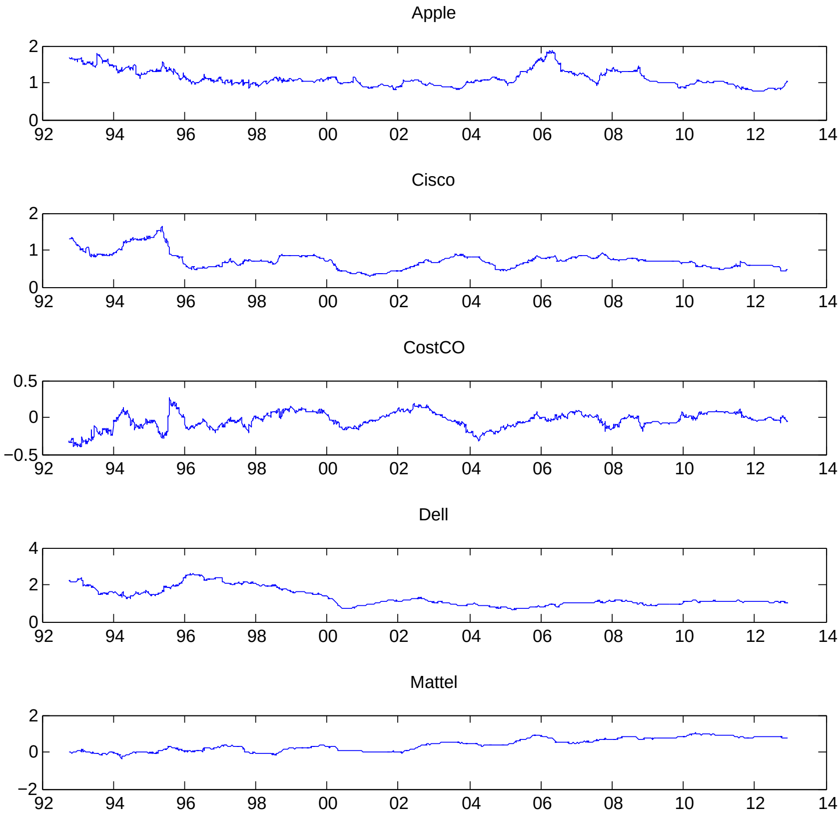


Figure A.4: OLS- NASDAQ stocks' systematic risk

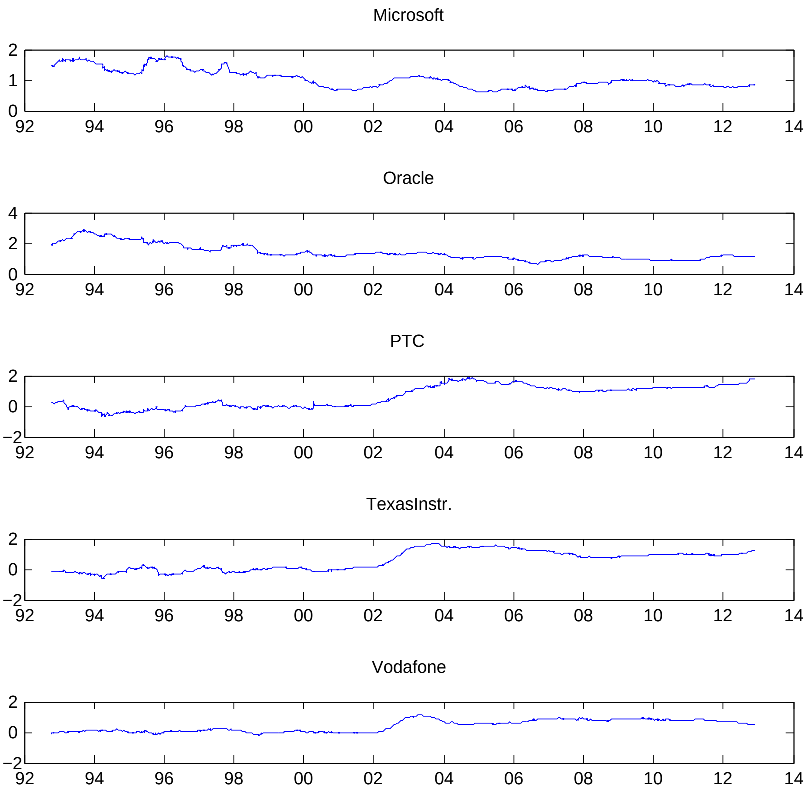


Figure A.5: DCC MGARCH- NYSE stocks' systematic risk

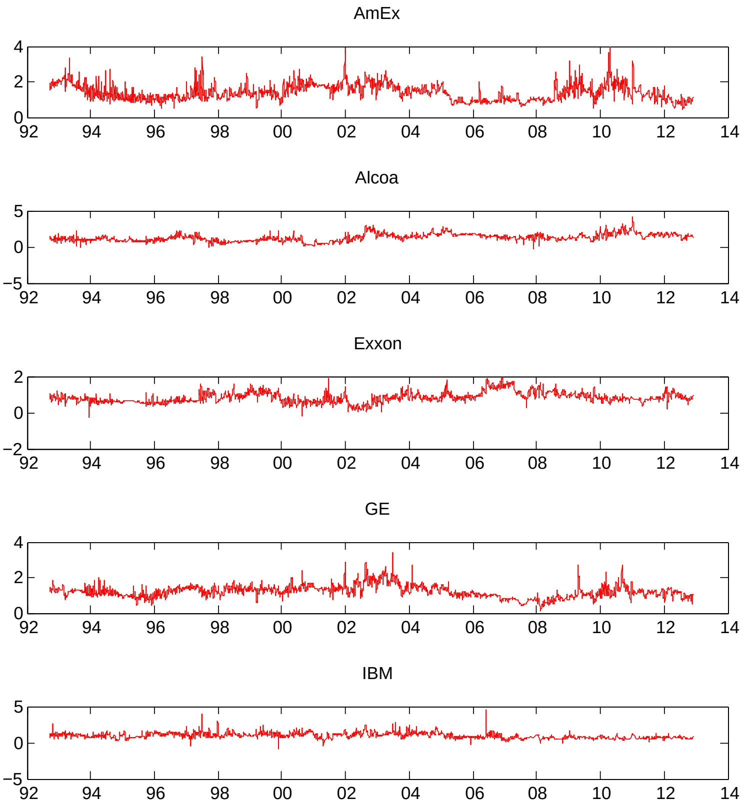


Figure A.6: DCC MGARCH- NYSE stocks' systematic risk

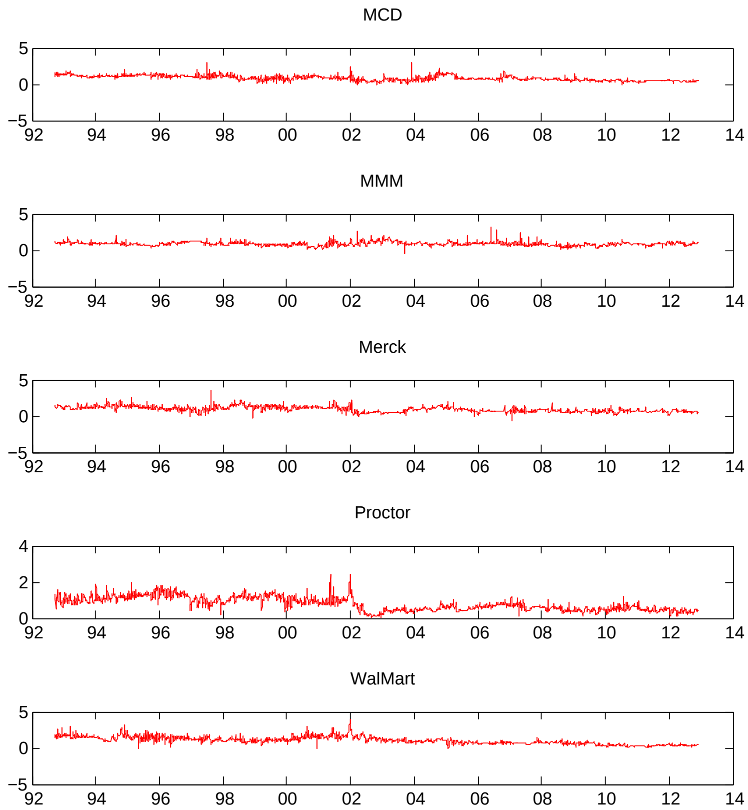


Figure A.7: DCC MGARCH- NASDAQ stocks' systematic risk

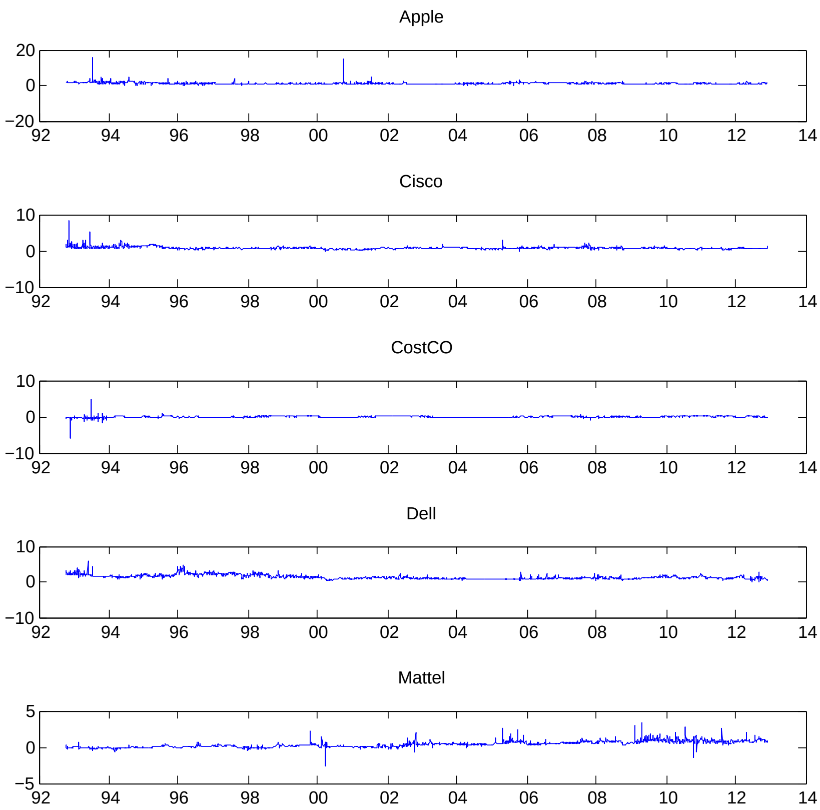


Figure A.8: DCC MGARCH- NASDAQ stocks' systematic risk

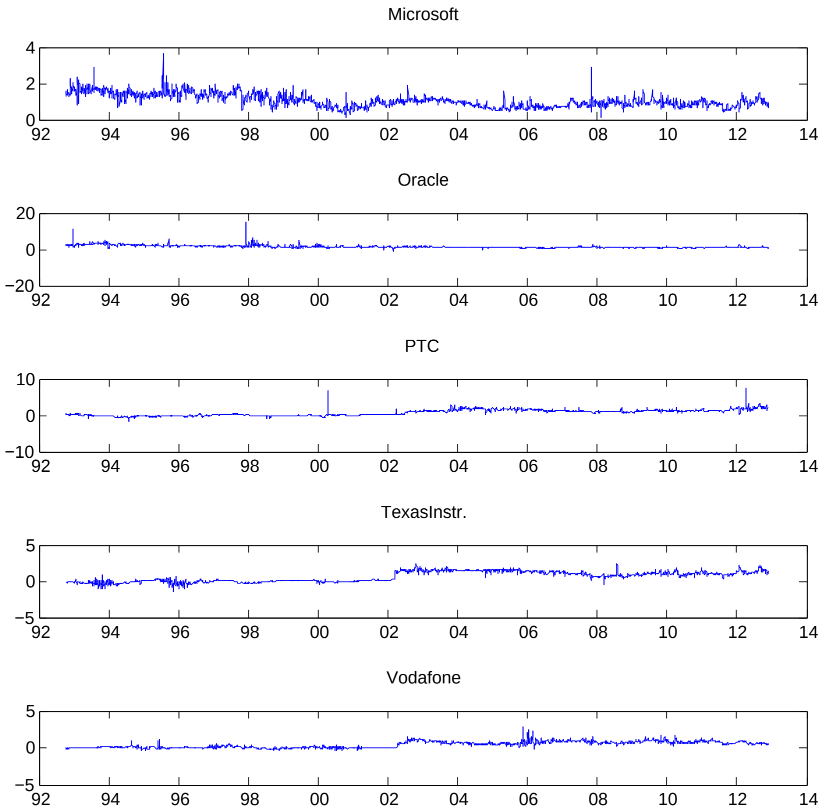


Figure A.9: SSF- NYSE stocks' systematic risk

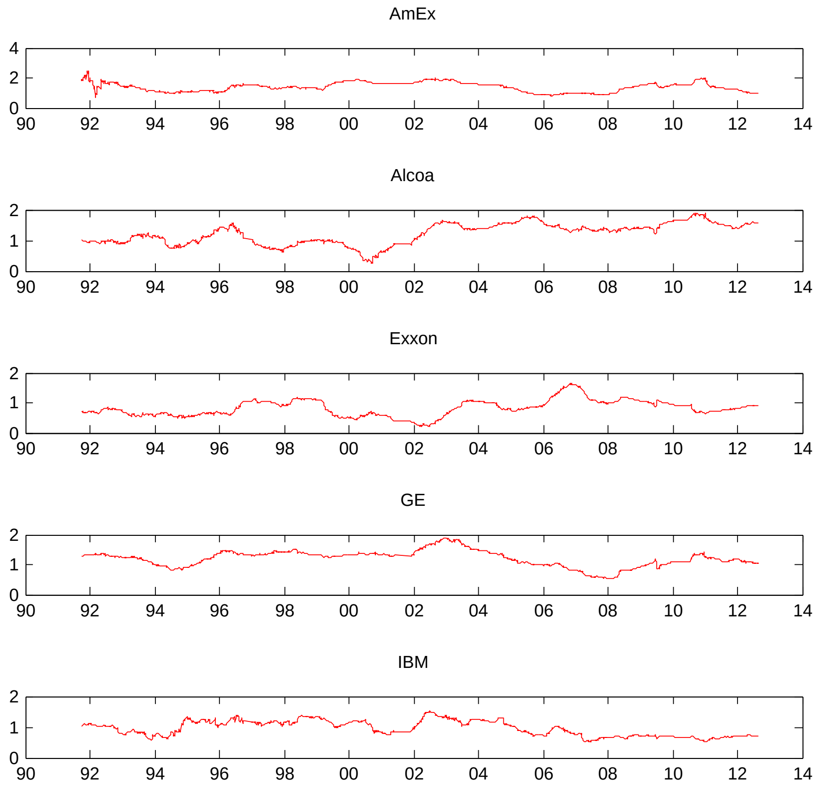


Figure A.10: SSF- NYSE stocks' systematic risk

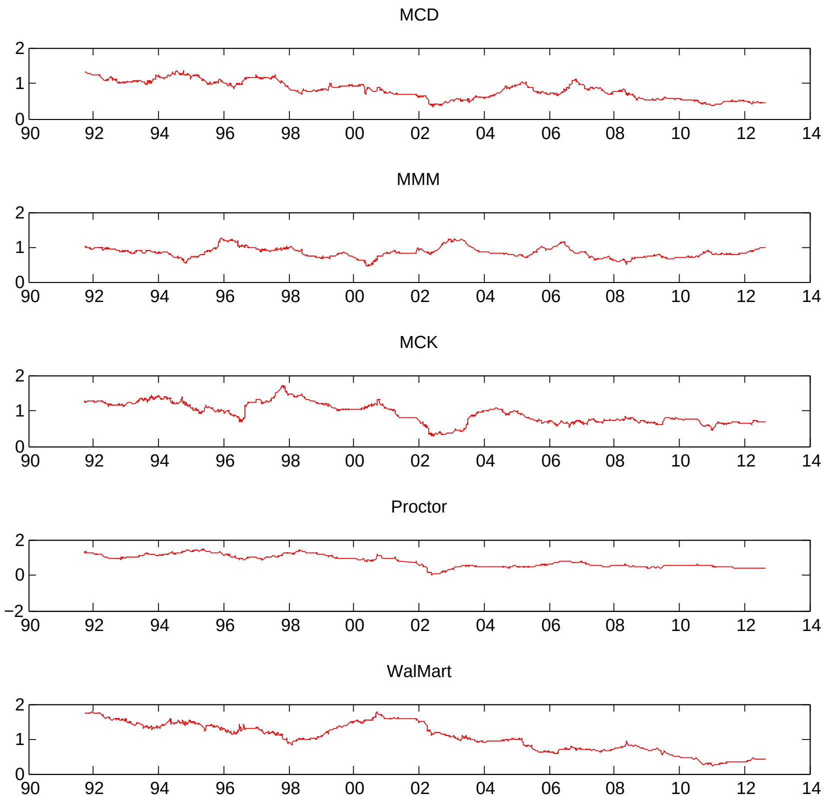


Figure A.11: SSF- NASDAQ stocks' systematic risk

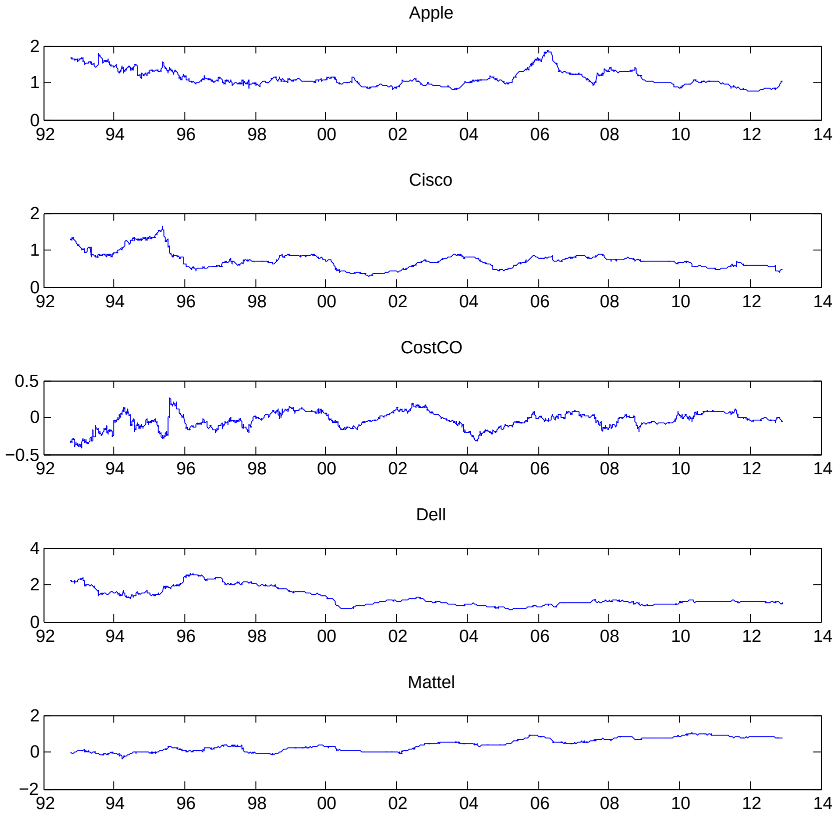


Figure A.12: SSF- NASDAQ stocks' systematic risk

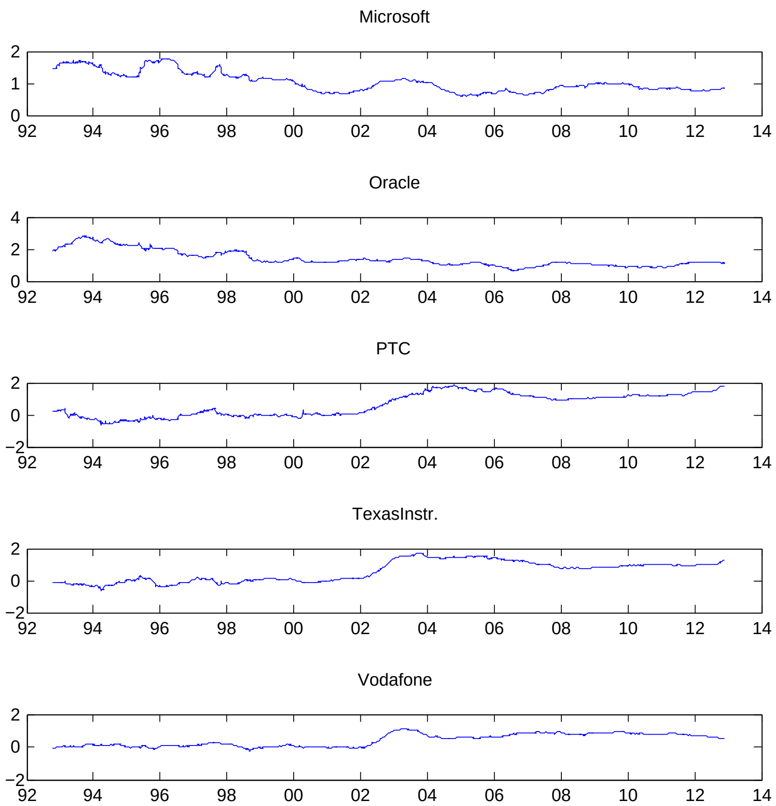


Figure A.13: NYSE portfolios' systematic risk

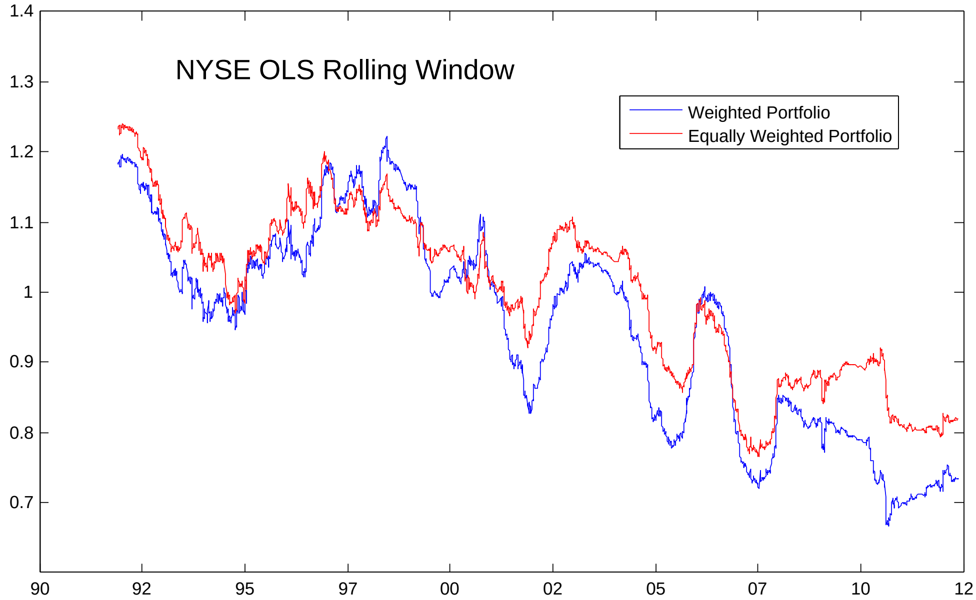


Figure A.14: NYSE portfolios' systematic risk

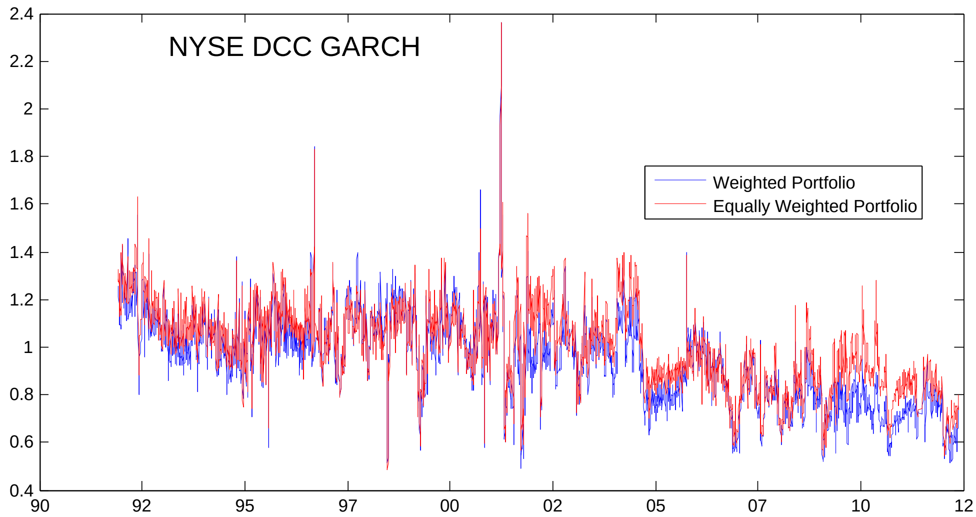


Figure A.15: NYSE portfolios' systematic risk

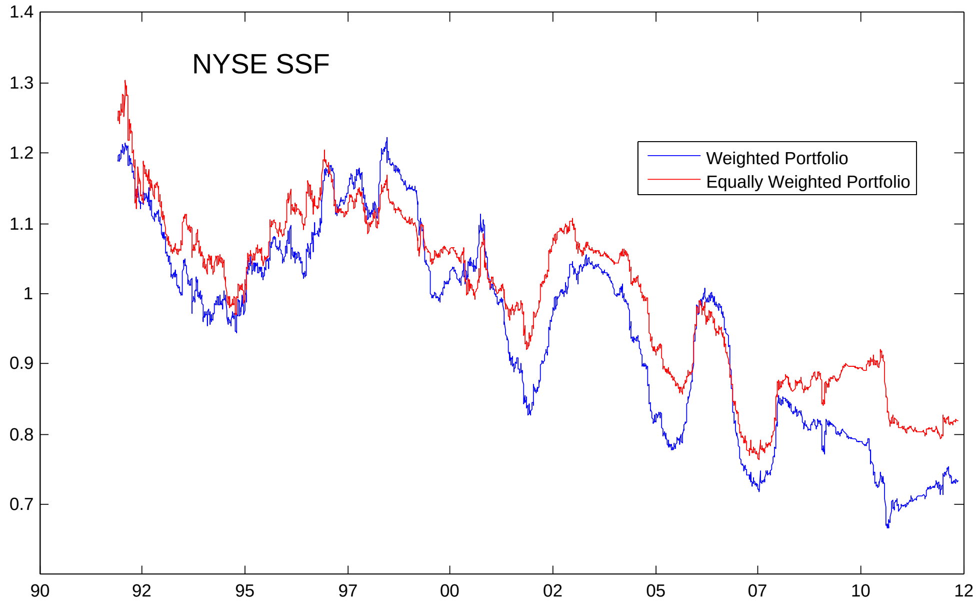


Figure A.16: NASDAQ portfolios' systematic risk

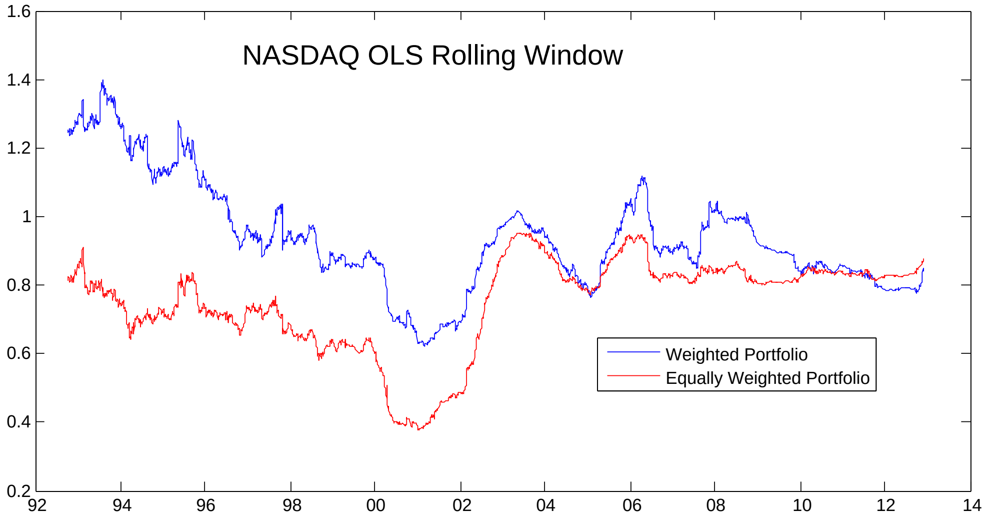


Figure A.17: NASDAQ portfolios' systematic risk

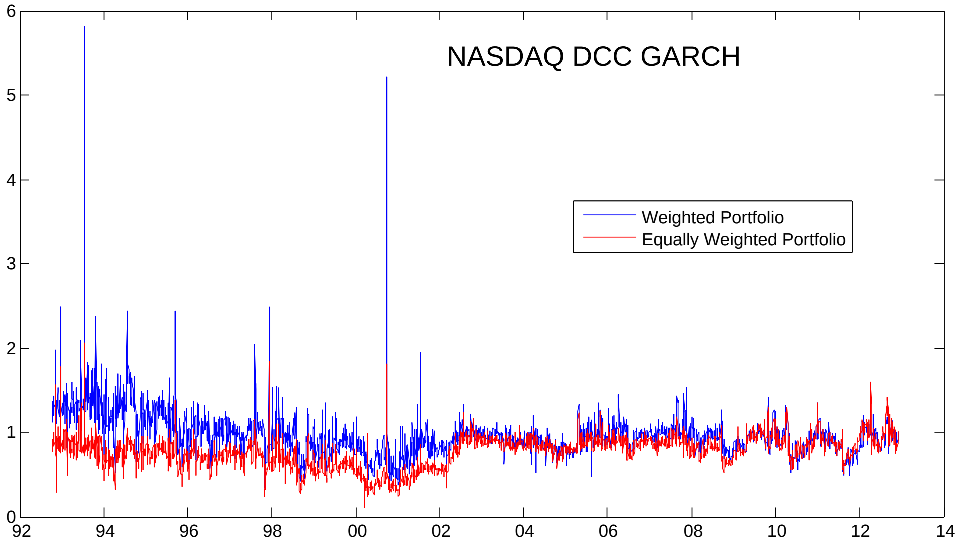
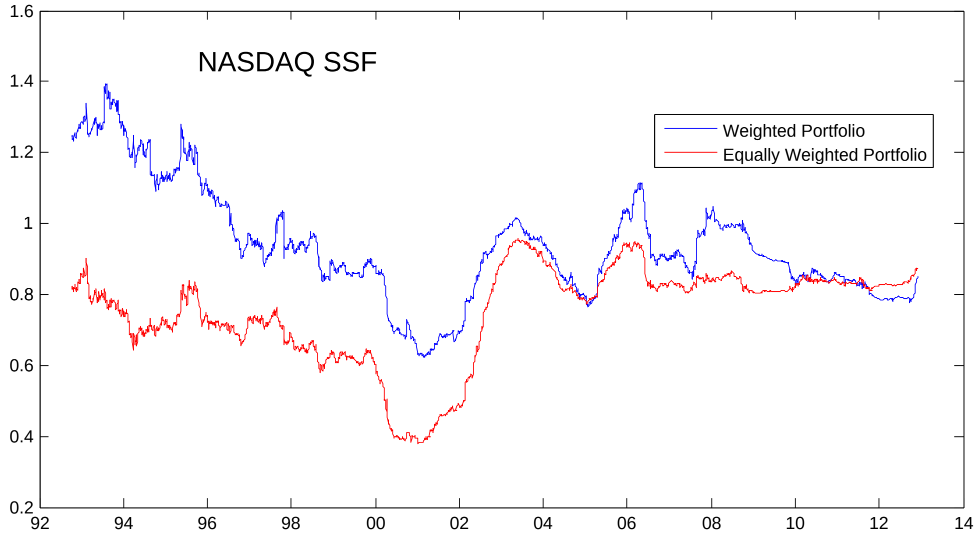


Figure A.18: NASDAQ portfolios' systematic risk



Master Thesis Proposal

Author	Bc. Daniel Šopov, MSc.
Supervisor	Petra Andrlíková, MSc.
Proposed topic	Modern way of calculation of CAPM coefficient: Beta hedging application

Topic characteristics This thesis aspires to find the best method to hedge the systematic risk within CAPM model framework evaluated on different stock indexes and particular companies' stocks as European FTSE 100 and NYSE Composite and Citigroup respectively.

Moreover, the whole research will collect data for last 20 years and by using various methods we will try to estimate time varying parameter Beta of the popular CAPM model. The considered approaches are: popular OLS estimation on a rolling window sample; Betas based on GARCH estimation of variance and covariance; and state space CAPM estimation.

For each time-varying Beta estimation, we calculate evolution of the value of Beta-hedged portfolio on extending samples, thus we simulate performance in real-life application. Based on the resulting portfolio values, we are able to discriminate amongst these approaches to calculate Beta. Afterwards our time series will be divided into two parts, the period before the financial crises in 2008 and after, where assume that this step will help us with considering the most powerful method when hedging the systematic risk.

Hypotheses

- 1) OLS is the most accurate method in hedging the risk of portfolio.

– 2) GARCH is the most accurate method in hedging the risk of portfolio.

– 3) State space CAPM estimation is the most accurate method among the rest in hedging the risk of portfolio.

Methodology In this thesis we will use variety of tests. Furthermore for our analysis we are going to use couple of methods. Firstly, we will run a series of 60 months rolling window OLS estimation, so that we will get time varying parameter. Secondly, we will use GARCH, which should be appropriate tool for our research and lastly we will program in Matlab toolbox SSF model, which should be the last method to provide us of significant results.

Outline

1. Introduction
2. Methodology
3. Empirical Part
4. Conclusion

Bibliography

– R.S.Tsay, (2005): Analysis of Financial Time series, second edition, Wiley-interscience

– S.J. Taylor (2005): Asset Price Dynamics, Volatility, and Prediction, Princeton University Press

– S.J. Taylor (2007): Modelling Financial Time series, second edition, World Scientific

– R.Engle (2001): GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics, Journal of Economic Perspectives, pp. 157-168.

– S.J. Koopman (2012): Time series analysis by state space methods, books.google.com

– P.J. Brockwell (2005): Time Series Analysis, Wiley Online Library